14. *The Mode of Development of Thermo-Remanent Magnetism in Igneous Rocks. (II)*

By Takesi NAGATA,

Earthquake Research Institute.

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1. Introduction.

In the preliminary note[1] to this study, the general phenomenological characters of thermo-remanent magnetism in igneous rocks in a weak magnetic field were described. Summarizing the conclusions arrived at in the previous study, we found that the thermo-remanent magnetization $\vec{J}_{t,H}(t')$, caused during cooling from t to t' in a magnetic field of $\vec{H}$, is given by

$$\vec{J}_{t,H}(t') = \vec{H} \cdot \int_{t'}^{t} P(t)\,dt\,, \tag{1}$$

where $P(t)$ is a function of temperature alone, having the characteristics

$$P(t \geqq t_c) = 0, \quad P(t_0) = \text{maximum}\,, \tag{2}$$

and

$$\frac{\partial}{\partial \tau} P(t) = 0,$$

where τ is the time required for cooling through unit temperature of the range. As will be seen from eq. (2), if the upper limit of the integration on the right-hand side of eq. (1), t, is higher than t_c, the thermo-remanent magnetism caused during cooling from t to 0 in $\vec{H}$ is always constant and equal to that caused during cooling from t_c to 0 in the same field. Hence, this remanent magnetization is called the saturated thermo-remanent magnetization in H, being denoted by $J_{t_c,H}$. Then,

$$\vec{J}_{t_c,H} = \vec{H} \cdot \int_{0}^{t_c} P(t)\,dt\,, \tag{3}$$

1) T. NAGATA, *Bull. Earthq. Res. Inst.*, 19 (1941), 49.

so long as the intensity of H is small, while, on the other hand, permanent magnetization $J_{t,H}(t)$ disappears with increase in temperature in a magnetic field of H', following the law

$$\frac{\partial}{\partial t}\Big[J_{t,H}(t')\Big] = -H \cdot P(t) \,, \qquad (4)$$

where H on the right-hand side of eq. (4) denotes the intensity of the magnetic field applied during development of the thermo-remanent magnetization $J_{t,H}(t')$, the magnetic field $\vec{H}'$ applied during the disappearance of that permanent magnetization owing to increase in temperature affecting only slightly its mode of disappearance.

For convenience, the characteristic quantities in $P(t)$, given in eqs. (1) and (2), namely, t_0 and t_c, are here called the transition temperature and critical temperature in thermo-remanent magnetism respectively. The figure of characteristic function $P(t)$ of a number of rock specimens has a single peak for temperature, that is, there is only one transition temperature in a $P(t)$ curve, although every specimen has a value that is peculiar to itself. Such a relatively simple form of $P(t)$ as that just mentioned will be called the fundamental form of $P(t)$. It was also found that some rock specimens have not only one, but two, or sometimes three, transition points for temperature, their $P(t)$ curve being presumed to be the result of superposition of two or three fundamental $P(t)$ curves, each of which, as mentioned above, has a rather simple form. This conclusion may be expressed by

$$P(t) = \sum_j P_j(t) \,,$$

where

$$P_j(t_{0j}) = \text{maximum} \,.$$

It is presumed that the law given by eq. (1) also holds in the process of development of thermo-remanent magnetism in these rocks that have more than two transition temperatures. In the first part of this report, experimental proof that this presumption is correct will now be shown.

The previous studies on the development of thermo-remanent magnetism mainly concerned with the phenomenological mode of its development, although it may not be possible to make clear the physical mechanism of its causation from the results described alone, since this phenomenon of development of thermo-remanent magnetism seems to be peculiar to material that are made up of complex aggregates of various kinds of micro-crystals, such as rocks. So long as the present experimental data go, it is not found in the usual ferro-magnetic substances of polycrystaline structure, nor in those of a single crystal. Therefore, the physical characteristics of igneous rocks like ferro-magnetic substances should be thoroughly understood before the phenomenon of thermo-remanent magnetism can be physically interpreted. As the first step in this study, the mode of development of thermo-remanent magnetism was compared with that of change in magnetic susceptibility with temperature, thetransition and critical temperatures in the former especially being related to the Curie-point temperature in the latter.

Relations (1) to (4) were established as the general mode of thermo-remanent magnetism in a weak magnetic field. Near the end of this report, we shall examine the dependency of thermo-remanent magnetization on the intensity of the applied magnetic field, without the restriction regarding the "weak" magnetic field.

Although we have not yet reached our final conclusion on the physical mechanism of development of thermo-remanent magnetism, the results obtained in the present study will add to our knowledge of the physical conditions of the earth's crust.

2. The general mode of development of thermo-remanent magnetism.

The rock specimens examined in the present study were ejecta from Volcano Huzi (Fuji), the petrographical and chemical characters of which were studied in considerable detail by H. Tsuya,[2] according to whom, these rocks are

No. 1. Lava of Hōei crater, olivine-basalt

2) H. TSUYA, *Bull. Earthq. Res. Inst.*, **15** (1937), 215.

Chemical Composition of Huzi ejecta (after H. Tsuya).

Specimen	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
SiO_2	49·60	49·60	50·28	50·64	50·66	51·05	51·09	51·30	63·84
Al_2O_3	16·90	16·14	18·30	18·58	18·25	18·35	17·62	18·74	15·82
Fe_2O_3	5·40	3·67	4·50	3·04	4·78	2·76	2·64	1·83	0·95
FeO	6·65	9·90	6·89	7·29	5·72	7·72	8·42	8·34	5·02
MgO	5·92	4·79	3·80	5·58	4·94	4·63	5·09	4·80	1·67
CaO	10·03	8·80	9·76	10·00	9·93	9·90	9·68	9·76	4·88
Na_2O	2·48	2·90	2·87	2·64	2·78	2·81	2·80	2·55	3·88
K_2O	0·58	0·93	0·94	0·61	0·77	0·81	0·76	0·71	2·12
H_2O+	0·50	0·55	0·20	0·20	0·38	0·40	0·28	0·22	0·45
H_2O-	0·12	0·14	0·08	0·06	0·13	0·11	0·06	0·06	—
TiO_2	1·40	1·97	1·78	1·15	1·38	1·41	1·38	1·43	0·87
P_2O_5	0·20	0·31	0·34	0·16	0·25	0·24	0·26	0·29	0·22
MnO	0·21	0·23	0·20	0·17	0·17	0·18	0·21	0·28	0·17
Total	100·05	99·93	99·94	100·12	100·19	100·37	100·29	100·31	90·89

No. 2. Tawarano-taki lava, olivine-basalt (almost aphyric)
No. 3. Makuiwa lava 1, olivine-basalt
No. 4. Makuiwa lava 2, two-pyroxene-bearing olivine-basalt
No. 5. Makuiwa lava 3, augite-bearing olivine-basalt
No. 6. Makuiwa lava 4, Hypersthene-augite-olivine-basalt
No. 7. Ejecta from Hōei crater, 1707, augite-bearing-olivine-basalt
No. 8. Aokigahara lava, two-pyroxene-olivine-basalt
No. 9. Karasu-iwa lava, aphanitic andesite, (pitchstone)

First, we measured the residual magnetization after partial magnetization during cooling[3] of these rocks. That is to say, while a specimen was cooling down, a magnetic field of $\vec{H}$ was applied, but only within the temperatnre range from t_{i+1} to t_i, while in other temperature ranges the external magnetic field was always kept at zero. The specimen that was cooled down to 0°C shows a residual permanent magnetization, which we shall call here *partial magnetization during cooling*, denoting it by $\varDelta J_{t_i,H}$, while $\varDelta t = t_{i+1} - t_i$. The apparatus used in this experiment was the same as that used in the former studies, the temperature range of $\varDelta t$ in the present measurement being always taken as 50 degrees.

