7. *A Note on Regional Anomaly and Secular Variation in Geomagnetism.**

By Takesi NAGATA,

Geophysical Institute, Tokyo Imperial University.

(Read Oct. 16, 1941.—Received Dec. 20, 1941.)

1. Introduction.

So far a number on theories of the causation of the earth's magnetism have been proposed by investigators who, in these theories, discussed chiefly the possibility of causation of the main "dipole field" of geomagnetism, the regional anomaly being frequently assumed to be a secondery effect, such, for example, as the magnetization of the upper part of the earth, magnetically induced by the main dipole magnetic field. As to secular variation in geomagnetism, similar hypotheses have been proposed, and developed in details especially by Nippoldt,[1] Haalck,[2] and Koenigsberger.[3] The chief objection to this theory, as advanced by Bildingmaier,[4] was that the magnetic permeability of the earth's crust—the upper part of the easth—is, as a whole, not so large as expected from the observed regional anomaly in geomagnetism, According to the writer's opinion, however, magnetization of igneous rocks up to 14.5 Gauss/cm^3 is not absolutely impossible, seeing that the saturation magnetization of magnetite, the principal mineral responsible for magnetism of rock, is about 480 Gauss, while the average percentage by weight to magnetite in igneous rocks is 5%, or 3% by volume. Putting aside the problem of the possibility of a highly magne tized earth's crust, we shall examine here whether or not a suitable distribution of magnetically permeable mass under the earth's surface can be responsible for the field of regional anomaly, as also the change in the former to that of secular variation, and further, if it were so, what distribution would be reasonable. According to the theories of

* The third report of "The Relation between Magnetic Anomaly and the Corresponding Subterranean Structure"

1) A. NIPPOLDT, *Terr. Mag.*, 26 (3921), 99.
2) H. HAALCK, *Gerl. Beitr. Geophys.*, 52 (1938), 243; *Z. S. Geophys.*, 8 (1932), 154.
3) J. G. KÖNIGSBERGER, *Z. S. Geophys.*, 8 (1932), 322.
4) F. BILDINGMAIER, *Phys. Z. S.* 11 (1910), 12 16; 12 (1911), 445; 926.

the authorities just mentioned, the regional anomaly of geomagnetism, namely, the residual after subtracting the main dipole field from the total magnetic field, is assumed here to be due to the magnetization induced in a magnetically permeable layer of thickness d and of susceptibility κ, owing to the main dipole magnetic field, secular variation being assumed to be due to its variation. Under this assumption, the general relation between the magnetic fieid on the earth's surface and the corresponding distribution of $\kappa\times d$ of the assumed layer will be established in the present study. The method used here is an extension of that described in the writer's previous study[5] on the relation between the local magnetic anomaly and the corresponding subterranean structure to the case of spherical surface distribution.

2. Theory.

As is well known, the geomagnetic potential W of that part whose origin is within the earth is generally expressed by

$$W=a\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}P_n^m(\cos\theta)\{g_n^m\cos m\phi+h_n^m\sin m\phi\}\,, \tag{1}$$

where a denotes the earth's radius. If, in eq. (1), we take the axis of $\theta=0$ as that agreeing with the direction of the centred dipole, then the potential due to the centred dipole is given by

$$\frac{W_c}{a}=g_1^0\frac{a^2}{r^2}P_1^0(\cos\theta), \tag{2}$$

while the residual parts corresponding to the regional anomaly is given by

$$\frac{W_r}{a}=\frac{a^2}{r^2}P_1{}^1(\cos\theta)\{g_1{}^1\cos\phi+b_1{}^1\sin\phi\}$$

$$+\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}P_n^m(\cos\theta)\ \{g_n^m\cos m\phi+h_n^m\sin m\phi\}\,, \tag{3}$$

in which the zero degree term, corresponding to $\dfrac{a}{r}$, is omitted. Next, as mentioned in the introduction, we assume that the potential of regional anomaly given by eq. (3) is due to secondary magnetization

5) T. NAGATA, *Bull. Earthq. Res. Inst.*, **16** (1939), 550; *Proc. Imp. Acad. Japan*, **14** (1938), 176.

of a magnetically permeable subterranean layer having the form of a nearly spherical shell, polarized by the dipole field given by eq. (2), the product of susceptibility κ and thickness d of the permeable layer being expressed by

$$\kappa d=\sum_{n=0}^{\infty}\sum_{m=0}^{n}P_n^m(\cos\theta)\cdot\{a_n^m\cos m\phi+b_n^m\sin m\phi\}\,. \tag{4}$$

In the case of secular variation, the term κd in eq. (4) will be interpreted as the change in the susceptibility or in the thickness of the permeable layer.

For simplicity, this permeable layer is assumed to be a thin spherical shell of constant thickness d, susceptibility κ alone varying from point to point. This approximation, however, will be permitted in the present problem, since the thickness of the earth's crust (meaning the magnetically permeable layer having a temperature lower than the Curie-point of magnetite, namely, 580°C, or at most, that of iron, 760°C) is only 20∼40 *km*—a value negligible compared with the radius of the earth. Besides, the reliability of this assumption has already been proved in a number of actual examples in connexion with a similar model of an infinitely wide plane,[6] as well as in Tsuboi's studies[7] of gravity anomaly made under the same assumption.

