

## 20. *Thermodynamical Origin of the Earth's Core. I.*

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### 1. *Introduction.*

Since no transverse wave is transmitted through the earth's core and since the earth tide with a period of half a lunar day is unduly large, it is fairly probable that the core in question has no rigidity, it being thus fluid<sup>1)</sup>. There, however, remains the question whether or not this fluid core is in a liquid state of relatively low temperature or in a gaseous state of rather high temperature. Although Lorenz,<sup>2)</sup> using the idea of gas energy, recently showed that the state may be gaseous, owing to the numerous assumptions made, it is impossible to conclude straightway from his result that the earth's core must be gaseous.

Furthermore, even if the core be liquid, if originally it had condensed from the gas within the core, condensation would have occurred at a period when the atomic numbers of the mixture of gases were very low, as will presently be shown—an almost improbable condition in the light of the recently developed theory of the nuclear transmutation of elements. For the condition that the core shall still be liquid, condensation should occur in a perfectly gaseous sphere of very low density and of relatively high atomic numbers<sup>3)</sup>.

At all events, the question whether the earth's core is gaseous or liquid, and also whether in the case of a liquid core, condensation occurred straightway from a large gaseous sphere separated from the sun, or from a special condition of gas, is very difficult to answer. Thus, in the present paper, we shall discuss only the gaseous state of the core and compare it with other physically possible states of the same core, with consequently a less conclusive answer to the question under consideration.

### 2. *Mathematical conditions for a polytropic gas in the core.*

It has already been shown that the equation of a spherically dis-

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1) H. JEFFREYS, *The Earth*, 2nd ed. (1929), 239.

2) H. LORENZ, *Z. f. Geophys.*, 15 (1939), 371-379.

3) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, 17 (1939), 525-538, 675-684, 18 (1940), 27-40.

tributed polytropic gas that is subjected to gravitational forces, is that of Poisson<sup>4)</sup>, namely

$$\frac{d^2\phi_1}{dr^2} + \frac{2}{r} \frac{d\phi_1}{dr} + \alpha^2 \phi_1^n = 0, \quad (1)$$

with

$$\alpha^2 = 4\pi G / \{(n+1)\kappa\}^n, \quad (2)$$

where  $\phi_1$  is the gravitational potential,  $G$  the gravitational constant, namely  $6.66 \cdot 10^{-8}$  in C. G. S. units, and  $\kappa$ ,  $n$  are constants showing the relation between pressure  $P_1$  and density  $\rho_1$  in the form

$$P_1 = \kappa \rho_1^{1+1/n}. \quad (3)$$

When  $\phi_1$  is obtained from (1), the distributions of density, temperature, and pressure can be determined, using the relations

$$\rho_1 = \{\phi_1 / (n+1)\kappa\}^n, \quad T = \beta \mu \phi_1 / (n+1)\mathfrak{R}, \quad P_1 = \rho_1 \phi_1 / (n+1), \quad (4), (5), (6)$$

respectively, where  $\mathfrak{R}$  is the universal gas constant  $8.26 \cdot 10^7$ ,  $\mu$  the atomic weight in terms of the hydrogen atom, and  $\beta$  the ratio of the gas pressure to the whole pressure.

The relation between the increment of pressure and gravitational attraction at any radius  $r$ , including the part of the rock shell is

$$\frac{dP}{dr} = \rho \frac{d\phi}{dr} \equiv - \frac{\rho G M_r}{r^2}, \quad (7)$$

where  $\rho$  is the density at that radius, and  $M_r$  the spherical mass within the same radius. Integrating (7), it is possible to get the expression for the pressure with such arbitrary constants as have been determined, using the condition of continuity in pressure at every boundary and the additional condition that the total mass of the core is known.

