8. *Possibility of Optical Explanation of Visible Wave-motion of Ground.*

By Yasuo SATÔ,

Earthquake Research Institute.

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1. Introduction. We have heard a number of reports from epicentral regions of great earthquakes that the wave-motions of ground or some structures such as roofs of buildings were noticed by the naked eyes[1]. Of course these reports are not necessarily trustworthy, and some of them are concerned with the objects on the ground such as trees, grass and so on[2]. But even if we exclude these cases, it is rather hasty to abandon all the other reports, considering them as illusion, for we have now few data and it is difficult to intorduce a decisive opinion. Therefore, while on one hand we should endeavor to collect correct and sufficient data, it is desirable to try to explain the fact reasonably. Now, as an attempt of this we will investigate the possibility of optical explanation of the fact; that is to say, if the compressional waves in the air, caused by the vertical movement of ground surface, may make the air density change and consequently the optical path of this region may have some curvature.

In the following discussion, we will assume the wave front horizontal and the problem to be two dimensional; and x-axis as horizontal and y-axis as vertical.

2. Relation between air-density and refractive index. According to Pernter-Exner[3], the relation between refractive index n and air density δ is expressed as $n = 1+c'\delta$ where c' is a small constant, somewhat dependent to the wave-length of light, for which we may adopt an average value ·000293 for white light.

Next, using the notations $\rho = \rho_0+\rho'$, $p = p_0+p'$, where p, ρ are pressure and density respectively and the suffix "o" means the average

(1) C. E. DUTTON, "Earthquakes" (1904) P. 13. W. H. HOBBS, "Earthquakes" (1907) P. 85. "MONTESSUS DE BALLORE, La Science Séismologique," P. 436. N. G. HECK, "Earthquakes" (1936) P. 21.

(2) N. H. HECK, loc. cit. T. MATUZAWA, "Disin." **9** (1937), P. 469.

(3) PERNTER-EXNER, "Meteorologische Optik." 2nd ed. P. 58 See also FUJIWHARA, KITAOKA; "Kisyô-kôgaku" (Iwanai, Buturigaku) P. 5.

value, and assuming the frequency (in one second) as ω, and the wave legth λ, substitute the expressions $V = V_0 \sin 2\pi(y/\lambda - \omega t)$, $\rho' = \rho_0' \sin 2\pi(y/\lambda - \omega t)$, in the fundamental equation of sound waves; then we have $\rho_0'/\rho_0 = v_0/(\lambda\omega) = v_0/V$, where, V is the velocity of sound wave. Therefore the rate of variation of density is equal to the ratio of maximum particle velocity to the propagation velocity of sound. For the present purpose we may take $\delta = \rho/\rho_0$, therefore we have

$$\delta = 1 + \varepsilon \sin 2\pi(y/\lambda - \omega t),\ \varepsilon = v_0/V \ldots\ldots\ldots\ldots\ldots \quad \text{(A)}$$

On the other hand, v_0 is determined by the vertical displacement of ground surface, so that ε is a small quantity in the case of real earthquakes. Substituting (A) into the expression of n, we have

$$n = (1+c')\{1 + K \sin 2\omega(y/\lambda - \omega t)\},\ K = c'\varepsilon/(1+c') \ldots\ldots\ldots \quad \text{(B)}$$

3. Optical path. From Fermat's principle, equation of optical path is $\delta T = \delta \int_{QO} \frac{ds}{c} = 0 \cdots (C)$, where c is the light velocity and Q, O represents the position of the object and the eye respectively. (See Fig. 1). Using refraction index, we may write $c = c_0/n$, where c_0 is the light velocity in vacuum. Now, we adopt as the unit of length $\lambda/2\pi$, and unit of time $1/2\pi\omega$, and use ξ, η and τ instead of x, y and t; then we have the equation

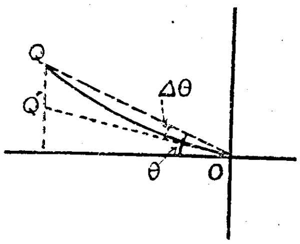

Fig. 1.

