19. *Dynamical Absorption of the Energy of Rayleigh-waves and Love-waves by Weak Surface Layers.*

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1. *A short review of the last investigation on the vibration damping of seismic waves by a weak layer.*

In our last investigation,[1] by assuming the existence of vibrators of relatively simple type resting on a semi-infinite elastic body, it was shown that a surface soil layer could serve as a dynamic damper to seismic surface waves, provided that the layer is of fairly large thickness and of small elasticities, with certain viscosities. The attenuation coefficient of the boundary waves in the soil layer of given viscosity that rests on a semi-infinite subjacent elastic medium without viscosity, is much greater than the coefficient of Rayleigh-waves transmitted through a semi-infinite body having the same elasticities as those of the subjacent medium and having the same viscosities as those of the soil layer.

Let M be the mass of the layer per unit surface area, c the modulus of elasticity of the layer, and p the frequency of the waves. When the surface layer acts as a dynamic damper, the attenuation coefficient usually increases with increase in Mp^2/c, tending to infinity at $Mp^2/c=1$, while the velocity of transmission diminishes with increase in Mp^2/c. In any case, the attenuation coefficient is always proportional to the coefficient of viscosity and the square of the vibrational frequency of the waves. There is no wave for $Mp^2/c>1$, the reason for which is that, if $Mp^2/c>1$, the amplitude of the possible waves tends to infinity with increasing depth in the lower medium, the total energy of the waves integrated through depth being then infinite.

As a matter of fact, the assumption of the existence of simple vibrators resting on a semi-infinite body is merely to serve as a working hypothesis. The condition that no wave exists for $Mp^2/c>1$, results from the same assumption. We are now in a position to discuss the

1) K. SEZAWA and K. KANAI, "The Action of Soil Layers and of the Ocean as Dynamic Dampers to Seismic Surface Waves, with Notes on a few Previous Papers," *Bull. Earthq. Res. Inst.*, 18 (1940), 150–168.

case in which the soil layer is a continuous visco-elastic medium, which is likely to fit in with the real conditions.

The cases discussed in the present paper are the transmission of Love-waves and that of Rayleigh-waves in a stratified body. As last mentioned, the surface layer in the present condition is a continuous and visco-elastic medium. The lower layer is also assumed to be a continuous and visco-elastic medium, but for ascertaining the damping quality of the visco-elastic surface layer, we shall specially assume, in our numerical calculations, that either the upper or lower layer is non-viscid.

2. *Expression for the attenuation coefficient of Rayleigh-waves transmitted through a stratified visco-elastic body.*

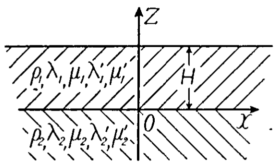

Fig. 1.

Let the densities, elastic constants, and viscous constants in the upper layer and lower layer be $\rho_1, \lambda_1, \mu_1, \lambda_1', \mu_1'$; $\rho_2, \lambda_2, \mu_2, \lambda_2', \mu_2'$, respectively. The dilatations and rotations in the two layers are then

$$\left.\begin{aligned}\Delta_1&=A_1e^{i(pt-fx+r_1z)}+B_1e^{i(pt-fx-r_1z)},\\ \varpi_1&=C_1e^{i(pt-fx+s_1z)}+D_1e^{i(pt-fx-s_1z)},\end{aligned}\right\}\tag{1}$$

$$\Delta_2=A_2e^{i(pt-fx)+r_2z},\quad \varpi_2=C_2e^{i(pt-fx)+s_2z},\tag{2}$$

where

$$\left.\begin{aligned}&r_1^2=h_1^2-f^2,\quad s_1^2=k_1^2-f^2,\quad r_2^2=f^2-h_2^2,\quad s_2^2=f^2-k_2^2,\\ &h_1^2=\frac{\rho_1p^2}{(\lambda_1+2\mu_1)+ip(\lambda_1'+2\mu_1')},\quad k_1^2=\frac{\rho_1p^2}{\mu_1+ip\mu_1'},\\ &h_2^2=\frac{\rho_2p^2}{(\lambda_2+2\mu_2)+ip(\lambda_2'+2\mu_2')},\quad k_2^2=\frac{\rho_2p^2}{\mu_2+ip\mu_2'}.\end{aligned}\right\}\tag{3}$$

The components of displacements can be found from the expressions for $\Delta_1, \varpi_1, \Delta_2, \varpi_2$, last given, in the same way as in the case of non-viscid media. Substituting the solutions thus obtained in the boundary conditions at $z=0$ and $z=H$, where H is the thickness of the layer, we get the relation

$$\begin{aligned}&\frac{4r_1s_1}{f^2}\left(2-\frac{k_1^2}{f^2}\right)\eta-\frac{r_1s_1}{f^2}\left\{4\vartheta+\left(2-\frac{k_1^2}{f^2}\right)^2\zeta\right\}\cos r_1H\cos s_1H\\ &\quad+\frac{r_1}{f}\varphi\left\{\frac{4r_2s_1^2}{f^3}+\frac{s_2}{f}\left(2-\frac{k_1^2}{f^2}\right)^2\right\}\cos r_1H\sin s_1H\\ &\quad+\frac{s_1}{f}\varphi\left\{\frac{4s_2r_1^2}{f^3}+\frac{r_2}{f}\left(2-\frac{k_1^2}{f^2}\right)^2\right\}\sin r_1H\cos s_1H\\ &\quad+\left\{\frac{4r_1^2s_1^2}{f^4}\zeta+\left(2-\frac{k_1^2}{f^2}\right)^2\vartheta\right\}\sin r_1H\sin s_1H=0,\end{aligned}\tag{4}$$