The solutions of (1), which are generally very complex, can be obtained by a graphical method. The special cases,  $n=1$  and  $n=5$ , may however be represented by simple mathematical expressions. Since the value of  $n$  in almost any actual case lies in the range between  $n=1$  and  $n=5$ , calculation of the two cases should indicate, qualitatively, the nature of the polytropic condition of the gas. It is known that the gravitational potential satisfying (1) is expressed by

$$\phi_1 = A \sin \alpha r / r, \quad \phi_1 = A / \sqrt{1 + (\alpha' r)^2 / 3} \quad [\alpha' = \alpha A^2] \quad (8), (9)$$

for  $n=1$  and  $n=5$ , respectively.

### 3. Numerical data and calculation.

For determining the masses in the rocky shell and the core, the

<sup>4)</sup> A. S. EDDINGTON, *The Internal Constitution of the Stars* (1926), 79; E. A. MILNE, *Handbuch der Astrophysik*, 3 (1930), 183.

results due to Bullen<sup>5)</sup> and Jeffreys<sup>6)</sup> will be used. Bullen considered an outer shell of about 350 km thick and an inner shell lying between the outer shell in question and the core. Whereas in the outer shell the density distribution is almost linear with radius, in the inner shell the same distribution is nearly parabolic. Jeffreys, assuming that the outer shell is about 474 km thick, modified Bullen's result to a certain extent. If we take Jeffreys values, the empirical expressions for density distribution may be of the forms

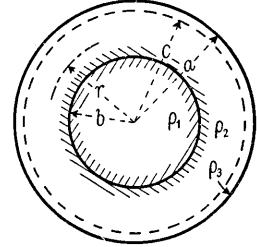


Fig. 1.

$$\rho_3 = 8.67 - 8.44 \cdot 10^{-9}r, \quad (6371 - 474 < 10^5 r \text{ cm} < 6371) \quad (10)$$

$$\rho_2 = 6.13 - 5.45 \cdot 10^{-18}r^2, \quad (6371 - 2900 < 10^5 r \text{ cm} < 6371 - 474) \quad (11)$$

from which the total mass in the whole crust is  $M = 4.070 \cdot 10^{27}$  gms. Since the total mass of the earth is  $M = 5.985 \cdot 10^{27}$  gms, the total mass of the core is  $M_c = 1.915 \cdot 10^{27}$  gms.

For convenience, we shall furthermore write,

$$a = 6371 \text{ km}, \quad b = 6371 - 2900 = 3471 \text{ km}, \quad c = 6371 - 474 = 5897 \text{ km}, \quad (12)$$

so that the part,  $b < r < a$ , is the rocky shell and that,  $r < b$ , is the core.

For ascertaining the density distribution in the core as affected by the density distribution in the rocky shell, we shall discuss four cases, namely, I.  $n=1$ ,  $\rho_2 = \rho_3 = \text{constant}$  ( $=4.485$ ); II.  $n=5$ ,  $\rho_2 = \rho_3 = \text{constant}$  ( $=4.485$ ); III.  $n=1$ ,  $\rho_2, \rho_3$  as in (10), (11); IV.  $n=5$ ,  $\rho_2, \rho_3$  as in (10), (11).

*Case I.*  $n=1$ ,  $\rho_2 = \rho_3 = \text{constant}$  ( $=4.485$ ). In this case,  $\phi_1 = A \sin \alpha r/r$ ,  $\rho_1 = \phi_1/2\kappa$ . In the rocky shell, since

$$\begin{aligned} M_r &= \frac{4\pi\rho_2}{3}(r^3 - b^3) + \int_0^b 4\pi\rho_1 r_1^2 dr_1 \\ &= \frac{4\pi\rho_2}{3}(r^3 - b^3) + \frac{2\pi A}{\kappa} \left( \frac{\sin \alpha b}{\alpha^2} - \frac{b \cos \alpha b}{\alpha} \right), \end{aligned} \quad (13)$$

$$\frac{dP_2}{dr} = \rho_2 \frac{d\phi_2}{dr} = -\frac{\rho_2 G M_r}{r^2}, \quad P_2 = -\int \frac{\rho_2 G M_r}{r^2} dr, \quad (14)$$

we get

$$P_2 = -\frac{4\pi G}{3} \rho_2^2 \left( \frac{r^2}{2} + \frac{b^3}{r} \right) + \frac{2\pi G \rho_2}{\kappa} A \left( \frac{\sin \alpha b}{\alpha^2} - \frac{b \cos \alpha b}{\alpha} \right) \frac{1}{r} + B, \quad (15)$$

where  $A, B$  are arbitrary constants. In the core, on the other hand,

$$P_1 = \frac{\rho_1 \phi_1}{n+1} = \frac{A^2 \sin^2 \alpha r}{4\kappa r^2}. \quad (16)$$