Now, let the spherical surface A of radius a, in Fig. 1, be the earth's surface, and B of radius b the thin permeable spherical shell. Take, then, the centred dipole $\vec{J}$ as that corresponding to the main magnetic field given by eq, (2). Let W be the potential due to magnetization of B, polarized by magnetic field due to $\vec{J}$ (where the potential does not contain that magnetie field directly due to $\vec{J}$). Similarly, let V be the potential due to magnetization of B, polarized by the field due to the centred single charge of magnetism σ, where potential V also does not contain the part directly due to σ. It can then be easily proved that between potential W due to magnetization of a continuous mass polarized by the field of a dipole $\vec{J}$ and that due to magnetization of the same mass polarized by the field of a single charge σ situated at the same point as J, there is the general

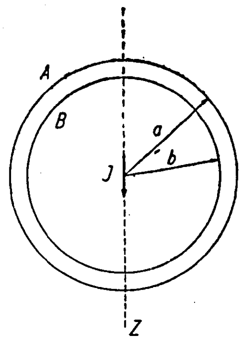


Fig. 1.

6) T. NAGATA, *loc. cit.*

7) C. TSUBOI, *Bull. Earthq. Res. Inst.*, **15** (1937), 636; **16** (1938), 273; **17** (1939), 350; 385.

relation[8)]

$$W=\frac{J}{\sigma}\left(\frac{\partial V}{\partial z}\right)_A, \tag{5}$$

where z denotes the direction of the dipole $\vec{J}$, while $\left(\frac{\partial}{\partial z}\right)_A$ gives the differentiation at "Aufpunkt." The relation given by eq. (5) is an extension of the well known Poisson's law[9)] between the gravitational and the magnetic potentials.

In the present case, taking the positive direction of the ζ axis as coinciding with that of the magnetic North pole, we get the following relation in place of eq. (5),

$$W=-\frac{J}{\sigma}\frac{\partial V}{\partial \zeta}, \tag{5'}$$

because the direction of the centred dipole is opposite the ζ-direction. It will be seen, on the other hand, that in the case of the centred single charge, the permeable layer B is polarized into a radial direction of the sphere, the intensity of magnetization at any point on B being proportional to magnitude $\kappa \times d$ at the respective point; that is to say, the magnetized shell B in this case may be substituted by a magnetic

8) Let δV be the potential at an external point (x, y, z) due to magnetization of an elemental volume δv at (x', y', z') in a continuous mass v, polarized by the magnetic field due to a single charge σ. Then

$$\delta V=\sigma\frac{(\vec{I}(x',y',z')\cdot\vec{r})}{r^3}, \quad r^2=(x-x')^2+(y-y')^2+(z-z')^2.$$

Let δW be the potential due to magnetization of the same volume, polarized by the field due to dipole $\vec{J}$, having a direction that is coincident with z which lies at the same point as σ. Then

$$\delta W=\lim_{\Delta z'\to 0}\sigma\left[\frac{(\{\vec{I}(x',y',z')-\vec{I}(x',y',z'+\Delta z')\}\cdot\vec{r})}{r^3}\right]$$

$$=\lim_{\Delta z'\to 0}J\frac{\dfrac{(\vec{I}(x',y',z')\cdot\vec{r})}{r^3}-\dfrac{(\vec{I}(x',y',z'+\Delta z')\cdot\vec{r})}{r^3}}{\Delta z'}=-J\cdot\frac{1}{\sigma}\frac{d}{dz'}\delta V,$$

since $\sigma\cdot\Delta z'=J$. And as $\frac{d}{dz'}\delta V=-\frac{d}{dz}\delta V$, $\delta W=\frac{J}{\sigma}\frac{d}{dz}\delta V$.

Since δV has no singular point, if (x, y, z) be taken in that space outside of v, we finaly get

$$W=\int_v \delta W dv=\frac{J}{\sigma}\int_v\frac{d}{dz}\delta V\cdot dv=\frac{J}{\sigma}\frac{d}{dz}\int_v \delta V\cdot dv=\frac{J}{\sigma}\frac{d}{dz}V.$$

9) See, for example, B. GUTENBERG. "Lehrbuch der Geophysik." pp. 520.

