10. *A Study on the Overturning of Rectangular Columns in the case of the Nankai Earthquake on December 21, 1946.*

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INTRODUCTION.

It is a well-known expedience in Japan that the magnitude of the acceleration of great earthquakes were commonly deduced from the ratio of *height* to *width* of overturned grave-stone which are rectangular column in shape[1]. But it is at times found that the grave-stone which has great height in comparison to the width, which in appearance is apt to be overturned, does not. On the contary one which has great width, in appearance hard to be overturned, is overturned. In this paper, we will study these phenomena theoretically and will exploit these results to deduce the magnitudes of the Nankai Earthquake of 1946 at two places; Kainan-Shi in Wakayama Prefecture and Kinomoto-Shi in Miye Prefecture. The latter data were kindly offered to us by Dr. H. Kawasumi and Mr. Y. Sato.

THEORETICAL CONSIDERATIONS.

(1) CONDITIONS OF OVERTURNING.

For simplicity, we consider the rocking or overturning of rectangular column in two dimensions. When a column, ABCD, is moved to AB′C′D′ which is the critical declination for overturning caused by the ground motion, the center of gravity, G, moves to G′ where it is vertically above the point A in Fig. 1. The work of the center of gravity, G, to be done during the displacement GG′ is given by

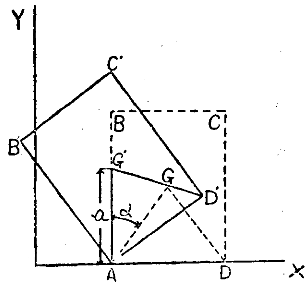

Fig. 1.

$$W_G = Mga - Mga\cos\alpha = Mga(1-\cos\alpha), \qquad (1)$$

1) F. Omori, *Rep. Imp. Earthq. Comm.* **32** (1900), 19-33. H. Kimura and K. Iida, *Jishin* **6** (1934), 125. R. Ikegami and F. Kishinouye, *Bull. Earthq. Res. Inst.* **24** (1946), 11-18.

in which M is the mass of the column, g is the acceleration of gravity, a is the length of AG and α is the angle of $\angle$ GAB.

Next, when a column is moved by a ground motion, $\epsilon \sin p(t+\delta t)$, the equation of motion is given by

$$(a^2+k^2)\frac{d^2\theta}{dt^2} = -ag\sin(\alpha-\theta)+a\epsilon p^2 \sin p(t+\delta t)\cos(\alpha-\theta), \qquad (2)$$

in which k is the moment of inertia at G and δt is the time-lag.

In this paper, vertical motions were neglected, for the Nankai Earthquake occurred off the coast of Kii and the vertical component of the earthquake motions were very small as compared with the horizontal motions.

It is difficult to obtain the general solution of the equation (2), so we attempt to obtain an approximate solution of equation (2).

We took approximately,

$$\sin(\alpha-\theta) \fallingdotseq \alpha-\theta, \quad \cos(\alpha-\theta) \fallingdotseq 1.$$

Then the solution of the equation is obtain as follows;

$$\alpha-\theta = A_1 e^{nt} + A_2 \bar{e}^{nt} + \frac{m}{n^2+p^2}\sin p(t+\delta t), \qquad (3)$$

where $n^2 = \dfrac{ag}{a^2+k^2}$, $m = \dfrac{a\epsilon p^2}{a^2+k^2}$, and A_1 and A_2 are arbitrary constants.

Suppose at $t=0$; $\theta=0$, $\dfrac{d\theta}{dt}=0$,

so constants A_1 and A_2 are determined as follows,

$$A_1 = \frac{1}{2}\left\{\alpha - \frac{mp}{n(n^2+p^2)}\right\}, \quad A_2 = \frac{1}{2}\left\{\alpha + \frac{mp}{n(n^2+p^2)}\right\}.$$

The above initial conditions may be not reasonable for $\dfrac{d\theta}{dt}$ must be finite at $t=0$ or $\left(\dfrac{d\theta}{dt}\right)_0 \neq 0$. But we cannot solve the equation from the given conditions. Therefore we assumed $\left(\dfrac{d\theta}{dt}\right)_0 = 0$ and then consider the effects as the velocity increases gradually from zero.

From equation (3), the work of the ground motion done by the rocking motion of the column, dW_D is given by

$$dW_D = -(a^2+k^2)M\frac{d^2\theta}{dt^2}d\theta.$$

Therefore, the work in a quarter period of the ground motion is given by

$$W_D = -(a^2+k^2)M\int_0^{\frac{\pi}{2p}} \frac{d^2\theta}{dt^2}\frac{d\theta}{dt}dt\,; \qquad (4)$$

When we substitute the solution (3) in equation (4), we obtain

$$W_D = \frac{a^2M}{6}\left[\frac{m^2p^2}{(n^2+p^2)^2}\left(10+\cosh\frac{n\pi}{p}\right)-\alpha^2n^2\left(2-\cosh\frac{n\pi}{p}\right)-\frac{2mn\alpha p}{n^2+p^2}\sinh\frac{n\pi}{p}\right], \qquad (5)$$

in which δt is neglected as $\delta t \fallingdotseq 0$.