where

$$\left.\begin{aligned}
&\varphi=\frac{(\mu_1+ip\mu_1')k_2^2k_1^2}{(\mu_2+ip\mu_2')f^4},\ \zeta=\frac{4r_2s_2}{f^2}\left(\frac{\mu_1+ip\mu_1'}{\mu_2+ip\mu_2'}-1\right)^2-\alpha^2,\\
&\eta=\frac{2r_2s_2}{f^2}\left(\frac{\mu_1+ip\mu_1'}{\mu_2+ip\mu_2'}-1\right)\beta-\alpha\gamma,\ \vartheta=\frac{r_2s_2}{f^2}\beta^2-\gamma^2,\\
&\alpha=\frac{2(\mu_1+ip\mu_1')}{\mu_2+ip\mu_2'}-2+\frac{k_2^2}{f^2},\ \beta=\frac{\mu_1+ip\mu_1'}{\mu_2+ip\mu_2'}\left(2-\frac{k_1^2}{f^2}\right)-2,\\
&\gamma=\frac{\mu_1+ip\mu_1'}{\mu_2+ip\mu_2'}\left(2-\frac{k_1^2}{f^2}\right)-2+\frac{k_2^2}{f^2}.
\end{aligned}\right\}\tag{5}$$

For obtaining the dispersion equation and the expression for the attenuation coefficient, we write

$$f=f_1-if_2,\tag{6}$$

$2\pi/f_1$ and f_2 being the wave length and the attenuation coefficient for the epicentral distance, respectively. For solving the problem, we shall assume that

$$\left.\begin{aligned}
&p\mu_1'\ll\mu_1,\quad p\mu_2'\ll\mu_2,\quad f_2\ll f_1,\\
&p(\lambda_1'+2\mu_1')\ll(\lambda_1+2\mu_1),\quad p(\lambda_2'+2\mu_2')\ll(\lambda_2+2\mu_2),
\end{aligned}\right\}\tag{7}$$

which condition would exist almost in any practical case, when it is possible to neglect any value of f_2/f_1 that is higher than its second order quantity. Putting (6) in (4), we get then

$$\begin{aligned}
&4\eta_1(2-\phi_1)\sqrt{(\phi_1-1)(\psi_1-1)}-\sqrt{(\phi_1-1)(\psi_1-1)}\{4\vartheta_1+\zeta_1(2-\phi_1)^2\}\cos R_1\cos S_1\\
&\quad+\varphi_1\sqrt{\psi_1-1}\{4(\phi_1-1)\sqrt{1-\psi_2}+(2-\phi_1)^2\sqrt{1-\phi_2}\}\cos R_1\sin S_1\\
&\quad+\varphi_1\sqrt{\phi_1-1}\{4(\psi_1-1)\sqrt{1-\phi_2}+(2-\phi_1)^2\sqrt{1-\psi_2}\}\sin R_1\cos S_1\\
&\quad+\{4\zeta_1(\phi_1-1)(\psi_1-1)+\vartheta_1(2-\phi_1)^2\}\sin R_1\sin S_1=0,
\end{aligned}\tag{8}$$

$$\begin{aligned}
&2\eta_1(2-\phi_1)\{\psi_1(\phi_1-1)(\delta-\lambda_{01})+\phi_1(\psi_1-1)(\delta-\mu_{01})\}\\
&\quad+4(\phi_1-1)(\psi_1-1)\{\eta_1\phi_1(\mu_{01}-\delta)+\eta_2(2-\phi_1)\}\\
&\quad+\{4\vartheta_1+\zeta_1(2-\phi_1)^2\}[(\phi_1-1)(\psi_1-1)(R_2\sin R_1\cos S_1\\
&\qquad+S_2\cos R_1\sin S_1)-\frac{1}{2}\cos R_1\cos S_1\{\psi_1(\phi_1-1)(\delta-\lambda_{01})\\
&\qquad+\phi_1(\psi_1-1)(\delta-\mu_{01})\}]\\
&\quad-(\phi_1-1)(\psi_1-1)\cos R_1\cos S_1\{4\vartheta_2+2\phi_1\zeta_1(2-\phi_1)(\mu_{01}-\delta)+\zeta_2(2-\phi_1)^2\}\\
&\quad+\{4(\phi_1-1)\sqrt{1-\psi_2}+(2-\phi_1^2)\sqrt{1-\phi_2}\}\sqrt{\phi_1-1}\Big[\Big\{\frac{1}{2}\varphi_1\psi_1(\delta-\lambda_{01})\\
&\qquad+\varphi_2(\psi_1-1)\Big\}\cos R_1\sin S_1+\varphi_1(\psi_1-1)(S_2\cos R_1\cos S_1-R_2\sin R_1\sin S_1)\Big]\\
&\quad+\varphi_1(\psi_1-1)\sqrt{\phi_1-1}\cos R_1\sin S_1\Big\{\frac{2\psi_2(\phi_1-1)}{\sqrt{1-\psi_2}}(\lambda_{02}-\delta)+4\phi_1\sqrt{1-\psi_2}(\delta-\mu_{01})
\end{aligned}$$

$$+\frac{\phi_2(2-\phi_1)^2}{2\sqrt{1-\phi_2}}(\mu_{02}-\delta)-2\phi_1(2-\phi_1)\sqrt{1-\phi_2}(\delta-\mu_{01})\}$$

$$+\sqrt{\psi_1-1}\{4(\psi_1-1)\sqrt{1-\phi_2}+(2-\phi_1)^2\sqrt{1-\psi_2}\}\Big[\Big\{\frac{1}{2}\varphi_1\phi_1(\delta-\lambda_{01})$$

$$+\varphi_2(\phi_1-1)\Big\}\sin R_1\cos S_1+\varphi_1(\phi_1-1)(R_2\cos R_1\cos S_1-S_2\sin R_1\sin S_1)\Big]$$

