9. *The Action of Soil Layers and of the Ocean as Dynamic Dampers to Seismic Surface Waves, and Notes on a few Previous Papers.*

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1. *The action of a weak layer as vibration damper to seismic waves.*

A few years back we showed a model demonstrating a dynamic damper to the seismic vibration of a structure[1]. After publishing the report of that investigation, our attention was called to the possibility of ocean water of relatively small depth serving as a damper in the same way. Since, as is well known, the viscosity of water is not great, the action of water in the direction mentioned would seem to be very feeble. On the other hand, soil layers on the earth's crust, notwithstanding their small elasticities, have relatively large viscosities, owing to which condition it is likely that they could act as vibration dampers to seismic waves.

Although bodily waves that are reflected several times on the surface layers can be damped by the viscous action of the surface soil, since the treatment of such a case involves mathematically ambiguities, we shall at present refrain from discussing it. In the case of boundary waves transmitted along the surface, however, notwithstanding certain mathematical complexities involved in its treatment, since the problem is theoretically determinate, the present investigation will be restricted to that case. Although the damping of waves transmitted along the ocean bottom is obviously small, yet for the sake of confirmation, this special case is also treated here.

For the case of the soil layers, the transmission of Love-type waves and that of Rayleigh-type waves are studied, whereas for the case of the ocean bottom, from the nature of things, it is sufficient to consider Rayleigh-type waves.

As to the nature of the surface soil, the layer is replaced by an oscillating layer of certain mass and elasticities, with some viscous damping. The lower layer, on the other hand, is assumed to be a perfectly elastic medium, but without viscosity. Although the lower

1) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **15** (1937), 23-32, 598-613.

layer may also be viscous, since, in such condition, the damping quality of the visco-elastic surface layer is indiscernible, we shall specially use the condition that the lower layer is non-viscid.

2. *The action of soil layer as a dynamic damper to Love-type boundary waves.*

In a previous paper[2], we considered the transmission of Love-type waves on the surface on which masses are distributed. It was assumed, for convenience, that the elastic force between the masses and the lower medium is infinitely great. In the present case, since the masses should act as a dynamic damper, the elastic force under consideration may be finite.

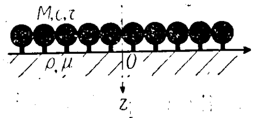

Fig. 1

Let M be the mass per unit area on the upper surface and c, τ the moduli of the elastic and damping forces that are resisting the horizontal movement of the mass M relative to the lower medium. Let also ρ, μ be the density and rigidity of the lower medium. If U, u be the horizontal displacements of the mass M and the lower medium, respectively, at right angles to the direction of transmission of the waves, then the equations of the vibratory motions of mass M and the lower medium become

$$M\frac{\partial^2 U}{\partial t^2}+\tau\frac{\partial}{\partial t}(U-u)+c(U-u)=0, \tag{1}$$

$$\rho\frac{\partial^2 u}{\partial t^2}=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{2}$$

in which

$$\mu\frac{\partial u}{\partial z}+\tau\frac{\partial}{\partial t}(U-u)+c(U-u)=0. \tag{3}$$

The solution of (2) and the form of U are

$$U=Ae^{i\alpha t-ifx}, \quad u=Be^{i\alpha t-ifx-sz}, \tag{4}$$

where

$$s^2=f^2-k^2, \quad k^2=\rho\alpha^2/\mu. \tag{5}$$

Using relations (1), (3), we have

$$\{\mu sc-M\alpha^2(\mu s+c)\}+i\alpha\tau(\mu s-M\alpha^2)=0. \tag{6}$$

It should be borne in mind that, as already said, the lower medium is non-viscid. When $c=\infty$, (6) becomes $\mu s=M\alpha^2$, that is, $\sqrt{1-k^2/f^2}-k^2M/f\rho=0$, which is virtually the same as (14) of the previous paper.[3]

To get the velocity of transmission and the attenuation coefficient of the waves, we put

2) K. Sezawa and K. Kanai, *Bull. Earthq. Res. Inst.*, 18 (1940), 1–10.

3) *loc. cit.* 2), 8.

$$f = f_1 - if_2\,; \tag{7}$$

when $2\pi/f$ is the wave length and f_2 the attenuation coefficient for the epicentral distance. For solving the problem, we shall assume that the attenuation coefficient is not very great, from which it is possible to neglect any value of f_2/f_1 that is higher than its second order quantity. Putting (7) in (6), we get then

$$\frac{f_1}{k} = \sqrt{\left(\frac{\dfrac{M\alpha^2}{c}\dfrac{c}{\mu k}}{1-\dfrac{M\alpha^2}{c}}\right)^2 + 1}, \qquad \frac{f_2}{k} = \frac{\alpha\tau}{c}\frac{M\alpha^2}{c}\frac{\dfrac{f_1^2}{k^2}-1}{\dfrac{f_1}{k}\left(1-\dfrac{M\alpha^2}{c}\right)}. \tag{8), (9}$$

The first equation gives the velocity of transmission α/f_1 and the second the damping factor $e^{-f_2 x}$ of the waves with epicentral distance. The results of calculation are shown in Figs. 2–7.