According to the definition of the characteristic function $P(t)$, it is given by

$$P(t) = \frac{1}{H} \cdot \lim_{\Delta t \to 0} \frac{\varDelta J_{t_i,H}}{\varDelta t} . \tag{5}$$

Actually, the $P(t) \sim t$ relations shown in Table I and in Fig. 1 were obtained with the aid of the approximate relation

$$P\left(t_i + \frac{1}{2}\varDelta t\right) \fallingdotseq \frac{1}{H} \cdot \frac{\varDelta J_{t_i,H}}{\varDelta t} . \tag{6}$$

As will clearly be seen from the numerals in Fig. 1, the $P(t) \sim t$ curves of some rock specimens (Nos. 1, 3, 4, and 5) have the standard type mode, while the others (Nos. 2, 6, 7, 8, and 9) have the complex mode, which is interpreted as the superposition of two fundamental curves of $P(t)$. The characteristic quantities of thermo-remanent magnetism, i. e. the transition temperature t_0, the critical temperature t_c, the height of the peak in the $P(t)$ curve $P(t_0)$, the specific intensity of the saturated thermo-remanence J_{t_c}, and the coefficient of thermo-remanent magnetism $Q_t \equiv J_{t_c}/\chi$, are given in Table III, where is also shown the temperature range (2ϑ) between the half-value points of $P(t_0)$—a measure for expressing the sharpness of the $P(t)$ curve,—which is expressed by

3) T. NAGATA, *Bull. Earthq. Res. Inst.*, 19 (1941), 66.

$$2\vartheta = t_2 - t_1, \qquad t_2 > t_0 > t_1,$$

$$P(t_1) = P(t_2) = \frac{1}{2} P(t_0) \,. \tag{7}$$

Taking a few typical specimens, the thermo-remanent magnetization $J_{t,H}(0)/H$, directly obtained in the experiment, were compared with the values of $\int_0^t P(t)\,dt \left(\simeq \sum_{t_i=0}^{t-\Delta t} \Delta J_{t_i,H} \right)$ of the same specimens, the latter being calculated from the experimental data given in Table I, while $-\frac{1}{H} \cdot \frac{\partial}{\partial t} J_{t,H}(0)$ was also compared with $P(t)$.[4]

As will be seen from Fig. 2 and Table II, eq. (1) seems to hold approximately in any case, regardless of whether $P(t)$ belongs to the standard type or not. We may, therefore, say that eq. (1) holds in the general case for the mode of causation of thermo-remanent magnetism in igneous rocks; that is, if

$$P(t) = \sum_j P_j(t) \,, \tag{8}$$

Table I. Values of $P(t)$ of Huzi ejecta.

$P(t) = \frac{\Delta J}{\Delta t \cdot H}$ (e.m.u.)

Range in Temp. \ Specimen	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
100∼ 50°C	0·28 $\times 10^{-5}$	0·26 $\times 10^{-5}$	0·20 $\times 10^{-5}$	0·26 $\times 10^{-5}$	0·36 $\times 10^{-5}$	0·24 $\times 10^{-5}$	0·08 $\times 10^{-5}$	0·40 $\times 10^{-5}$	0·20 $\times 10^{-5}$
150∼100	0·18 ″	0·36 ″	0·08 ″	0·30 ″	0·40 ″	0·32 ″	2·38 ″	3·08 ″	0·42 ″
200∼150	0·28 ″	0·28 ″	0·18 ″	0·42 ″	0·46 ″	0·32 ″	6·66 ″	6·74 ″	0·42 ″
250∼200	0·42 ″	2·66 ″	0·12 ″	0·42 ″	0·40 ″	3·98 ″	11·68 ″	3·18 ″	7·08 ″
300∼250	0·44 ″	1·02 ″	0·50 ″	0·88 ″	0·26 ″	4·74 ″	9·74 ″	2·34 ″	3·28 ″
350∼300	1·24 ″	0·98 ″	0·24 ″	0·10 ″	0·46 ″	0·88 ″	5·68 ″	0·98 ″	0·32 ″
400∼350	2·34 ″	0·62 ″	0·66 ″	0·52 ″	0·30 ″	0·96 ″	5·36 ″	0·84 ″	0·42 ″
450∼400	1·90 ″	0·80 ″	0·66 ″	0·16 ″	0·90 ″	0·88 ″	6·72 ″	1·02 ″	0·26 ″
500∼450	4·98 ″	2·08 ″	4·18 ″	2·12 ″	4·02 ″	1·60 ″	8·52 ″	3·50 ″	0·32 ″
550∼500	10·04 ″	4·64 ″	8·36 ″	3·60 ″	7·98 ″	5·08 ″	15·10 ″	4·16 ″	1·96 ″
600∼550	3·74 ″	5·28 ″	5·62 ″	1·40 ″	8·44 ″	4·50 ″	10·36 ″	3·08 ″	0·88 ″
650∼600	0·34 ″	0·28 ″	0·24 ″	0·06 ″	0·58 ″	0·16 ″	1·28 ″	0·18 ″	0·08 ″
700∼650	0·18 ″	0·06 ″	0·08 ″	0·16 ″	0·26 ″	0·22 ″	0·28 ″	0·00 ″	0·10 ″
750∼700	—	—	—	—	—	—	0·42 ″	—	—

4) The method of determining the value of $J_{t,H}(0)$ was already described in the previous paper. T. NAGATA, *loc. cit.*

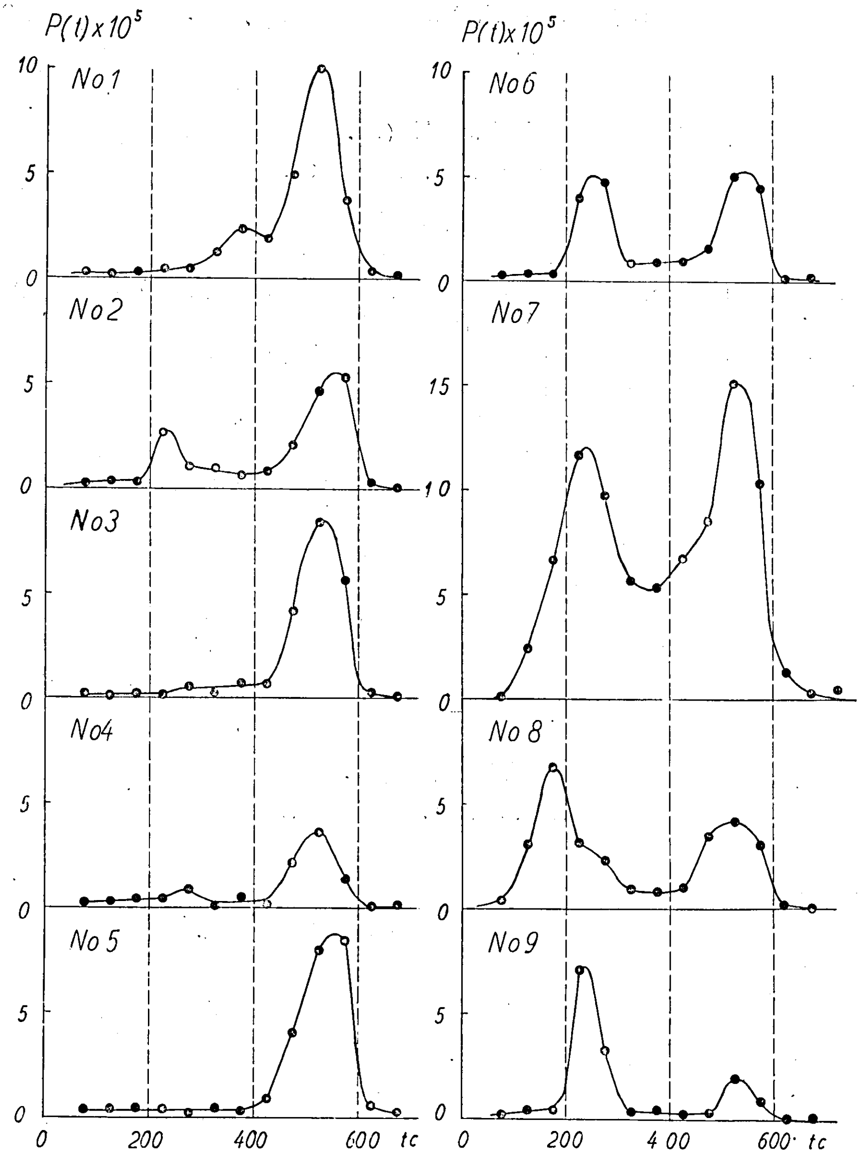

Fig. 1. Characteristic function $P(t)$ of thermo-remanent magnetism of Huzi ejecta.

the thermo-remanent magnetization in H is given by

$$\vec{J}_{t,H}(t) = \vec{H} \cdot \int_{t'}^{t} P(t)\,dt = \vec{H} \cdot \sum_{j} \int_{t'}^{t} P_j(t)\,dt . \tag{9}$$

Table II (a).