$$+\varphi_1(\phi_1-1)\sqrt{\psi_1-1}\sin R_1\cos S_1\Big\{\frac{2\phi_2(\psi_1-1)}{\sqrt{1-\phi_2}}(\mu_{02}-\delta)+4\psi_1\sqrt{1-\phi_2}(\delta-\lambda_{01})$$

$$+\frac{\psi_2(2-\phi_1)^2}{2\sqrt{1-\psi_2}}(\lambda_{02}-\delta)-2\phi_1(2-\phi_1)\sqrt{1-\psi_2}(\delta-\mu_{01})\Big\}$$

$$+\sqrt{(\phi_1-1)(\psi_1-1)}\{4\zeta_1(\phi_1-1)(\psi_1-1)+\vartheta_1(2-\phi_1)^2\}(R_2\cos R_1\sin S_1$$

$$+S_2\sin R_1\cos S_1)$$

$$+\sqrt{(\phi_1-1)(\psi_1-1)}\sin R_1\sin S_1[4\zeta_1\{\psi_1(\phi_1-1)(\delta-\lambda_{01})$$

$$+\phi_1(\psi_1-1)(\delta-\mu_{01})\}+4\zeta_2(\phi_1-1)(\psi_1-1)+\vartheta_2(2-\phi_1)^2$$

$$+2\vartheta_1\phi_1(2-\phi_1)(\mu_{01}-\delta)]=0, \qquad (9)$$

where

$$\alpha_1=2\mu_0-2+\phi_2, \qquad \alpha_2=2\mu_0\mu_{01}-(2\mu_0+\phi_2)\mu_{02}+\phi_2\delta,$$

$$\beta_1=\mu_0(2-\phi_1)-2, \qquad \beta_2=\mu_0\{2\mu_{01}+(\phi_1-2)\mu_{02}-\phi_1\delta\},$$

$$\gamma_1=\mu_0(2-\phi_1)-2+\phi_2, \quad \gamma_2=2\mu_0\mu_{01}+\{\mu_0(\phi_1-2)-\phi_2\}\mu_{02}+(\phi_2-\mu_0\phi_1)\delta,$$

$$\varphi_1=\mu_0\phi_1\phi_2, \qquad \varphi_2=2\mu_0\phi_1\phi_2(\delta-\mu_{02}),$$

$$\zeta_1=4(\mu_0-1)^2\sqrt{(1-\phi_2)(1-\psi_2)}-\alpha_1^2,$$

$$\zeta_2=\frac{2}{\sqrt{(1-\phi_2)(1-\psi_2)}}[(\mu_0-1)^2\{\psi_2(1-\phi_2)(\lambda_{02}-\delta)+\phi_2(1-\psi_2)(\mu_{02}-\delta)\}$$

$$+4\mu_0(\mu_0-1)(1-\phi_2)(1-\psi_2)(\mu_{01}-\mu_{02})-\alpha_1\alpha_2\sqrt{(1-\phi_2)(1-\psi_2)}],$$

$$\eta_1=2\beta_1(\mu_0-1)\sqrt{(1-\phi_2)(1-\psi_2)}-\alpha_1\gamma_1,$$

$$\eta_2=\beta_1(\mu_0-1)\Big\{\psi_2\sqrt{\frac{1-\phi_2}{1-\psi_2}}(\lambda_{02}-\delta)+\phi_2\sqrt{\frac{1-\psi_2}{1-\phi_2}}(\mu_{02}-\delta)\Big\}$$

$$+2\sqrt{(1-\phi_2)(1-\psi_2)}\{\beta_1\mu_0(\mu_{01}-\mu_{02})+\beta_2(\mu_0-1)\}-\alpha_1\gamma_2-\gamma_1\alpha_2,$$

$$\vartheta_1=\beta_1^2\sqrt{(1-\phi_2)(1-\psi_2)}-\gamma_1^2,$$

$$\vartheta_2=\beta_1^2\Big\{\frac{\psi_2}{2}\sqrt{\frac{1-\phi_2}{1-\psi_2}}(\lambda_{02}-\delta)+\frac{\phi_2}{2}\sqrt{\frac{1-\psi_2}{1-\phi_2}}(\mu_{02}-\delta)\Big\}$$

$$+2\beta_1\beta_2\sqrt{(1-\phi_2)(1-\psi_2)}-2\gamma_1\gamma_2,$$

$$R_1=f_1H\sqrt{\psi_1-1}, \qquad R_2=\frac{f_1H}{2\sqrt{\psi_1-1}}(\delta-\psi_1\lambda_{01}),$$

$$S_1=f_1H\sqrt{\phi_1-1}, \qquad S_2=\frac{f_1H}{2\sqrt{\phi_1-1}}(\delta-\phi_1\mu_{01}),$$

$$\delta=\frac{2f_2}{f_1},\ \mu_{01}=\frac{\mu_1'}{\mu_1}p,\ \mu_{02}=\frac{\mu_2'}{\mu_2}p,\ \lambda_{01}=\frac{\lambda_1'+2\mu_1'}{\lambda_1+2\mu_1}p,$$

$$\lambda_{02}=\frac{\lambda_2'+2\mu_2'}{\lambda_2+2\mu_2}p,\ \phi_1=\frac{\rho_1 p^2}{\mu_1 f_1^2},\ \phi_2=\frac{\rho_2 p^2}{\mu_2 f_1^2},\ \psi_1=\frac{\rho_1 p^2}{(\lambda_1+2\mu_1)f_1^2},$$

$$\psi_2=\frac{\rho_2 p^2}{(\lambda_2+2\mu_2)f_1^2},\ \mu_0=\frac{\mu_1}{\mu_2}. \tag{10}$$

Equation (8) gives the velocity of transmission p/f_1 $(=\sqrt{\mu_1\phi_1/\rho_1}=\sqrt{\mu_2\phi_2/\rho_2})$ and equation (9) the attenuation factor $e^{-f_2 x}$. Although equation (8) can be solved by the trial and error method, since the equation is virtually the same as that we had used for getting the velocity of dispersive Rayleigh-waves[2], no further calculation is now required, not at any rate for the condition of stratification discussed previously. The determination of $f_2(=\delta f_1/2)$ from (9) can, however, be attained by merely substituting the results of (8) in (9).