Fig. 2 represents the quantity f_1/α (reciprocal of the velocity) with abscissa $M\alpha^2/c$ for the given values of $M\alpha/\sqrt{\mu\rho}$, namely, $M\alpha/\sqrt{\mu\rho}=0{\cdot}1$, $0{\cdot}3$, and Fig. 3 the same quantity with the same abscissa for the given values of $c/\alpha\sqrt{\mu\rho}$, namely, $c/\alpha\sqrt{\mu\rho}=0{\cdot}1, 1, 10$. There is no value of f_1/α for $M\alpha^2/c>1$. Although, from the form of (8), it is likely that f_1/α can exist beyond $M\alpha^2/c=1$, since $(M\alpha^2/c)(c/\mu k)/(1-M\alpha^2/c)$ in (8) is virtually the same as s, it follows that if $M\alpha^2/c>1$, then s becomes negative, in which case the amplitude of the waves tends to infinity with increasing depth in the lower medium, the reason that there is no wave existent for $M\alpha^2/c>1$ becoming thus obvious.

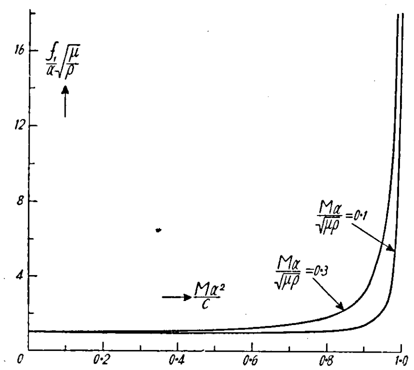

Fig. 2. Reciprocal of velocity of Love-type waves.

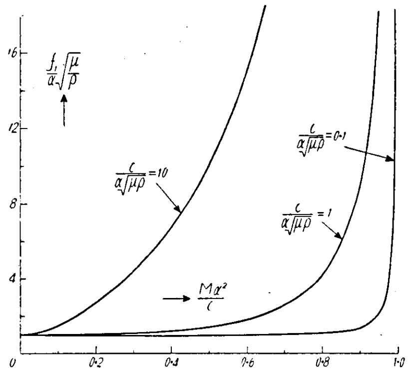

Fig. 3. Reciprocal of velocity of Love-type waves.

The curves corresponding to the reciprocals of those in Figs. 2, 3 are plotted in Figs. 4, 5. The reason for there being no wave for $M\alpha^2/c>1$ is the same as that just mentioned. From the nature of the problem, Fig. 4 represents the variation in velocity with c for a given M and Fig. 5 that with M for a given c.

It will be seen from these figures that with increase in $M\alpha^2/c$, the velocity of transmission, generally, diminishes, tending to zero at $M\alpha^2/c=1$. If we consider the effect of M and that of c on the velocity of transmission, separately, the nature of the problem becomes clearer. If M be given, the velocity increases with increase in c, whereas if c be given, the velocity diminishes with increase in M.

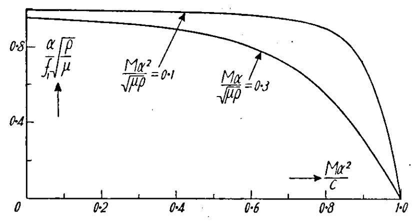

Fig. 4. Velocity of Love-type waves.

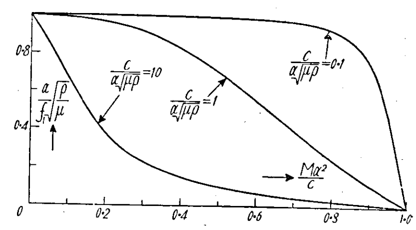

Fig. 5. Velocity of Love-type waves.

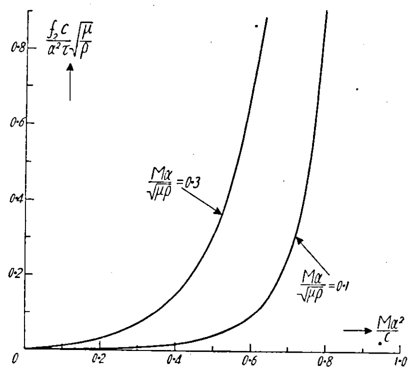

Fig. 6. The attenuation coefficient of Love-type waves.

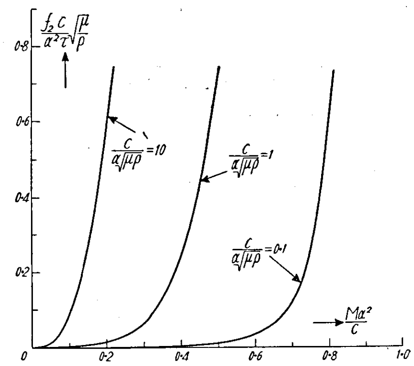

Fig. 7. The attenuation coefficient of Love-type waves.

We shall next consider the variation in the attenuation coefficients, as shown in Figs. 6, 7. Fig. 6 represents the attenuation coefficient varying with c for a given M and Fig. 7 that varying with M for a given c.

It will be seen from these figures that with increase in $M\alpha^2/c$, the coefficient generally increases, tending to infinity at $M\alpha^2/c=1$. If the effects of M and of c were considered separately, it would be found that if M is given, the coefficient diminishes with increase in c, whereas if c is given, the coefficient increases with increase in M.

From the above results, it holds that, in accord with the increase or decrease in the velocity of transmission due to change in either M or c, the attenuation coefficient invariably decreases or increases.