Specimen No. 5. $H=0{\cdot}96$ Oe.

Range of Temp.	$\frac{\Delta J}{H}$	$\frac{\partial J}{H \cdot \partial t} \times \Delta t$	$\Sigma \frac{\Delta J}{H}$	$\frac{J}{H}$	Temp.
100~ 50°C	$0{\cdot}20 \times 10^{-3}$	—	$0{\cdot}20 \times 10^{-3}$	$0{\cdot}36 \times 10^{-3}$	100°C
150~100	0·20 〃	$0{\cdot}02 \times 10^{-3}$	0·40 〃		150
200~150	0·23 〃	0·02 〃	0·63 〃	0·39 〃	200
250~200	0·20 〃	0·22 〃	0·83 〃		250
300~250	0·13 〃	0·22 〃	0·96 〃	0·83 〃	300
350~300	0·23 〃	0·01 〃	1·19 〃	0·84 〃	350
400~350	0·15 〃	0·64 〃	1·34 〃	1·48 〃	400
450~400	0·45 〃	1·13 〃	1·79 〃	2·61 〃	450
500~450	2·01 〃	2·64 〃	3·80 〃	5·25 〃	500
550~500	3·99 〃	4·55 〃	7·79 〃	9·80 〃	550
600~550	4·22 〃	3·76 〃	12·01 〃	12·56 〃	600
650~600	0·29 〃	−0·16 〃	12·30 〃	12·40 〃	650
700~650	0·13 〃	0·08 〃	12·43 〃	12·48 〃	700

Table II (b).

Specimen No. 9. $H=0{\cdot}96$ Oe.

Range of Temp.	$\frac{\Delta J}{H}$	$\frac{\partial J}{H \cdot \partial t} \times \Delta t$	$\Sigma \frac{\Delta J}{H}$	$\frac{J}{H}$	Temp.
100~ 50°C	$0{\cdot}10 \times 10^{-3}$	$0{\cdot}12 \times 10^{-3}$	$0{\cdot}10 \times 10^{-3}$	$0{\cdot}12 \times 10^{-3}$	100°C
150~100	0·21 〃	0·13 〃	0·31 〃	0·25 〃	150
200~150	0·21 〃	1·62 〃	0·52 〃	1·87 〃	200
250~200	3·54 〃	3·01 〃	4·06 〃	4·88 〃	250
300~250	1·64 〃	0·98 〃	5·70 〃	5·86 〃	300
350~300	0·16 〃	0·10 〃	5·86 〃	5·96 〃	350
400~350	0·21 〃	0·23 〃	6·07 〃	6·19 〃	400
450~400	0·13 〃	0·08 〃	6·20 〃	6·27 〃	450
500~450	0·16 〃	0·17 〃	6·36 〃	6·44 〃	500
550~500	0·98 〃	0·53 〃	7·34 〃	6·97 〃	550
600~550	0·44 〃	0·83 〃	7·78 〃	7·80 〃	600
650~600	0·04 〃	0·05 〃	7·82 〃	7·85 〃	650
700~650	0·05 〃	−0·05 〃	7·87 〃	7·80 〃	700

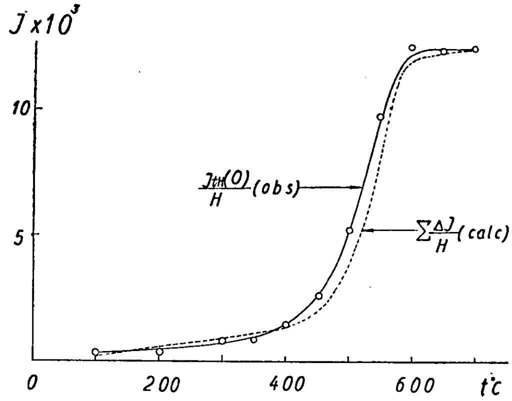

Fig. 2a. Comparison of $J_{t,H}(0)/H$ with $\frac{1}{H}\Sigma \Delta J_{t,H}$. Specimen No. 5.

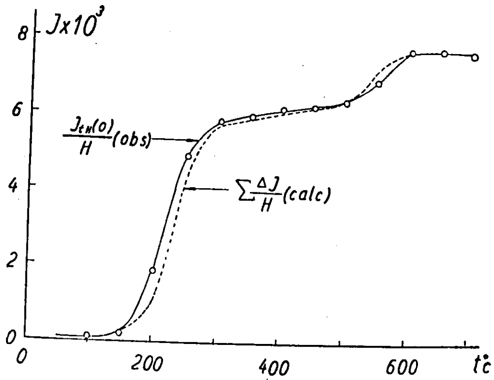

Fig. 2b. Comparison of $J_{t,H}(0)/H$ with $\frac{1}{H}\Sigma \Delta J_{t,H}$. Specimen No. 9.

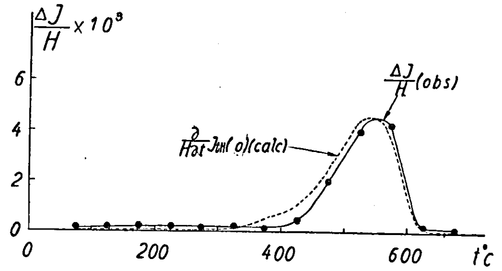

Fig. 2c. Comparison of $\frac{\Delta J_{t,H}}{H}$ with $\frac{1}{H}\cdot\frac{\partial}{\partial t}J_{t,H}(0)\times\Delta t$. Specimen No. 5.

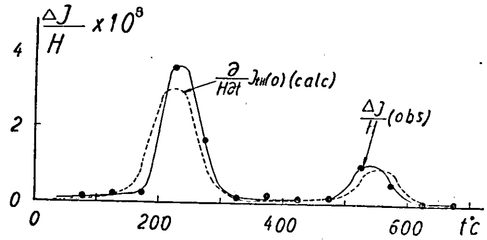

Fig. 2d. Comparison of $\frac{\Delta J_{t,H}}{H}$ with $\frac{1}{H}\cdot\frac{\partial}{\partial t}J_{t,H}(0)\times\Delta t$. Specimen No. 9.

3. Relation between the mode of change in magnetic susceptibility and that of thermo-remanent magnetism.

The method of measuring the change in susceptibility of rocks with temperature, together with some results of measurement, were already described in the writer's previous paper.[5] In his instrument for measuring this, the susceptibility of the rock specimen in a vacuum furnace is measured by means of the ballistic method, with an error of 1∼3 percent, in a magnetic field of 1·5∼2·2 Oersteds, where, as he has frequently noted, the induced magnetization $\vec{H}\cdot\chi(t)$, which is reversible with respect to the applied magnetic field, is rigidly separated from the permanent compoment of magnetization that is due to the thermo-remanent magnetism $J_{t_i,H}(t)$.

5) T. NAGATA, *Bull. Earthq. Res. Inst.*, **19** (1941), 579.

Table III. Characteristic quantities of thermo-remanent magnetism of Huzi ejecta.