3. *Absorption of the energy of Rayleigh-waves by a weak surface layer.*

We shall take, for example, such a condition of stratification that $\rho_1=\rho_2(\equiv\rho)$, $\lambda_1=\mu_1$, $\lambda_2=\mu_2$, $\mu_2/\mu_1=20$. Then, from the results of the previous paper last given, the dispersion curves are of the type shown in Fig. 2. The orbital motion of the waves indicated by the broken line in the figure is invariably the same as that of gravitational waves, that of the waves indicated by the full line being reversed.

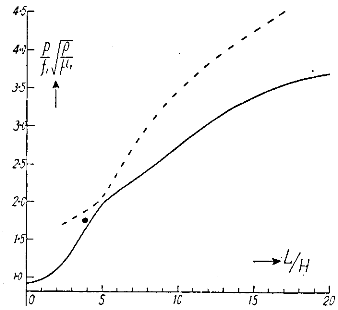

Fig. 2. Dispersion curves for Rayleigh-waves. $\rho_1=\rho_2$, $\lambda_1=\mu_1$, $\lambda_2=\mu_2$, $\mu_2/\mu_1=20$.

We shall next consider the attenuation coefficients of the waves for the condition that corresponds to Fig. 2, the relations between the coefficients of viscosity being $\lambda_1'=\mu_1'$, $\lambda_2'=\mu_2'$. For ascertaining the nature of wave absorption by the surface layer, we have calculated two conditions with respect to the viscosities; the first in which (i) $\mu_2'=0$, $\mu_1'(\equiv\mu')\neq 0$ and the second (ii) $\mu_1'=0$, $\mu_2'(\equiv\mu')\neq 0$. The results of calculation using (9), are shown in Figs. 3, 4 and in Tables I, II. Fig. 3 and the tables represent the variation in the attenuation coefficient f_2 with wave length L for

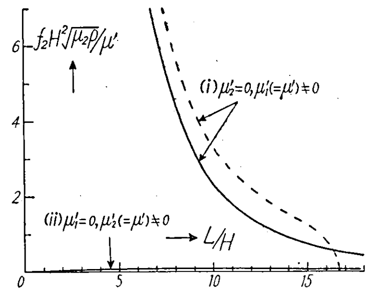

Fig. 3. Variation of attenuation coefficient f_2 with L for a given H. $\rho_1=\rho_2\equiv\rho$, $\lambda_1=\mu_1$, $\lambda_2=\mu_2$, $\lambda_1'=\mu_1'$, $\lambda_2'=\mu_2'$. $\mu_2/\mu_1=20$. Ordinates of the extended parts of the curves are shown in Table I, II.

2) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 237–244.

a given layer thickness H and Fig. 4 that with layer thickness H for a given wave length L; the full lines and broken lines in these figures correspond to the dispersion curves shown by full line and broken line, respectively, in Fig. 2.

It will be seen from Figs. 3, 4 that, for any case of dispersion curve, the attenuation coefficient of waves in media with condition (i) $\mu_2'=0$, $\mu_1'\neq 0$, greatly exceeds that of waves in media with condition (ii) $\mu_1'=0$, $\mu_2'\neq 0$, for a wide range of L or of H, indicating that the damping of the waves is pronounced if the weak surface layer is viscous.

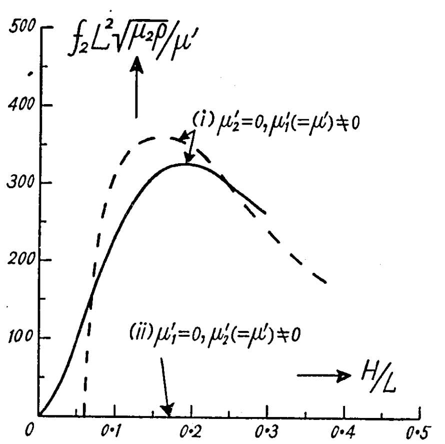

Fig. 4. Variation of attenuation coefficient f_2 with H for a given L. $\rho_1=\rho_2\equiv\rho$, $\lambda_1=\mu_1$, $\lambda_2=\mu_2$, $\mu_2/\mu_1=20$, $\lambda_1'=\mu_1'$, $\lambda_2'=\mu_2'$.

Table I. Ordinates for full lines in Fig. 3.

L/H	0	4·17	5·14	11·7	16·0	29·2
$\mu_2'=0$, $\mu_1'\neq 0$	∞	17·4	10·7	1·47	0·592	0·032
$\mu_1'=0$, $\mu_2'\neq 0$	0	0·072	0·076	0·075	0·064	0·051

Table II. Ordinates for broken lines in Fig. 3.

L/H	2·66	4·72	8·00	12·3	16·7
$\mu_2'=0$, $\mu_1'\neq 0$	24·1	15·4	5·14	2·11	0
$\mu_1'=0$, $\mu_2'\neq 0$	0·033	—	—	0·106	0·071

Furthermore, since the attenuation factor is that of the exponential function of the attenuation coefficient, the damping of the waves of case (i) $\mu_2'=0$, $\mu_1'\neq 0$, far exceeds that of the waves of case (ii) $\mu_1'=0$, $\mu_2'\neq 0$.