Since, furthermore, the ordinate in any one of Figs. 6, 7 is $f_2 c\sqrt{\mu/\rho}/\alpha^2\tau$, the attenuation coefficient is always proportional to the coefficient of viscosity τ and the square of vibrational frequency α, whence it follows that the boundary waves of Love-type can be well damped provided the density and the viscosity of the surface soil layer (and also the frequency of the waves) are as great as possible and the elasticity of that layer is, on the other hand, as small as possible.

3. *The action of a soil layer as a dynamic damper to Rayleigh-type boundary waves.*

In the previous paper[4] we considered also the transmission of Rayleigh-type waves along the surface on which masses are distributed. The restriction in the previous paper that the elastic force between the masses and the lower medium shall be infinitely great, will now be removed.

The meanings of the constants are almost similar to those in the preceding section. In the present case, $M, M', c, c', \tau, \tau'$ represent the effective masses on the surface for horizontal and vertical motions, and moduli of elastic and damping resistances to horizontal and vertical movements of such masses relative to the lower medium for Rayleigh-type motion. Let ρ, λ, μ be the elastic constants of the lower medium. If U, V, u, v are horizontal and vertical displacements of the masses M, M' and the lower medium, respectively; then the equations of the vibratory motions of the M, M' and the lower medium are expressed by

$$M\frac{\partial^2 U}{\partial t^2}+\tau\frac{\partial}{\partial t}(U-u)+c(U-u)=0, \tag{10}$$

$$M'\frac{\partial^2 V}{\partial t^2}+\tau'\frac{\partial}{\partial t}(V-v)+c'(V-v)=0, \tag{11}$$

$$\rho\frac{\partial^2\Delta}{\partial t^2}=(\lambda+2\mu)\left(\frac{\partial^2\Delta}{\partial x^2}+\frac{\partial^2\Delta}{\partial z^2}\right), \quad \rho\frac{\partial^2\varpi}{\partial t^2}=\mu\left(\frac{\partial^2\varpi}{\partial x^2}+\frac{\partial^2\varpi}{\partial z^2}\right), \qquad (12),(13)$$

with the conditions that

$$\mu\left(\frac{\partial u}{\partial z}+\frac{\partial v}{\partial x}\right)+\tau\frac{\partial}{\partial t}(U-u)+c(U-u)=0, \tag{14}$$

$$\lambda\Delta+2\mu\frac{\partial v}{\partial z}+\tau'\frac{\partial}{\partial t}(V-v)+c'(V-v)=0, \tag{15}$$

$$\Delta=\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial z}, \quad 2\varpi=\frac{\partial v_2}{\partial x}-\frac{\partial u_2}{\partial z}. \tag{16}$$

The solutions of (12), (13) and the forms of U, V are

$$\left.\begin{aligned} u_1&=\frac{if}{h^2}Ae^{i\alpha t-ifx-rz}, & v_1&=\frac{r}{h^2}Ae^{i\alpha t-ifx-rz}\\ u_2&=-\frac{s}{k^2}Be^{i\alpha t-ifx-sz}, & v_2&=\frac{if}{k^2}Be^{i\alpha t-ifx-sz}, \end{aligned}\right\} \tag{17}$$

4) loc. cit. 2).

$$U=Ce^{i\alpha t-ifx}, \quad V=De^{i\alpha t-ifx}, \tag{18}$$

where $u=u_1+u_2$, $v=v_1+v_2$ and

$$r^2=f^2-h^2, \quad s^2=f^2-k^2, \quad h^2=\frac{\rho\alpha^2}{\lambda+2\mu}, \quad k^2=\frac{\rho\alpha^2}{\mu}. \tag{19}$$

Substituting (17), (18) in relations (10), (11), (14), (15), we finally get

$$\begin{aligned} mm'R-m(1+i\varepsilon')\{m'n'rk^3+R\}-m'(1+i\varepsilon)\{mnsk^3+R\} \\ +(1+i\varepsilon)(1+i\varepsilon')[m'n'k^2\{mn(rs-f^2)+rk\}+mnsk^3+R]=0, \end{aligned} \tag{20}$$

where

$$\left.\begin{aligned} R=(f^2+s^2)^2-4f^2rs, \quad m=\frac{M\alpha^2}{c}, \quad m'=\frac{M'\alpha^2}{c'}, \\ n=\frac{c}{k\mu}, \quad n'=\frac{c'}{k\mu}, \quad \varepsilon=\frac{\alpha\tau}{c}, \quad \varepsilon'=\frac{\alpha\tau'}{c'}. \end{aligned}\right\} \tag{21}$$

It should be remembered that the lower medium is non-viscid.

