1. *Transmission of Arbitrary Elastic Waves from a Spherical Source, Solved with Operational Calculus. III.*

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1. *Introduction.*

In the previous paper,[1] the problem of transmission of arbitrary elastic waves generated from a spherical source with spheroidal vibration of order $n=2$, was solved. Although the case discussed in that paper was that for the vibration resulting from pressure change at the origin, in the present paper the problem will be extended to the case for the vibration arising from change in shearing force at the same origin. The types of shearing force are quite similar to those in the previous investigation, namely, that of rectangular form and that of sine form, both beginning from a quiescent state.

2. *General equations for the waves generated from a spherical source under shearing forces.*

The integral expressions of displacements of the waves for any n are the same as those shown in equations (1), (2) in the previous paper. Let the normal stress at the spherical source, $r=a$, be zero and the tangential stress there be distributed spheroidally, but changing with time in the form $f(t)$. It is then possible to write

$$\lambda\Delta+2\mu\frac{\partial u}{\partial r}=0, \tag{1}$$

$$\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial \theta}\right)=\frac{dP_n(\cos\theta)}{d\theta}f(t) \tag{2}$$

at $r=a$, where u, v are normal and transverse displacements as shown in the previous paper.

Let $g(z)$ be the operational form of $f(t)$, that is to say,

1) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **19** (1941), 417~442.

$$g(z)=z\int_0^\infty e^{-zt}f(t)dt. \qquad (3)$$

It is then possible to write

$$\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial \theta}\right)_{r=a}=\frac{dP_n(\cos\theta)}{d\theta}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{zt}}{z}g(z)dz. \qquad (4)$$

Substituting the expressions of displacements in (1), (4), we get

$$B_n=-\left(\frac{c_2}{c_1}\right)^2\frac{R}{n(n+1)P}A_n, \qquad (5)$$

$$(\widehat{r\theta}_{r=a}=)\frac{dP_n(\cos\theta)}{d\theta}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{zt}A_n}{z}\Omega dz=\frac{dP_n(\cos\theta)}{d\theta}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{zt}g(z)}{z}dz, \qquad (6)$$

where

$$\left.\begin{aligned}
&\Omega=\frac{\mu}{Pz^2c_1^2a^{5/2}n(n+1)}\left\{RS+2n(n+1)PQ\right\},\\
&Q=(n-1)H^{(1)}_{n+1/2}(izac_1)-izac_1H^{(1)}_{n+3/2}(izac_1),\\
&S=2(1-n^2)H^{(1)}_{n+1/2}(izac_2)\\
&\qquad+izac_2(2n+1)H^{(1)}_{n+3/2}(izac_2)+(zac_2)^2H^{(1)}_{n+5/2}(izac_2),\\
&R=\left\{(\lambda/2\mu)(zac_1)^2+n(n-1)\right\}H^{(1)}_{n+1/2}(izac_1)\\
&\qquad-izac_1(2n+1)H^{(1)}_{n+3/2}(izac_1)-(zac_1)^2H^{(1)}_{n+5/2}(izac_1),\\
&P=(n-1)H^{(1)}_{n+1/2}(izac_2)-izac_2H^{(1)}_{n+3/2}(izac_2).
\end{aligned}\right\} \qquad (7)$$

From (6) it is possible to write

$$A_n=\frac{g(z)}{\Omega}, \qquad (8)$$

from which we have

$$\Delta=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{zt}}{z}\frac{g(z)}{\Omega}\frac{H^{(1)}_{n+1/2}(izrc_1)}{\sqrt{r}}P_n(\cos\theta)dz \qquad (9)$$

for any value of r and similar expressions for 2ϖ, u, v.

3. *The expressions for $n=2$, $\lambda=\mu$.*

When $n=2$ and $\lambda=\mu$, the expressions for $u=u_1+u_2$, $v=v_1+v_2$ reduce to

$$
\left.
\begin{aligned}
u_1 &= \frac{-4\sqrt{3}\,a^3}{\mu c_2^2 r^4}\,\frac{P_2(\cos\theta)}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{g(z)P'}{z^3\Omega'}e^{z\{t-c_1(r-a)\}} \\
&\qquad\cdot\left\{(zrc_2)^3+4\sqrt{3}\,(zrc_2)^2+27(zrc_2)+27\sqrt{3}\right\}dz\,, \\
v_1 &= \frac{12a^3}{\mu c_2^2 r^4}\,\frac{dP_2(\cos\theta)}{d\theta}\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{g(z)P'}{z^3\Omega'}e^{z\{t-c_1(r-a)\}} \\
&\qquad\cdot\left\{(zrc_2)^2+3\sqrt{3}\,(zrc_2)+9\right\}dz\,, \\
u_2 &= \frac{2\sqrt{3}\,a^3}{\mu c_2^2 r^4}\,\frac{P_2(\cos\theta)}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{g(z)R'}{z^3\Omega'}e^{z\{t-c_2(r-a)\}} \\
&\qquad\cdot\left\{(zrc_2)^2+3(zrc_2)+3\right\}dz\,, \\
v_2 &= \frac{a^3}{\sqrt{3}\,\mu c_2^2 r^4}\,\frac{dP_2(\cos\theta)}{d\theta}\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{g(z)R'}{z^3\Omega'}e^{z\{t-c_2(r-a)\}} \\
&\qquad\cdot\left\{(zrc_2)^3+3(zrc_2)^2+6(zrc_2)+6\right\}dz\,,
\end{aligned}
\right\}\qquad (10)
$$

where

$$
\left.
\begin{aligned}
P' &= (zac_2)^3+5(zac_2)^2+12(zac_2)+12, \\
R' &= \sqrt{3}\,(zac_2)^4+13(zac_2)^3+37\sqrt{3}\,(zac_2)^2+216(zac_2)+216\sqrt{3}\,, \\
\Omega' &= \left\{zac_2+(0{\cdot}8033-1{\cdot}0406i)\right\}\left\{zac_2+(0{\cdot}8033+1{\cdot}0406i)\right\} \\
&\quad\cdot\left\{zac_2+(3{\cdot}648-1{\cdot}4410i)\right\}\left\{zac_2+(3{\cdot}648+1{\cdot}4410i)\right\} \\
&\quad\cdot\left\{zac_2+(1{\cdot}8015-4{\cdot}1852i)\right\}\left\{zac_2+(1{\cdot}8015+4{\cdot}1852i)\right\} \\
&= \prod^{s}\left\{zac_2+(p_s-iq_s)\right\}\left\{zac_2+(p_s+iq_s)\right\}.
\end{aligned}
\right\}\qquad (11)
$$

The roots of $\Omega'=0$ are the same as those of $\Omega_0=0$ in the previous paper.

4. *The special case,* $f(t)=p_0$ *for* $t>0$ and $f(t)=0$ for $t<0$.