5) K. E. BULLEN, *M. N. R. A. S. Geophys. Suppl.*, 3 (1936) 395-401.

6) H. JEFFREYS, *ibid.*, 4 (1937), 50-61.

The boundary conditions are

$$P_2=0, \quad P_2=P_1 \quad (17), (18)$$

at  $r=a$ , and  $r=b$ , respectively. Substituting (15), (16) in (17), (18), and remembering that the value of  $M_r$  in (13) at  $r=b$  is

$$M_r = M_c = \frac{2\pi A}{\kappa} \left( \frac{\sin \alpha b}{\alpha^2} - \frac{b \cos \alpha b}{\alpha} \right) = 1.915 \cdot 10^{27} \text{ gms}, \quad (19)$$

it is possible to determine  $A$  and  $\alpha$  (namely  $\kappa$ ). Calculation by the method of trial and error gives

$$\left. \begin{aligned} A &= 1.025 \cdot 10^{30} \text{ C. G. S.}, \\ \alpha &= 4.95 \cdot 10^{-9} \text{ C. G. S. } (\kappa = 1.708 \cdot 10^{10} \text{ C. G. S.}) \end{aligned} \right\} \quad (20)$$

*Case II.*  $n=5$ ,  $\rho_2 = \rho_3 = \text{constant}$  ( $=4.485$ ). In this case,  $\phi_1 = A/\sqrt{1+(\alpha'r)^2/3}$ ,  $\rho_1 = (\phi_1/6\kappa)^5$ . In the rocky shell,

$$\begin{aligned} M_r &= \frac{4\pi\rho_2}{3}(r^3 - b^3) + M_c = \frac{4\pi\rho_2}{3}(r^3 - b^3) + \int_0^b 4\pi\rho_1 r_1^2 dr_1 \\ &= \frac{4\pi\rho_2}{3}(r^3 - b^3) + \frac{4\pi A^5 b^3}{3(6\kappa)^5(1 + \alpha'^2 b^2/3)^{\frac{3}{2}}}, \end{aligned} \quad (21)$$

the pressure in the same shell being obtained in the same way as in Case I. The pressure in the core is

$$P_1 = \frac{\rho_1 \phi_1}{n+1} = \frac{A^6}{6(6\kappa)^5(1 + \alpha'^2 r^2/3)^3}. \quad (22)$$

Using the boundary conditions  $P_2=0$ ,  $P_2=P_1$  at  $r=a$ ,  $r=b$ , respectively, and the condition

$$M_c = \frac{4\pi A^5 b^3}{3(6\kappa)^5(1 + \alpha'^2 b^2/3)^{\frac{3}{2}}} = 1.915 \cdot 10^{27} \text{ gms}, \quad (23)$$

it is possible to find the constants

$$\left. \begin{aligned} A &= 1.30 \cdot 10^{12} \text{ C. G. S.}, \\ \alpha &= 2.16 \cdot 10^{-33} \text{ C. G. S. } (\kappa = 1.18 \cdot 10^{11} \text{ C. G. S.}) \end{aligned} \right\} \quad (24)$$

in the same way as in Case I.