It is particularly interesting that the broken line of case (i) in Figs. 3 and 4 ends with zero ordinate at the abscissa, and that the second dispersion curve vanishes. Since, at this abscissa, Rayleigh-waves (of M_2 kind) change in type from surface to non-surface, the damping of the waves then depends mainly on the property of the medium in the subjacent layer (without viscosity), the reason for the attenuation coefficient tending to zero being now obvious.

We shall next consider the variation in the attenuation coefficient with changes in wave length alone or in layer thickness alone. In the first place, Fig. 3 indicates that for $L/H \to 0$, although in case (i) $\mu_2'=0$, $\mu_1'\neq 0$, the attenuation coefficinet tends to infinity; in case (ii) $\mu_1'=0$, $\mu_2'\neq 0$, the same coefficient vanishes. The reason for this is that in case (i) the whole disturbance of zero wave length in the viscous medium, viscous resistance is infinitely great, whereas in case (ii), the same disturbance being in non-viscous medium, damping resistance vanishes.

Figs. 3, 4 also show that whereas the attenuation coefficient in case (i) $\mu_2'=0$, $\mu_1'\neq 0$, is maximum for an intermediate value of H for a given wave length, in case (ii) $\mu_1'=0$, $\mu_2'\neq 0$, the coefficient is maximum for an intermediate value of L. This is a very important feature of the problem, its explanation being as follows.

A few years back[3),4)], we ascertained that the amplitude of dispersive Rayleigh-waves or Love-waves in non-viscous media assumes a maximum value for a certain thickness of the layer—a feature resembling the resonance condition of the standing vibration of that layer. In view of the general nature of standing resonance vibration, it is likely that the damping of Rayleigh-waves or Love-waves transmitted in viscous media would be maximum for such thickness of the layer as corresponds to the condition resembling resonance, the occurrence of maxima of the attenuation coefficients in Figs. 3, 4 being thus quite probable. Although it is also possible for the curves of condition (i) in Fig. 3 or for the curves of condition (ii) in Fig. 4 to have their maxima, owing to the general steepness of the curves, there is no real maximum in these curves.

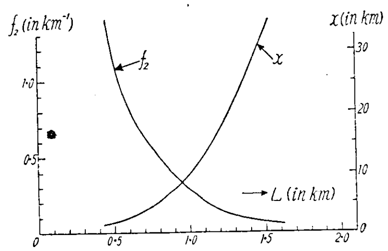

Fig. 5. The attenuation coefficient f_2 (in km^{-1}) and focal distance x in km at which wave amplitude becomes 1/10 its original value. $H=100$m, $\mu_1'=10^8$ C.G.S., $\mu_2'=0$, $\rho_1=\rho_2=2{\cdot}5$, $\sqrt{\mu_2/\rho_2}=4$ km/sec., $\mu_2/\mu_1=20$

Finally, for ascertaining the numerical value of the attenuation coefficient, we shall assume, for example, $H=100$m, $\mu_2'=0$, $\mu_1'(=\mu')=10^8$ C. G. S., $\rho_1=\rho_2=2{\cdot}5$, $\sqrt{\mu_2/\rho_2}=4$ km/sec, $\mu_2/\mu_1=20$; and calculate the attenuation coefficient f_2 in km^{-1} for various lengths, as well as the focal distance in km at which the amplitude of the waves diminishes specially to one-tenth of that at the origin (waves being transmitted in one direction), the results of which are shown in Fig. 5.

This figure shows that even if the coefficient of viscosity μ_1' were

3) K. SEZAWA and K. KANAI, "Relation between the Thickness of a Surface Layer and the Amplitudes of Love-waves", *Bull. Earthq. Res. Inst.*, **15** (1937), 579-581.

4) *Ditto*, "Relation between the Thickness of a Surface Layer and the Amplitudes of Dispersive Rayleigh-waves", *Bull. Earthq. Res. Inst.*, **15** (1937), 845-859.

as small as 10^8 C. G. S., the amplitude of the Rayleigh-waves transmitted through the layer under consideration would be one-tenth its original value for such focal distance as a certain multiple of the wave length.

4. *The expression for the attenuation coefficient of Love-waves transmitted through a stratified visco-elastic body.*

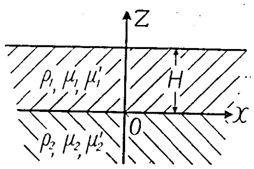

Fig. 6.

Let the densities, elastic constants, and the viscous constants in the upper and lower layers be ρ_1, μ_1, μ_1'; ρ_2, μ_2, μ_2', respectively. The displacements in these two layers are then respectively

$$\left.\begin{aligned} v_1 &= A_1 e^{i(pt-fx+s_1z)} + B_1 e^{i(pt-fx-s_1z)}, \\ v_2 &= A_2 e^{i(pt-fx)+s_2z}, \end{aligned}\right\} \tag{11}$$

where

$$s_1^2 = \frac{\rho_1 p^2}{\mu_1 + ip\mu_1'} - f^2 \quad s_2^2 = f^2 - \frac{\rho_2 p^2}{\mu_2 + ip\mu_2'}. \tag{12}$$