No.	Rock	χ_0	t_0	θ	$\bar{\theta}$	J_{t_c}	$P(t^0)$	2ϑ	Q_t	Fe_2O_3	FeO	TiO_2	$\frac{Il}{Mt+Il}$	Mt
1	olivine-basalt	2·25 $\times 10^{-3}$	520°C (380)	580°C (390)	460°C (350)	12·1 $\times 10^{-3}$	10·1 $\times 10^{-3}$ (2·3 ″)	80° (80)	5·4	5·40 %	6·65 %	1·40 %	0·26	7·87 %
2	olivine-basalt	1·82 ″	540 / 240	580 / 300	530 / 235	9·7 ″	5·5 ″ / 2·7 ″	80° / 85°	5·3	3·67	9·90	1·97	0·42	5·33
3	olivine-basalt	1·28 ″	520	580	500	10·5 ″	8·4 ″	110°	8·3	4·50	6·89	1·78	0·34	6·48
4	Two-pyroxene-bearing-olivine-basalt	1·29 ″	525	580	495	5·2 ″	4·0 ″	100°	4·0	3·04	7·29	1·15	0·33	4·40
5	Augite-bearing-olivine-basalt	1·55 ″	545	590	515	12·4 ″	8·9 ″	100°	8·0	4·78	5·72	1·38	0·27	6·95
6	Hypersthene-augite-olivine-basalt	1·31 ″	535 / 250	580 / 320	500 / 285	11·3 ″	5·4 ″ / 4·8 ″	75° / 65°	8·6	2·76	7·72	1·41	0·41	3·94
7	Augite-bearing-olivine-basalt	0·70 ″	530 / 230	580 / 310	520 / 260	41·7 ″	11·7 ″ / 15·2 ″	140° / 125°	59·5	2·64	8·42	1·38	0·40	3·94
8	Two-pyroxene-olivine-basalt	0·91 ″	520 / 190	585 / 290	530 / 195	14·8 ″	4·2 ″ / 6·8 ″	125° / 90°	16·4	1·83	8·34	1·43	0·52	2·55
9	Aphanitic-andesite	0·40 ″	530 / 240	585 / 300	530 / 230	7·9 ″	2·1 ″ / 7·2 ″	60° / 60°	22·2	0·95	5·02	0·87	0·55	1·39

Table III. Specific magnetic susceptibility of Huzi ejecta at various temperatures.

Specimen / Temp.	No. 7.		No. 8.		No. 9.	
	H	C	H	C	H	C
20°C	1·86×10⁻³	1·77×10⁻³	0·94×10⁻³	0·95×10⁻³	0·40×10⁻³	6·40×10⁻³
40	1·97 〃	1·83 〃	1·15 〃	1·10 〃	0·43 〃	0·43 〃
60	2·17 〃	1·93 〃	1·42 〃	1·39 〃	0·46 〃	0·47 〃
80	2·32 〃	2·05 〃	1·66 〃	1·63 〃	0·49 〃	0·49 〃
100	2·53 〃	2·13 〃	1·93 〃	1·77 〃	0·52 〃	0·53 〃
120	2·69 〃	2·25 〃	2·06 〃	1·91 〃	0·55 〃	0·57 〃
140	2·86 〃	2·40 〃	2·14 〃	1·96 〃	0·60 〃	0·62 〃
160	2·92 〃	2·54 〃	2·11 〃	1·89 〃	0·63 〃	0·65 〃
180	2·95 〃	2·53 〃	1·72 〃	1·66 〃	0·67 〃	0·66 〃
200	2·69 〃	2·41 〃	1·29 〃	1·26 〃	0·69 〃	0·67 〃
220	2·38 〃	2·11 〃	0·97 〃	1·08 〃	0·56 〃	0·56 〃
240	2·00 〃	1·79 〃	0·81 〃	0·82 〃	0·36 〃	0·37 〃
260	1·63 〃	1·51 〃	0·68 〃	0·62 〃	0·28 〃	0·25 〃
280	1·38 〃	1·20 〃	0·61 〃	0·47 〃	0·21 〃	0·18 〃
300	1·27 〃	1·01 〃	0·53 〃	0·44 〃	0·15 〃	0·13 〃
320	1·19 〃	0·99 〃	0·50 〃	0·43 〃	0·13 〃	0·13 〃
340	1·14 〃	0·93 〃	0·49 〃	0·42 〃	0·12 〃	0·12 〃
360	1·11 〃	0·86 〃	0·48 〃	0·42 〃	0·12 〃	0·13 〃
380	1·05 〃	0·83 〃	0·48 〃	0·40 〃	0·13 〃	0·12 〃
400	0·96 〃	0·78 〃	0·48 〃	0·40 〃	0·11 〃	0·12 〃
420	0·89 〃	0·70 〃	0·47 〃	0·39 〃	0·12 〃	0·12 〃
440	0·79 〃	0·63 〃	0·46 〃	0·39 〃	0·12 〃	0·12 〃
460	0·70 〃	0·60 〃	0·43 〃	0·38 〃	0·12 〃	0·12 〃
480	0·57 〃	0·48 〃	0·39 〃	0·36 〃	0·11 〃	0·12 〃
500	0·48 〃	0·41 〃	0·34 〃	0·34 〃	0·11 〃	0·10 〃
520	0·38 〃	0·30 〃	0·31 〃	0·28 〃	0·09 〃	0·10 〃
540	0·19 〃	0·16 〃	0·21 〃	0·18 〃	0·06 〃	0·06 〃
560	0·10 〃	0·08 〃	0·10 〃	0·09 〃	0·03 〃	0·03 〃
580	0·01 〃	0·00 〃	0·01 〃	0·01 〃	0·01 〃	0·00 〃
600	0·00 〃	0·00 〃	0·00 〃	0·00 〃	0·00 〃	0·00 〃
620	0·00 〃	0·00 〃	0·00 〃	−0·01 〃	0·00 〃	−0·01 〃
640	0·00 〃	0·00 〃	−0·01 〃	0·00 〃	−0·01 〃	0·01 〃
660	−0·01 〃		0·00 〃		0·00 〃	

Since the observed values of $\chi(t)$ of a number of specimens of Huzi ejecta have already been reported, these values of the remaining other specimens alone are shown in Table IV and Fig. 3. Seeing that the $\chi(t) \sim t$ relations in the heating and cooling processes are nearly the same, we may say that the change in susceptibility of Huzi ejecta with temperature is almost reversible with respect to temperature variation.

On the other hand, we have already found that the susceptibility of some specimens change monotonously with change in temperature,

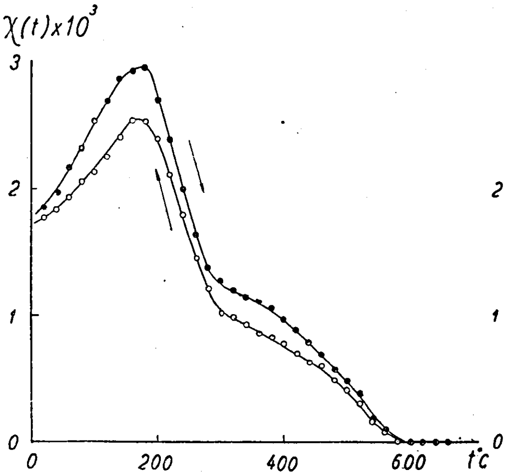

Fig. 3 a. Change in susceptibility with temperature.
Specimen No. 2. $H=2{\cdot}15$ Oe.

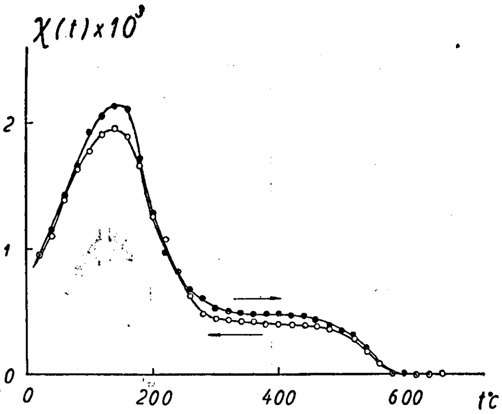

Fig. 3 b. Change in susceptibility with temperature.
Specimen No. 8. $H=2{\cdot}15$ Oe.