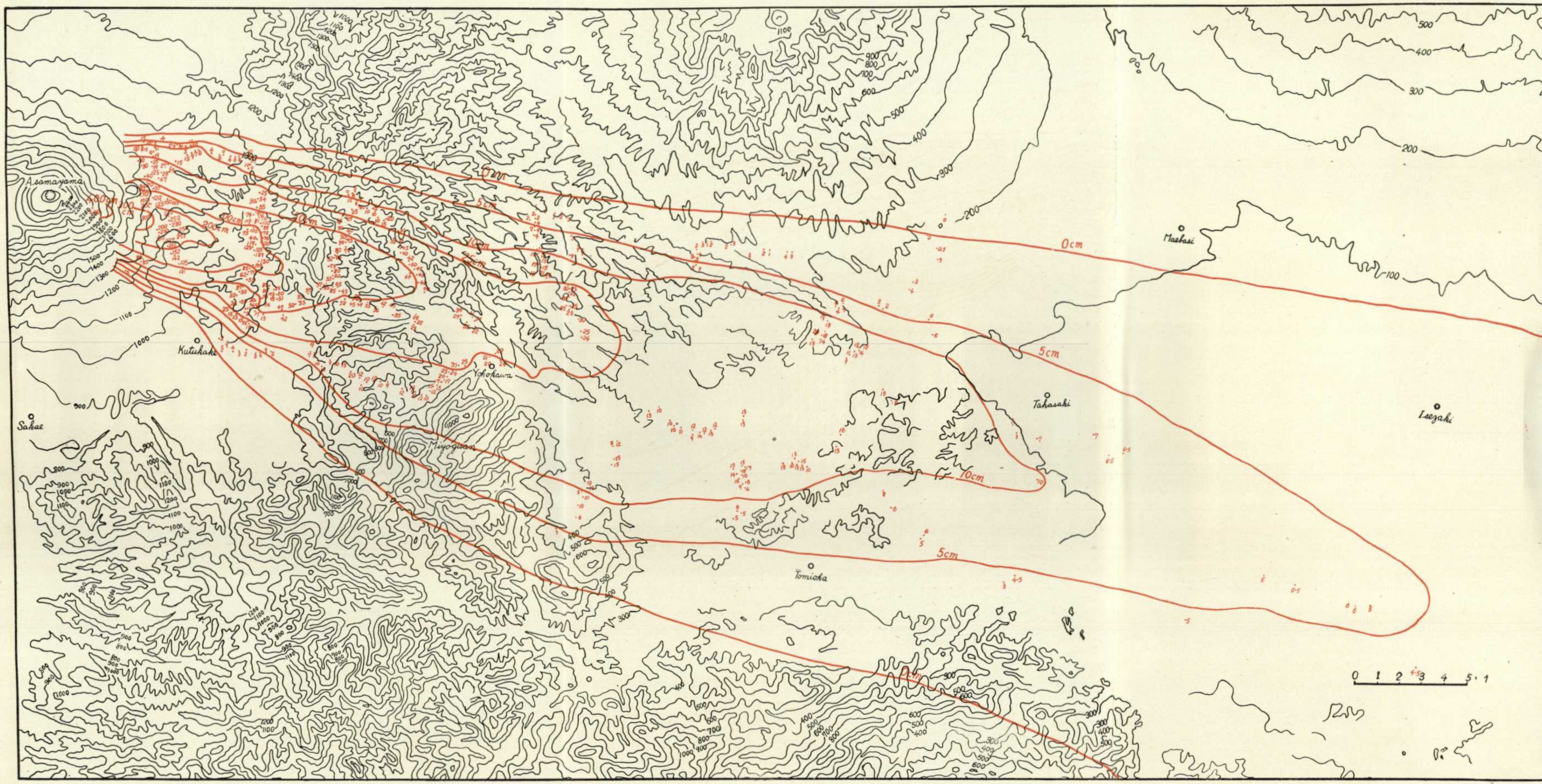


Fig. 2. Distribution of the Temmei pumice. numero (red) = thickness (in cemtimetre) of the pumice layer.

Fig. 9 a.

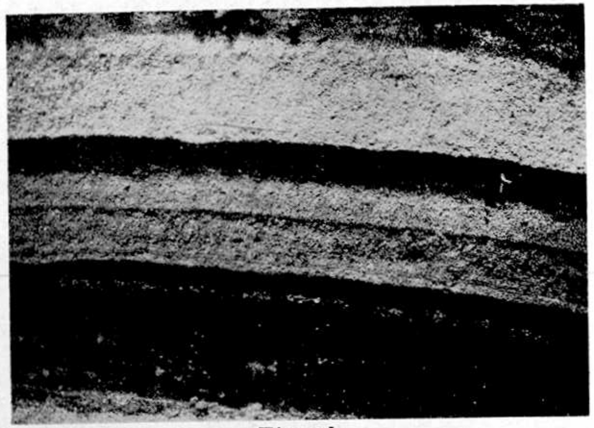


Fig. 9 b.

a, b. The Temmei pumice layer near Sengataki (Karuizawa), distant 9 km from the crater.
The upper white layer is the Temmei pumice.

Fig. 10 a.

Fig. 10 b.

a, b. The Temmei pumice layer near Matuida, distant 24 km from the crater.

(震研彙報 第二十號 圖版 水上)

a

The smoke 17 seconds after th explosion.

b

25 seconds after the explosion.

(震研彙報 第二十號 圖版 水上)

c

35 seconds after the explosion.

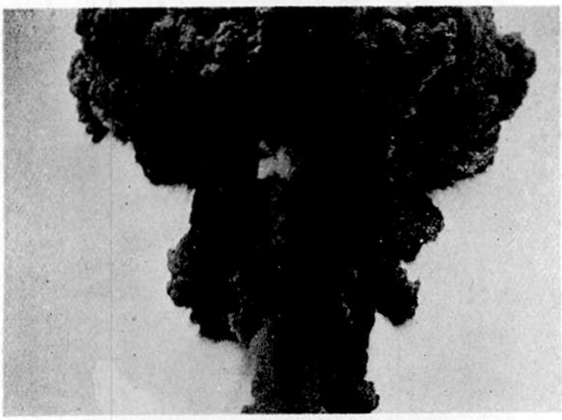
Fig. 11. Smoke of explosion March 7, 1936.

d

60 seconds after the explosion.

e

120 seconds after the explosion.



f

Cauliflower shape smoke of explosion.

Fig. 11. Smoke of explosion March 7, 1936.