*Case III.*  $n=1$  and density distributions  $\rho_3$ ,  $\rho_2$  as in (10), (11). Gravitational potential  $\phi_1$  and pressure  $P_1$  in the core in this case are the same as in Case I. For the inner rocky shell,  $b < r < c$ ,

$$M_r = \int_b^r 4\pi\rho_2 r_1^2 dr_1 + M_c, \quad (25)$$

and for the outer rocky shell,  $c < r < a$ ,

$$M_r = \int_c^r 4\pi\rho_3 r_1^2 dr_1 + \int_b^c 4\pi\rho_2 r_1^2 dr_1 + M_c. \quad (26)$$

Substituting (10), (11) in the respective terms of these expressions, we get

$$\left. \begin{aligned} \int_b^r 4\pi\rho_2 r_1^2 dr_1 &= 4\pi \{2.043(r^3 - b^3) - 1.09 \cdot 10^{-18}(r^5 - b^5)\}, \\ \int_b^c 4\pi\rho_2 r_1^2 dr_1 &= 3.283 \cdot 10^{27}, \\ \int_c^r 4\pi\rho_3 r_1^2 dr_1 &= -4.235 \cdot 10^{27} + 3.632 \cdot 10r^3 - 2.652 \cdot 10^{-8}r^4. \end{aligned} \right\} \quad (27)$$

Using (7), it is possible to find the expressions for the pressures  $P_2$ ,  $P_3$  in the layers  $b < r < c$ ,  $c < r < a$ , respectively. The solution for the core is the same as in Case I. The boundary conditions are  $P_3 = 0$ ,  $P_3 = P_2$ ,  $P_2 = P_1$  at  $r = a$ ,  $r = c$ ,  $r = b$ , respectively. The constancy of the mass  $M_c$  in the core is the same as in the preceding sections. Using these conditions, we finally get

$$\left. \begin{aligned} A &= 1.09 \cdot 10^{20} \text{ C. G. S.}, \\ \alpha &= 4.84 \cdot 10^{-9} \text{ C. G. S. } (\kappa = 1.79 \cdot 10^{10} \text{ C. G. S.}). \end{aligned} \right\} \quad (28)$$

*Case IV.*  $n = 5$  and density distribution  $\rho_3$ ,  $\rho_2$  as in (10), (11). The gravitational potential  $\phi_1$  and pressure  $P_1$  in the core in this case are the same as in Case II. The values of  $M_r$  for the two rock shells are shown in (25)-(27). Using the conditions given in Case III, we get

$$\left. \begin{aligned} A &= 1.36 \cdot 10^{12} \text{ C. G. S.}, \\ \alpha &= 1.91 \cdot 10^{-23} \text{ C. G. S. } (\kappa = 1.24 \cdot 10^{11} \text{ C. G. S.}). \end{aligned} \right\} \quad (29)$$

Since the values of  $A$  and  $\alpha$  have been determined, it is possible, in every case, to get the distribution of  $\phi_1$  in the core, using equations (8), (9). When  $\phi_1$  is thus determined, the distributions of density  $\rho_1$  and temperature  $T$  within the core are ascertained with the aid of expressions (4), (5).

**4. Distributions of density and pressure within the core.**

The numerical calculation in the previous section gives the density distribution within the core; the results of calculation for Cases I, II, III, IV are shown in Fig. 2.

It will be seen that if  $n = 1$ , the density distribution for the case of  $n = 1$ ,  $\rho_3 = \rho_2 = 4.485$  differs somewhat from that for the case in which

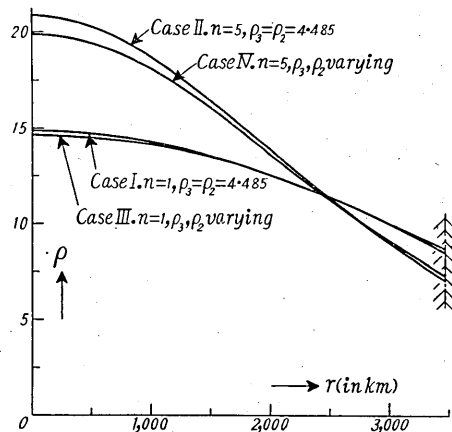


Fig. 2. Density distribution within the core.

