33. *The Seismic Vibration of a Saw-tooth Roof Structure.*

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The problem of seismic resistance of a saw-tooth roof structure has not yet been attacked mathematically. Although Taniguchi[1] reported damage to such a structure in the Tango Earthquake of 1927, no analytical study of such damage has not yet been made. The present paper is an attempt to ascertain the free vibration of a saw-tooth roof structure, as shown Fig. 1. In order to simplify the problem, an infinite number of spans is assumed. Furthermore, it is assumed, for simplicity, that the masses are concentrated at the panel points, in consequence of which the mass for horizontal movement will differ from that for vertical movement, the respective masses in question then being m_1, m'_1, m_2, m'_2. Let l_1, l'_1, l_2, l_3, h be the length of lower column, length of upper column, beam span, length of roof member, and the total length of the vertical members, respectively; let also E_1I_1, E_2I_2, 0; E_1a_1, 0, E_3a_3 be the bending and longitudinal stiffnesses of the columns, beam, and roof member, respectively. The lateral movements y_1, y'_1, y_2, 0 and the longitudinal displacements z_1, z'_1, 0, z_3 of the respective members just mentioned satisfy differential equations of types

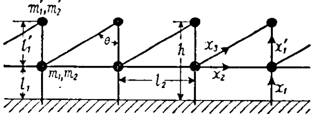

Fig. 1.

$$\frac{\partial^4 y}{\partial x^4}=0, \quad \frac{\partial^2 z}{\partial x^2}=0. \tag{1}$$

The solutions of these equations are

$$\left.\begin{aligned} y_1&=A_1+B_1x_1+C_1x_1^2+D_1x_1^3, & z_1&=\alpha_1+\beta_1x_1,\\ y'_1&=A'_1+B'_1x'_1+C'_1x_1'^2+D'_1x_1'^3, & z'_1&=\alpha'_1+\beta'_1x'_1,\\ y_2&=A_2+B_2x_2+C_2x_2^2+D_2x_2^3, & z_3&=\alpha_3+\beta_3x_3. \end{aligned}\right\} \tag{2}$$

The boundary conditions are written

$$x_1=0\,; \qquad y_1=0, \quad z_1=0, \quad \frac{dy_1}{dx_1}=0, \qquad (3), (4), (5)$$

1) T. TANIGUCHI, *Bull. Earthq. Res. Inst.*, 3 (1927), 133.

$$x_1=l_1,\ x_1'=0,\ x_2=l_2,\ x_{22}=0,\ x_3=0;$$

$$y_1=y_1',\quad y_2=y_{22}=-z_1=-z_1',\quad z_3=z_1\cos\theta+y_1\sin\theta, \tag{(6), (7), (8), (9), (10)}$$

$$\frac{dy_1}{dx_1}=\frac{dy_1'}{dx_1'}=\frac{dy_2}{dx_2}=\frac{dy_{22}}{dx_{22}}, \tag{(11), (12), (13)}$$

$$E_1I_1\left(\frac{d^2y_1}{dx_1^2}-\frac{d^2y_1'}{dx_1'^2}\right)+E_2I_2\left(\frac{d^2y_2}{dx_2^2}-\frac{d^2y_{22}}{dx_{22}^2}\right)=0, \tag{14}$$

$$E_1I_1\left(\frac{d^3y_1'}{dx_1'^3}-\frac{d^3y_1}{dx_1^3}\right)-E_3a_3\frac{dz_3}{dx_3}\sin\theta=m_1p^2y_1, \tag{15}$$

$$E_2I_2\left(\frac{d^3y_{22}}{dx_{22}^3}-\frac{d^3y_2}{dx_2^3}\right)+E_1a_1\left(\frac{dz_1'}{dx_1'}-\frac{dz_1}{dx_1}\right)+E_3a_3\frac{dz_3}{dx_3}\cos\theta=m_2p^2y_2, \tag{16}$$

$$x_1'=l_1',\ x_3=l_3;\quad z_3=y_1'\sin\theta+z_1'\cos\theta,\quad E_1I_1\frac{d^2y_1'}{dx_1'^2}=0, \tag{(17), (18)}$$

$$-E_1I_1\frac{d^3y_1'}{dx_1'^3}+E_3a_3\frac{dz_3}{dx_3}\sin\theta=m_1'p^2y_1', \tag{19}$$

$$E_1a_1\frac{dz_1'}{dx_1'}+E_3a_3\frac{dz_3}{dx_3}\cos\theta=m_2'p^2z_1'. \tag{20}$$