(震研彙報 第二十號 圖版 水上)

double shell, on which the distribution of intensity of magnetization is given by

$$M(\theta,\phi)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}P_n^m(\cos\theta)\{\alpha_n^m\cos m\phi+\beta_n^m\sin m\phi\} \tag{6}$$

Now, since

$$M=\frac{\sigma}{b^2}\kappa d, \tag{7}$$

eq. (6) becomes

$$M(\theta,\phi)=\frac{\sigma}{b^2}\sum_{n=0}^{\infty}\sum_{m=0}^{n}P_n^m(\cos\theta)\{a_n^m\cos m\phi+b_n^m\sin m\phi\}, \tag{8}$$

or

$$\alpha_n^m=\frac{\sigma}{b^2}a_n^m,\qquad \beta_n^m=\frac{\sigma}{b^2}b_n^m, \tag{9}$$

whence the magnetic potential due to this magnetic double shell is given by

$$\left.\begin{aligned}V_+&=\sum_n\sum_m\left(\frac{b}{r}\right)^{n+1}P_n^m(\cos\theta)\{A_n^m\cos m\phi+B_n^m\sin m\phi\},\ r\geqq b,\\ V_-&=\sum_n\sum_m\left(\frac{r}{b}\right)^{n}P_n^m(\cos\theta)\{A'^m_n\cos m\phi+B'^m_n\sin m\phi\},\ r\leqq b,\end{aligned}\right\} \tag{10}$$

the boundery conditions on $r=b$ being given by

$$\left.\begin{aligned}4\pi(M)_{r=b}&=(V_+)_{r=b}-(V_-)_{r=b},\\ \left(\frac{\partial V_+}{\partial r}\right)_{r=b}&=\left(\frac{\partial V_-}{\partial r}\right)_{r=b}\end{aligned}\right\} \tag{11}$$

From eqs. (7), (10), (11) we get

$$A_n^m=\frac{4\pi n}{2n+1}\alpha_n^m,\qquad B_n^m=\frac{4\pi n}{2n+1}\beta_n^m$$

or

$$V_+=\frac{4\pi\sigma}{b^2}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{n}{2n+1}\left(\frac{b}{r}\right)^{n+1}P_n^m(\cos\theta)\{a_n^m\cos m\phi+b_n^m\sin m\phi\} \tag{12}$$

On the other hand, since $\zeta=r\cos\theta$,

$$\frac{\partial}{\partial\zeta}=\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}. \tag{13}$$

Then from eqs. (5′), (12), (13), the magnetic potential W due to magnetization of permeable shell B, polarized by the centred dipole J, is given by

$$W=\frac{4\pi J}{b^2}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{b^{n+1}}{r^{n+2}}\cdot\frac{n}{2n+1}\left\{(n+1)P_n^m\cos\theta+\frac{dP_n^m}{d\theta}\sin\theta\right\}\times$$
$$\cdot\{a_n^m\cos m\phi+b_n^m\sin m\phi\} \tag{14}$$

Putting the following recurrence formulae into eq. (14)

$$\cos\theta\cdot P_n^m=\frac{\sqrt{(n+1)^2-m^2}}{2n+1}P_{n+1}^m+\frac{\sqrt{n^2-m^2}}{2n+1}P_{n-1}^m,$$

$$\sin\theta\cdot\frac{dP_n^m}{d\theta}=\frac{n\sqrt{(n+1)^2-m^2}}{2n+1}P_{n+1}^m-\frac{(n+1)\sqrt{n^2-m^2}}{2n+1}P_{n-1}^m,$$

we get

$$W=\frac{4\pi J}{b^2}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{n}{2n+1}\sqrt{(n+1)^2-m^2}\,\frac{b^{n+1}}{r^{n+2}}P_{n+1}^m(\cos\theta)\times$$
$$\{a_n^m\cos m\phi+b_n^m\sin m\phi\} \tag{15}$$

Then, n, in eq. (15) being replaced by $n-1$, the distribution of potential value on the earth's surface $r=a$ is given by

$$W=\frac{4\pi J}{b^2}\sum_{n=1}^{\infty}\sum_{m=0}^{n-1}\frac{b^n}{a^{n+1}}\sqrt{n^2-m^2}\,\frac{n-1}{2n-1}P_n^m(\cos\theta)\times$$
$$\{a_{n-1}^m\cos m\phi+b_{n-1}^m\sin m\phi\} \tag{16}$$

On the other hand, the distribution of magnetic potential on the earth's surface, corresponding to the regional anomaly or to the secular variation in geomagnetism, can be calculated from the observed values of the geomagnetic forces, it being usually expressed by

$$\frac{W}{a}=P_1^1(\cos\theta)(g_1^1\cos\phi+h_1^1\sin\phi)+\sum_{n=2}^{\infty}\sum_{m=0}^{n}P_n^m(\cos\theta)\times$$
$$\cdot(g_n^m\cos m\phi+h_n^m\sin m\phi) \tag{17}$$

Then, comparing eqs. (16), (17), term by term, we have

$$\left.\begin{aligned} g_n^m&=4\pi J\frac{b^{n-2}}{a^{n+2}}\cdot\frac{n-1}{2n-1}\sqrt{n^2-m^2}\,a_{n-1}^m,\\ h_n^m&=4\pi J\frac{b^{n-2}}{a^{n+2}}\cdot\frac{n-1}{2n-1}\sqrt{n^2-m^2}\,b_{n-1}^m.\end{aligned}\right\}\begin{pmatrix} n\geqq 2\\ n-1\geqq m\geqq 0\end{pmatrix} \tag{18}$$

Inversely, if g_n^m and h_n^m are known, a_n^m and b_n^m are given by

$$\left.\begin{aligned} a_{n-1}^{m} &= \frac{a^2b^2}{4\pi J}\left(\frac{a}{b}\right)^n \frac{2n-1}{(n-1)\sqrt{n^2-m^2}} g_n^m, \\ b_{n-1}^{m} &= \frac{a^2b^2}{4\pi J}\left(\frac{a}{b}\right)^n \frac{2n-1}{(n-1)\sqrt{n^2-m^2}} h_n^m, \quad (n\geqq 2,\, n-1\geqq m\geqq 0) \end{aligned}\right\} \tag{19}$$

An important result derived from this calculation is that

$$g_n^n = h_n^n = 0 \qquad (n\geqq 1)\,, \tag{20}$$

that is to say, provided the regional anomaly is assumed to be due to secondary magnetization of the earth's crust, polarized by the centred dipole field, the coefficients of sectorial terms in the surface harmonic expansion of the potential must be zero, regardless of the way in which $\kappa \times d$ is distributed in the assumed crust.