To simplify the problem, we assume that

$$M=M', \quad c=c', \quad \tau=\tau',$$

that is,

$$m=m', \quad n=n', \quad \varepsilon=\varepsilon' \ (\ll 1). \tag{22}$$

In order to get the velocity of transmission and the attenuation coefficient of the waves, we write

$$f=f_1-if_2, \tag{23}$$

$2\pi/f_1$ and f_2 being the wave length and the attenuation coefficient for the epicentral distance, respectively. Then, assuming that $f_2/f_1\ll 1$, we get

$$\begin{aligned} \left(1-\frac{M\alpha^2}{c}\right)^2\left\{\left(\frac{2f_1^2}{k^2}-1\right)^2-4\frac{f_1^2}{k^2}\sqrt{\left(\frac{f_1^2}{k^2}-\frac{h^2}{k^2}\right)\left(\frac{f_1^2}{k^2}-1\right)}\right\} \\ +\frac{M\alpha^2}{c}\frac{c}{k\mu}\left(1-\frac{M\alpha^2}{c}\right)\left(\sqrt{\frac{f_1^2}{k^2}-\frac{h^2}{k^2}}+\sqrt{\frac{f_1^2}{k^2}-1}\right) \\ +\left(\frac{M\alpha^2}{c}\right)^2\left(\frac{c}{k\mu}\right)^2\left\{\sqrt{\left(\frac{f_1^2}{k^2}-\frac{h^2}{k^2}\right)\left(\frac{f_1^2}{k^2}-1\right)}-\frac{f_1^2}{k^2}\right\}=0, \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{f_2}{k}=\frac{\frac{\alpha\tau}{c}}{\Phi}\Bigg[\left(1-\frac{M\alpha^2}{c}\right)\left(1+\frac{M\alpha^2}{c}\right)\left\{\left(\frac{2f_1^2}{k^2}-1\right)^2-4\frac{f_1^2}{k^2}\sqrt{\left(\frac{f_1^2}{k^2}-\frac{h^2}{k^2}\right)\left(\frac{f_1^2}{k^2}-1\right)}\right\} \\ +\frac{M\alpha^2}{c}\frac{c}{k\mu}\left(\sqrt{\frac{f_1^2}{k^2}-\frac{h^2}{k^2}}+\sqrt{\frac{f_1^2}{k^2}-1}\right) \\ +\left(\frac{M\alpha^2}{c}\right)^2\left(\frac{c}{k\mu}\right)^2\left\{\sqrt{\left(\frac{f_1^2}{k^2}-\frac{h^2}{k^2}\right)\left(\frac{f_1^2}{k^2}-1\right)}-\frac{f_1^2}{k^2}\right\}\Bigg], \end{aligned} \tag{25}$$

where

$$\Phi=\frac{f_1}{k}\left[4\left(1-\frac{M\alpha^2}{c}\right)\left(1+\frac{M\alpha^2}{c}\right)\left\{2\left(\frac{2f_1^2}{k^2}-1\right)-2\sqrt{\left(\frac{f_1^2}{k^2}-\frac{h^2}{k^2}\right)\left(\frac{f_1^2}{k^2}-1\right)}\right.\right.$$

$$\left.-\frac{f_1^2}{k^2}\left(\sqrt{\frac{\frac{f_1^2}{k^2}-1}{\frac{f_1^2}{k^2}-\frac{h^2}{k^2}}}+\sqrt{\frac{\frac{f_1^2}{k^2}-\frac{h^2}{k^2}}{\frac{f_1^2}{k^2}-1}}\right)\right\}$$

$$+\frac{M\alpha^2}{c}\frac{c}{k\mu}\left(1-\frac{M\alpha^2}{c}\right)\left(\frac{1}{\sqrt{\frac{f_1^2}{k^2}-\frac{h^2}{k^2}}}+\frac{1}{\sqrt{\frac{f_1^2}{k^2}-1}}\right)$$

$$\left.+\left(\frac{M\alpha^2}{c}\right)^2\left(\frac{c}{k\mu}\right)^2\left\{\sqrt{\frac{\frac{f_1}{k^2}-1}{\frac{f_1^2}{k_2}-\frac{h^2}{k^2}}}+\sqrt{\frac{\frac{f_1^2}{k^2}-\frac{h^2}{k^2}}{\frac{f_1^2}{k^2}-1}}-2\right\}\right]. \qquad (26)$$

Equation (24) gives wave length $2\pi/f_1$, corresponding to frequency α, and that in (25) the attenuation coefficient f_2 of the waves for epicentral distance.

When $c=\infty$, (24) reduces to

$$-\left\{\frac{4rs}{f_1^2}-\left(1+\frac{s^2}{f_1^2}\right)^2\right\}+\frac{M\alpha^2}{\mu}\frac{k^2}{f_1^3}\left(\frac{r}{f_1}+\frac{s}{f_1}\right)-\left(\frac{M\alpha^2}{\mu}\right)^2\frac{1}{f_1^2}\left(1-\frac{rs}{f_1}\right)=0.$$

By writing $\alpha^2/\mu=k^2/\rho, f_1=f,$ ......, the above equation becomes

$$\left(\frac{k}{f}\right)^4\left(\frac{fM}{\rho}\right)^2\left(1-\frac{rs}{f^2}\right)-\left(\frac{k}{f}\right)^4\frac{fM}{\rho}\left(\frac{r}{f}+\frac{s}{f}\right)+\left\{\frac{4rs}{f^2}-\left(1+\frac{s^2}{f^2}\right)^2\right\}=0, \qquad (27)$$

which is the same as that in (7) of the previous paper.

Fig. 8 represents the quantity f_1/α (reciprocal of the velocity) with

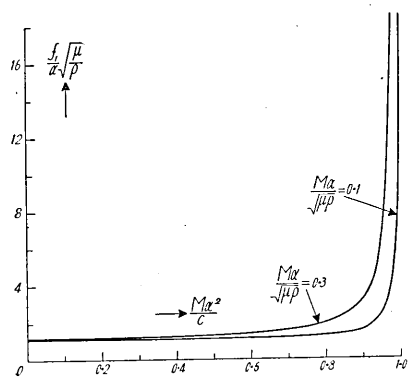

Fig. 8. Reciprocal of velocity of Rayleigh-type waves.