In this case, $g(z)=p_0$ for $t>0$, $g(z)=0$ for $t<0$. Thus from (10) we have

$$
u_1=u_{01}+u_{f1},\quad u_2=u_{02}+u_{f2},\quad v_1=v_{01}+v_{f1},\quad v_2=v_{02}+v_{f2}, \qquad (12)
$$

where

$$\left.\begin{aligned}
u_{01}&=\frac{-a^5p_0P_2(\cos\theta)}{2116\mu r^4}\left[\left\{2208\left(\frac{r}{a}\right)^2-4968\left(\frac{r}{a}\right)-729\right\}\right.\\
&\left.\quad+4968\sqrt{3}\left(\frac{r}{a}-1\right)\left\{\frac{t-c_1(r-a)}{ac_2}\right\}+7452\left\{\frac{t-c_1(r-a)}{ac_2}\right\}^2\right],\\
v_{01}&=\frac{3a^5p_0}{2116\mu r^4}\frac{dP_2(\cos\theta)}{d\theta}\left[\left\{184\left(\frac{r}{a}\right)^2-552\left(\frac{r}{a}\right)-81\right\}\right.\\
&\left.\quad+552\sqrt{3}\left(\frac{r}{a}-1\right)\left\{\frac{t-c_1(r-a)}{ac_2}\right\}+828\left\{\frac{t-c_1(r-a)}{ac_2}\right\}^2\right],\\
&\qquad\qquad\qquad\qquad\left[\left\{t-c_1(r-a)\right\}>0\right]\\
u_{01}&=v_{01}=0,\qquad\qquad\qquad\left[\left\{t-c_1(r-a)\right\}<0\right]
\end{aligned}\right\}\tag{13}$$

$$\left.\begin{aligned}
u_{02}&=\frac{3a^5p_0P_2(\cos\theta)}{1058\mu r^4}\left[\left\{828\left(\frac{r}{a}\right)^2-2484\left(\frac{r}{a}\right)+925\right\}\right.\\
&\left.\quad+2484\left(\frac{r}{a}-1\right)\left\{\frac{t-c_2(r-a)}{ac_2}\right\}+1242\left\{\frac{t-c_2(r-a)}{ac_2}\right\}^2\right],\\
v_{02}&=\frac{-a^5p_0}{1058\mu r^4}\frac{dP_2(\cos\theta)}{d\theta}\left[\left\{1242\left(\frac{r}{a}\right)^2-2484\left(\frac{r}{a}\right)+925\right\}\right.\\
&\left.\quad+2484\left(\frac{r}{a}-1\right)\left\{\frac{t-c_2(r-a)}{ac_2}\right\}+1242\left\{\frac{t-c_2(r-a)}{ac_2}\right\}^2\right],\\
&\qquad\qquad\qquad\qquad\left[\left\{t-c_2(r-a)\right\}>0\right]\\
u_{02}&=v_{02}=0,\qquad\qquad\qquad\left[\left\{t-c_2(r-a)\right\}<0\right]
\end{aligned}\right\}\tag{14}$$

$$\left.\begin{aligned}
u_{f1}&=\frac{-8\sqrt{3}a^5p_0}{\mu r^4}P_2(\cos\theta)\sum_{s=1}^{3}\sqrt{\frac{(P_s^2+P_s'^2)(U_{1s}^2+U_{1s}'^2)}{(Z_s^2+Z_s'^2)(\Omega_s^2+\Omega_s'^2)}}e^{-\frac{p_s}{ac_2}\{t-c_1(r-a)\}}\\
&\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_1(r-a)\right\}+\tan^{-1}\frac{P_s'}{P_s}+\tan^{-1}\frac{U_{1s}'}{U_{1s}}-\tan^{-1}\frac{Z_s'}{Z_s}-\tan^{-1}\frac{\Omega_s'}{\Omega_s}\right],\\
v_{f1}&=\frac{24a^5p_0}{\mu r^4}\frac{dP_2(\cos\theta)}{d\theta}\sum_{s=1}^{3}\sqrt{\frac{(P_s^2+P_s'^2)(V_{1s}^2+V_{1s}'^2)}{(Z_s^2+Z_s'^2)(\Omega_s^2+\Omega_s'^2)}}e^{-\frac{p_s}{ac_2}\{t-c_1(r-a)\}}\\
&\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_1(r-a)\right\}+\tan^{-1}\frac{P_s'}{P_s}+\tan^{-1}\frac{V_{1s}'}{V_{1s}}-\tan^{-1}\frac{Z_s'}{Z_s}-\tan^{-1}\frac{\Omega_s'}{\Omega_s}\right],
\end{aligned}\right\}\tag{15}$$

$$\left[\left\{t-c_1(r-a)\right\}>0\right]$$

$$u_{f1}=v_{f1}=0, \qquad \left[\left\{t-c_1(r-a)\right\}<0\right]$$

$$u_{f2}=\frac{4\sqrt{3}\,a^5p_0}{\mu r^4}P_2(\cos\theta)\sum_{s=1}^{3}\sqrt{\frac{(R_s^2+R_s'^2)(U_{2s}^2+U_{2s}'^2)}{(Z_s^2+Z_s'^2)(\Omega_s^2+\Omega_s'^2)}}\,e^{-\frac{p_s}{ac_2}\{t-c_2(r-a)\}}$$

$$\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_2(r-a)\right\}+\tan^{-1}\frac{R_s'}{R_s}+\tan^{-1}\frac{U_{2s}'}{U_{2s}}-\tan^{-1}\frac{Z_s'}{Z_s}-\tan^{-1}\frac{\Omega_s'}{\Omega_s}\right],$$

$$v_{f2}=\frac{-2a^5p_0}{\sqrt{3}\,\mu r^4}\,\frac{dP_2(\cos\theta)}{d\theta}\sum_{s=1}^{3}\sqrt{\frac{(R_s^2+R_s'^2)(V_{2s}^2+V_{2s}'^2)}{(Z_s^2+Z_s'^2)(\Omega_s^2+\Omega_s'^2)}}\,e^{-\frac{p_s}{ac_2}\{t-c_2(r-a)\}} \tag{16}$$

$$\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_2(r-a)\right\}+\tan^{-1}\frac{R_s'}{R_s}+\tan^{-1}\frac{V_{2s}'}{V_{2s}}-\tan^{-1}\frac{Z_s'}{Z_s}-\tan^{-1}\frac{\Omega_s'}{\Omega_s}\right],$$

$$\left[\left\{t-c_2(r-a)\right\}>0\right]$$

$$u_{f2}=v_{f2}=0, \qquad \left[\left\{t-c_2(r-a)\right\}<0\right]$$

in which

$$P_s=p_s(3q_s-p_s)+5(p_s^2-q_s^2)+12(1-p_s)\,,$$

$$P_s'=q_s\left\{(3p_s^2-q_s^2)-10p_s+12\right\},$$

$$R_s=\sqrt{3}\,(p_s^4-6p_s^2q_s^2+q_s^4)+13p_s(3q_s^2-p_s^2)$$
$$+37\sqrt{3}\,(p_s^2-q_s^2)+216(\sqrt{3}-p_s)\,,$$

$$R_s'=q_s\left\{4\sqrt{3}\,p_s(q_s^2-p_s^2)+13(3p_s^2-q_s^2)-74\sqrt{3}\,p_s+216\right\},$$

$$Z_s=p_s(3q_s^2-p_s^2)\,,\quad Z_s'=q_s(3p_s^2-q_s^2)\,,$$

$$\Omega_s=6p_s\left\{(p_s^2-q_s^2)(3q_s^2-p_s^2)+2q_s^2(3p_s^2-q_s^2)\right\}$$
$$+\left(25+\frac{65}{\sqrt{3}}\right)(p_s^4-6p_s^2q_s^2+q_s^4)+\left(232+\frac{164}{\sqrt{3}}\right)p_s(3q_s^2-p_s^2)$$
$$+\left(339+\frac{1107}{\sqrt{3}}\right)(p_s^2-q_s^2)-\left(882+\frac{1104}{\sqrt{3}}\right)p_s+552\left(1+\frac{1}{\sqrt{3}}\right),$$

$$\Omega_s'=q_s\left[6\left\{2p_s^2(p_s^2-3q_s^2)+(p_s^2-q_s^2)(3p_s^2-q_s^2)\right\}-4p_s(p_s^2-q_s^2)\left(25+\frac{65}{\sqrt{3}}\right)\right. \tag{17}$$

$$+\left(232+\frac{164}{\sqrt{3}}\right)(3p_s^2-q_s^2)-2p_s\left(339+\frac{1107}{\sqrt{3}}\right)+\left(882+\frac{1104}{\sqrt{3}}\right)\Bigg],$$

$$U_{1s}=\left(\frac{r}{a}\right)^3p_s\left(3q_s^2-p_s^2\right)+4\sqrt{3}\left(\frac{r}{a}\right)^2\left(p_s^2-q_s^2\right)-27\left(\frac{r}{a}\right)p_s+27\sqrt{3},$$