Using the boundary conditions at $z=0$ and $z=H$, where H is the thickness of the layer, we get

$$\tan s_1 H = \frac{(\mu_2 + ip\mu_2')s_2}{(\mu_1 + ip\mu_1')s_1}. \tag{13}$$

Writing

$$f = f_1 - if_2, \tag{14}$$

and putting

$$p\mu_1' \ll \mu_1, \quad p\mu_2' \ll \mu_2, \quad f_2 \ll f_1, \tag{15}$$

(13) can be decomposed into

$$\tan(f_1 H\sqrt{\phi_1 - 1}) - \frac{1}{\mu_0}\sqrt{\frac{1-\phi_2}{\phi_1 - 1}} = 0, \tag{16}$$

$$\begin{aligned} &\tau_1\{(2-\phi_1)(1-\phi_2) + \mu_0\phi_1(\phi_1-1)\sqrt{1-\phi_2} f_1 H \sec^2(f_1 H\sqrt{\phi_1 - 1})\} \\ &\quad + \tau_2(\phi_1 - 1)(2-\phi_2) \\ &\quad - 2\delta\{(\phi_1 - \phi_2) + \mu_0(\phi_1 - 1)\sqrt{1-\phi_2} f_1 H \sec^2(f_1 H\sqrt{\phi_1 - 1})\} = 0, \end{aligned} \tag{17}$$

where

$$\left.\begin{aligned} &\phi_1 = \frac{\rho_1 p^2}{\mu_1 f_1^2}, \quad \phi_2 = \frac{\rho_2 p^2}{\mu_2 f_1^2}, \quad \tau_1 = \frac{p\mu_1'}{\mu_1}, \quad \tau_2 = \frac{p\mu_2'}{\mu_2}, \\ &\qquad \delta = \frac{f_2}{f_1}, \quad \mu_0 = \frac{\mu_1}{\mu_2}. \end{aligned}\right\} \tag{18}$$

Equation (16) gives the velocity of transmission $p/f_1 (=\sqrt{\mu_1\phi_1/\rho_1} = \sqrt{\mu_2\phi_2/\rho_2})$ and equation (17) the attenuation factor $e^{-f_2 x}$. Since equation (16) is virtually the same as the dispersion equation of Love-waves transmitted through non-viscous media, its solution can be obtained in the usual way. Substituting the value of f_1 thus obtained in equation (17), it is possible to determine the value of f_2.

5. *Absorption of the energy of Love-waves by a weak surface layer.*

With a view to getting the numerical values of the attenuation coefficient of Love-waves transmitted along a weak surface layer, we shall take three conditions of rigidity ratio, namely, $\mu_2/\mu_1=2$, 5, 20, the ratio of densities being unity, namely $\rho_1=\rho_2(=\rho)$. The dispersion curves are shown in Fig. 7.

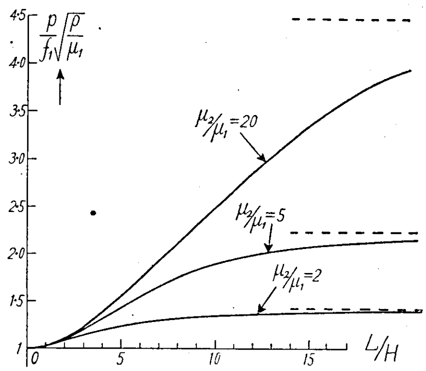

Fig. 7. Dispersion curves for Love-waves. $\rho_1=\rho_2$, $\mu_2/\mu_1=2$, 5, 20.

The attenuation coefficients for the cases now under consideration can be obtained with the aid of the formulae in the preceding section. The calculations for the variation in attenuation coefficient f_2 with change in wave length L for a layer of given thickness H are shown in Figs. 8, 9, 10, and in Table III, IV, V. For ascertaining the nature of wave absorption by the surface layer, two conditions with respect to the viscosities, namely, (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq 0$, (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq 0$, have been studied. As in the case of Rayleigh-waves, the attenuation coefficient of Love-waves in media with condition (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq 0$, is very much greater than that of the same waves in media with condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq 0$ for a wide range of L, showing that if the weak surface layer is viscous, damping of the waves is marked. As in Rayleigh-waves, although in case (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq 0$, the attenuation coefficient tends to infinity for $L/H\to 0$, in case (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq 0$, the same coefficient vanishes for $L/H\to 0$, the reason for which is also the same as that in the case of Rayleigh-

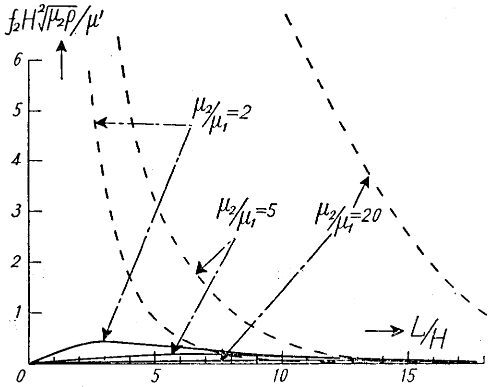

Fig. 8. Variation of attenuation coefficient f_2 with L for a given H. $\rho_1=\rho_2\equiv\rho$. Broken lines represent condition (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq 0$ and full lines condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq 0$. Ordinates of extended parts of broken lines and ordinates of full lines in magnified scale are shown in Figs. 9, 10, respectively, and in Table III, IV, V.

waves[5]. Figs. 8, 10 show too that in condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$, the attenuation coefficient becomes maximum for an intermediate value of wave length. As shown in Section 3, this arises from the fact that the nature of the waves is in resonance-like condition for such wave length. The reason for there being no maximum of ordinates for condition (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq0$, is that the inclination of the curves in this condition is very steep.

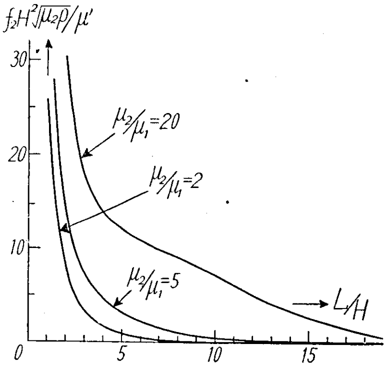

Fig. 9. Curves showing extended parts of broken lines in Fig. 8.