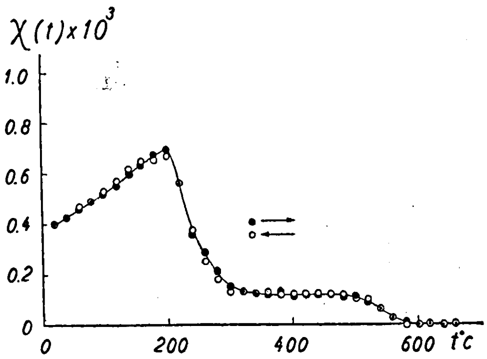

Fig. 3 c. Change in susceptibility with temperature.
Specimen No. 9. $H=2{\cdot}74$ Oe.

while others change stepwise with temperature, like those given in Fig. 3. Hence, like the mode of development of thermo-remanent magnetism, the mode of change in susceptibility of rocks with temperature is assumed given, in the case, by the superposition of general a few fundamental modes $\chi_j(t)$, each of which has a simple functional form almost the same as that of the usual ferro-magnetic substances. That is to say,

$$\chi(t)=\sum_j \chi_j(t) ,$$

where to each of the separated curves $\chi_j(t)$, a mean Curie-point $\bar{\theta}_j$,

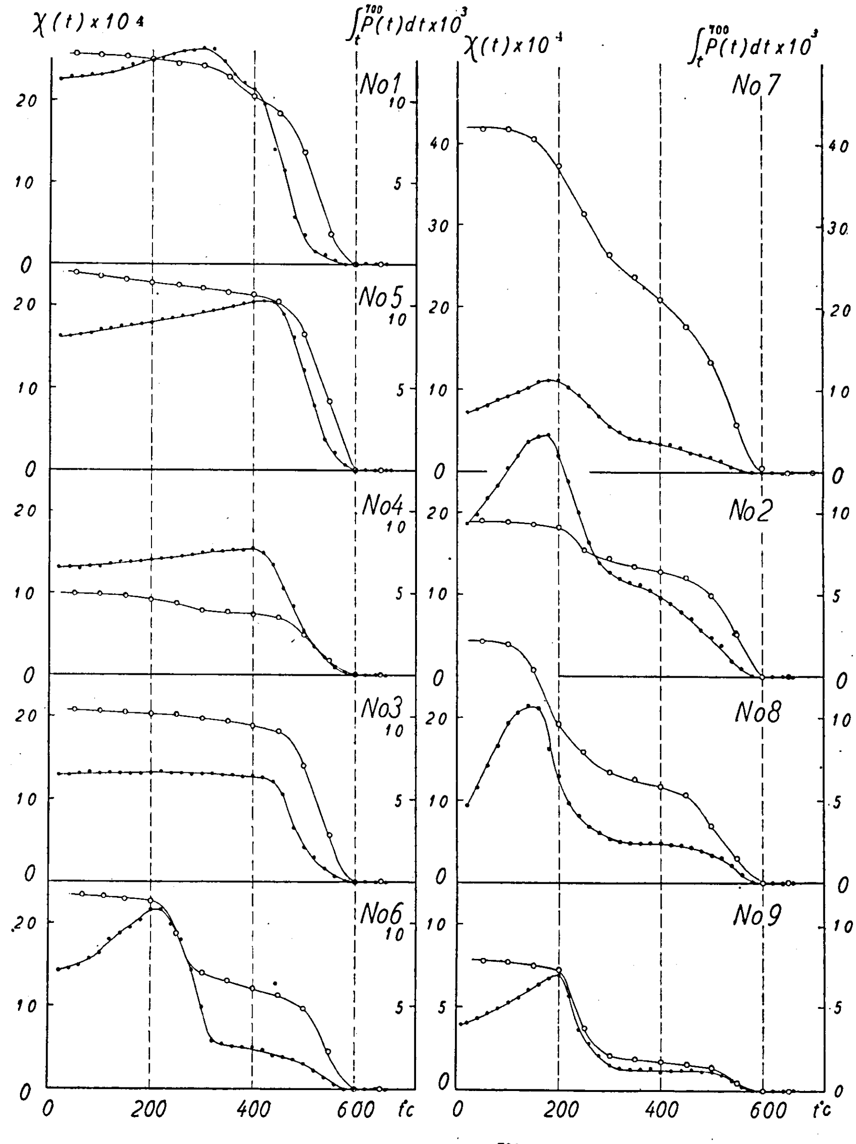

Fig. 4. Comparison of $\chi(t)$ with $\int_t^{700} P(t)\,dt$ of Huzi ejecta.

●—●, specific susceptibility $\chi(t)$.

○—○, thermo-remanent magnetism $\int_t^{700} P(t)\,dt$.

and an apparent Curie-point θ_j, are assumed to correspond, $\bar{\theta}_j$ and θ_j being given respectively by

$$-\left[\frac{\partial}{\partial t}\chi_j(t)\right]_{t=\bar{\theta}_j}=\text{maximum},$$

$$\chi_j(\theta_j)=0. \tag{11}$$

The observed values of $\bar{\theta}_j$ and θ_j of Huzi ejecta are given in Table III, where the specific susceptibility at room temperature, the largest values of susceptibility at various temperatures in the range from 0°C to the highest apparent Curie-temperature, and the temperature at which it is the largest, are shown.

Now, in order to compare these results with the modes of development of thermo-remanent magnetism, the $\chi(t)$ curves in the heating process and the $J_{t_c}(t)\left(=\int_t^{t_c}P(t)\,dt\right)$ curves of the same specimens are shown together in Fig. 4, while transition temperature t_{0j}, the mean Curie-temperature $\bar{\theta}_j$, and apparent Curie-temperature θ_j, are graphically shown in Fig. 5. in the form of spectra with respect to temperature.

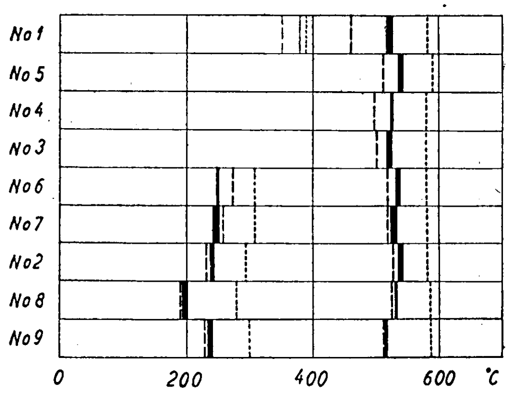

Fig. 5. Characteristic temperatures of ejecta from Volcano Huzi.

Full line: transition temperature t_0,
Dotted line: apparent Curie-point $\bar{\theta}$,
Broken line: mean Curie-temperature θ.

Comparing the mode of $P(t)$ with that of $\chi(t)$, or t_{0j} with $\bar{\theta}_j$ and θ_j, we find that the standard mode of $P(t)$ corresponds to that of $\chi(t)$, and the complex mode in the former to that of the latter, each separated curve $\chi_j(t)$ in susceptibility corresponding to each $P_j(t)$ in thermo-remanence. In other words, a transition temperature t_{0j} in thermo-remanent magnetism always corresponds to a mean Curie-point $\bar{\theta}_j$, and to an apparent Curie-point θ_j, and further the temperature of t_{0j} is nearly the same as those of $\bar{\theta}_j$ and θ_j, although the former does not exactly agree with $\bar{\theta}_j$ and θ_j. This correspondence will be seen more clearly in Fig. 5.

Since the error in determining the temperature of the specimen in both cases of measuring $P(t)$ and $\chi(t)$ amounted to 5∼10 degrees, no rigorous values of t_{0j}, $\bar{\theta}_j$, and θ_j have yet been obtained. Generally speaking, however, the transition temperature is 40～80 degrees lower than the apparent Curie-temperature.

It may thus be concluded that the thermo-remanent magnetism is developed during cooling from the apparent Curie-temperature through a temperature range of about 100 degrees; or in other words, during the time that the magnitude of the saturation magnetization of the specimen is increasing immediately after it has been transformed into the ferro-magnetic state from the para-magnetic state.

Dealing now specially with the modes of $P(t)$ and $\chi(t)$ of Huzi ejecta, we may say that they belong either to the standard mode or to that constituted of the two fundamental modes, one of which is the standard mode, while the other has its transition point and its apparent Curie-point at about 200～250°C and 290～320°C respectively, whence it may be presumed that the ferro-magnetic minerals in Huzi ejecta consist of two mineral groups of different chemical composition. One of them, which corresponds to the standard mode of $P(t)$ and $\chi(t)$, will have nearly the same composition as magnetite, since its apparent Curie-point almost agrees with that of pure magnetite, in contrast to which the chemical composition of the other group of ferro-magnetic minerals, which corresponds to the mode of $\chi_j(t)$ (an apparent Curie-temperature of which is 290～320°C), differs from that of magnetite, for which reason the corresponding modes of $\chi_j(t)$ and of $P_j(t)$ will be called here the *extraordinary* modes. Our experiments show that the extraordinary mode in $P(t)$ is nearly zero in some specimens of Huzi ejecta, while in a few others it is clearly prominent compared with the standard one, showing that in various rocks from Volcano Huzi, the amount of ferro-magnetic minerals corresponding to the extraordinary mode varies from zero to a certain fairly large value.