$$U'_{1s}=q_s\left(\frac{r}{a}\right)\left\{\left(\frac{r}{a}\right)^2(3p_s^2-q_s^2)-8\sqrt{3}\left(\frac{r}{a}\right)p_s+27\right\},$$

$$V_{1s}=\left(\frac{r}{a}\right)^2(p_s^2-q_s^2)-3\sqrt{3}\left(\frac{r}{a}\right)p_s+9,$$

$$V'_{1s}=q_s\left(\frac{r}{a}\right)\left\{3\sqrt{3}-2\left(\frac{r}{a}\right)p_s\right\},$$

$$U_{2s}=\left(\frac{r}{a}\right)^2(p_s^2-q_s^2)-3p_s\left(\frac{r}{a}\right)+3,\quad U'_{2s}=q_s\left(\frac{r}{a}\right)\left\{3-2p_s\left(\frac{r}{a}\right)\right\},$$

$$V_{2s}=\left(\frac{r}{a}\right)^3p_s(3q_s^2-p_s^2)+3\left(\frac{r}{a}\right)^2(p_s^2-q_s^2)-6\left(\frac{r}{a}\right)p_s+6,$$

$$V'_{2s}=q_s\left(\frac{r}{a}\right)\left\{\left(\frac{r}{a}\right)^2(3p_s^2-q_s^2)-6p_s\left(\frac{r}{a}\right)+6\right\}.$$

The expressions for displacements at the spherical boundary $r=a$ are expressed by

$$\left.\begin{aligned}
&u_{01}\Big/\frac{ap_0}{\mu}P_2(\cos\theta)=\frac{-1}{2116}\left\{7452\left(\frac{t}{ac_2}\right)^2-3489\right\},\\
&v_{01}\Big/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=\frac{3}{2116}\left\{828\left(\frac{t}{ac_2}\right)^2-449\right\},\\
&u_{02}\Big/\frac{ap_0}{\mu}P_2(\cos\theta)=\frac{3}{1058}\left\{1242\left(\frac{t}{ac_2}\right)^2-731\right\},\\
&v_{02}\Big/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=\frac{-1}{1058}\left\{1242\left(\frac{t}{ac_2}\right)^2-317\right\},\qquad [t/ac_2>0]\\
&u_{01}=v_{01}=u_{02}=v_{02}=0,\qquad [t/ac_2<0]
\end{aligned}\right\}\quad(18)$$

$$u_{f1}\Big/\frac{ap_0}{\mu}P_2(\cos\theta)=-8\sqrt{3}\sum_{s=1}^{3}M_s\sqrt{U_{1s,a}^2+U'^2_{1s,a}}\,e^{-\frac{p_st}{ac_2}}$$

$$\cdot\cos\left\{\frac{q_st}{ac_2}+\mu_s+\tan^{-1}\frac{U'_{1s,a}}{U_{1s,a}}\right\},$$

$$\left.\begin{aligned}
&v_{f1}\Bigg/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=24\sum_{s=1}^{3}M_s\sqrt{V_{1s,a}^2+V_{1s,a}'^2}\,e^{-\frac{p_st}{ac_2}}\\
&\qquad\qquad\cdot\cos\left\{\frac{q_st}{ac_2}+\mu_s+\tan^{-1}\frac{V'_{1s,a}}{V_{1s,a}}\right\},\\
&u_{f2}\Bigg/\frac{ap_0}{\mu}P_2(\cos\theta)=4\sqrt{3}\sum_{s=1}^{3}N_s\sqrt{U_{2s,a}^2+U_{2s,a}'^2}\,e^{-\frac{p\,t}{ac_2}}\\
&\qquad\qquad\cdot\cos\left\{\frac{q_st}{ac_2}+\nu_s+\tan^{-1}\frac{U'_{2s,a}}{U_{2s,a}}\right\},\\
&v_{f2}\Bigg/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=\frac{-2}{\sqrt{3}}\sum_{s=1}^{3}N_s\sqrt{V_{2s,a}^2+V_{2s,a}'^2}\,e^{-\frac{p\,t}{ac_2}}\\
&\qquad\qquad\cdot\cos\left\{\frac{q_st}{ac_2}+\nu_s+\tan^{-1}\frac{V'_{2s,a}}{V_{2s,a}}\right\},\\
&\qquad\qquad\qquad\qquad[t/ac_2>0]\\
&u_{f1}=v_{f1}=u_{f2}=v_{f2}=0,\qquad\qquad [t/ac_2<0]
\end{aligned}\right\}\quad(19)$$

where

$$\left.\begin{aligned}
&U_{1s,a}=p_s(3q_s^2-p_s^2)+4\sqrt{3}\,(p_s^2-q_s^2)+27(\sqrt{3}-p_s),\\
&U'_{1s,a}=q_s\left\{(3p_s^2-q_s^2)-8\sqrt{3}\,p_s+27\right\},\\
&V_{1s,a}=(p_s^2-q_s^2)-3\sqrt{3}\,p_s+9,\qquad V'_{1s,a}=q_s\left\{3\sqrt{3}-2p_s\right\},\\
&U_{2s,a}=(p_s^2-q_s^2)+3(1-p_s),\qquad U'_{2s,a}=q_s\left\{3-2p_s\right\},\\
&V_{2s,a}=p_s(3q_s^2-p_s^2)+3(p_s^2-q_s^2)+6(1-p_s),\\
&V'_{2s,a}=q_s\left\{(3p_s^2-q_s^2)+6(1-p_s)\right\},\\
&M_s=\sqrt{\frac{P_s^2+P_s'^2}{(Z_s^2+Z_s'^2)(\Omega_s^2+\Omega_s'^2)}},\qquad N_s=\sqrt{\frac{R_s^2+R_s'^2}{(Z_s^2+Z_s'^2)(\Omega_s^2+\Omega_s'^2)}},\\
&\mu_s=\tan^{-1}\frac{P'_s}{P_s}-\tan^{-1}\frac{Z'_s}{Z_s}-\tan^{-1}\frac{\Omega'_s}{\Omega_s},\\
&\nu_s=\tan^{-1}\frac{R'_s}{R_s}-\tan^{-1}\frac{Z'_s}{Z_s}-\tan^{-1}\frac{\Omega'_s}{\Omega_s}.
\end{aligned}\right\}\quad(20)$$

The displacements at a relatively large r can be expressed by

$$\left.\begin{aligned}
&u_{01}\Big/\frac{a^3p_0P_2(\cos\theta)}{r^2\mu}=\frac{-24}{23},\qquad v_{01}\Big/\frac{a^3p_0}{r^2\mu}\frac{dP_2(\cos\theta)}{d\theta}=\frac{6}{23},\\
&\qquad\qquad\left[\left\{t-c_1(r-a)\right\}>0,\quad \frac{t-c_1(r-a)}{ac_2}\leqslant\frac{r}{a}\right]\\
&u_{01}=v_{01}=0,\qquad \left[\left\{t-c_1(r-a)\right\}<0\right]\\
&u_{02}\Big/\frac{a^3p_0P_2(\cos\theta)}{r^2\mu}=\frac{54}{23},\qquad v_{02}\Big/\frac{a^3p_0}{r^2\mu}\frac{dP_2(\cos\theta)}{d\theta}=\frac{-27}{23},\\
&\qquad\qquad\left[\left\{t-c_2(r-a)\right\}>0,\quad \frac{t-c_2(r-a)}{ac_2}\leqslant\frac{r}{a}\right]\\
&u_{02}=v_{02}=0,\qquad \left[\left\{t-c_2(r-a)\right\}<0\right]
\end{aligned}\right\}\quad(21)$$