Substituting (2) in (3)$\sim$(20), we get the frequency equation of free vibrations as follows:

$$MN+2\vartheta\xi\eta'^2\sin^2\theta RS=0, \tag{21}$$

where

$$\left.\begin{aligned}
M=&\gamma_1\gamma_1'\eta^3\eta'^2\left\{12\zeta\eta\eta'+\phi(3+\eta')\right\}-12\gamma_1\eta^3(3\zeta\eta+\phi)\\
&-12\gamma_1'\left\{3\zeta\eta(\eta^3+4\eta'^3)+\phi\right\}+36(12\zeta\eta+\phi),\\
N=&\gamma_2\gamma_2'\psi\eta\eta'-\gamma_2\xi\eta(\vartheta\eta'\cos^2\theta+\psi)-\gamma_2'\xi(\vartheta\eta\eta'\cos^2\theta+\psi)\\
&+\xi^2(\vartheta\eta'\cos^2\theta+\psi),\\
R=&-2\gamma_1\eta^3\left\{3\zeta\eta\eta'+\phi(1+2\eta)\right\}-\gamma_1'\eta^2\left\{6\zeta\eta\eta'(1+5\eta')\right.\\
&\left.+\phi(1+2\eta)(2+\eta')\right\}+6\left\{12\zeta\eta\eta'+\phi(1+2\eta)\right\},\\
S=&\gamma_2\gamma_2'\eta\eta'-\gamma_2\xi\eta-\gamma_2'\xi+\xi^2,
\end{aligned}\right\} \tag{22}$$

$$\gamma_1=\frac{m_1h^3p^2}{E_1I_1},\quad \gamma_1'=\frac{m_1'h^3p^2}{E_1I_1},\quad \gamma_2=\frac{m_2h^3p^2}{E_1I_1},\quad \gamma_2'=\frac{m_2'h^3p^2}{E_1I_1},\quad \xi=\frac{a_1h^2}{I_1},\quad \zeta=\frac{E_2I_2}{E_1I_1},$$

$$\vartheta=\frac{E_3a_3}{E_1a_1},\quad \eta=\frac{l_1}{h},\quad \eta'=\frac{l_1'}{h},\quad \phi=\frac{l_2}{h},\quad \psi=\frac{l_3}{h}. \tag{23}$$

Equation (21) gives four vibrational frequencies, two of them corresponding to motions of horizontal type and the others to those of vertical type.

If we assume, as is usually done, that the inertia masses of the movements in the vertical sense are the same as those in the horizontal sense, we may put $\gamma_1=\gamma_2\equiv\gamma\,(m_1=m_2\equiv m)$, $\gamma_1'=\gamma_2'\equiv\gamma'\,(m_1'=m_2'\equiv m')$.

With a view to ascertaining the quantitative nature of the seismic properties of the roof members, we selected certain cases with numerical conditions.

(i) First, let $\gamma=\gamma'$, $\xi(=a_1h^2/I_1)=8000$, $\vartheta(=E_3a_3/E_1a_1)=1$, $\phi(=l_2/h)=1$ be given. The numerical calculation of the vibrational frequencies for various inclinations θ of the roof members for two cases of ζ $(=E_2I_2/E_1I_1)$, i.e. $\zeta=0$, 1 are shown in Table I and plotted in Fig. 2.

Table I. The values of $p\sqrt{\frac{mh^3}{E_1I_1}}$ for various θ.

θ	45°	47°	50°	52°	60°	70°
$E_2I_2/E_1I_1=0$	43·95	—	8·459	6·205	3·13	2·257
	108·3	—	86·7	85·7	80·6	73·5
	∞	—	208	190·4	178·5	206·7
	∞	—	269	250	235·4	248
$E_2I_2/E_1I_1=1$	43·95	19·3	8·833	—	3·562	2·398
	108·3	90·6	87·2	—	80·56	73·50
	∞	—	230	—	182·2	197·6
	∞	—	291	—	240·8	242

θ	80°	85°	86°	88°	89°	90°
$E_2I_2/E_1I_1=0$	1·52	—	1·327	—	1·247	1·224
	67·8	—	65·0	—	63·6	63·3
	242	—	254	—	301	∞
	312	—	482	—	959	∞
$E_2I_2/E_1I_1=1$	1·780	1·564	—	1·666	—	2·208
	67·8	65·4	—	64·1	—	63·25
	217	244	—	284	—	∞
	311	429	—	680	—	∞

It will be seen that since there are four freedoms of vibration, there are four frequencies for every inclination of the roof members.

Since curves *I, I′* represent mainly vibrations of horizontal type and curves *II, II′* those of vertical type, frequencies of horizontal type are always fairly less than those of vertical type. Curves *I, II* are replot-

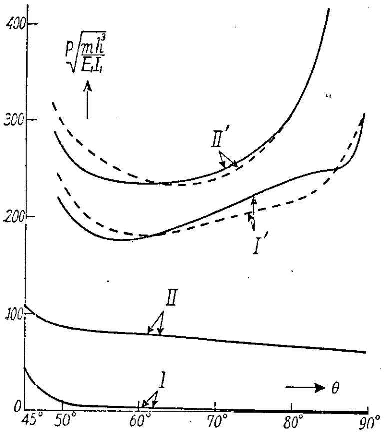

Fig. 2. Vibration frequencies of the case $\xi(=a_1h^2/I_1)=8000$, $\vartheta(=E_3a_3/E_1a_1)=1$, $\phi(=l_2/h)=1$. Full line: $\zeta(=E_2I_2/E_1I_1)=0$. Broken line: $\zeta(=E_2I_2/E_2I_1)=1$.