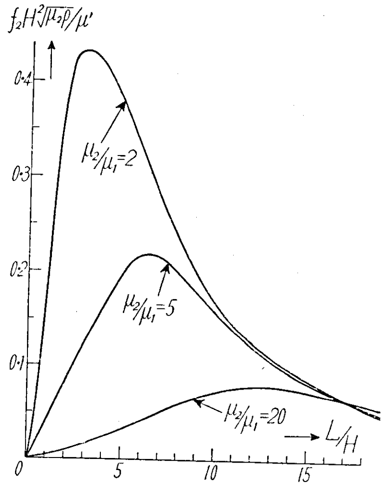

Fig. 10. Curves showing full lines in Fig. 8, on magnified scale.

Table III. Ordinates in Figs. 12, 13. $\mu_2/\mu_1=2$.

L/H	1·31	2·30	3·70	5·93	8·36	17·5
$\mu_2'=0,\ \mu_1'\neq0$	17·6	5·87	2·05	0·53	0·132	0·0084
$\mu_1'=0,\ \mu_2'\neq0$	0·270	0·419	0·419	0·331	0·219	0·061

Table IV. Ordinates in Figs. 12, 13. $\mu_2/\mu_1=5$.

L/H	1·32	2·39	4·25	6·94	8·81	10·6	12·8
$\mu_2'=0,\ \mu_1'\neq0$	28·9	10·7	4·27	1·61	0·73	0·34	0·14
$\mu_1'=0,\ \mu_2'\neq0$	0·055	0·097	0·173	0·215	0·181	0·149	0·112

Table V. Ordinates in Figs. 12, 13. $\mu_2/\mu_1=20$.

L/H	1·32	2·00	4·13	7·42	9·20	11·8	18·1	27·0
$\mu_2'=0,\ \mu_1'\neq0$	58·6	30·2	13·6	9·55	7·89	5·23	0·978	0·059
$\mu_1'=0,\ \mu_2'\neq0$	0·006	0·010	0·025	0·055	0·068	0·078	0·060	0·028

5) loc. cit. 3), 4).

The variation of attenuation coefficient f_2 with change in layer thickness H for a given wave length L is shown in Figs. 11, 12, 13.

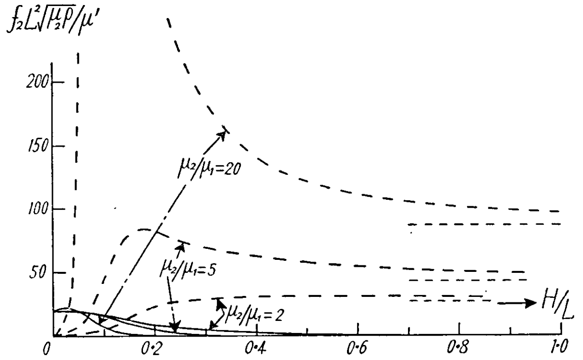

Fig. 11. Variation of attenuation coefficient f_2 with H for a given L. $\rho_1=\rho_2\equiv\rho$. Broken lines represent condition (i) $\mu_2'=0$, $\mu_1'(=\mu)\neq0$ and full lines condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$. Ordinates of extended parts of broken lines and ordinates of full lines in magnified scale are shown in Figs. 12, 13, respectively.

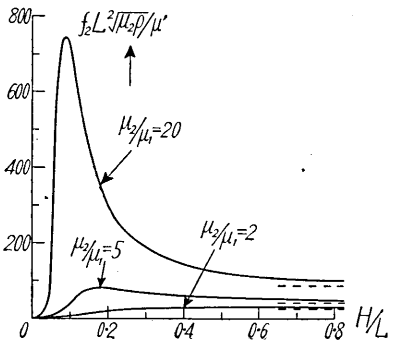

Fig. 12. Curves showing extended parts of broken lines in Fig. 11.

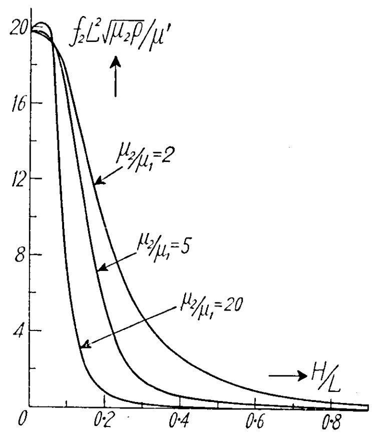

Fig. 13. Curves showing full lines in Fig. 11, on magnified scale.

The conditions with respect to the viscosities in the media are also (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq0$ and (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$. In this case, as in Rayleigh-waves, the attenuation coefficient f_2 for condition (i) is maximum for an intermediate value of layer thickness H, for the same reasons as that last given.

6. *Absorption of the energy of Love-waves by a dense layer.*

The possibility of Love-waves being transmitted along a dense layer instead of a weak layer is well known, the dispersion and the attenuation equation for which are the same as those in (16), (17). We shall now compare the damping quality of Love-waves arising from a weak layer and that arising from a dense layer.

The cases considered are such that (I) $\rho_2/\rho_1=1/2$, $\mu_2/\mu_1=1$, and (II) $\rho_2/\rho_1=1$, $\mu_2/\mu_1=2$; the dispersion curves for these two cases are shown in Fig. 14. The calculation for the variation in attenuation coefficient f_2 with change in wave length L for a given layer thickness H, and that with change in layer thickness H for a given wave length L, are shown in Figs. 15, 16, respectively.

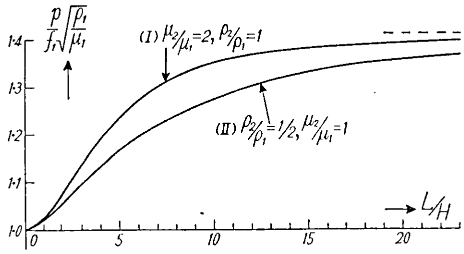

Fig. 14. Dispersion curves for Love-waves in two case: (I) $\rho_2=\rho_1$, $\mu_2/\mu_1=2$, (II) $\mu_2/\mu_1=1$ $\rho_2/\rho_1=1/2$.