$$\left.\begin{aligned}
&u_{f1}\Big/\frac{a^2p_0P_2(\cos\theta)}{r\mu}=-8\sqrt{3}\sum_{s=1}^{3}M_s\sqrt{U_{1s,\infty}^2+U_{1s,\infty}'^2}\,e^{-\frac{p_s}{ac_2}\{t-c_1(r-a)\}}\\
&\qquad\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_1(r-a)\right\}+\mu_s+\tan^{-1}\frac{U_{1s,\infty}'}{U_{1s,\infty}}\right],\\
&v_{f1}\Big/\frac{a^3p_0}{r^2\mu}\frac{dP_2(\cos\theta)}{d\theta}=24\sum_{s=1}^{3}M_s\sqrt{V_{1s,\infty}^2+V_{1s,\infty}'^2}\,e^{-\frac{p_s}{ac_2}\{t-c_1(r-a)\}}\\
&\qquad\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_1(r-a)\right\}+\mu_s+\tan^{-1}\frac{V_{1s,\infty}'}{V_{1s,\infty}}\right],\\
&\qquad\qquad\left[\left\{t-c_1(r-a)\right\}>0\right]\\
&u_{f1}=v_{f1}=0,\qquad \left[\left\{t-c_1(r-a)\right\}<0\right]\\
&u_{f2}\Big/\frac{a^3p_0P_2(\cos\theta)}{r^2\mu}=4\sqrt{3}\sum_{s=1}^{3}N_s\sqrt{U_{2s,\infty}^2+U_{2s,\infty}'^2}\,e^{-\frac{p_s}{ac_2}\{t-c_2(r-a)\}}\\
&\qquad\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_2(r-a)\right\}+\nu_s+\tan^{-1}\frac{U_{2s,\infty}'}{U_{2s,\infty}}\right],\\
&v_{f2}\Big/\frac{a^2p_0}{r\mu}\frac{dP_2(\cos\theta)}{d\theta}=\frac{-2}{\sqrt{3}}\sum_{s=1}^{3}N_s\sqrt{V_{2s,\infty}^2+V_{2s,\infty}'^2}\,e^{-\frac{p_s}{ac_2}\{t-c_2(r-a)\}}\\
&\qquad\cdot\cos\left[\frac{q_s}{ac_2}\left\{t-c_2(r-a)\right\}+\nu_s+\tan^{-1}\frac{V_{2s,\infty}'}{V_{2s,\infty}}\right],\\
&\qquad\qquad\left[\left\{t-c_2(r-a)\right\}>0\right]
\end{aligned}\right\}\quad(22)$$

$$u_{f2}=v_{f2}=0\,, \qquad\qquad \left[\left\{t-c_2(r-a)\right\}<0\right] \Bigg)$$

where

$$\left.\begin{aligned} U_{1s,\infty}&=p_s(3q_s^2-p_s^2)\,, & U'_{1s,\infty}&=q_s(3p_s^2-q_s^2)\,,\\ V_{1s,\infty}&=p_s^2-q_s^2, & V'_{1s,\infty}&=-2p_sq_s,\\ U_{2s,\infty}&=p_s^2-q_s^2, & U'_{2s,\infty}&=-2p_sq_s,\\ V_{2s,\infty}&=p_s^2(3q_s^2-p_s^2)\,, & V'_{2s,\infty}&=q_s(3p_s^2-q_s^2)\,. \end{aligned}\right\} \qquad (23)$$

The results of numerical calculation for u_{01}, u_{02}, u_{f1}, u_{f2}, v_{01}, v_{02}, v_{f1}, v_{f2} and also for $u_{01}+u_{f1}$, $u_{02}+u_{f2}$, $v_{01}+v_{f1}$, $v_{02}+v_{f2}$, all at $r=a$, are shown in Figs. 1, 2. It will be seen that, as in the case of pressure change, with time increase the displacements of forced waves, whether

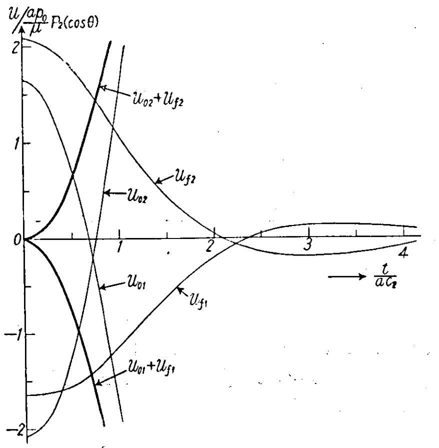

Fig. 1. Displacements for u's at $r=a$. $f(t)=p_0$ for $t>0$, $f(t)=0$ for $t<0$.

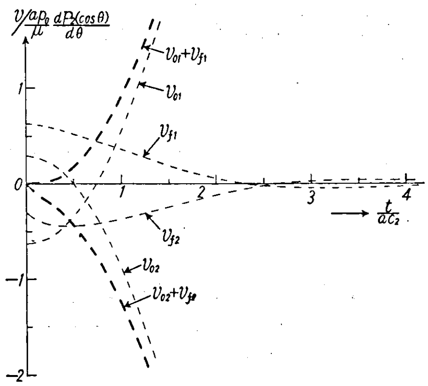

Fig. 2. Displacements for v's at $r=a$. $f(t)=p_0$ for $t>0$, $f(t)=0$ for $t<0$.

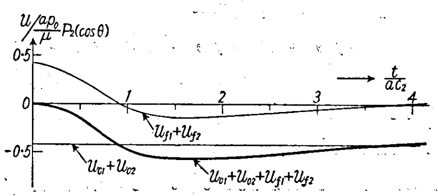

Fig. 3. Resultant displacements for u's at $r=a$. $f(t)=p_0$ for $t>0$, $f(t)=0$ for $t<0$.

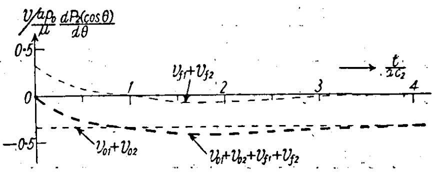

Fig. 4. Resultant displacements for v's at $r=a$. $f(t)=p_0$ for $t>0$, $f(t)=0$ for $t<0$.

in dilatational waves or in distortional waves, augment without limit, whereas the displacements of free waves diminish. Thus, the resultant displacements $u_{01}+u_{f1}$, $u_{02}+u_{f2}$, $v_{01}+v_{f1}$, $v_{02}+v_{f2}$ also diverge with time.

The resultant displacements of forced waves, namely, $u_{01}+u_{02}$, $v_{01}+$

v_{02} never diverge. The values of $u_{01}+u_{02}$, $v_{01}+v_{02}$, together with $u_{f1}+u_{f2}$, $v_{f1}+v_{f2}$, $u_{01}+u_{02}+u_{f1}+u_{f2}$, $v_{01}+v_{02}+v_{f1}+v_{f2}$, are shown in Figs. 3, 4. It will be seen that $u_{01}+u_{02}$, $v_{01}+v_{02}$ are constant for any t, and that $u_{01}+u_{02}+u_{f1}+u_{f2}$, $v_{01}+v_{02}+v_{f1}+v_{f2}$ tend to constant values $u_{01}+u_{02}$, $v_{01}+v_{02}$ in question, for $t\rightarrow\infty$. The values of $u_{01}+u_{02}$, $v_{01}+v_{02}$ agree with the change in shearing force at the spherical boundary, at least apparently.

The results of numerical calculation for u_{01}, v_{01}, u_{f1}, v_{f1}, $u_{01}+u_{f1}$, $v_{01}+v_{f1}$, and also for u_{02}, v_{02}, u_{f2}, v_{f2}, $u_{02}+u_{f2}$, $v_{02}+v_{f2}$, all at $r\rightarrow\infty$, are shown in Figs. 5, 6. It should be borne in mind that although the order in $1/r$ for u_{01}, v_{01}, v_{f1}, u_{02}, u_{f2}, v_{02} is higher than that for u_{f1}, v_{f2}, all curves are plotted, for convenience, with the same scale for ordinate. Thus,

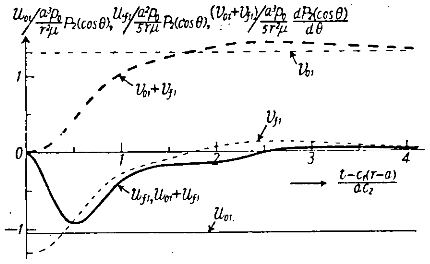

Fig. 5. Displacements for longitudinal waves at $r=\infty$. $f(t)=p_0$ for $t>0$, $f(t)=0$ for $t<0$.

