

2. *On the Problem of Instabilities of Higher Orders in a Seismometer. II.*

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1. *Introduction.*

In our previous paper,¹⁾ various vibrational frequencies at which a seismometer becomes unstable was ascertained both mathematically and experimentally. It was shown that if the damping of a seismometer is great, it is not likely that instability of any order can occur unless the vibration amplitude of the ground is extremely large. In the same paper, the problem was restricted to that condition in which a seismometer becomes unstable, and was not concerned with the vibrational state, whether in the stable or the unstable condition. Since the nature of the instability in any case can generally be ascertained when the whole range of the stability condition has been investigated, our present problem will also be extended to such a range, although the order of instability particularly in experiments is for such part as is near 1/2.

2. *Mathematical formulae.*

The mathematical solution for instability in the case of approximately sinusoidal ground motion was obtained in the previous paper. The damping factor σ for one complete cycle may be represented by the equation

$$\sigma^2 - 2\sigma e^{-2\pi \frac{\zeta}{p}} \cosh 2\pi\mu - e^{-4\pi \frac{\zeta}{p}} = 0, \quad (1)$$

where

$$\left| \cos \pi \omega_1 \cos \pi \omega_2 - \frac{\omega_1^2 + \omega_2^2}{2\omega_1 \omega_2} \sin \pi \omega_1 \sin \pi \omega_2 \right| = \cosh 2\pi\mu. \quad (2)$$

In solving equation (1), the damping factor is of the type

1) K. KANAI and K. SEZAWA, *Bull. Earthq. Res. Inst.*, 18 (1940), 483~496.

$$\sigma = e^{-2\pi \frac{\kappa}{p} \pm 2\pi \mu}, \quad (3)$$

so that the damping coefficient of the seismometer in the case of ground movements at right angles to the direction of the pendulum oscillation, is

$$\kappa' = \kappa - \rho p, \quad (4)$$

the coefficient therefore depending on the frequency of the ground movements. The critical condition that $\kappa' = 0$, is shown in Fig. 1.

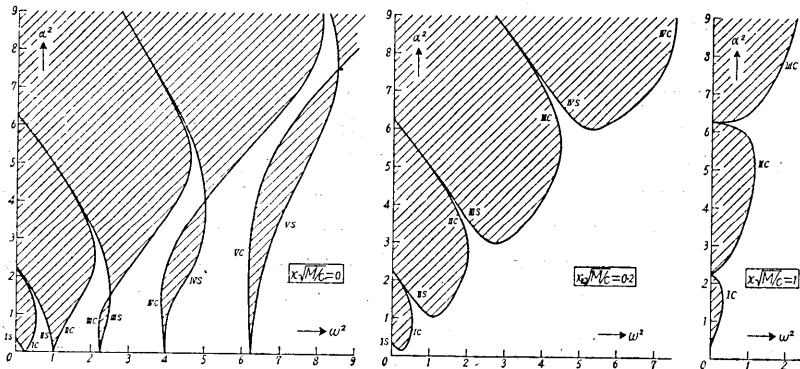


Fig. 1. Curves showing criticals of stability.

Let M , κ , c , L be the mass, logarithmic decrement in the quiescent state, vibrational stiffness ($c\gamma$ being the restitutive force), and the equivalent pendulum length, respectively. If

$$\frac{8A}{\pi^2 L} \ll 1 \quad (\alpha^2 \ll \omega^2), \quad \kappa^2 \frac{M}{c} \ll 1 \quad (\epsilon^2 \ll \omega^2), \quad (5)$$

the damping coefficient κ' in the vibratory state is approximately

$$\left. \begin{aligned} \kappa' &= \kappa - \frac{32\sqrt{2} A}{\pi^2 L T_0} && \text{for } 1/2 \text{ order stability,} \\ \kappa' &= \kappa - \frac{4}{\pi} \sqrt{2\left(1 - \frac{1}{\pi}\right)} \frac{A}{L T_0} && \text{for } 2/2 \text{ order stability,} \\ \kappa' &= \kappa - \frac{16\sqrt{17} A}{27\pi^2 L T_0} && \text{for } 3/2 \text{ order stability,} \end{aligned} \right\} \quad (6)$$

and so on.