$\rho_3, \rho_2$  varies if  $n=5$ ; the distributions under consideration also differs slightly for the two cases. Since, furthermore, the density distribution with radius of the polytropic state  $n=5$ , is more varying, it is possible to say that the state  $n=5$  represents a condition in which the moment of inertia of the earth is diminished. It is likely that the density distribution in almost any case of polytropy lies between the two systems of curves in Fig. 2, namely, the curves corresponding to Cases I, III ( $n=1$ ) and those corresponding to Cases II, IV ( $n=5$ ).

From Jeffrey's investigation, the density distribution within the earth's core is somewhat similar to curves between Cases I, III and II, IV. Besides, since the most probable polytropic condition of the gas is near  $n=2$  or 3, this assumption of the gaseous condition of the earth's core seems justifiable.

Since the distributions of density and the gravitational potential have already been found, it is now possible to get the distribution of pressure within the core by means of (6); the results of calculation for Cases I, II, III, IV are shown in Fig. 3. Thus, when  $n=1$ , the pressure at the centre of the earth is  $3.82 \cdot 10^6$  atm. and that at the outer boundary of the core is  $1.35 \cdot 10^6$  atmospheres.

If it were assumed that the formula for the velocity of compressional waves through a fluid, namely,  $c = \sqrt{\gamma P_1 / \rho_1}$ , applies to the waves through the core, the velocity in question would become 7.27 km/sec at the earth's centre and 5.6 km/sec at the outer boundary of the core. From Gutenberg's estimation, the velocity of longitudinal waves through the core is 10.9 km/sec at the earth's centre and 8.5 km/sec at the outer boundary of the core, whence it follows that the velocity of the waves in our case is only two-thirds that actually observed. The difference under consideration may arise either from the fact that in the core with a fluid under enormous pressure, the formula  $c = \sqrt{\gamma P_1 / \rho_1}$  would be greatly modified, as in the case of elasticity under initial pressure, or from the fact that the value of  $n$  of the core gas is rather less than 1. From our present knowledge, it is impossible to ascertain this qualitatively. All we can say is that, even should the core be gaseous, the velocity of the bodily waves

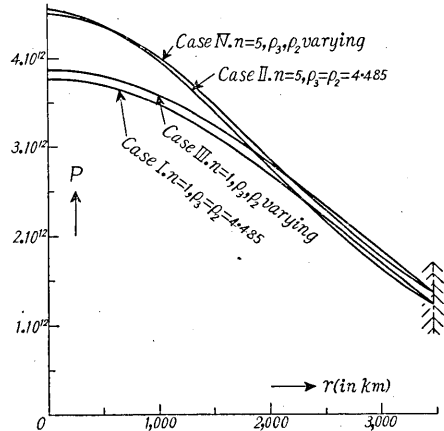


Fig. 3. Pressure distribution within the core.

through the core is of the same order as that actually observed.

**5. Distribution of temperature within the core.**

The numerical calculation shown in Section 3 also gives the distribution of temperature within the core; the results of calculation for Cases I, II, III, IV under the assumption that  $\mu=8.5$  for a working hypothesis<sup>7)</sup>, are shown in Fig. 4.

As shown in Fig. 4, the condition  $n=5$  is nearly isothermal, the temperature distribution in that condition being approximately uniform. The same distribution for  $n=1$  is, on the other hand, of that type in which the temperature tends to diminish largely with radius. The change in temperature distribution in the core with change in the density distribution in the rock shell is not very marked, provided index  $n$  be given. At all events, the temperature within the core is as much as  $10^4$  K.

The temperature distribution shown in Fig. 4 was determined on the assumption that  $\mu=8.5$  for a working hypothesis. If the value of  $\mu$  in question were

7) The assumption of  $\mu=8.5$  from certain reasoning is due to H. LORENZ, *loc. cit.* 2).