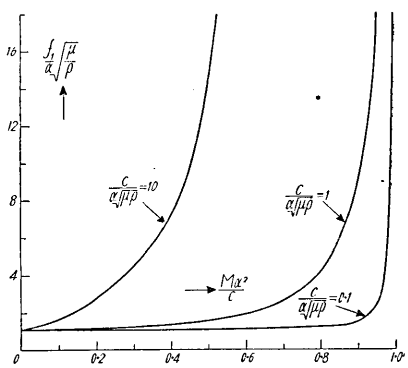

Fig. 9. Reciprocal of velocity of Rayleigh-type waves.

abscissa $M\alpha^2/c$ for given values of $M\alpha/\sqrt{\mu\rho}$, namely, $M\alpha/\sqrt{\mu\rho}=0{\cdot}1, 0{\cdot}3$, and Fig. 9 the same quantity with the same abscissa for given values of $c/\alpha\sqrt{\mu\rho}$, namely, $c/\alpha\sqrt{\mu\rho}=0{\cdot}1, 1, 10$. In the present case, too, there is no value of f_1/α for $M\alpha^2/c>1$, for reason similar to that in the case of Love-type waves.

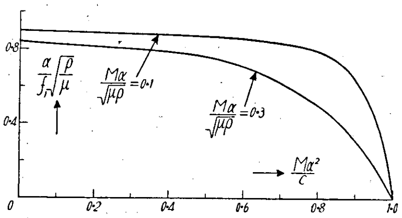

Fig. 10. Velocity of Rayleigh-type waves.

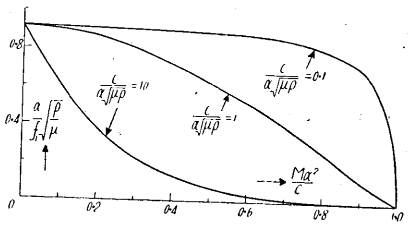

Fig. 11. Velocity of Rayleigh-type waves.

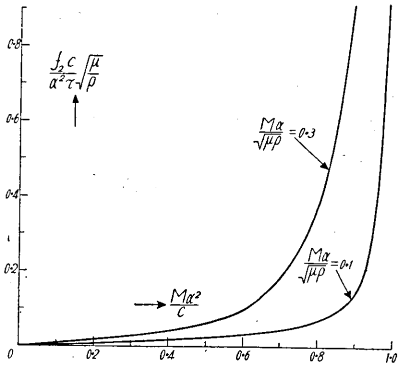

Fig. 12. The attenuation coefficient of Rayleigh-type waves.

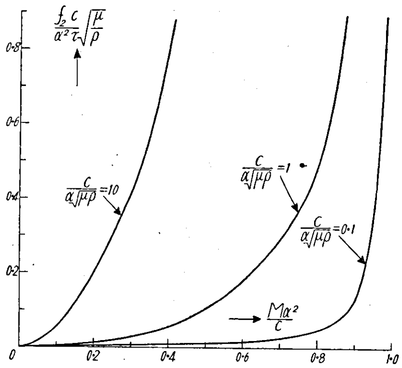

Fig. 13. The attenuation coefficient of Rayleigh-type waves.

The curves corresponding to the reciprocals of those in Figs. 8, 9 are plotted in Figs. 10, 11. The reason that no wave exists for $M\alpha^2/c>1$, is obvious from the fact that the condition is like that in the case of Love-type waves. Fig. 10 represents the variation in the velocity with c for a given M and Fig. 11 that with M for a given c.

It will be seen from these figures that, as in the case of Love-type waves, the velocity of transmission diminishes with $M\alpha^2/c$ and also with M alone, but increases with increase in c alone.

The variations in the attenuation coefficients are shown in Figs. 12, 13. Fig. 12 represents the attenuation coefficient varying with c for a given M and Fig. 13 that varying with M for a given c.

It will be seen from these figures that with increase in $M\alpha^2/c$, the coefficient generally increases, tending to infinity at $M\alpha^2/c=1$. Furthermore, when M is given, the coefficient diminishes with increase in

c and, when c is given, the same coefficient increases with increase in M.

From these conditions, it appears that in agreement with increase or decrease in the velocity of transmission (due to change in either one of M or c), the attenuation coefficient invariably decreases or increases. Figs. 12, 13 also show that the attenuation coefficient is always proportional to the coefficient of viscosity and the square of the vibrational frequency of the waves. It holds that the boundary waves of Rayleigh-type can also be damped well provided the density and the viscosity of the surface soil layer (and also the frequency of the waves) are as large as possible and the elasticity of that layer is as small as possible.

Comparing Figs. 12, 13 with Figs. 6, 7, it is possible to say that the attenuation coefficient for Love-type waves does not differ much, at any rate in its order, from that for Rayleigh-type waves.

4. *Damping effect of ocean water on the transmission of Rayleigh-type waves.*

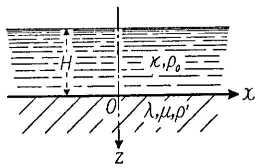

Fig. 14.