In both these figures condition (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq0$, and condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$ are indicated by broken lines and full lines, respectively.

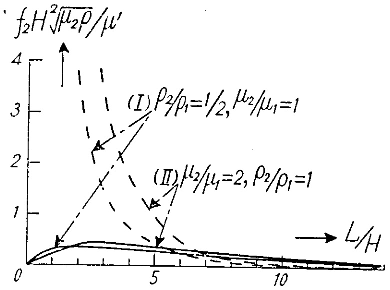

Fig. 15. Variation of attenuation coefficient f_2 with L for a given H in two cases, (I) $\mu_2/\mu_1=1$, $\rho_2/\rho_1=1/2$, (II) $\rho_2=\rho_1$ $\mu_2/\mu_1=2$. Broken lines represent condition (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq0$, and full lines condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$.

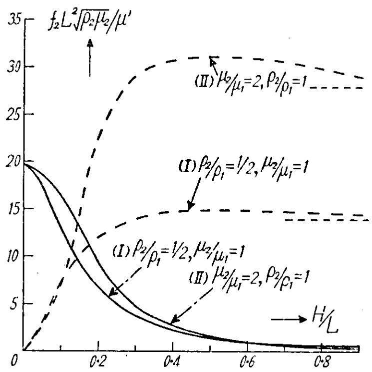

Fig. 16. Variation of attenuation coefficient f_2 with H for a given L in two cases (I) $\mu_2/\mu_1=1$ $\rho_2/\rho_1=1/2$, (II) $\rho_2=\rho_1$, $\mu_2/\mu_1=2$. Broken lines represent condition (i) $\mu_2'=0$, $\mu_1'(=\mu')\neq0$ and full lines conditions (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$.

It will be seen from these figures that every nature of the attenuation coefficient in the case of Love-waves transmitted along a dense layer is, qualitatively, quite similar to that in the case of Love-waves transmitted along a weak layer. Thus, the attenuation coefficient of Love-waves in condition (ii) $\mu_1'=0$, $\mu_2'(=\mu')\neq0$, is maximum for an intermediate value of wave length, while the same coefficient in condition (ii) $\mu_2'=0$, $\mu_1'(=\mu')\neq0$, is maximum for an intermediate value of layer thickness, which condition holds whether the surface layer is of the nature of low elasticity or that of high density.

7. *Summary and concluding remarks.*

It was ascertained mathematically that a weak or dense surface layer serves as a dynamic damper to Rayleigh-waves or Love-waves transmitted along that layer. The attenuation coefficient of any surface waves, in the case of the surface layer being viscous, is much greater than that in the case of the subjacent layer being viscous.

If the surface layer is viscous and the subjacent layer is non-viscid, the attenuation coefficient is maximum for an intermediate thickness of layer, but if the surface layer is non-viscid and the subjacent layer is viscous, the same coefficient is maximum for an intermediate wave length. This arises from the condition of our previous result that there exists a resonance-like feature in the wave transmission for an intermediate wave length or for an intermediate layer thickness.

In the case of Rayleigh-waves, particularly, the attenuation coefficient of those waves of the second kind vanishes at such wave length as that in which the waves change from surface type to non-surface type.

The present investigation shows that, with increase in epicentral distance, seismic surface waves transmitted through a region covered with fairly thick loam would be damped to a certain extent, whereas the same waves transmitted through a rocky surface would scarcely be damped even should the amplitudes of the waves on such surface be originally rather small.

Another problem of practical importance is how to avoid the seismic disturbances due to a machinary working and surface traffic. Since, as shown by many authors, these disturbances are almost surface waves, the best way to avoid them would be to cover the ground to a certain area surrounding their source with a weak soil layer of certain thickness.

In conclusion, we wish to express our thanks to Messrs. Watanabe and Kodaira, who assisted us greatly in our calculations. We are also indebted to the Officials for Scientific Research in the Ministry of Education for financial aid (Funds for Scientific Research) granted for a series of investigation, of which this study is a part.

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19. 軟地表層の存在によつて起るレーレー波及ラブ波の勢力の力學的吸收

地震研究所 妹澤克惟
金井清

この前の報告でこの研究と同じ種類の事柄を問題を簡單化したものについて述べて置いたが，玆では地表層其他の狀態を一層具體化し，且つ純粋のレーレー波及ラブ波について波動勢力の吸收作用を示した.

地表層の彈性が少ない程，その密度が大きい程，且つその粘性が大きい程波動勢力の吸收がよく行はれる．下層の粘性は少し位大きくても殆ど効果がない.

表面層に粘性があつて下層にそれがない場合には，地表層の厚さが中位のときに波動減衰作用が

極大になるし，反對に表面層に粘性がなくて下層にそれがある場合には，波長が中位のときに波動減衰作用が極大になる．之は我々が以前に述べた様に表面波には適當の波長で共振に類似の現象があるといふ事柄に關聯してあり得る現象である．

尙，レーレー波の場合には，特にその第 2 種の波について波の性質が表面性から非表面性に變る様な波長に於て，波動減幅の指數が零となる．

この研究から，相當の厚さの地表土によつて掩はれてをる地域では表面波がよく減衰する事がわかる．尤もその様な地域ではその波の振幅は最初には特に大きいけれども．

次に實際問題に大切な事柄として，工場や交通機關からの振動を防ぐには，この研究から次の様にすればよい事がわかる．即ち，振動源の周圍の相當の面積に渉り，適當の厚さの土で地表を掩ふのが最も有效であるといふ事である．