When, especially, damping in the quiescent state is critical, namely, $\kappa \sqrt{M/c} = 1$, the value of κ' near the $1/2$ order instability becomes

$$\left. \begin{aligned} \kappa' < 0 & \text{ for } A/L > 1.233, \\ 0 < \kappa' < \kappa & \text{ for } 0.4394 < A/L < 1.233, \\ \kappa' = \kappa + ic & \text{ for } A/L < 0.4394, \end{aligned} \right\} (7)$$

the numerical values of κ' being shown in Fig. 2.

3. *Experimental investigation, and its result compared with that of the mathematical calculation.*

The vibration table shown in the previous paper was also used in the present experiment. The vibration period T_0 was always kept at that of the $1/2$ order instability. Since there were four models, the period in question was also of four kinds, the amplitude of the table, A , being also kept constant in each model. The logarithmic decrement in the quiescent state, however, varied over a wide range. The conditions of the model and the table in each case are shown in Table I. Although κ_1 shown in the same table represents the damping coefficient of the pendulum without the vibration table, for such a value of κ_1 the pendulum on the vibration table is in a state of critical stability, that is, state $\kappa' = 0$.

Table I.

Case	T_0 (sec)	M (kg)	L (cm)	A (cm)	κ_1 (sec ⁻¹)
A	0.187	1.41	10.7	0.16	0.37
B	0.208	1.12	7.75	0.17	0.48
C	0.188	15.05	11.0	0.185	0.41
D	0.188	15.05	11.0	0.35	0.78

If, in experiments with table vibration, the condition of the seismometer is stable, the free oscillation of that pendulum decays with time, but if, on the contrary, the condition is unstable, the amplitude of the same oscillation augments, the feature of the latter oscillation being however quite regular with an envelop of the form of a diverging exponential curve. Photographic records of two cases of B, namely, case B1 and case B2, are shown, as examples, in Figs. 3, 4. The table movements corresponding to these two cases are shown in Figs. 5, 6. When the table is not moving, the oscillation of the seismometer is quite stable for both cases as shown in Figs. 7, 8.

The vibration experiments were conducted for a wide range in the

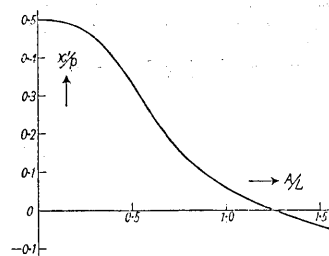


Fig. 2. The relation between κ'/p and A/L for $\kappa\sqrt{M}/c=1$.

value of κ in each case, the results being shown in Fig. 9. The condition of the points plotted on the positive side of the ordinate is stable, whereas that of the points plotted on the negative side is unstable.

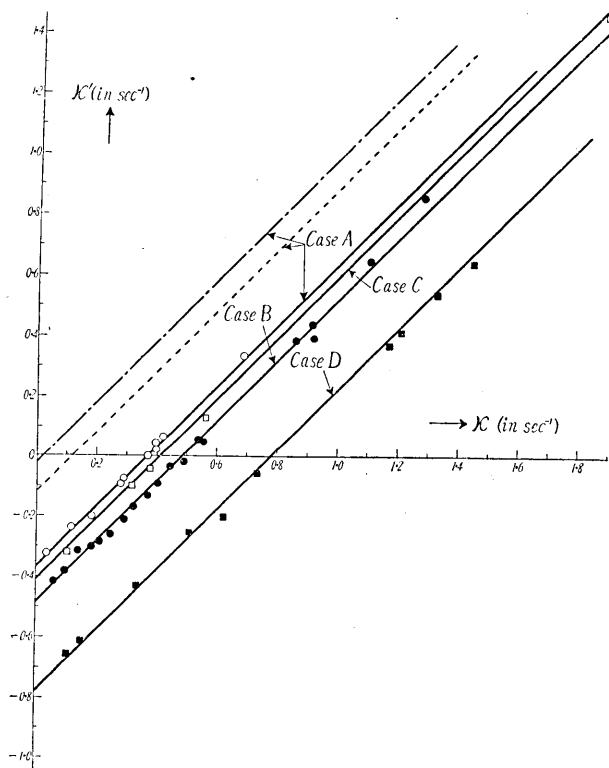


Fig. 9. Variation of κ' with κ .