This conclusion may be a rather natural result, if we take into consideration that the potential that is symmetric with respect to the magnetic equator cannot be derived from any distribution of κd, magnetized by the antisymmetric field due to the centred dipole having the direction perpendicular to the equator plane.

Thus, in our model, the induced magnetization of permeable shell B given by eq. (6) or (8) is responsible for the regional anomaly in geomagnetism except that part of it which is expressed by

$$W'' = a\sum_{n=1}^{\infty} P_n^n(\cos\theta)\,(g_n^n \cos n\phi + h_n^n \sin n\phi)\,, \tag{21}$$

whereas the distribution of $M(\theta, \phi)$ on a $r=b$ shell, corresponding to the potential

$$W' = a\sum_{n=2}^{\infty}\sum_{m=0}^{n-1} P_n^m(\cos\theta)\,(g_n^m \cos m\phi + h_n^m \sin m\phi)\,, \tag{22}$$

can be uniquely determined from the given value of the potential shown in eq. (22) with the aid of the relation of (8) and (19), provided the amount of a/b is suitably assumed.

3. Actual analysis.

Before applying the present method to an actual problem of geomagnetism, we assume the following conditions, the second and third assumptions in which seem to hold approximately in the earth, provided the first is permissible.

(i) The earth's magnetic field is given by the sum of the field due to the centred dipole and that due to secondery magnetization of a spherical permeable shell, corresponding to the earth's crust.

(ii) Since the permeable layer is sufficiently thin, and its susceptibility is not large, the demagnetizing factor in the magnetization of the shell is negligible.

(iii) Magnetization of the permeable layer does not affect the direction of the centred dipole nor its intensity; that is, the centred dipole consists of perfectly rigid magnetization.

In Table I are shown the values of g_n^m and h_n^m obtained by F. Dyson and H. Furner[10] from the geomagnetic chart for 1920, given by the Hydrographic Department of the British Admiralty. Although these

Table I. Values of Coefficents of Spherical harmonic expansion of geomagnetic potential for 1920. (After Dyson and Furner.)

g_n^m

n \ m	0	1	2	3	4	5	6
1	− 0·3095	− 0·0226					
2	− 0·0089	+ 0·0299	+ 0·0144				
3	+ 0·0102	− 0·0157	+ 0·0118	+ 0·0076			
4	+ 0·0088	+ 0·0068	+ 0·0079	− 0·0040	+ 0·0018		
5	− 0·0023	+ 0·0026	+ 0·0018	− 0·0004	− 0·0009	+ 0·0000	
6	+ 0·0007	+ 0·0011	+ 0·0001	− 0·0020	− 0·0006	+ 0·0003	− 0·0006

h_n^m

n \ m	1	2	3	4	5	6
1	+ 0·0592					
2	− 0·0124	+ 0·0084				
3	− 0·0044	+ 0·0012	+ 0·0024			
4	+ 0·0025	− 0·0015	− 0·0013	− 0·0003		
5	− 0·0015	+ 0·0001	+ 0·0001	− 0·0013	0·0000	
6	+ 0·0002	+ 0·0009	− 0·0004	− 0·0003	0·0000	− 0·0004

values largely consist, as is well known, of parts that are of internal origin, and the residual part of external origin, we take here, following the opinion of Dyson and Furner, those g_n^m and h_n^m as being due to internal origin alone, because a reliable estimate of the external-origin part is very difficult, chiefly owing to observational errors as well as in computation, and also because the part of external origin is very small compared with that of internal origin, the former, according to L. A. Bauer,[11] being only a few percent of the latter.

We shall now try to assume that the direction of the centred

10) F. DYSON and H. FURNER, *Geophys. Supl. N. N. R. A. S.*, 1 (1923), 76.

11) L. A. BAUER, *Terr. Mag.*, 28 (1623), 1.

dipole, corresponding to the earth's primary magnetic field, coincides with the earth's axis of rotation, the dipole pointing to the earth's South pole. This assumption is derived as a logical consequence of the theory that the primary geomagnetic field is caused by an electric charge rotating together with the earth, or by gyromagnetic effect, the former proposed by W. Sutherland,[12] G. Angenheister,[13] W. F. G. Swann and A. Longacre,[14] T. Schlomka,[15] and H. Haalck,[16] and the latter by S. J. Barnett.[17] If so, according to our calculation, the sectorial terms g_n^n, h_n^n, ($n \geqq 1$), in the expression of geomagnetic potential on the earth's surface ought to disappear, whereas g_n^n and h_n^n in Table I are not zero, nor are they small compared with the other