Let the depth and viscosity of the water of the ocean be H and τ, and the pressure at z be p, with density ρ. The horizontal and vertical displacements of water, u, v, must satisfy the equations

$$\left.\begin{aligned}\rho_0\frac{\partial^2 u}{\partial t^2}&=-\frac{\partial p}{\partial x}+\tau\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)\\ \rho_0\frac{\partial^2 v}{\partial t^2}&=-\frac{\partial p}{\partial z}+\tau\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g,\end{aligned}\right\} \tag{28}$$

the static and dynamic parts in ρ and those in p being separated as below

$$\rho=\rho_0+\rho_1,\quad p=p_0+p_1. \tag{29}$$

The equation of continuity is

$$\frac{D\rho}{Dt}+\rho_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}\right)=0. \tag{30}$$

The relation between pressure and density is

$$Dp/Dt=c^2D\rho/Dt, \tag{31}$$

where $c^2=\kappa/\rho_0=(dp/d\rho)_{\rho=\rho_0}$. From (29), (30), (31), we get

$$\frac{\partial p_1}{\partial t}+\rho_0 c^2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}\right)+\rho_0 g\frac{\partial v}{\partial t}=0, \tag{32}$$

in which such approximation as

$$\frac{Dp}{Dt}\approx\frac{\partial p}{\partial t}+\frac{\partial p}{\partial(z+v)}\frac{\partial(z+v)}{\partial t}=\frac{\partial p_1}{\partial t}+\frac{\partial p_0}{\partial z}\frac{\partial v}{\partial t} \tag{33}$$

is used. Eliminating p_1, ρ_0 between (28), (30), (32), we obtain

$$\left.\begin{aligned}\rho_0\frac{\partial^3 u}{\partial t^3}&=\frac{\partial}{\partial x}\left\{\rho_0 c^2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}\right)+\rho_0 g\frac{\partial v}{\partial t}\right\}+\tau\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right),\\ \rho_0\frac{\partial^3 v}{\partial t^3}&=\frac{\partial}{\partial z}\left\{\rho_0 c^2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}\right)+\rho_0 g\frac{\partial v}{\partial t}\right\}+\tau\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right).\end{aligned}\right\}\tag{34}$$

By introducing the potential ϕ, satisfying

$$\frac{\partial u}{\partial t}=-\frac{\partial\phi}{\partial x},\quad \frac{\partial v}{\partial t}=-\frac{\partial\phi}{\partial z},\tag{35}$$

the equations in (34) are equivalent to

$$\frac{\partial^2\phi}{\partial t^2}=\left(c^2+\frac{\tau}{\rho_0}\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right)+g\frac{\partial\phi}{\partial z}.\tag{36}$$

For solving (36), we put $\phi=\phi_0 e^{i\alpha t-ifx}$, when the equation then reduces to

$$\left(c^2+\frac{i\alpha\tau}{\rho_0}\right)\frac{\partial^2\phi_0}{\partial z^2}+g\frac{\partial\phi_0}{\partial z}+\phi_0\left\{\alpha^2-f^2\left(c^2+\frac{i\alpha\tau}{\rho_0}\right)\right\}=0,\tag{37}$$

the solution of (37) therefore being

$$\phi=(Ae^{sz}+Be^{-sz})e^{i\alpha t-ifx+rz},\tag{38}$$

where

$$r=\frac{g}{2\left(c^2+\dfrac{i\alpha\tau}{\rho_0}\right)},\quad s^2=\frac{g^2-4\left(c^2+\dfrac{i\alpha\tau}{\rho_0}\right)\left\{\alpha^2-f^2\left(c^2+\dfrac{i\alpha\tau}{\rho_0}\right)\right\}}{4\left(c^2+\dfrac{i\alpha\tau}{\rho_0}\right)^2}.\tag{39}$$

We shall next consider the movement of the ground. Let ρ', λ, μ be the density and the elastic constants of the ground. From the elastic relations

$$\left.\begin{aligned}&\rho'\frac{\partial^2\Delta}{\partial t^2}=(\lambda+2\mu)\left(\frac{\partial^2\Delta}{\partial x^2}+\frac{\partial^2\Delta}{\partial z^2}\right),\quad \rho'\frac{\partial^2\varpi}{\partial t^2}=\mu\left(\frac{\partial^2\varpi}{\partial x^2}+\frac{\partial^2\varpi}{\partial z^2}\right),\\ &\Delta=\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial z},\quad 2\varpi=\frac{\partial v_2}{\partial x}-\frac{\partial u_2}{\partial z},\\ &\Delta=Ce^{-r'z+i\alpha t-ifx},\quad 2\varpi=De^{-s'z+i\alpha t-ifx},\\ &r'^2=f^2-h^2,\quad s'^2=f^2-k^2,\quad h^2=\rho'\alpha^2/(\lambda+2\mu),\quad k^2=\rho'\alpha^2/\mu,\end{aligned}\right\}\tag{40}$$

we get the expressions for the displacement of the ground as follows:

$$\left.\begin{aligned}u_1&=\frac{if}{h^2}Ce^{-r'z+i\alpha t-ifx},\quad v_1=\frac{r'}{h^2}Ce^{-r'z+i\alpha t-ifx},\\ u_2&=\frac{-s'}{k^2}De^{-s'z+i\alpha t-ifx},\quad v_2=\frac{if}{k^2}De^{-s'z+i\alpha t-ifx},\end{aligned}\right\}\tag{41}$$

the horizontal and vertical displacements of the ground being u_1+u_2, v_1+v_2, respectively.