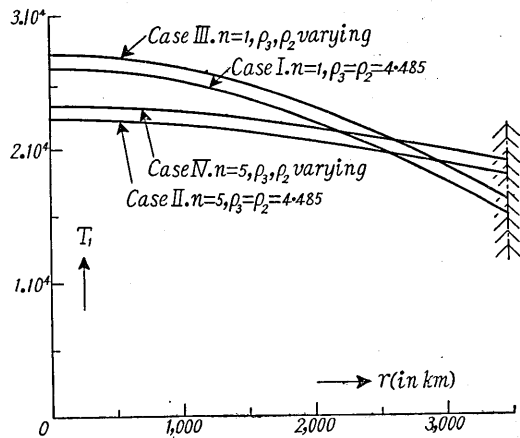


Fig. 4. Temperature distribution within the core, it being assumed that  $\mu=8.5$ .

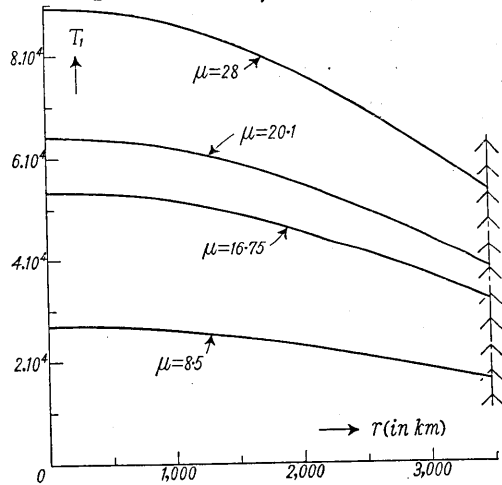


Fig. 5. Temperature distribution within the core for different values of  $\mu$  in Case III;  $n=1, \rho_3, \rho_2$  varying.

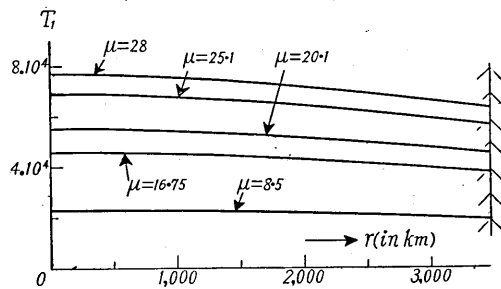


Fig. 6. Temperature distribution within the core for different values of  $\mu$  in Case IV;  $n=5, \rho_3, \rho_2$  varying.

changed, the temperature distribution would also change; the results of calculation for Cases III, IV are shown in Figs. 5, 6.

It will be seen from these figures that the value of the temperature at any point in the core is proportional to the value of  $\mu$ , whence it follows that if the gas in the core is of a high atomic number, say, that of iron or nickel, the temperature in it will be as high as  $10^5$  K, whereas if the gas is of a low atomic number, say, that of proton or helium, the temperature in question may be of the order of  $10^3$  K. This condition is of some interest in the light of the recently developed physics concerning the nuclear transmutation of elements, some remarks on which will be made in the next section.

It should be remembered that since the pressure is very high, the effect of ionisation on the present problem for the earth is negligible. Since, furthermore, Elsasser<sup>8)</sup> gave recently a special explanation of the origin of the earth's magnetic field, no restriction to temperature in the core is needed.

**6. Atomic weights of elements composing the core gas and the nuclear transformation of these elements.**

From the results of the preceding section, it was found that if the earth's core were gaseous, the atomic weight of each element composing the gas would assume any value in the restricted sense that the temperature in the core should vary in agreement with the change in atomic weight in question.

On the other hand, judging from the recent investigations of Gamow<sup>9)</sup> and Bethe<sup>10)</sup>, it is possible for thermo-nuclear transformation of relatively light elements, say, atomic number 9 or 10, to exist at such temperature as  $10^7$  K. The higher the atomic number, the more the temperature for the nuclear reaction increases. With a view to ascertaining the radiation energy liberated in the nuclear transformation of light elements, we shall

Table I.