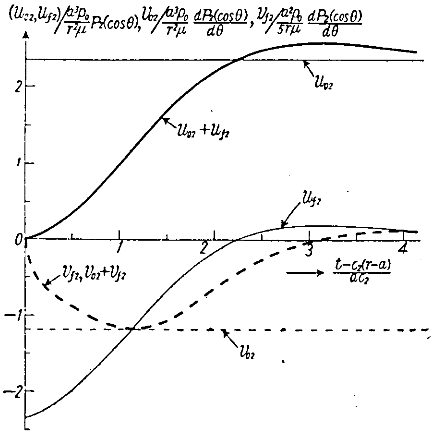

Fig. 6. Displacements for transverse waves at $r=\infty$. $f(t)=p_0$ for $t>0$, $f(t)=0$ for $t<0$.

the curves for $u_{01}+u_{f1}$, $v_{02}+v_{f2}$ are identical respectively with those for u_{f1}, v_{f2}. The condition of divergence and convergence for the forced waves is the same as that shown in the previous paper.

The initial motions at $r\rightarrow\infty$ for $u_{01}+u_{f1}$, $v_{01}+v_{f1}$, $u_{02}+u_{f2}$, $v_{02}+v_{f2}$, are, respectively, in good accordance with those at $r=a$ for $u_{01}+u_{f1}$, $v_{01}+v_{f1}$, $u_{02}+u_{f2}$, $v_{02}+v_{f2}$. The motions corresponding to these curves begin from zero displacements. All these curves are indicated with thick lines (full lines for u_s' and broken lines for v_s'). It is likely that the initial motions of the displacements $u_{01}+u_{f1}$ and $v_{02}+v_{f2}$ at $r=a$ as well as at $r\rightarrow\infty$ are invariably in the sense of change in shearing force at the spherical origin.

Finally, it appears that the displacements represented by $u_{01}+u_{02}+u_{f1}+u_{f2}$, $v_{01}+v_{02}+v_{f1}+v_{f2}$ at $r=a$, agree with change in shearing force at the spherical origin, for initial motion as well as for motion at any time. The curves for these cases are also drawn with thick lines (full lines for u'_s and broken lines for v'_s).

5. *The special case, $f(t)=p_0\sin\beta t$ for $t>0$ and $f(t)=0$ for $t<0$.*

In this case $g(z)=z\beta/(z^2+\beta^2)$ for $t>0$, $g(z)=0$ for $t<0$. Thus, from (10) we obtain

$$\left.\begin{aligned}
u_1&=\frac{-4\sqrt{3}\,a^3\beta}{\mu c_2^2 r^4}\,\frac{P_2(\cos\theta)}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{P' e^{z\{t-c_1(r-a)\}}}{z^2(z^2+\beta^2)\,\Omega'}\\
&\qquad\cdot\left\{(zrc_2)^3+4\sqrt{3}\,(zrc_2)^2+27(zrc_2)+27\sqrt{3}\right\}dz,\\
v_1&=\frac{12a^3\beta}{\mu c_2^2 r^4}\,\frac{dP_2(\cos\theta)}{d\theta}\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{P' e^{z\{t-c_1(r-a)\}}}{z^2(z^2+\beta^2)\,\Omega'}\\
&\qquad\cdot\left\{(zrc_2)^2+3\sqrt{3}\,(zrc_2)+9\right\}dz,\\
u_2&=\frac{-2\sqrt{3}\,a^3\beta}{\mu c_2^2 r^4}\,\frac{P_2(\cos\theta)}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{R' e^{z\{t-c_2(r-a)\}}}{z^2(z^2+\beta^2)\,\Omega'}\\
&\qquad\cdot\left\{(zrc_2)^2+3(zrc_2)+3\right\}dz,\\
v_2&=\frac{-a^3\beta}{\sqrt{3}\,\mu c_2^2 r^4}\,\frac{dP_2(\cos\theta)}{d\theta}\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{R' e^{z\{t-c_2(r-a)\}}}{z^2(z^2+\beta^2)\,\Omega'}\\
&\qquad\cdot\left\{(zrc_2)^3+3(zrc_2)^2+6(zrc_2)+6\right\}dz.
\end{aligned}\right\}\quad(24)$$

We shall write

$$\left.\begin{aligned}
u_1&=u_{01}+u'_{01}+u_{f1}, & u_2&=u_{02}+u'_{02}+u_{f2},\\
v_1&=v_{01}+v'_{01}+v_{f1}, & v_2&=v_{02}+v'_{02}+v_{f2},
\end{aligned}\right\}\quad(25)$$

where $u_{01}+u'_{01}$, $u_{02}+u'_{02}$, $v_{01}+v'_{01}$, $v_{02}+v'_{02}$, are the parts of u_1, u_2, v_1, v_2 that correspond to forced waves and u_{f1}, u_{f2}, v_{f1}, v_{f2} the parts that correspond to free waves.

The results of calculation show that

$$\left.\begin{aligned}
u_{01}&=\frac{-54a^5p_0P_2(\cos\theta)}{23\mu r^4\zeta}\left[3\left\{\frac{t-c_1(r-a)}{ac_2}\right\}+\sqrt{3}\left(\frac{r}{a}-1\right)\right],\\
v_{01}&=\frac{18a^5p_0}{23\mu r^4\zeta}\,\frac{dP_2(\cos\theta)}{d\theta}\left[3\left\{\frac{t-c_1(r-a)}{ac_2}\right\}+\sqrt{3}\left(\frac{r}{a}-1\right)\right],\\
&\qquad\qquad\left[\left\{t-c_1(r-a)\right\}>0\right]
\end{aligned}\right\}\quad(26)$$

$$u_{01}=v_{01}=0\,, \qquad \left[\left\{t-c_1(r-a)\right\}<0\right]$$

$$\left.\begin{aligned}
u_{02}&=\frac{162a^5p_0P_2(\cos\theta)}{23\mu r^4\zeta}\left\{\frac{t-c_2(r-a)}{ac_2}+\frac{r}{a}-1\right\},\\
v_{02}&=\frac{-54a^5p_0}{23\mu r^4\zeta}\frac{dP_2(\cos\theta)}{d\theta}\left\{\frac{t-c_2(r-a)}{ac_2}+\frac{r}{a}-1\right\},\\
&\qquad\qquad\left[\left\{t-c_2(r-a)\right\}>0\right]\\
u_{02}&=v_{02}=0\,, \qquad \left[\left\{t-c_2(r-a)\right\}<0\right]
\end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned}
u'_{01}&=\frac{4\sqrt{3}\,a^5p_0P_2(\cos\theta)}{\mu r^4\zeta}\sqrt{\frac{(P_\beta^2+P_\beta'^2)(U_{1\beta}^2+U_{1\beta}'^2)}{\Omega_\beta^2+\Omega_\beta'^2}}\\
&\cdot\sin\left[\beta\left\{t-c_1(r-a)\right\}+\tan^{-1}\frac{P'_\beta}{P_\beta}+\tan^{-1}\frac{U'_{1\beta}}{U_{1\beta}}-\tan^{-1}\frac{\Omega'_\beta}{\Omega_\beta}\right],\\
v'_{01}&=\frac{-12a^5p_0}{\mu r^4\zeta^2}\frac{dP_2(\cos\theta)}{d\theta}\sqrt{\frac{(P_\beta^2+P_\beta'^2)(V_{1\beta}^2+V_{1\beta}'^2)}{\Omega_\beta^2+\Omega_\beta'^2}}\\
&\cdot\sin\left[\beta\left\{t-c_1(r-a)\right\}+\tan^{-1}\frac{P'_\beta}{P_\beta}+\tan^{-1}\frac{V'_{1\beta}}{V_{1\beta}}-\tan^{-1}\frac{\Omega'_\beta}{\Omega_\beta}\right],\\
&\qquad\qquad\left[\left\{t-c_1(r-a)\right\}>0\right]\\
u'_{01}&=v'_{01}=0\,, \qquad \left[\left\{t-c_1(r-a)\right\}<0\right]
\end{aligned}\right\} \quad (28)$$