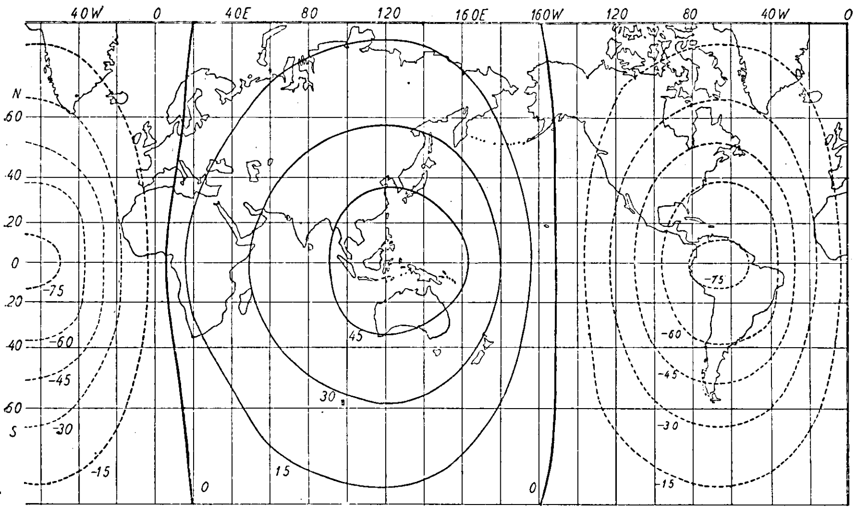


Fig. 2. Distribution of geomagnetic potential, for which the earths crust (magnetizible layer) can not be responsible.

$W'' = a\sum_n P_n^n(\cos\theta)(g_n^n\cos n\phi + h_n^n\sin n\phi)$ unit$=10^2\gamma$

terms. We may thus say that the secondary magnetization of the earth's crust, polarized by the primary magnetic field set up by the model at a centred dipole that is coaxial with the rotation axis of the

12) W. Sutherland, *Terr. Mag.*, **5** (1900), 73; **8** (1904) 167; **13** (1908), 155.

13) G. Angenheister, *Nachr. Gesel. Wiss. Göttingen.*, (1924), 229. *Phys. Z. S.* **26** (1925), 307.

14) W. F. G. Swann, *Phil. Mag.*, **3** (1927), 1088; W. F. G. Swann and A. Longacre, *Journ. Franklin Inst.*, **206** (1928), 421.

15) T. Schlomka, *Z. S. Geophys.*, **9** (1933), 99.

16) H. Haalck. *Z. S. Geophys.*, **12** 1936), 112; *Gerl. Beitr. Geophys.* **52** (1938), 243.

17) S. T. Barnett, *Phys. Z. S.*, **35** (1934), 203.

earth, cannot be responsible for the whole of the regional anomaly in geomagnetism, athough the former may be responsible for part of the latter.

By substituting Dyson and Furner's values of g_n^m and h_n^m in eq. (21), that part of geomagnetic potential on the earth's surface for which the earth's crust cannot be responsible was obtained, with results as shown in Fig. 2.

In this result, the term corresponding to $P_1^1(\cos\theta)$ is naturally the most important.

Assuming, next, that the residual potential given by eq. (22) is entirely due to induced magnetization of the assumed earth's crust, it is possible to calculate the distribution of $\kappa\times d$ in the crust from Dyson aud Furner's values by means of eqs. (19) and (4). Putting $a=6360\,km$, $a-b=30\,km$, and $J=8{\cdot}1\times10^{25}$ e. m. u., we get the actual values of a_n^m and b_n^m, as shown in Table II. The distribution of $\kappa\times d$

Table II. Values of coefficients of spherical harmonic expansion of κd

a_n^m

n \ m	0	1	2	3	4	5
1	−2142	+8320				
2	+1371	−2237	+2129			
3	+ 832	+ 663	+ 862	− 571		
4	− 169	+ 194	+ 143	+ 37	− 109	
5	+ 54	+ 67	+ 6	− 138	− 48	+ 33

b_n^m

n \ m	1	2	3	4	5
1	−3450				
2	− 627	+ 216			
3	+ 243	− 164	− 186		
4	− 113	+ 8	+ 10	− 159	
5	− 13	+ 57	− 27	− 24	0

(unit$=10^3$ e. m. u.)

in the assumed permeable shell, obtained by means of synthesis of the values of a_n^m and b_n^m with the aid of eq. (4), is given in Fig. 3, where the constant value independent of position (i. e. the coefficient of zero degree harmonic) is omitted, since it is impossible to estimate its magnitude by the present method. It will be noted that, in this result, the earth's upper layer under the Pacific Ocean has a much lower permeability compared with that under Europe, America, and the

Atlantic Ocean. Further, it may be while worth to note here that although the omitted part of the geomagnetic potential, shown in Fig. 2, is positive in the Pacific region, it is negative in the other regions. However, whether there is any reasonable relation between these two or not, is a question for the future to answer. As to the magnitude of $\kappa \times d$, shown in Fig. 3, it must be mentioned that the difference between its maximum and minimum values is remarkably large, amounting to about 2×10^7 e. m. u.; that is, taking $d = 50$ *km*, we get $\kappa_{max} - \kappa_{min} = 4$ e. m. u. Needless to say, such susceptibility is far beyond the magnitude of the rock's susceptibility that has been experimentally obtained so far.[18] It is also much larger than the "magnetizibility" of igneous rocks, i. e. the sum of the susceptibility and thermo-remanent magnetization, expressed mathematicaly[19] by $\kappa + \int_t^{t_i} P(t)\,dt$, so far as data[20] obtained by experimental