Reaction	Erg./gr. sec.
$^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e +$	2.2
$^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + h\nu$	$1 \cdot 10^{17}$
$^3\text{H} + ^1\text{H} \rightarrow ^4\text{He} + h\nu$	$3 \cdot 10^{18}$
$^6\text{Li} + ^1\text{H} \rightarrow ^4\text{He} + ^3\text{He}$	$1 \cdot 10^{10}$
$^7\text{Li} + ^1\text{H} \rightarrow 2^4\text{He}$	$5 \cdot 10^{15}$
$^9\text{Be} + ^1\text{H} \rightarrow ^6\text{Li} + ^4\text{He}$	$3 \cdot 10^{13}$
$^{10}\text{B} + ^1\text{H} \rightarrow ^{11}\text{C} + h\nu$	$3 \cdot 10^6$
$^{11}\text{B} + ^1\text{H} \rightarrow 3^3\text{He}$	$3 \cdot 10^{11}$
$^{11}\text{C} + ^1\text{H} \rightarrow ^{12}\text{N} + h\nu$	1.1
$^{12}\text{C} + ^1\text{H} \rightarrow ^{13}\text{N} + h\nu$	180
$^{13}\text{C} + ^1\text{H} \rightarrow ^{14}\text{N} + h\nu$	$3.6 \cdot 10^4$
$^{14}\text{N} + ^1\text{H} \rightarrow ^{15}\text{O} + h\nu$	30
$^{15}\text{N} + ^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He}$	$4.5 \cdot 10^5$
$^{16}\text{O} + ^1\text{H} \rightarrow ^{17}\text{F} + h\nu$	$7 \cdot 10^{-5}$
$^{19}\text{F} + ^1\text{H} \rightarrow ^{18}\text{O} + 4\text{He}$	55
$^{22}\text{Ne} + ^1\text{H} \rightarrow ^{23}\text{Na} + h\nu$	$7 \cdot 10^{-5}$
$^{26}\text{Mg} + ^1\text{H} \rightarrow ^{27}\text{Al} + h\nu$	$8 \cdot 10^{-9}$
$^{30}\text{Si} + ^1\text{H} \rightarrow ^{31}\text{P} + h\nu$	$3 \cdot 10^{-22}$

8) W. M. ELSASSER, *Phys. Rev.*, **55** (1939), 489.

9) G. GAMOW, *Phys. Rev.*, **53** (1938), 59, **55** (1939), 718; *Nature*, **144** (1939), 575, 620.

10) H. BETHE, *Phys. Rev.*, **55** (1939), 434.



here reproduce some of the numerical results shown in Gamow's paper, as in Table I. The reactions occurring in the present sun (considered by Gamow) are shown underlined. These reactions are thus of carbon-nitrogen cycles. Since from this table, the temperature in the interior of the sun is of the order of  $10^7\text{K}$ , the temperature for the nuclear reactions of the heavier elements, say,  $\mu > 20$ , would be much higher than  $10^7\text{K}$ .

From our present investigation, the temperature of the core, even should that core remain gaseous, would not greatly exceed  $10^3\text{K}$ , from which condition it holds that no nuclear reaction exists in the present core. The elements in the core gas had formed at a certain stage of the primitive earth. And since the temperature of the core gas now under consideration is too low, even if the core were gas, the atomic numbers of the elements composing it would not greatly exceed those of the elements in the sun<sup>11</sup>.

**7. The condition of the core of it were liquid.**

In the previous section, it was shown that it is scarcely possible for the core to be a mixture of gases of high atomic numbers. Were the atomic numbers fairly high, then since the temperature is also so high as to exceed the critical of condensation that can be indicated by VAN DER WAAL's equation in thermodynamics, it would scarcely be possible for the heavy metals like nickel or steel to condense. Since, in the light of nuclear transformation, it is inconceivable that such heavy metals could condense immediately from a gas of low atomic number, condensation, if that were possible, would have occurred in the very early stage of the earth, that is, when it was a large gaseous sphere.

For the metallic core of the earth to condense from a large gaseous sphere, there is still the drawback that the gas is a mixture of elements to be condensed, the theory of nuclear transformation being disregarded in this case. Condensation in this sense will be developed in the condition shown in our previous papers<sup>12</sup>, that is, in that of the temperature in the outer part of the gaseous sphere relative to the condensing point in that part.