In order to see the relation between the mode of $P(t)$, or of $\chi(t)$, and the chemical composition, if any, the values of $P(t_{oe.})/\left\{P(t_{os.})+P(t_{oe.})\right\}$ were plotted against the ratios $FeO:Fe_2O_3$; TiO_2, as shown in Fig. 6, where $t_{oe.}$ and $t_{os.}$ denote the transition temperatures of the extraordinary and standard modes respectively. It will be seen from this figure that the extraordinary mode of thermo-remanent magnetism grows as the relative amount of Fe_2O_3 decreases.

On the other hand, taking into consideration the fact that the minerals concerned with the magnetic behaviour of rocks will be limited to magnetite ($FeO \cdot Fe_2O_3$,), ilmenite ($FeO \cdot TiO_2$), and their solid solutions,[6] the values of $P(t_{oe.})/\left\{P(t_{oe.})+Pt_{os.})\right\}$ were again plotted

6) See for examples; M. KAMIYAMA, *Journ. Geol. Soc. Japan*, 36 (1629), 12; H. L. ALLING; "*Interpretive Petrology of the Igneous Rocks.*" (1936), pp. 140.

against the ratio of ilmenite to magnetite in norms, i. e. $Il/(Mt+Il)$, as shown in Fig. 7.

It is then clear from Fig. 7 that the extraordinary mode in $P(t)$ begins to appear when the relative content of ilmenite just exceeds the value, say, $Il/(Il+Mt)=0{\cdot}30$, and that it grows as the amount of Il increases. As to the mode of $\chi_j(t)$, it was also found that, qualitatively speaking, its extraordinary mode begins to appear when the content of ilmenite exceeds the same ratio as that just mentioned, from which it may be concluded that the extraordinary mode in $\chi(t)$ as well as in $P(t)$ is closely related to the relative amount of Il, or TiO_2, in the ferro-magnetic minerals contained in the rocks. It should be noted, however, that this effect of ilmenite on the magnetic behaviour of rocks may not be due to the amount of pure ilmenite in the rocks, but to the presence of solid solutions of ilmenite and magnetite, or to that of a more complex mineral, composed of Fe_2O_3, FeO, and TiO_2, since the appearance of the extraordinary mode depends on the ratio of $Il/(Il+Mt)$, or that of Il/Mt, and not on the amount of Il itself, which is nearly the same in all specimens, as shown in Table III and in Fig. 6, and since the intensity of magnetization of ilmenite is very small compared with that of magnetite, the former being only about 1/20 that of the latter.[7]

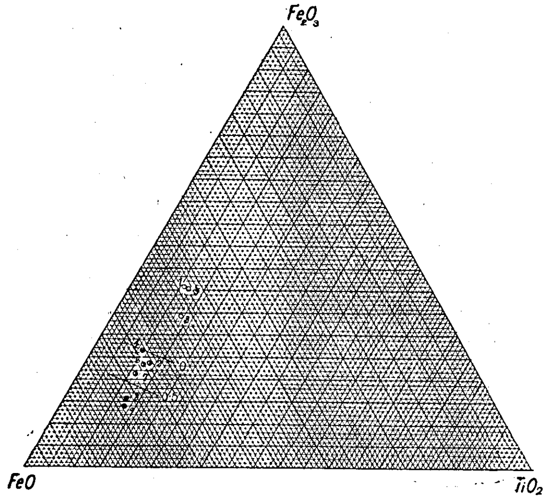

Fig. 6. Relation between the value of $P(t_{oe.})/\{P(t_{os.})+P(t_{oe.})\}$ and the ratio Fe_2N_3: FeO: TiO_2.

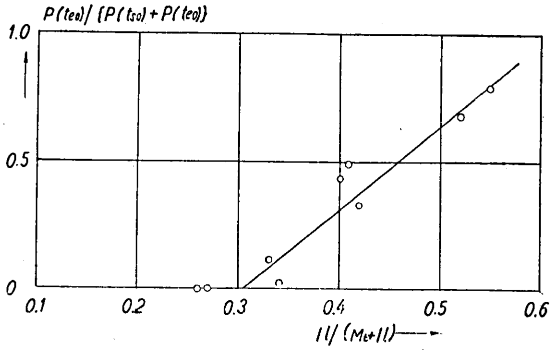

Fig. 7. Relation between the value of $P(t_{oe.})/\{P(t_{os.})+P(t_{oe.})\}$ and the ratio $Il/(Mt+Il)$.

7) J. G. KÖNIGSBERGER, *Terr. Mag.*, 43 (1938), 119.

4. The dependency of thermc-remanent magnetization on the magnetic field applied.

That the intensity of thermo-remanent magnetization at any stage during the process of its development is proportional to that of the magnetic field applied, provided the last-named is not intense—say,

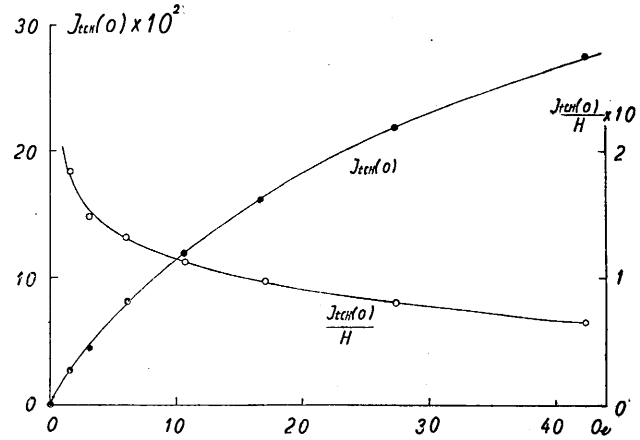

Fig. 8 a. Relation between the intensity of saturated thermo-remanent magnetism and that of the applied magnetic field. (Miyake-sima, Yoridai-no-sawa lava).

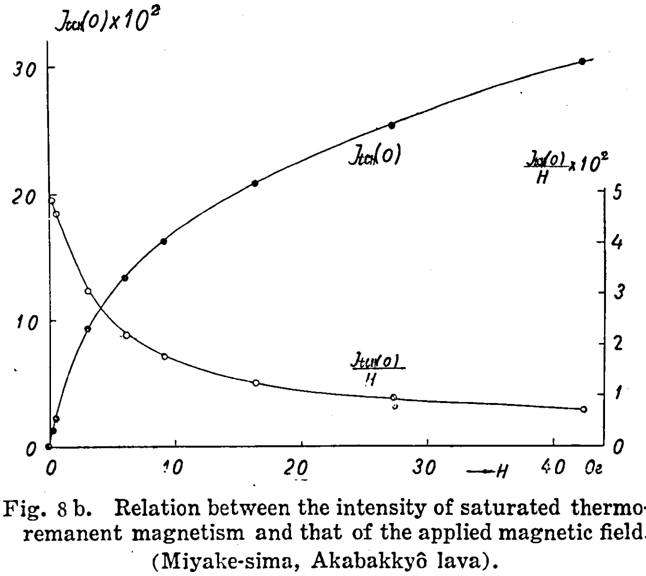

Fig. 8 b. Relation between the intensity of saturated thermo-remanent magnetism and that of the applied magnetic field. (Miyake-sima, Akabakkyô lava).

smaller than 1 Oersted—has been already proved experimentally. So long as we are concerned only with the magnetic behaviour of igneous rocks in the earth's magnetic field (0·35∼0·6 Oe.), the various laws of thermo-remanent magnetism established under the limitation that the magnetic field shall be weak, will be sufficiently useful.

On the other hand, because the general relation between the intensity of thermo-remanence and that of the applied magnetic field is required for the purpose of clearing the physical mechanism of development of this phenomenon, the intensity of saturated thermo-remanent magnetization in a magnetic field H of various intensities was measured, where H varies from 0·3 to 40 Oe. In the actual experiment, the rock specimen in a vacuum furnace, non-magnetic and non-inductive, set co-axially with a large circular solenoid, was slowly cooled, the specimen after cooling from 700°C to 0°C in H having the corresponding saturated thermo-remanent magnetization $J_{t_c,H}$.

Table V. Saturated thermo-remanent magnetism.