$$\delta T = \delta \frac{1}{c_0}(1-c') \int_{-a}^{0} \{1 + K \sin(\eta - \tau)\}\{1 + (d\eta/d\xi)^2\}^{\frac{1}{2}} d\xi = 0 \ldots\ldots\ldots \quad \text{(D)}$$

This is solved rigorously by the aid of elliptic integral. But the main object of the present paper exists in getting the apparent change of position of objects, so we will at first treat the case in which the light velocity varies according to the formula $c \propto 1 - K'\eta \ldots\ldots\ldots$ (E)

3. 1. Relation between two velocity distributions. Now we will assume the form of integrand in (C) as $f(\xi, \eta, \eta')$. At first we take $\eta = \phi_0(\xi)$ as the equation of optical path, and $\eta = \phi_0(\xi) + \epsilon\, \psi(\xi)$ as another path that lies near $\eta = \phi_0(\xi)$. Here ϵ is a small constant and $\psi(\xi)$ vanishes at the points Q and O. Denoting the reciprocal of velocity as $\sigma(\eta)$, we have the following expression; $f(\xi, \eta, \eta') = \sigma(\eta)(1 + \eta'^2)^{\frac{1}{2}}$. Introduce this into (C), and we have

$$\delta T = -\int_{QO} \psi(\xi)(1+\phi'^2)^{-\frac{3}{2}} F(\xi, \eta) d\xi, \quad F(\xi, \eta) \equiv \sigma(\eta)\phi''(\xi) - \sigma'(\eta)\{1+\phi'(\xi)^2\}^2 \quad \text{(D)}'$$

If the velocity is given by the formula $\sigma(\eta) = \sigma_0(\eta) \equiv 1 + c\sin(\eta - \tau)$, and the corresponding path is determined by the equation $\eta = \phi_0(\xi)$, then from (D);

$$\phi_0'' = [K\cos(\eta-\tau)/\{1+K\sin(\eta-\tau)\}](1-\phi_0'^2) \quad \text{(F)}$$

Next, we will consider another kind of medium in which $\sigma(\eta)$ is given by

$$\sigma(\eta) = \sigma_1(\eta) \equiv 1/(1-K'\eta), \quad K' \ll 1,$$

and express the path of this case by the equation $\eta = \phi_1(\xi)$. Our present purpose is to detect what difference may exist between two functions $\phi_0(\xi)$ and $\phi_1(\xi)$. Substituting (F) into (D)$'$

$$\begin{aligned} F(\xi, \eta) &= (1-K'\eta)^{-2}\{1+K\sin(\eta-\tau)\}^{-1}[(1-K'\eta)K\cos(\eta-\tau) \\ &\qquad -K'\{1+K\sin(\eta-\tau)\}](1+\phi_0'^2) \\ &< \quad \text{,,} \qquad \text{,,} \qquad [(1+K'\cdot 2\pi)K \quad -K'\{1-K'\}](1+\phi_0'^2) \end{aligned}$$

Now if we assume $K' = K(1+8K)$, then $[(1+K'\cdot 2\pi)K - K'\{1-K'\}] < 0$, therefore $F(\xi, \eta)$ is negative without regard to ξ. To make δT as small as possible we ought to take $\psi(\xi)$ always negative. This means that the optical path in a medium $\sigma(\eta) = \sigma_1(\eta) = 1/\{1-K(1+8K)\eta\}$, lies always in the lower side of $\eta = \phi_0(\xi)$ which represents a path in a medium $\sigma(\eta) = \sigma_0(\eta)$. On the other hand, when $-\frac{\pi}{2} \leq \eta - \tau \leq \frac{\pi}{2}$, the curve $\eta = \phi_0(\xi)$ lies in the lower side of a straight line that passes Q and O. Therefore the deflection angle, in the former case, is larger than in the latter.