On the other hand, it is also possible to get mathematically the relation between κ and κ' , using the expression (6), the results of which are shown by a series of straight lines. The broken line and the chain line in the same figure correspond to the 2/2 order and the 3/2 order instabilities in case A. It will be seen from this figure that the mathematical and the experimental results agree fairly well. Although no experiment was made for the 2/2 order and the 3/2 order instabilities, we feel confident that a similar result could be obtained if the investigation were extended to such cases.

Equation (7) shows that the seismometer cannot be unstable unless the vibration amplitude of the ground is abnormally large. At all events, in view of the results in (6), (7), and also in Fig. 9, it is now possible to conclude that a seismometer of Ewing-type or a similar pendulum could be made an oscillatory stable one, provided a certain viscous damp-



Fig. 3. B 1. $\kappa' = 0.393$.

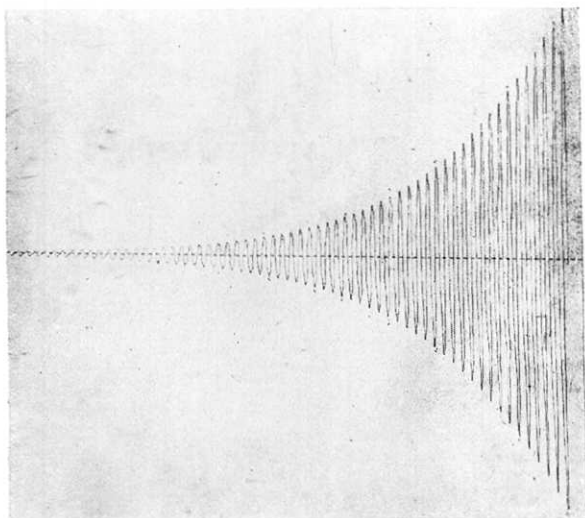


Fig. 4. Case B 2. $\kappa' = 0.381$.



Fig. 5. Case B 1.



Fig. 6. Case B 2.

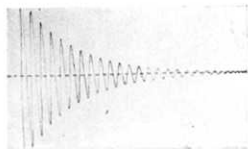


Fig. 7. Case B 1.
 $\kappa = 0.919$.

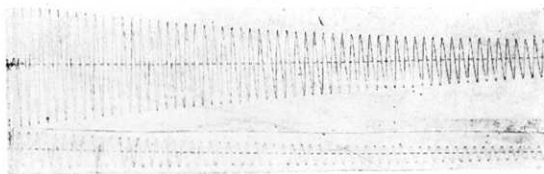


Fig. 8. B 2. $\kappa = 0.0976$.

ing were added to that pendulum, although its sensitivity would change with difference in the vibration amplitude of the ground at right angles to the direction of the pendulum oscillation.

4. *Concluding remarks.*

The damping coefficient of a seismometer when there are ground movements at right angles to the direction of the pendulum oscillation, was investigated both for stable and unstable conditions. The results of our mathematical investigation agreed well with the experimental. It was concluded that although the instability of a seismometer can be prevented by merely changing its damper, it is impossible to keep invariably the same sensitivity if there were such ground movements as just mentioned.

In conclusion, we wish to express our thanks to Mr. Kodaira, who assisted us greatly in our work. We also wish to express our thanks to the officials of the Division of Scientific Research, in the Ministry of Education, for financial aid (Funds for Scientific Research) granted us for a series of our investigations, of which this study is a part.

2. 地震計に於ける高次の不安定について

地震研究所 { 金 井 清
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地震計に於ける高次の不安定の問題の研究の續きとして、振子の振動方向に直角の向きに土地の振動がある場合の振子の振動減衰係数を安定の場合及不安定の場合につき研究した。數理的研究の結果は實驗的研究の結果とよく一致した。結論として地震計の不安定はその粘性的制振器を強くする事によつていくらでも免れる事が出来るけれども、土地のかくの如き振動がある場合にその感度を常に一定に保つのは不可能である事がわかつたのである。