(a) Specimen S-N-59
(Miyake-sima Akabakkyô-lava)

H	$J_{t_c,H}(0)$	$J_{t_c,H}(0)/H$
oe. 1·51	$2·67\times10^{-2}$	$1·84\times10^{-1}$
3·02	4·44 〃	1·47 〃
6·04	8·04 〃	1·33 〃
10·6	11·9 〃	1·13 〃
16·6	16·1 〃	0·97 〃
27·2	21·9 〃	0·81 〃
42·3	27·5 〃	0·65 〃

(b) Specimen S-N-60
(Miyake-sima Yoridai-no-sawa lava)

H	$J_{t_c,H}(0)$	$J_{t_c,H}(0)/H$
oe. 0·28	$1·36\times10^{-2}$	$4·91\times10^{-2}$
0·46	2·22 〃	4·62 〃
3·02	9·33 〃	3·09 〃
6·04	13·35 〃	2·21 〃
9·06	16·2 〃	1·78 〃
16·3	20·7 〃	1·26 〃
27·2	25·2 〃	0·93 〃
42·3	30·1 〃	0·71 〃

As shown in Fig. 8, the intensity of $J_{t_c,H}$ of two rock specimens[8] were plotted against the magnitude of H, where the values of $J_{t_c,H}/H$ are also plotted. As will be seen from this figure, although $J_{t_c,H}$ increases as H increases, the former is not proportional to the latter, $J_{t_c,H}/H$ decreasing with increase of H.

Since the remanent magnetization $J_{t_c,H}$ ought to approach a certain constant value in the case that H is sufficiently large, it follows that

$$\lim_{H\to\infty} J_{t_c,H}=J_0. \tag{12}$$

In the present experiment, however, the largest value of H applied seems to be considerably smaller than that obtained by saturation magne-

8) These rock specimens are the lava ejected from Volcano Miyake-sima in 1940.

tization J_0. On the other hand, from the result obtained in the former study, it is required that

$$J_{t_c,H} \doteqdot HJ_{t_c}, \text{ provided } H \ll 1. \tag{3'}$$

The general formula, then, expressing the relation between $J_{t_c,H}$ and H throughout the whole range from 0 to infinity, may not be simple, since it must not only fit all values obtained in the present measurements, but must also satisfy the conditions of eqs. (12) and (3′).

In a certain range of H, say 0~10 Oe., however, the following formula will probably be preferred, namely,[9]

$$J_{t_c,H} = J_{t_c}\frac{(1+k)H}{1+kH}\,.^{10)} \tag{13}$$

It should be noted here that determination of the pure effect of thermo-remanent magnetization is almost impossible in a strong magnetic field, because ordinary remanent magnetism, which does not depend on heat treatment, appears in this case.

At any rate, it may be concluded that the effect of the appearance of thermo-remanent magnetization is greater according as the external magnetic field is small.

5. Conclusion.

From the present study, the development of thermo-remanent magnetism in igneous rocks can be interpreted as their irreversible magnetization at a temperature immediately below their Curie-point temperature. On the other hand, from the mode of change in susceptibility with temperature, it may be presumed that the ferro-magnetic minerals in the rock consist of a large number of microcrystals of various chemical compositions. For example, in a rock specimen having the standard mode of $\chi(t)$, the chemical composition of every micro-crystal of the ferro-magnetic minerals may be almost the same as that of magnetite, although it may differ slightly the one from the other. In other words,

9) J_{t_c} is defined as $J_{t_c,H}(0)$ corresponding to $H=1$.

10) It was proved experimentally that, in the range of H where eq. (13) holds, there also holds approximately the law of development of thermo-remanent magnetism that

$$J_{t_i,H}(t) = F(H)\int_t^{t_i} P(t)\,dt,$$

where

$$F(H) = (1+k)\frac{H}{1+kH}\,.$$

by letting s and $f(s)$ respectively denote a measure of chemical composition and the quantity of the microcrystal that has s, then the total amount of ferro-magnetic minerals S in unit volume of rock is given by

$$S=\int_{s_1}^{s_2} f(s)ds, \tag{14}$$

where s_1 and s_2 are the limiting values of possible distribution of s; the apparent mode of change in susceptibility with temperature, then, being given by

$$\chi(t)=\int_{s_1}^{s_2} f(s)\chi(s,t)ds, \tag{15}$$

where $\chi(s,t)$ denotes the susceptibility of an elemental micro-crystal, being a function of s as of t. Here, the mode of $\chi(s,t)$ appears to be almost the same as that of the usual ferro-magnetic substance, its Curie-point being uniquely determined as a function of s.

Since, further, the distribution function $f(s)$ would have nearly a parabolic form, as shown schematically in Fig. 9, the mode of $\chi(t)$ given by eq. (15) exactly agrees with that of the actual igneous rock obtained in our experiments, where the mean Curie-point $\bar{\theta}$, expressed by $-\left[\frac{\partial}{\partial t}\chi(t)\right]_{t=\bar{\theta}}=$maximum, is naturally derived from eq. (15), while the apparent Curie-point is interpreted as the highest in group $\theta(s)$.

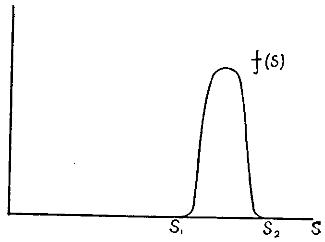

Fig. 9.

If the mechanism is really as that just mentioned, a possible explanation of the development of thermo-remanent magnetization during cooling in a magnetic field may be as follows.

In the process of cooling in a magnetic field, a part of the elemental domain in every micro-crystal contained in a rock takes, irreversibly, the direction of easiest magnetization nearest that of the applied magnetic field at a temperature just below the Curie-point peculier to it, where the number and dimensions of these domains may be a function of the chemical composition s, the dimension d of the micro-crystal, and H, the former two being probably connected with this phenomenon through the effect of internal stress in the micro-crystal due to s and d.

Then, $\phi(H,d,s)$ and $I(t,s)$ denote the total volume of the elemental domains satisfying the condition just-mentioned of the micro-crystal and the intensity of saturation magnetization of the domains respectively, the magnetization of such a micro-crystal of unit volume being given by $I(t,s)\phi(H,d,s)$, where the saturation magnetization $I(t,s)$ increases with decrease of temperature, being zero when the temperature is exactly $\theta(s)$.

The remanent magnetization $M(t)$ of the whole rock specimen at temperature t, then, is given by

$$M(t)=\int_{s_1}^{s_2} I(t,s)\phi(H,d,s)f(s)ds\,. \tag{16}$$

In eq. (16), since $\theta=\theta(s)$, we may put

$$I(t,s)=J(t,\theta),\quad \phi(H,d,s)=F(H)\cdot\psi(d,\theta)$$

and

$$f(s)=g(\theta)\cdot\frac{d\theta}{ds}, \tag{17}$$

with the result that

$$M(t)=F(H)\cdot\int_{\theta_1}^{\theta_2} J(t,\theta)\psi(d,\theta)g(\theta)d\theta\,, \tag{18}$$

where

$$\theta_1=\theta(s_1),\ \text{and}\ \theta_2=\theta(s_2).$$

Seeing that the saturated thermo-remanent magnetization measured in the present experiment was defined as the residual magnetization after cooling a rock specimen from t_c to 0°C in a magnetic field of H, we get

$$M(0)=F(H)\cdot\int_{\theta_1}^{\theta_2} J(0,\theta)\psi(d,\theta)g(\theta)d\theta$$

$$=F(H)\cdot\int_{0}^{t_c} J(0,\theta)\psi(d,\theta)g(\theta)d\theta=J_{t_c,H}, \tag{19}$$

since $0\leqq\theta_1\leqq\theta_2\leqq t_c$, and $g(\theta)=0$ when $\theta\leqq\theta_1$, or $\theta\geqq\theta_2$.

The thermo-remanent magnetism which developed during cooling from t to t' in a magnetic field H is given also by

$$J_{t,H}(t)=F(H)\cdot\int_{t'}^{t} J(0,\theta)\psi(d,\theta)g(\theta)d\theta\,, \tag{20}$$

where $t=\theta=\theta(s)$, and $t'=\theta'=\theta(s')$.