Now, the boundary conditions are

$$p=0 \tag{42}$$

at $z=-H$, and

$$v=v_1+v_2, \tag{43}$$

$$\mu\left\{\frac{\partial(u_1+u_2)}{\partial z}+\frac{\partial(v_1+v_2)}{\partial x}\right\}=\tau\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial z}+\frac{\partial v}{\partial x}\right), \tag{44}$$

$$\lambda\Delta+2\mu\frac{\partial(v_1+v_2)}{\partial z}=-\rho_0\left(\frac{\partial\phi}{\partial t}+gv\right) \tag{45}$$

at $z=0$.

Substituting (35), (38), (41) in these conditions, we get

$$\frac{k^2r'\rho_0}{\mu}\mathrm{sh}sH(g^2r^2-g^2s^2+2g\alpha^2r+\alpha^4)]$$
$$+\{(f^2+s'^2)^2-4f^2r's'\}\{(gr^2-gs^2+\alpha^2r)\mathrm{sh}sH+\alpha^2s\mathrm{ch}sH\}$$
$$+2\frac{i\alpha\tau}{\mu}f^2\{2r's'-(f^2+s'^2)\}\{(gr^2-gs^2+\alpha^2r)\mathrm{sh}sH+\alpha^2s\mathrm{ch}sH\}=0. \tag{46}$$

Now, we shall write

$$f=f_1-if_2, \tag{47}$$

and assume that the attenuation coefficient f_2 is not very large, when (46) then reduces to

$$2m\phi^3\sqrt{1-\frac{\mu}{\lambda+2\mu}\phi^2}\{b\phi^2(2a^2+b)-a^2\}\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)$$
$$+b\left\{(2-\phi^2)^2-4\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}\right\}\left\{a(3b\phi^2-2)\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)\right.$$
$$\left.+\phi bQ\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right\}=0, \tag{48}$$

$$\frac{f_2}{k}=\frac{\varepsilon}{4n\Phi}\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}\left[2nm\phi^4kH(a^2-2b)\sqrt{1-\frac{\mu}{\lambda+2\mu}\phi^2}\right.$$
$$\cdot\{\phi^2b(2a^2+b)-a^2\}\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)$$
$$+8mna^2b\phi^5Q\sqrt{1-\frac{\mu}{\lambda+2\mu}\phi^2}\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)$$
$$+nb\phi\left\{(2-\phi^2)^2-4\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}\right\}\left\{6ab\phi Q\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)\right.$$
$$+akH(3b\phi^2-2)(a^2-2b)\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)$$
$$\left.+b\phi kHQ(a^2-2b)\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)+2b\phi^2(a^2-2b)\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right\}$$

$$+4bQ\left\{2\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}-(2-\phi^2)\right\}\left\{a(2-3b\phi^2)\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)\right.$$
$$\left.\left.-b\phi Q\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right\}\right], \quad (49)$$

where

$$\Phi=\phi^3\sqrt{1-\phi^2}\{a^2-\phi^2b(2a^2+b)\}\left\{m\phi Q\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)\right.$$
$$\left.+2m\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)kH\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right\}$$
$$+2a^2m\phi^4\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)\sqrt{1-\phi^2}Q\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)$$
$$+2\phi bQ\left\{4-3\phi^2+\phi^2\frac{\mu}{\lambda+2\mu}(2\phi^2-3)\right.$$
$$\left.-2(2-\phi^2)\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}\right\}\left\{a(3b\phi^2-2)\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)\right.$$
$$\left.+b\phi Q\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right\}+b\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}\left\{(2-\phi^2)^2\right.$$
$$\left.-4\sqrt{\left(1-\frac{\mu}{\lambda+2\mu}\phi^2\right)(1-\phi^2)}\right\}$$
$$\cdot\left\{2a\phi Q\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)+a(2-3b\phi^2)kH\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right.$$
$$\left.\left.-bkH\phi Q\mathrm{sh}\left(\frac{kH}{2\phi}Q\right)-2\phi^2b\mathrm{ch}\left(\frac{kH}{2\phi}Q\right)\right\}\right], \quad (50)$$

symbols $a, b, m, \ldots.$ in (48), (49), (50) indicating that

$$\left.\begin{aligned} &a=\frac{g}{c^2k}, \quad b=\frac{\rho_0\mu}{\rho'\kappa}, \quad m=\frac{\rho_0}{\rho'}, \quad n=\frac{\mu}{\kappa}, \quad \varepsilon=\frac{\alpha\tau}{\kappa}, \\ &\phi=\frac{k}{f_1}, \quad Q=\sqrt{4+\phi^2(a^2-4b)}. \end{aligned}\right\} \quad (51)$$

Equation (48) gives the velocity of transmission α/f_1 and (49) the attenuation coefficient f_2.

As a working hypothesis, let

$\rho'/\rho_0=2{\cdot}6, \quad \sqrt{\mu/\rho'}=4$ km/sec, $\quad \lambda=\mu, \quad \tau=0{\cdot}018$ C. G. S.,

$\mu=4{\cdot}10^{11}$ C. G. S., $\quad \rho_0=1, \quad c=1430$ m/sec.