$$\left.\begin{aligned}
u'_{02}&=\frac{-2\sqrt{3}\,a^5P_2(\cos\theta)}{\mu r^4\zeta^2}\sqrt{\frac{(R_\beta^2+R_\beta'^2)(U_{2\beta}^2+U_{2\beta}'^2)}{\Omega_\beta^2+\Omega_\beta'^2}}\\
&\cdot\sin\left[\beta\left\{t-c_2(r-a)\right\}+\tan^{-1}\frac{R'_\beta}{R_\beta}+\tan^{-1}\frac{U'_{2\beta}}{U_{2\beta}}-\tan^{-1}\frac{\Omega'_\beta}{\Omega_\beta}\right],\\
v'_{02}&=\frac{a^5p_0}{\sqrt{3}\,\mu r^4\zeta^2}\frac{dP_2(\cos\theta)}{d\theta}\sqrt{\frac{(R_\beta^2+R_\beta'^2)(V_{2\beta}^2+V_{2\beta}'^2)}{\Omega_\beta^2+\Omega_\beta'^2}}\\
&\cdot\sin\left[\beta\left\{t-c_2(r-a)\right\}+\tan^{-1}\frac{R'_\beta}{R_\beta}+\tan^{-1}\frac{V'_{2\beta}}{V_{2\beta}}-\tan^{-1}\frac{\Omega'_\beta}{\Omega_\beta}\right],
\end{aligned}\right\} \quad (29)$$

$$\left.\begin{aligned} & && \left[\left\{t-c_2(r-a)\right\}>0\right] \\ & u'_{02}=v'_{02}=0\,, && \left[\left\{t-c_2(r-a)\right\}<0\right] \end{aligned}\right)$$

where

$$\left.\begin{aligned} &\zeta=\beta ac_2, \qquad P_\beta=12-5\zeta^2, \qquad P'_\beta=\zeta(12-\zeta^2)\,, \\ &R_\beta=\sqrt{3}\,(\zeta^4-37\zeta^2+216)\,, \qquad R'_\beta=\zeta(216-13\zeta^2)\,, \\ &\Omega_\beta=-\zeta^6+(58+41/\sqrt{3}\,)\zeta^4-(441+552/\sqrt{3}\,)\zeta^2+552\,, \\ &\Omega'_\beta=\zeta\left\{(5+13/\sqrt{3}\,)\zeta^4-(113+123\sqrt{3}\,)\zeta^2+184(3+\sqrt{3}\,)\right\}, \\ &U_{1\beta}=27\sqrt{3}-4\sqrt{3}\left(\frac{\zeta r}{a}\right)^2, \qquad U'_{1\beta}=\frac{\zeta r}{a}\left\{27-\left(\frac{\zeta r}{a}\right)^2\right\}, \\ &V_{1\beta}=9-\left(\frac{\zeta r}{a}\right)^2, \qquad V'_{1\beta}=3\sqrt{3}\,\frac{\zeta r}{a}\,, \\ &U_{2\beta}=3-\left(\frac{\zeta r}{a}\right)^2, \qquad U'_{2\beta}=3\frac{\zeta r}{a}\,, \\ &V_{2\beta}=6-3\left(\frac{\zeta r}{a}\right)^2, \qquad V'_{2\beta}=\frac{\zeta r}{a}\left\{6-\left(\frac{\zeta r}{a}\right)^2\right\}. \end{aligned}\right\} \tag{30}$$

The forms of free waves are the same as those shown in the previous section, with restriction that

$$\left.\begin{aligned} Z_s&=\frac{1}{\zeta}\left\{(p_s^4-6p_s^2q_s^2+q_s^4)+\zeta^2(p_s^2-q_s^2)\right\}, \\ Z'_s&=\frac{-2p_sq_s}{\zeta}\left\{2(p_s^2-q_s^2)+\zeta^2\right\}. \end{aligned}\right\} \tag{31}$$

The displacements at the spherical boundary $r=a$ for the special case $\beta ac_2=1$ are expressed by

$$\left.\begin{aligned} &u_{01}\Bigg/\frac{ap_0P_2(\cos\theta)}{\mu}=-7{\cdot}0435\frac{t}{ac_2}\,, \\ &v_{01}\Bigg/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=2{\cdot}3478\frac{t}{ac_2}\,, \\ &u_{02}\Bigg/\frac{ap_0P_2(\cos\theta)}{\mu}=7{\cdot}0435\frac{t}{ac_2}\,, \end{aligned}\right\} \tag{32}$$

$$v_{02}\Big/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=-2{\cdot}3478\frac{t}{ac_2}, \qquad \left[\frac{t}{ac_2}>0\right]$$

$$u_{01}=v_{01}=u_{02}=v_{02}=0, \qquad \left[\frac{t}{ac_2}<0\right]$$

$$\left.\begin{aligned}
&u'_{01}\Big/\frac{ap_0P_2(\cos\theta)}{\mu}=7{\cdot}5197\sin\left(\frac{t}{ac_2}-0{\cdot}21264\right),\\
&v'_{01}\Big/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=-2{\cdot}6118\sin\left(\frac{t}{ac_2}-0{\cdot}21484\right),\\
&u'_{02}\Big/\frac{ap_0P_2(\cos\theta)}{\mu}=-8{\cdot}1313\sin\left(\frac{t}{ac_2}-0{\cdot}23500\right),\\
&v'_{02}\Big/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=2{\cdot}1927\sin\left(\frac{t}{ac_2}-0{\cdot}18741\right), \qquad \left[\frac{t}{ac_2}>0\right]\\
&u'_{01}=v'_{01}=u'_{02}=v'_{02}=0, \qquad \left[\frac{t}{ac_2}<0\right]
\end{aligned}\right\}\quad(33)$$

$$\left.\begin{aligned}
u_{f1}\Big/\frac{ap_0P_2(\cos\theta)}{\mu}=&\,1{\cdot}883\,e^{-0{\cdot}8033\frac{t}{ac_2}}\cos\left\{1{\cdot}0406\frac{t}{ac_2}-0{\cdot}564\right\}\\
&-0{\cdot}01804\,e^{-1{\cdot}8015\frac{t}{ac_2}}\cos\left\{4{\cdot}1852\frac{t}{ac_2}-0{\cdot}0035\right\}\\
&+0{\cdot}03633\,e^{-3{\cdot}648\frac{t}{ac_2}}\cos\left\{1{\cdot}4410\frac{t}{ac_2}+1{\cdot}089\right\},\\
v_{f1}\Big/\frac{ap_0}{\mu}\frac{dP_2(\cos\theta)}{d\theta}=&-0{\cdot}6318\,e^{-0{\cdot}8033\frac{t}{ac_2}}\cos\left\{1{\cdot}0406\frac{t}{ac_2}-0{\cdot}484\right\}\\
&+0{\cdot}01875\,e^{-1{\cdot}8015\frac{t}{ac_2}}\cos\left\{4{\cdot}1852\frac{t}{ac_2}-1{\cdot}542\right\}\\
&+0{\cdot}01001\,e^{-3{\cdot}648\frac{t}{ac_2}}\cos\left\{1{\cdot}4410\frac{t}{ac_2}+1{\cdot}499\right\},\\
u_{f2}\Big/\frac{ap_0P_2(\cos\theta)}{\mu}=&-1{\cdot}972\,e^{-0{\cdot}8033\frac{t}{ac_2}}\cos\left\{1{\cdot}0406\frac{t}{ac_2}-0{\cdot}277\right\}\\
&+0{\cdot}03100\,e^{-1{\cdot}8015\frac{t}{ac_2}}\cos\left\{4{\cdot}1852\frac{t}{ac_2}+1{\cdot}427\right\}
\end{aligned}\right\}\quad(34)$$