Then comparing eqs. (19) and (20) with eqs. (3) and (1) respectively, we get

$$P(\theta)=J(0,\theta)\psi(d,\theta)g(\theta). \tag{21}$$

That is to say, $P(\theta)d\theta$ is given by the product of three quantities, their physical meaning being respectively, (1) the volume of those micro-crystals that have Curie-points at temperatures from θ to $\theta+d\theta$, (2) the volume percentage of those domains in the micro-crystal that take the direction of the applied magnetic field at temperature just below their respective Curie-point, and (3) the intensity of saturated magnetization of these domains at 0°C.

Needless to say, it is possible that the foregoing discussion merely offers a possible physical mechenism for the development of thermo-remanent magnetism, harmonizing with the various facts observed in the present experiment. In the writer's preliminary work[11] on thermo-remanent magnetism, however, the values corresponding to $M(t)$ at various temperatures from 0°C to t_c were obtained, from which we get $M(t)\sim M(0)$ as the first approximation, with the result that $J(t,\theta)\simeq J(0,\theta)$. This fact seems to contradict the result expected from the above-mentioned theory, that is $J(t,\theta)<J(0,\theta)$, provided $t>0$.

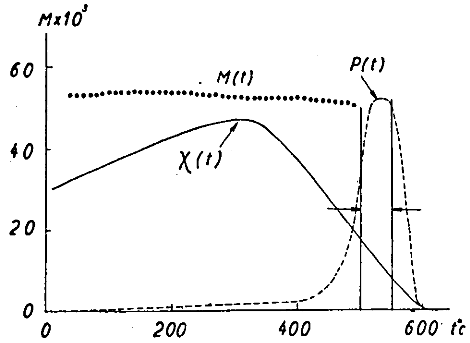

Fig. 10.

$M(t)$: intensity of magnetization during cooling in non-magnetic space, where the specimen was magnetized by cooling from 550°C to 500°C (the range between the two arrows) in a magnetic field.

$\chi(t)$; susceptibility on an arbitrary scale (H=2·09 Oe)

$P(t)$; characteristic function of thermo-remanent magnetism on an arbitrary scale (H=0 96 Oe)

For the purpose of ascertaining more clearly the relation between $M(t)$ and $M(0)$, we measured the intensity of magnetization of rock specimen at various temperatures during cooling from t' to zero in non-magnetic space, where the rock specimen had been magnetized, with cooling in magnetic field of H from t to t'. An actual example of the result of measurement is shown in Fig. 10, where the test-specimen was a piece from Yoridai-no-sawa lava in Miyake-sima (S–N–60), the $P(t)$ curve of which is remarkably sharp as shown by the dotted line in the same figure.

The intensity of magnetization of the rock specimen, which was cooled from 550°C to 500°C in a magnetic field of 15 Oe, was measured by means of an astatic magneto-

11) T. NAGATA, *Bull. Earthq. Res. Inst.*, 19 (1941), 66.

meter at various temperatures during cooling from 500°C to 0°C in non-magnetic space.

As will be clearly seen in Fig. 10, the intensity of magnetization changes very little with decrease in temperature, with the result that $M(t) \simeq M(0)$, in spite of the fact that the susceptibility $\chi(t)$ of the same specimen changes remarkably with decrease in temperature as shown by the full line in the same figure. We may, then, conclude that, so long as we assume that the saturation magnetization so changes with temperature as that in usual ferro-magnetic single crystals, the simple theory that the development of thermo-remanent magnetism is due to the irreversible rotation of elemental domains in the ferro-magnetic minerals in rock at temperature just below their peculiar Curie-point can scarcely be adopted. Although the physical mechanism of development of thermo-remanent magnetism in rocks has not yet been cleared, we may expect that it is probably due to the change in the crystal lattice itself in the ferro-magnetic minerals, or otherwise, owing to a certain physical condition. The saturation magnetization $J(t,\theta)$ in eq. (18) changes very little in the wide range of temperature from 0°C to 500°C (though the latter phenomenon seems to be hardly possible).

This problem is now being examined with careful attention and by the most accurate method possible. Upon ascertainig the physical conditions that determine the occurrence of $P(t)$ in rocks, it should be possible, according to its result, to estimate fairly quantitatively the development of thermo-remanent magnetism in the subterranean rocks of the earth's crust, and thus make clear the effect of the rock's magnetization on regional and local geomagnetic anomaly, as well as that of changes in magnetization on secular variation of the earth's magnetic field.

In conclusion, the writer wishes to express his sincere thanks to Prof. H. Tsuya, who kindly allowed the writer the use of his samples and petrographic data obtained by him, as well as for his valuable advices. The writer's hearty thanks are due also to Prof. C. Tsuboi and Prof. S. Kaya for their interest and encouragement throughout the present study, and to the Department of Education and the Hattori Hôkô Kai for grant received.

14. 火成岩に於ける熱殘留磁氣生成の機構に就いて (II)

地震研究所 永田 武

火成岩に於ける熱殘留磁氣生成過程及び生成條件に對する實驗的追求の一部である．序報に報告せる實驗結果によつて，一つの岩石試料を充分高温から冷却する際，温度 t から t' 迄冷却する間のみ磁場 $\vec{H}$ を與へ，他の温度區間は無磁場冷却をする時，殘留する磁氣を $\vec{J}_{t,H}(t')$ で表はせば

$$\vec{J}_{t,H}(t')=\vec{H}\cdot\int_{t'}^{t}P(t)\,dt \qquad (1)$$

である．但し，$P(t)$ は温度のみの函數で個々の岩石に特有の函數形を示す．

この報告に於いては更に次の諸點を明らかにした．

(1) 多くの岩石に於いては $P(t)$ 曲線は一つの極大を有する誤差函數に似た簡單な形を有するが，ある岩石に於いては，この様な簡單な標準曲線が二つ以上重疊してゐる場合がある．この場合に就いても (1) 式はやはり成立する．

(2) 帶磁率の温度による變化 $\chi(t)$ と熱殘留磁氣特性曲線 $P(t)$ とを比べて見ると，キユリー温度と熱殘留磁氣遷移温度とが互ひに對應する．即ち

$$\chi(t)=\sum_j\chi_j(t),\quad P(t)=\sum_j P_j(t)$$

とすると，$\chi_j(t)$ は $P_j(t)$ に對應し，その前者のキユリー温度 θ_j は，後者の遷移温度 t_{0j} と一對一對應を爲し，t_{0j} は θ_j より 60〜80° 低いのが一般である．

(3) $\chi(t)$ が温度と共に段階的に變化する事實は，岩石間の強磁性鑛物粒の成分が，大まかに見て幾つかの群に分けられる事を示すが，一つ一つの群に於いてもキユリー點が一定でなく，ある特定の温度の附近に殆んど連續的に集合してゐると思はれる．(第3圖)

(4) 特に富士火山噴出岩 9 個に就いて吟味して見ると，$P(t)=P_s(t)+P_e(t)$ であり t_{0s} は 510°〜540°C にあり，t_{0e} は 200〜240°C にある．之に對應して θ は 580〜600°C 及び 280〜300°C である．前者はどの試料にも表はれ，θ_s の値より見て Fe_3O_4 に近い化學成分を有する鑛物粒に起因すると思はれる．之に對して $P_e(t)$ は，岩石中の No m 値に於ける $Il/(Mt+Il)$ の比が 0·3 より大なるものゝみに表はれ，然も $P_e(t_{0e})/\{P_s(t_{0s})+P_e(t_{0e})\}$ は $Il/(Mt+Il)$ の增加と共に大なる．$\chi(t)$ についてもほゞ同樣の事が云はれる．從つて $P_s(t)$ はマグネタイトとイルメナイトとのある固溶體に起因すると思はれる．

(5) 要するに，熱殘留磁氣の生成は，岩石内の強磁性粒がキユリー點以下に冷却して強磁性狀態に入つた直後に於ける不可逆的磁化現象であるが，その生成條件は未だ充分には分つてゐない．

(6) 磁場の強さに對して弱磁場の範圍では (1) 式に示す如く J は H に比例するが，一般には H の代りに $F(H)$ とおかるべきであり，その形は第6圖に示す如くである．

(7) 最後に熱殘留磁氣生成の機構に關して一つの可能な現象論的解釋を行ひ，且つ若干の實驗的吟味を附加した．

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