It follows now that for the metallic core of the earth to be condensed in accord with the theory of nuclear transformation, the primitive gaseous earth should have once passed a stage of very high temperature and then greatly cooled with its body gaseous as a whole, which condition, however, would be rather difficult of realization.

11) S. Ono had long ago an idea that the earth's core may be gaseous; *Geophys. Mag.*, 1 (1927), 97-101.

12) K. SEZAWA and K. KANAI, *loc. cit.* 3).

From the foregoing considerations, we arrive at the conclusion that, with the theory of nuclear transformation of elements in mind it is likely that the earth's core is gaseous, with a temperature of the order of from  $10^3\text{K}$  to  $10^4\text{K}$ , whereas if this theory be disregarded, liquid metallic core may be possible.

8. *General Summary and concluding remarks.*

The question whether the earth's core is gaseous or liquid, and whether, in the case of a liquid core, condensation occurred straightway from a large primitive gaseous sphere that separated from the sun or from some special gaseous conditions is very difficult to answer. As an attempt to do so, we solved a problem of polytropic gas within the core. The density distribution in the rocky shell was assumed to be uniform in one case and to be that of Jeffreys in another. The polytropic conditions used in this paper are  $n=1$  and 5, the latter being somewhat similar to the isothermal condition. The velocity of the longitudinal waves is of the order of that of the actual core waves. Since the pressure is very high, the effect of ionisation is neglected in the present problem. The temperature in the core becomes of the order of  $10^3\text{K}$ , but varies proportionally with the atomic weights of the gases in the core. According to Elsasser, such high temperature of the core is not inconsistent with the existence of the earth's magnetic field.

With the theory of nuclear transformation of elements in mind, if the earth's core be gaseous, the elements composing that gas would be of relatively low atomic numbers for the reason shown in Section 6. It is impossible for any metallic core to condense immediately from the gas within the core. If the earth's core be metallic liquid, the metals must have condensed from a large gaseous sphere, in which case, disregarding the theory of nuclear transformation, the elements corresponding to the metals must have existed in a mixture in the gaseous sphere. For the metallic core of the earth to be condensed in accord with the theory of nuclear transformation, the primitive gaseous earth should have once passed a stage of very high temperature and then greatly cooled with its body gaseous as a whole—a condition rather difficult of realization.

20. 地球核の熱力學的成因について (第 1 報)

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地球核が氣體狀であらうか液體狀であらうかといふ問題, 及びそれが液體狀であるにしても, 太陽から分離せる大きな瓦斯球から直接液化したかそれとも別の瓦斯状のものから液化したかとい

ふ問題に対する解答は非常にむづかしい。その解答に近づく一つの試みとして地球核の中がポルトロープ瓦斯であるとする場合の問題を解いて見た。この論文に於けるポルトロープの條件は  $n=1$  と  $n=5$  の場合であるが、 $n=5$  の場合の状態は  $n=\infty$  の場合の状態に可なり似て来る。核瓦斯中の縦波の速度は実際の地球核を過る縦波の速度と同じ程度である。圧力が餘り高いから電離の影響はないものとしてよい。

核の中の温度は  $10^4\text{K}$  の程度であるが、瓦斯の原子量に比例して變る。Elsasser の研究によれば、この様な高い温度でも地球の磁場の存在と抵觸しない事になる。

原子核の轉換の理論を念頭に置いて考えると、若し地球核が氣體状であるとしたときには、本文の説明の理由によつてその瓦斯を組成する元素は比較的の低い原子番號でなければならぬ事になる。若し地球核が金屬液であるとしたときには、金屬は大きな瓦斯球から液化した事になる。この様に液化した場合には、原子核轉換の理論は考へずに金屬に相當する元素が最初から瓦斯球中に混じて来る必要がある。しかし金屬の液化が原子核轉換の理論に従て行はれた場合には、初期の瓦斯状地球が一度非常に高い温度状態を通過し、次に瓦斯状の儘の一體として非常に冷却したものでなければならぬ事になるが、これはむしろ實現不可能な條件である。