$$-0{\cdot}01998\, e^{-3{\cdot}643\frac{t}{ac_2}} \cos\left\{1{\cdot}4410\frac{t}{ac_2}+1{\cdot}466\right\},$$

$$v_{f2}\Big/\frac{ap_0}{\mu}\frac{dP_0(\cos\theta)}{d\theta}=0{\cdot}6498\, e^{-0{\cdot}8033\frac{t}{ac_2}} \cos\left\{1{\cdot}0406\frac{t}{ac_2}-0{\cdot}919\right\}$$

$$-0{\cdot}02049\, e^{-1{\cdot}8015\frac{t}{ac_2}} \cos\left\{4{\cdot}1852\frac{t}{ac_2}-2{\cdot}772\right\}$$

$$-0{\cdot}01551\, e^{-3{\cdot}643\frac{t}{ac_2}} \cos\left\{1{\cdot}4410\frac{t}{ac_2}+1{\cdot}226\right\}, \qquad \left[\frac{t}{ac_2}>0\right]$$

$$u_{f1}=v_{f1}=u_{f2}=v_{f2}=0. \qquad \left[\frac{t}{ac_2}<0\right]$$

The expressions for displacements at a relatively large r, that is, $r\to\infty$ are of the forms

$$u_{01}\Big/\frac{a^4p_0P_2(\cos\theta)}{r^3\mu}=-4{\cdot}0666, \qquad v_{01}\Big/\frac{a^4p_0}{r^3\mu}\frac{dP_2(\cos\theta)}{d\theta}=1{\cdot}3555, \qquad \left[\left\{t-c_1(r-a)\right\}>0\right]$$

$$u_{01}=v_{01}=0, \qquad \left[\left\{t-c_1(r-a)\right\}<0\right]$$

$$u_{02}\Big/\frac{a^4p_0P_2(\cos\theta)}{r^3\mu}=7{\cdot}0435, \qquad v_{02}\Big/\frac{a^4p_0}{r^3\mu}\frac{dP_2(\cos\theta)}{d\theta}=-2{\cdot}3478, \qquad \left[\left\{t-c_2(r-a)\right\}>0\right]$$

$$u_{02}=v_{02}=0, \qquad \left[\left\{t-c_2(r-a)\right\}<0\right] \qquad (35)$$

$$u'_{01}\Big/\frac{a^2p_0P_2(\cos\theta)}{r\mu}=-0{\cdot}15807\sin\left\{\frac{t-c_1(r-a)}{ac_2}+0{\cdot}77992\right\},$$

$$v'_{01}\Big/\frac{a^3p_0}{r^2\mu}\frac{dP_2(\cos\theta)}{d\theta}=-0{\cdot}27379\sin\left\{\frac{t-c_1(r-a)}{ac_2}+2{\cdot}35071\right\}, \qquad \left[\left\{t-c_1(r-a)\right\}>0\right]$$

$$
\left.\begin{aligned}
&u'_{01}=v'_{01}=0\,, && \Big[\{t-c_1(r-a)\}<0\Big]\\
&u'_{02}\bigg/\frac{a^3p_0P_2(\cos\theta)}{r^2\mu}=-2{\cdot}2552\sin\left\{\frac{t-c_2(r-a)}{ac_2}+1{\cdot}92380\right\},\\
&v'_{02}\bigg/\frac{a^2p_0}{r\mu}\frac{dP_2(\cos\theta)}{d\theta}=-0{\cdot}37604\sin\left\{\frac{t-c_2(r-a)}{ac_2}+0{\cdot}35300\right\},\\
& && \Big[\{t-c_2(r-a)\}>0\Big]\\
&u'_{02}=v'_{02}=0\,, && \Big[\{t-c_2(r-a)\}<0\Big]
\end{aligned}\right\}\quad(36)
$$

$$
\left.\begin{aligned}
&u_{f1}\bigg/\frac{a^2p_0P_2(\cos\theta)}{r\mu}\\
&\quad=0{\cdot}1439\,e^{-0{\cdot}8033\frac{t-c_1(r-a)}{ac_2}}\cos\left\{1{\cdot}0406\frac{t-c_1(r-a)}{ac_2}-0{\cdot}786\right\}\\
&\qquad+0{\cdot}06371\,e^{-1{\cdot}8015\frac{t-c_1(r-a)}{ac_2}}\cos\left\{4{\cdot}1852\frac{t-c_1(r-a)}{ac_2}-1{\cdot}465\right\}\\
&\qquad\quad+0{\cdot}1065\,e^{-3{\cdot}648\frac{t-c_1(r-a)}{ac_2}}\cos\left\{1{\cdot}4410\frac{t-c_1(r-a)}{ac_2}+1{\cdot}543\right\},\\
&v_{f1}\bigg/\frac{a^3p_0}{r^2\mu}\frac{dP_2(\cos\theta)}{d\theta}\\
&\quad=0{\cdot}1896\,e^{-0{\cdot}8033\frac{t-c_1(r-a)}{ac_2}}\cos\left\{1{\cdot}0406\frac{t-c_1(r-a)}{ac_2}+0{\cdot}126\right\}\\
&\qquad+0{\cdot}02419\,e^{-1{\cdot}8015\frac{t-c_1(r-a)}{ac_2}}\cos\left\{4{\cdot}1852\frac{t-c_1(r-a)}{ac_2}-0{\cdot}300\right\}\\
&\qquad\quad+0{\cdot}04696\,e^{-3{\cdot}648\frac{t-c_1(r-a)}{ac_2}}\cos\left\{1{\cdot}4410\frac{t-c_1(r-a)}{ac_2}+1{\cdot}919\right\},\\
& && \Big[\{t-c_1(r-a)\}>0\Big]\\
&u_{f1}=v_{f1}=0\,, && \Big[\{t-c_1(r-a)\}<0\Big]\\
&u_{f2}\bigg/\frac{a^3p_0P_2(\cos\theta)}{r^2\mu}\\
&\quad=2{\cdot}337\,e^{-0{\cdot}8033\frac{t-c_2(r-a)}{ac_2}}\cos\left\{1{\cdot}0406\frac{t-c_2(r-a)}{ac_2}-0{\cdot}428\right\}
\end{aligned}\right\}\quad(37)
$$

$$+0{\cdot}03816\, e^{-1{\cdot}805\frac{t-c_2(r-a)}{ac_2}} \cos\left\{4{\cdot}1852\frac{t-c_2(r-a)}{ac_2}+2{\cdot}091\right\}$$

$$-0{\cdot}04387\, e^{-3{\cdot}648\frac{t-c_2(r-a)}{ac_2}} \cos\left\{1{\cdot}4410\frac{t-c_2(r-a)}{ac_2}+1{\cdot}795\right\},$$

$$v_{f2}\Bigg/\frac{a^2p_0}{r\mu}\frac{dP_2(\cos\theta)}{d\theta}$$

$$=0{\cdot}5122\, e^{-0{\cdot}8033\frac{t-c_2(r-a)}{ac_2}} \cos\left\{1{\cdot}0406\frac{t-c_2(r-a)}{ac_2}-1{\cdot}340\right\}$$

$$-0{\cdot}02900\, e^{-1{\cdot}8015\frac{t-c_2(r-a)}{ac_2}} \cos\left\{4{\cdot}1852\frac{t-c_2(r-a)}{ac_2}-2{\cdot}216\right\}$$

$$-0{\cdot}02867\, e^{-3{\cdot}648\frac{t-c_2(r-a)}{ac_2}} \cos\left\{1{\cdot}4410\frac{t-c_2(r-a)}{ac_2}+1{\cdot}419\right\},$$

$$\left[\{t-c_2(r-a)\}>0\right]$$

$$u_{f2}=v_{f2}=0. \qquad \left[\{t-c_2(r-a)\}<0\right]$$

The results of numerical calculation for u_{01}, u'_{01}, u_{02}, u'_{02}, u_{f1}, u_{f2}, v_{01}, v'_{01}, v_{02}, v'_{02}, v_{f1}, v_{f2} and also for $u_{01}+u'_{01}$, $u_{01}+u'_{01}+u_{f1}$, $u_{02}+u'_{02}$, $u_{02}+u'_{02}+u_{f2}$, $v_{01}+v'_{01}$, $v_{01}+v'_{01}+v_{f1}$, $v_{02}+v'_{02}$, $v_{02}+v'_{02}+v_{f2}$, all at $r=a$, are shown in Figs. 7, 8. In this case, too, the feature of divergence and