One condition of the overturning of the column due to any given motion is obtained from equation (1) and (5).

Namely, when $W_G > W_D$ (6), the column will be not overturned and when $W_G < W_D$ (7), the column will be overturned.

(2) CONDITION OF BEGINNING OF THE ROCKING MOTION.

At equation (2), if we assume initial conditions $t = 0$; $\theta = 0$, $\frac{d^2\theta}{dt^2} = 0$, we obtain a condition at the begining of the rocking motion, then

$$\delta t = \frac{1}{p}\sin^{-1}\frac{g\tan\alpha}{\epsilon p^2}, \qquad (8)$$

In equation (8), in order that δt is significant, the following condition is necessary;

$$\frac{g\tan\alpha}{\epsilon p^2} < 1, \qquad (9)$$

Namely, the column will begin to rock if acted on by a ground motion that satisfies the condition (9).

APPLICATION TO THE PRACTICAL DATA.

The recent Nankai Earthquake occurred just before the newvyear, so the overturned grave-stones were unexpectedly restored in their places quickly for the new year festival, and data obtained were less than our expectations.

In this paper, we will apply the above results to the data obtained at Kainan-Shi and Kinomoto-Shi. (Table I).

TABLE I.

Place		Width	Height	α	a
Kainan-shi	Overturned	13 cm	43 cm	0.293	22.3 cm
	Non-overturned	24	115	0.218	55.3
Kinomoto-shi	Overturned	17	44	0.449	23.4
	Non-overturned	31	168	0.189	85.5

We applied equation (7) to the data of the overturned grave-stone and equation (8) to the non-overturned one in TABLE I and calculated the values of ε giving the values $p = 5, 6, 7, 8, 9, 10, 11, 12, 13$.

These results are shown in Fig. 2 and 4 respectively by curve I and II. The values of ε to be satisfied by equation (9), for these values of p, namely, the conditions to begin the motion, are shown by broken lines.

Furthermore, in order to indicate an acceleration, we calulated the values of p and ε from the following equation (10), in this case acceleration A was treated as parameter.

$$\varepsilon p^2 = A \tag{10}$$

The values of ε for various values of p are plotted in Fig. 2 and 4 by dotted lines.

(1) OVERTURING AT KAINAN-SHI.

In Fig. 2, when a ground motion that consisted of p and ε which are above the broken line, and furthermore exists between the hatched part, surrounded by curve I and II, is given, the high grave-stone is not overturned and the low one is overturned. Namely, at Fig. 2, assuming that a frequency of the ground motion, p, were nine (period $T = 0.70$ sec), we will deduce that the amplitude ε must be about from 5.2 to 5.5 cm in order that the above mentioned phenomena will occur.

At this time, we see from Fig. 2 that the acceleration is a little greater than 400 gal. Of course we can not uniquely determine the magnitude of the earthquake motion, but if we assume any one of p or ε or acceleration, we can deduce the motion at that place.

It is an interesting fact that if the frequency exceeds a certain limit, (period becomes larger), the above mentioned phenomena can not occur independent of amplitude. Namely, in this case, unless $p > 5.5$ or $\varepsilon < 12$ cm, the above mentioned phenomena does not occur. From the above mentioned facts, we can deduce that on firm ground where the vibration period of the ground are short these phenomena may offen occur, and on the contrary, on soft ground, any and every grave-stone may overturn.

The Sennen-ji where we got these data, stand at the foot of a mountain in Kainan city and we believe that this place is located on firm ground. Namely, we are willing to deduce that at this place, the damages had been caused by an earthquake which had a period of about 0.7 sec and an amplitude of from 5.2 to 5.5 cm, in other words by the earth motion which had an acceleration of about 430 gal.

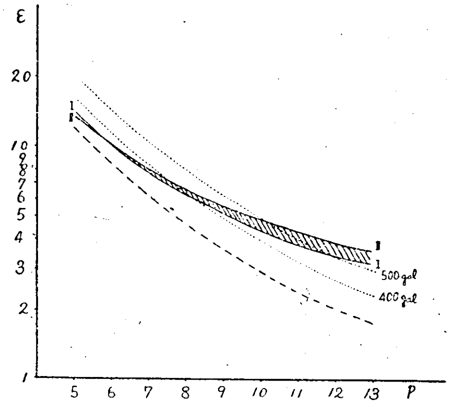

Fig. 2. Curve I and II show the limits of overturning and non-overturning of the grave-stone at Kainan-shi.
-------- shows the acceleration
— — — — — shows the beginning of the rocking motion.