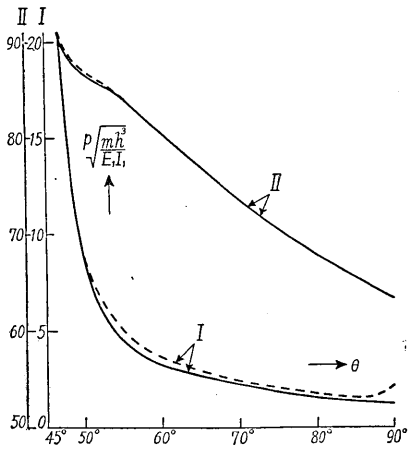

Fig. 3. Vibration frequencies of the case $\xi(=a_1h^2/I_1)=8000$, $\vartheta=(E_3a_3/E_1a_1)=1$, $\phi(=l_2/h)=1$. Full line: $\zeta(=E_2I_2/E_1I_1)=0$. Broken line: $\zeta(=E_2I_2/E_1I_1)=1$.

ted in Fig. 3 with different magnifying scales of ordinates. From Fig. 3, it will be seen that, beyond a certain inclination, namely, $\theta<80°$ in the present case, the change in E_2I_2/E_1I_1 is scarcely effective. It may be added that the vibrational frequencies of higher orders, namely, *I′*, *II′*, tend to be infinitely great at such angles as $\theta=0°$ and $90°$, which results from the condition that the masses are concentrated at the panel points.

(ii) We shall next assume that $\xi(=a_1h^2/I_1)=8000$, $\phi(=l_2/h)=1$, $\theta=60°$. The results of calculation for the vibrational frequencies for *I* (the lowest mode) for different ratios of $\vartheta(=E_3a_3/E_1a_1)$ are shown in

Table II. The values of $p\sqrt{\dfrac{mh^3}{E_1I_1}}$ for various $\vartheta(=E_3a_3/E_1a_1)$.

$\vartheta(=E_3a_3/E_1a_1)$	0	0·001	0·005	0·01	0·02	0·05	0·1	0·5	1·0	∞
$E_2I_2/E_1I_1=0$	1·688	—	2·676	2·858	2·981	3·071	3·101	3·129	3·131	3·133
$E_2I_2/E_1I_1=1$	2·519	2·749	—	3·288	—	3·494	—	3·550	3·564	3·564

Table II and plotted in Fig. 4. It will be seen that for such inclination of the roof as $\theta=60°$, a very slight stiffness of the roof member relative to that of the column, say, $\vartheta(=E_3a_3/E_1a_1)=0{\cdot}03$, is just as effective as in the case of $\vartheta(=E_3a_3/E_1a_1)=\infty$. Besides, in such a condition, stiffness of the beam does not contribute much to the stiffness of the entire structure.

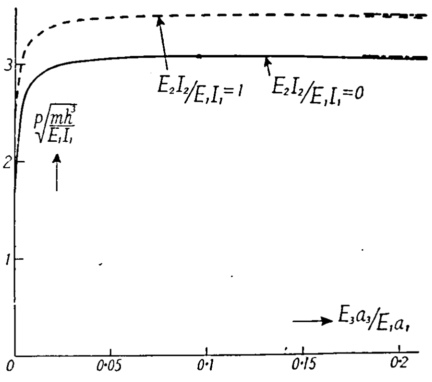

Fig. 4. Vibration frequencies for the lowest mode of the case $\xi(=a_1h^2/I_1)=8000$, $\phi(=l_2/h)=1$, $\theta=60°$.

At all events, it has been ascertained that the roof members in a saw-tooth roof structure is much more effective in resisting the seismic vibration of a structure, than the horizontal members of a rectangular framed structure.

In conclusion I wish to record my indebtedness to Professor Sezawa for many valuable suggestions in regard to this problem. I wish also to express my hearty thanks to Mr. Watanabe, for assistance in the numerical calculations.

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33. 鋸 歯 狀 小 屋 の 耐 震 性

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鋸歯狀小屋は採光上最も有効な構造として工場建築に盛んに使用されて居るが，此の構造の耐震的效果に關する理論的研究は今迄誰によつても成されて居らぬやうである．

種々の意味から，この問題を一度理論的に研究しておく必要があると考へて手を附けた．

この論文では，計算を簡單にする爲に張間が無限にあり，質量は各屋根材と柱の結合點に集中してゐるものとした．叉屋根材と柱は支持の狀態で，水平な梁がある場合と水平な梁がない場合の二通りに就いて計算を行つた．

計算の結果，屋根材の長さの方向の剛度の低い間はこの剛度を少しく增すと架構の振動數が著しく高くなり，屋根材の剛度が柱の長さの方向の剛度の 0·03 倍位になると，それ以上 この剛度を增しても架構の振動數は一定以上高くはならぬ，叉屋根材と柱の間の傾きが 80° 位以下になると梁の屈曲剛度は殆んど役に立たないことがわかつた．

要するに鋸歯狀小屋組が陸屋根，山形屋根等の小屋組よりも耐震的に效果のあることが理論的に確められた譯である．

尙，此の研究は妹澤先生の御指導によつて成されたもので，玆に深く感謝の意を表する．