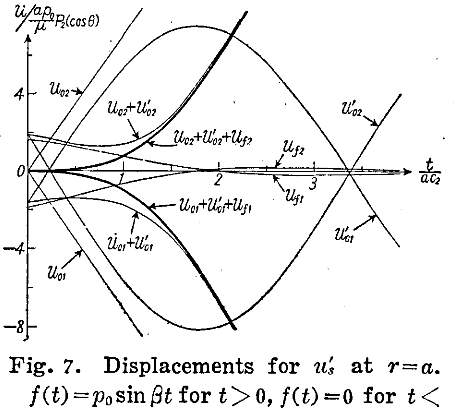

Fig. 7. Displacements for u's at $r=a$. $f(t)=p_0\sin\beta t$ for $t>0$, $f(t)=0$ for $t<0$, $\beta ac_2=1$.

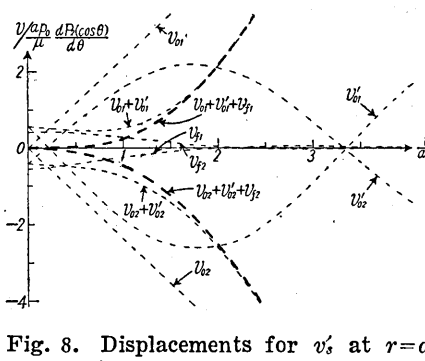

Fig. 8. Displacements for v's at $r=a$. $f(t)=p_0\sin\beta t$ for $t>0$, $f(t)=0$ for $t<0$, $\beta ac_2=1$.

convergence of displacements with time increase is quite similar to that in the case of pressure change in the spherical origin. The resultants of forced waves and those of free waves, together with those of both waves inclusive, namely, $u_{01}+u'_{01}+u_{02}+u'_{02}$, $v_{01}+v'_{01}+v_{02}+v'_{02}$, $u_{f1}+u_{f2}$, v_{f1}

$+v_{f2}$, $u_{01}+u'_{01}+u_{02}+u'_{02}+u_{f1}+u_{f2}$, $v_{01}+v'_{01}+v_{02}+v'_{02}+v_{f1}+v_{f2}$ are shown in Figs. 9, 10. As in the previous case, the displacements resulting from

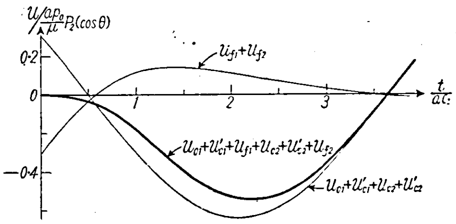

Fig. 9. Resultant displacements for u'_s at $r=a$. $f(t)=p_0\sin\beta t$ for $t>0$, $f(t)=0$ for $t<0$, $\beta ac_2=1$.

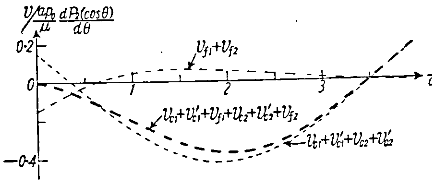

Fig. 10. Resultant displacements for v'_s at $r=a$. $f(t)=p_0\sin\beta t$ for $t>0$, $f(t)=0$ for $t<0$, $\beta ac_2=1$.

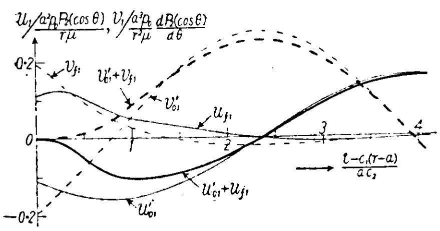

Fig. 11. Displacements for longitudinal waves at $r=\infty$. $f(t)=p_0\sin\beta t$ for $t>0$, $f(t)=0$ for $t<0$, $\beta ac_2=1$.

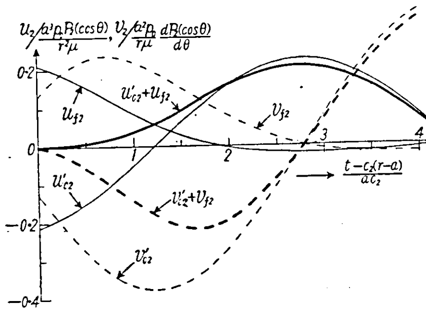

Fig. 12. Displacements for transverse waves at $r=\infty$. $f(t)=p_0\sin\beta t$ for $t>0$, $f(t)=0$ for $t<0$, $\beta ac_2=1$.

both free as well as forced waves, namely, $u_{01}+u'_{01}+u_{02}+u'_{02}+u_{f1}+u_{f2}$, $v_{01}+v'_{01}+v_{02}+v'_{02}+v_{f1}+v_{f2}$ at $r=a$ are in the sense of change in shearing force for initial motion as well as for motion at any time. The results of numerical calculation for u'_{01}, v'_{01}, u_{f1}, v_{f1}, $u'_{01}+u_{f1}$, $v'_{01}+v_{f1}$, and also for u'_{02}, v'_{02}, u_{f2}, v_{f2}, $u'_{02}+u_{f2}$, $v'_{02}+v_{f2}$, all at $r=\infty$, are shown in Figs. 11, 12. At all events, it is possible to conclude that the initial motions at $r=\infty$ for $u_{01}+u'_{01}+u_{f1}$, $v_{01}+v'_{01}+v_{f1}$, $u_{02}+u'_{02}+u_{f2}$, $v_{02}+v'_{02}+v_{f2}$ respectively, accord with those at $r=a$ for the same components of displacements. Of these initial motions, the components $u_{01}+u'_{01}+u_{f1}$, $v_{02}+v'_{02}+v_{f2}$, both at $r=a$ and at $r=\infty$, are also in the sense of change in shearing force at the spherical origin.

6. *Concluding remarks.*

The problem of transmission of arbitrary elastic waves from a spherical origin in the case of change in shearing force of the order $n=2$, is discussed. As to the type of exciting forces, two cases, namely that of rectangular type and that of sine type, both beginning from a quiescent state, have been solved. It has been found that the nature of the generated waves is quite similar to that in the case of pressure change at the spherical origin. It follows that if the type of the vibration at the origin were spheroidal, it would be almost impossible for the data of generated waves to ascertain whether the exciting force at the origin is of pressure type or of shearing force type.

In conclusion we wish to express our thanks to Mr. Watanabe, who assisted us greatly in our calculation. We also wish to thank the officials of the Division of Scientific Research, in the Ministry of Education, for financial aid (Scientific Research Expenditure) granted us for a series of investigation, of which this study is a part.

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1. 演算法を應用せる任意形彈性波の傳播の問題（第３報）

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金井清

球形源に於て $n=2$ の球狀振動が行はれるときに，源に働く力が剪應力である場合を數理的に取扱つた．發生する波の性質は源に壓力が働く場合と殆ど同じである事がわかつた．從て只今のやうな源があるときに波の性質から源に働く力が壓力型であるか剪應力型であるかを確めるのは殆ど不可能であるといふ結論に達した．