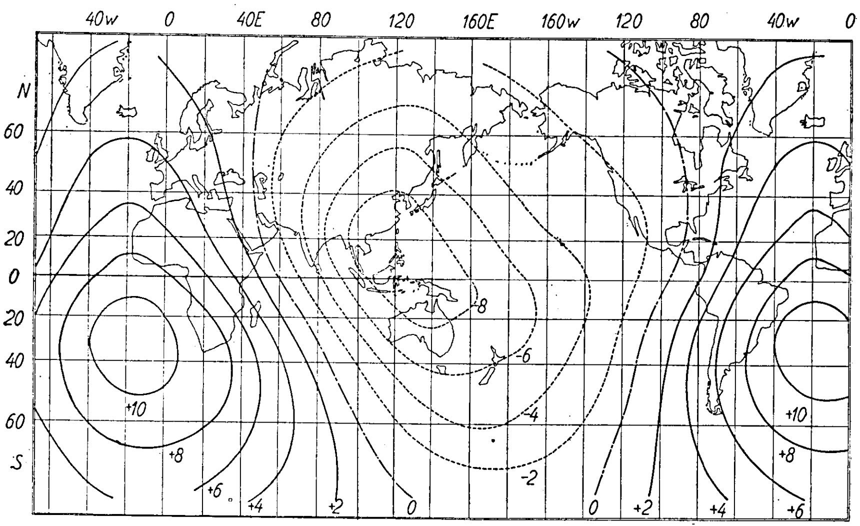


Fig. 3. Distribution of $\kappa \times d$ under the earth's surface.
Unit$=10^6$ e. m. u.

measurements at ordinary atmospheric pressure go. However, it will be again noted that the intensity of magnetization corresponding to $\kappa F = \kappa \times \frac{J}{b^3} \sqrt{1 + 3\cos^2\theta}$ $(0{\cdot}32\ \kappa \sim 0{\cdot}64\ \kappa$ e. m. u.) is fairly smaller than

18) H. Reich, *Handb. d. Exper. Phys.*, XXV, 3. pp. 28~31, Leipzig, 1930. T. Nagata, *Bull. Earthq. Res. Inst.*, **18** (1940), 102; **19** (1941), 304; 402.

19) F. Nagata, *Bull. Earthq. Res. Inst.*, **19** (1941), 49.

20) J. G. Königsberger, *Terr. Mag.*, **43** (1938), 119; 299. T. Nagata, *Bull. Earthq. Res. Inst.*, **18** (1940), 281.

that of average saturation magnetization of the igneous rocks composing the earth's crust, even if the maximum value of κ were 10 e.m.u.

Table III. The distribution of κd on the surface of the assumed spherical shell.

Lat. \ Long.	0	20E	40	60	80	100	120	140	160E
70 N	+ 231	+ 160	+ 54	− 70	− 181	− 264	− 305	− 302	− 270
50	+ 409	+ 269	+ 65	− 255	− 413	− 580	− 714	− 576	− 464
30	+ 542	+ 371	+ 115	− 214	− 561	− 809	− 872	− 778	− 622
10	+ 745	+ 541	+ 249	− 107	− 510	− 826	− 956	− 900	− 753
10 S	+ 955	+ 769	+ 489	+ 127	− 287	− 658	− 872	− 906	− 824
30	+1081	+ 939	+ 705	+ 384	+ 1	− 375	− 641	− 759	− 749
50	+1009	+ 917	+ 745	+ 501	+ 206	− 992	− 333	− 473	− 524
70	+ 701	+ 655	+ 561	+ 427	+ 269	+ 108	− 32	− 133	− 184

Lat \ Long.	180	160W	140	120	100	80	60	40	20W
70 N	− 221	− 168	− 113	− 53	+ 18	+ 98	+ 178	+ 236	+ 258
50	− 355	− 279	− 229	− 172	− 70	+ 88	+ 267	+ 408	+ 460
30	− 479	− 356	− 311	− 255	− 172	+ 71	+ 340	+ 553	+ 623
10	− 609	− 485	− 386	− 276	− 102	+ 164	+ 473	+ 721	+ 818
10 S	− 698	− 558	− 406	− 225	+ 15	+ 324	+ 650	+ 903	+1011
30	− 661	− 530	− 361	− 145	+ 133	+ 455	+ 767	+1000	+1105
50	− 490	− 398	− 253	− 54	+ 195	+ 469	+ 723	+ 913	+1010
70	− 185	− 172	− 53	+ 70	+ 218	+ 374	+ 518	+ 628	+ 695

(unit$=10^4$ e. m. u.)

4. Summary and Conclusion.

In this note, under the assumption that the primary main field of geomagnetism can be substituted by the centred dipole that is coaxial with the earth's rotation axis, the distribution of subterranean magnetically permeable mass, which is responsible for the regional anomaly on the earth's surface, was calculated. It was also shown that according to the above mentioned model, the actual geomagnetic potential contains the omitted part for which the subterranean mass cannot be responsible. However, as another approximation to the earth's main magnetic field, we can take the lastnamed as being due to the centred dipole that is coaxial with the so-called magnetic axis of the earth, so long as we assume *a priori* that it is so, although it will be devoid of any physical meaning.

If we use this model, the term corresponding to $P_1^1(\cos\theta)$, the dominant term in the omitted potential W'' given by eq. (21), naturally disappears.

A very similar method of analysis is applicable to the case of secular variation in geomagnetism, in which case the field of the centred dipole, coaxial with the magnetic axis, will be a sufficiently good approximation to the earth's permanent magnetic field.[21] The model of secondary magnetization of the earth's upper layer will probably be used rather in the case of secular variation in geomagnetism, seeing that the origin of secular variation may be traced to the upper part of the earth from the fact that the coefficients of higher degree terms in its spherical harmonic expansion are relatively large. as J. Bartels[22] has pointed out.

The geomagnetic co-latitude and longitude being taken as the coordinates, the actual computation for the regional anomaly and secular variation under this assumption is now in progress.