Tentatively, we will study how these grave-stones of two kinds move, when the ground motion which has $p = 9$ and $\varepsilon = 5.2$ cm is given.

For the purpose of mechanical integration, we transfer the equation (2) to the equation (11).

$$\left.\begin{aligned}\frac{d\theta}{dt} &= -\frac{ag}{a^2+k^2}\sin(\alpha-\theta)(t-t_0)+\frac{a\varepsilon p}{a^2+k^2}\cos(\alpha-\theta)\\ &\qquad\times[\cos p(t_0+\delta t)-\cos p(t+\delta t)]+\left(\frac{d\theta}{dt}\right)_0\\ \theta &= -\frac{ag}{a^2+k^2}\sin(\alpha-\theta)\frac{(t-t_0)^2}{2}+\frac{a\varepsilon}{a^2+k^2}\cos(\alpha-\theta)\\ &\qquad\times[p(t-t_0)\cos p(t_0+\delta t)\\ &\quad-\{\sin p(t+\delta t)-\sin p(t_0+\delta t)\}]+\left(\frac{d\theta}{dt}\right)_0(t-t_0)+\theta_0\end{aligned}\right\}\qquad(11)$$

By the equation (11), the relations of the angle θ of rotation of the grave-stones to the time t are calculated. These relations among θ and t are shown by Fig. 3. Namely, in Fig. 3, curve I shows the relations to the non-overturned grave-stone and curve II shows that to overturned one. When the ground motion which has $p = 9$ and $\varepsilon = 5.2$ cm is given, we can deduce that the grave-stone of great width, which is in appearance hard to overturn, will be susceptable to rocking more than one of great height.

The curve II of Fig. 3 shows that the grave-stone does not overturn by the first declination. But if we took into considerration, the side-slip of the grave-stone when it declines, we can deduce that the earthquake motiohs at Kainan-shi from the data correspond to the hatched part in Fig. 2. Moreover, in Fig. 3, the given ground motion is shown by the dotted line.

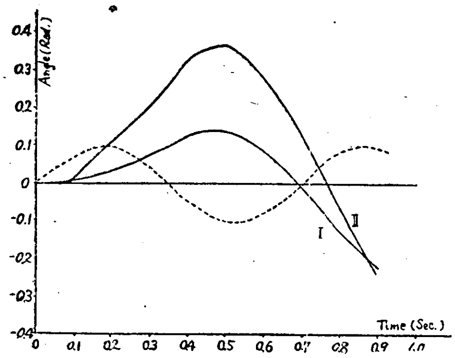

Fig. 3. Curve I shows the motion of the non-overturned grave-stone.
Curve II shows that of the overturned one.
......shows the given ground motion.

(2) OVERTURNING AT KINOMOTO-SHI.

From the data at Kinomoto we calculate in a similar manner by equations (6), (7), (9) and (10), and obtained Fig. 4 by plotting these values.

In this case, we can deduce the amplitude of the ground motion from the hatched part in Fig. 4. If we assume the period of the ground motion to be about 0.7 sec ($p = 9$), its amplitude would be between 5.80 to 5.87 cm (maximum acceleration is about 474 gal), and if the period were about 0.62 sec $p = 10$), the amplitude should be between 5.3 to 5.4 cm (maximum acceleration is about 535 gal).

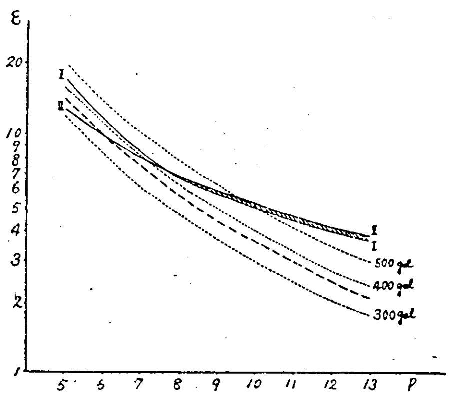

Fig. 4. Curve I and II show the limits of overturning and non-overturning of the grave-stone at Kinonoto-shi.

CONCLUSION.

To determine the magnitude of a great earthquake, it is necessary that proper seismographs are distributed in many places near the epicentre. But in the present circumstances we cannot expect such an ideal condition. Therefore, observations made of the rocking or overturning of comparatively regularly formed bodies, for instance, grave-stones or stone-lanterns, have been utilized to determine the magnitude. At the same time this was usually deduced simply from the ratio of height and width of the body. But we believed that the estimation of magnitude deduced solely from the ratio of height to width was not complete, for we found at times that the grave-stone which had great height in comparison with width did not overturn, and on the contrary one which had great width did overturn.

We investigated the method which deduced the magnitude of earthquake taking advantage of these special phenomena and obtained the above mentioned results.

In the future, we hope that this method will be applied to investigate the magnitude of earthquakes.