By the above theory we may see the possibility of using a medium, in which $\sigma(\eta) = \sigma_1(\eta) \equiv 1/(1-K'\eta)$, to get an upper bound of deflection angle; we shall see later that this gives also a good approximation of the largest value of it.

3.2. Estimation of deflection angle. Brachistchronic line in a medium $\sigma(\eta) = \sigma_1(\eta)$ is obtained by equating the expression $F(\xi, \eta)$ of $(D)'$ to zero.

$$(1-K'\eta)^{-1}\eta'' - K'(1-K'\eta')^{-2}(1+\eta'^2)^2 = 0$$

Integrating the expression we have the following solution easily, where L and M are integration constants. $K'(\xi^2+\eta^2) - 2\eta + 2L\xi - M = 0$. From a boundary condition $\xi = 0$, $\eta = 0$, we have at once $M = 0$, and

$$\{\xi+L/K'\}^2+\{\eta-1/K'\}^2=\{1+L^2\}/K'^2$$

This is an equation of circle whose radius is $(1+L^2)^{\frac{1}{2}}/K'$. Arbitrary constant L, in this expression is obtained by putting another boundary condition $\xi=-\alpha$, $\eta=\beta$. Thus $L=\{K'(\alpha^2+\beta^2)+2\beta\}/2\alpha$. This quantity is, as compared with unity, very small. Hence the radius of circle is approximately $1/K'$, which depends neither on α nor on β.

Deflection angle of optical path, denoted by $\varDelta\theta$ in Fig. 1, is obtained by the following process.

$$\tan\theta=\beta/\alpha,\ \tan(\theta+\varDelta\theta)=[-d\eta/d\xi]_{\xi=\eta=0}=\{K'(\alpha^2+\beta^2)+2\beta\}$$

$$\therefore\quad \varDelta\theta=\frac{1}{2}K'\alpha$$

Thus the deflection angle depends only on distance.

As time goes on, distrbution of air density becomes reversed and $\varDelta\theta$ changes its sign to become $-\frac{1}{2}K'\alpha$. Therefore, the total deflection angle amounts to $K'\alpha$. When the object point Q lies on the same horizon with O, β becomes zero and L becomes as $L=K'\alpha/2$. Maximum value of η, in this case, is $\eta_{\max}=\frac{3}{8}K'\alpha^2$, which is fairly small compared with unity; therefore we may legitimately adopt the following approximation when $|\eta-\tau|$ is small.

$$\sigma_0(\eta)\equiv1+K\sin(\eta-\tau)\fallingdotseq1/\{1-K(\eta-\tau)\}\propto1/\{1-K'\eta\}\equiv\alpha_1(\eta)$$

Thus the above obtained amount $K'\alpha$ gives not only an upper bound of deflection angle, but also a good approximation.

4. Numerical values in a practical case. Now, we will examine the possibility of the explanation by introducing numerical values adequate in the case of real earthquakes.

At first, we adopt the value $c'=\cdot000293$. Next, since ε is equal to v_0/V, taking a greatest value thinkable for v_0, we put $v_0=100$cm/sec. Then we have $\varepsilon=100/33,000\fallingdotseq\cdot003$, therefore

$$K=c'\varepsilon/(1+c')\fallingdotseq1\cdot0\times10^{-6},\ K\fallingdotseq K'$$

As the unit length we have used $\lambda/2\pi$, which nearly becomes, taking a frequency a little larger than 3 sec^{-1}, 15 metres.

On the other hand, the maximum total deflection angle is about $K'\alpha$. Therefore, for a distance 150 metres ($\alpha=10$), we have $|\varDelta\theta|_{\max}=$

10^{-5}. But this is too small to be detected by the naked eyes. Besides, the apparent displacement of the object point (QQ' in Fig. 1) is only about 1 mm., which is by far smaller than the real displacement of ground. So we may conclude that the visible wave motion of ground surface is no more easily explained by assuming optical effect. Even if we assume shorter sound waves, the deflection angle by no means increases. We have hitherto adopted a most favorable case for the optical explanation, but now we are to give up this intention.

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