In conclusion, the writer wishes to express his sincere thanks to Prof. C. Tsuboi for his advice and encouragement in the course of this study. His hearty thanks are due also to the Department of Education for the Science Research Grant received.

---

21) In the rough approximation that the permanent field is given by that due to the centred dipole that is coaxial with the earth's rotation axis, we get

$$\frac{\partial}{\partial t}(\kappa d) = -\left\{16{\cdot}9\cos\theta + \sin\theta(-22{\cdot}4\cos\phi + 69{\cdot}8\sin\phi)\right\} \times 10^{-3} \text{ e.m.u./year},$$

$$\frac{\partial}{\partial t}(W'') = 42\cos\theta + \sin\theta(-9\cos\phi + 12\sin\phi) + \sin^2\theta(11\cos 2\phi - 7\sin 2\phi) \ \gamma\text{/year},$$

from the actual values of secular variation during 1902–1920, which was analyzed by J. Bartels, the spherical harmonic coefficients obtained by him being

g_1^0	g_1^1	h_1^1	g_2^1	g_2^1	h_2^1	g_2^2	h_2^2
+42	−9	+12	−7	+8	−25	+13	−8

(unit = γ/year).

22) J. BARTELS, *Veröff. Preust. Meteor. Inst. Abhandl.*, 8 Nr. 2, (1935).

7. 地球磁場の地方異常並びに永年變化に就いて

地球物理學教室 永 田 武

地球の永久磁場の起因を說明せんとする試みの中に於いて，所謂地方的異常は，地殼乃至は地表に近い地球內上層部が主磁場によつて誘導磁化されてゐる事に起因すると解釋されてゐる場合が多い．特に最近 Haalck の如きは地球磁場の地方的異常の分布と大陸の分布との間には密接な關係があると論じてゐる．他方永年變化の起因に就いても同樣の考が用ひられ，地球上層部の帶磁狀態の變化に因つて永年變化が生ずるといふ考が屢々提出されてゐる．

然し地球表面に於ける地方的異常又は永年變化の分布と，之に對應すべき地球內上層部の帶磁狀態の分布との間の關係を量的に吟味した結果は未だ見當らない樣である．

この論文に於いては，先づ地球表面下任意の深さにある，地球半徑に比して厚さの薄い一つの球殼を以て帶磁層を代表せしめ，その球殼の帶磁率 κ と厚さ d との積の分布が任意の形の球面分布として與へられる場合に，一つの Centred dipole の磁場によつて與へられる地球の主磁場がその球殼を誘導磁化するとし，その球殼の帶磁による球殼外の磁氣ポテンシャルを求める問題を取扱つた．その結果，地球表面に於ける磁氣ポテンシャルの分布が與へられゝば，逆に前記球殼上の $\kappa\times d$ の分布を一義的に求める事が出來る．但し球殼の半徑は適當に假定しなくてはならない．

要するに重力異常から地下構造を求める坪井教授の方法，及びそれを擴張して筆者の與へた一樣な主磁場が存在する場合に局部的地磁氣異常から地下構造を求める方法等と同じ考を更に擴張したに過ぎない．

この方法を地球の永久磁場の問題に適用して見ると，若し主磁場に對應する Centred dipole が地球の廻轉軸と等しい方向を持つとするならば，（此の假定は，地球の主磁場が地球と共に廻轉する電氣の誘導に中るとする Sutherland, Angenheister, Haalck 流の考や，Gyromagnetic 効果に因るとする考を採用する際の當然の結論であるが，）地球磁氣ポテンシャルを極距離 θ 經度 ϕ を用ひて球函數に展開した時，一般に $P_n^n(\cos\theta)$．$(n\geqq 1)$ の係數は零でなくてはならぬ事になる．球殼の $\kappa\times d$ が如何なる分布をしても，磁氣ポテンシャルは $P_n^m(0\leqq m\leqq n-1)$ 諸項のみを含む級數によつて表はされる譯である．然るに實際の觀測値から，地球磁氣地方的異常のポテンシャルを展開した結果に於いて P_n^n の係數はかなり大きい．從つて地方的異常のすべてを主磁場による地殼の二次的誘導磁化に起因せしめる事は許されない．$\kappa\times d$ の二次的帶磁が責任を負ひ得ない磁氣ポテンシャル部分の地球表面に於ける分布は本文第 2 圖に示す如くである．

猶又，P_n^n 項を含を部分を取除いた殘りの磁氣ポテンシャル

$$W'=a\sum_{n=2}^{\infty}\sum_{m=0}^{n-1}P_n^m(\cos\theta)(g_n^m\cos m\phi+h_n^m\sin m\phi)$$

に對應する球殼の $\kappa\times d$ を求める事が出來るが，Dyson 及び Furner の分析結果を用ひ，$\kappa d(\theta,\phi)$ てを求めた結果は本文第 3 圖に示す如く，太平洋區域で小さく大西洋方面で大きい．$d=50\,km$ と假定する時 $\kappa_{max}-\kappa_{min}=4$ e.m.u. に達するが，この値は今迄知られてゐる．岩石の帶磁率，熱殘留磁氣の値に比して著しく大きい．但し磁鐵鑛 3% を含む岩石の飽和磁氣の强さは 15 e.m.u. で

あるから，この値に比べて前記の κ の値は絶對に不可能といふ大きさではない事には注意を要する．

永年變化に就いても同様の事が云はれる．但しこの場合は主磁場として地球磁軸を軸とする Centred dipole の磁場を用ひる方がより實際に近いと思はれるので目下計算中である．

この研究の遂行は文部省科學研究費による處が多い，記して謝意を表する．