Substituting these numerical values in the equations just given, we obtained the results shown in Figs. 15, 16, 17. The cases shown in these figures are $T(=2\pi/\alpha)$ in sec = 0·1, 1, 1000, respectively. Although the case in Fig. 17 is a rather hypothetical one, it is added for confirmation.

It will be seen that the velocity of transmission generally decreases with increase in sea depth. For a certain depth, the distribution of the wave amplitude in water changes from hyperbolic to sinusoidal, the velocity of the waves in such condition being shown with a broken straight line. This velocity is nearly the same as that of compressional waves in water, namely, $c=1430$ m/sec. The velocity for $H=0$ is naturally that for Rayleigh-waves transmitted through a semi-infinite body.

Curves f_2 in Figs. 15, 16, 17 represent the attenuation coefficients. The chain line in each case gives the ordinate under such a condition that the waves change from hyperbolic to sinusoidal. It will be seen that the increment of the coefficient becomes considerable at such an abscissa that the slope of the dispersion curve becomes steepest.

The question whether the attenuation coefficient of the waves is large or not, is of some interest. The results in these figures show that f_2 is as small as 10^{-10} (km^{-1}) in the cases of $T=0{\cdot}1$ sec and $T=1$ sec, and as small as 10^{-11}(km^{-1}) even in the case of $T=1$ sec, from which it holds that the waves of period $0{\cdot}1$ sec and of period 1 sec damp with an attenuation factor of the order $exp.\,(-10^{-10}x)$, x being in km. It is now obvious that sea water does not contribute to any damping of seismic waves.

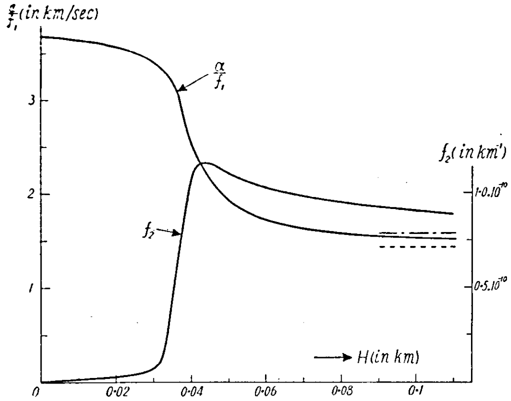

Fig. 15. Velocity and attenuation coefficient of waves in the ocean bottom; $T=0{\cdot}1$ sec.

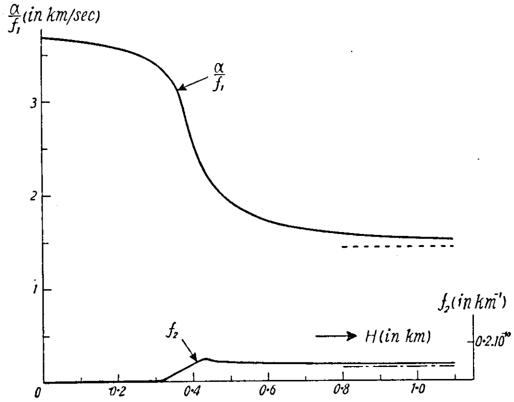

Fig. 16. Velocity and attenuation coefficient of waves in the ocean bottom; $T=1$ sec.

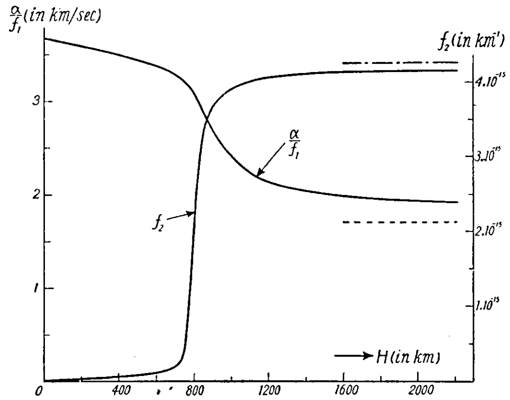

Fig. 17. Velocity and attenuation coefficient of waves in the ocean bottom; $T=1{,}000$ sec.

5. *Comparison of the effect of a soil layer, serving as a dynamic damper to the boundary waves, with the condition of surface waves with self-damping.*

With a view to ascertaining the quantitative magnitude of the damping action of a soil layer, let the thickness of that layer be $H=100$ m. As a working hypothesis, let the density, elasticities, and the viscosities of that layer be

$$\rho=2, \quad \lambda=\mu, \quad \lambda'=\mu', \quad \sqrt{\mu/\rho}=500\ \text{m/sec}, \quad \mu'=3\cdot10^8\ \text{C. G. S.}$$

Further, let the density and elasticities of the lower layer be

$$\rho=2\cdot5, \quad \lambda=\mu, \quad \lambda'=\mu'=0, \quad \sqrt{\mu/\rho}=4\ \text{km/sec},$$

the period of the waves being always $1\cdot133$ sec ($\alpha=5\cdot55\ \text{sec}^{-1}$). On the other hand, it is possible to write

$$M=\rho H, \quad c=\pi^2\mu/4H, \quad \tau=\mu'\pi^2/4H \tag{52}$$

for Love-type waves and for the horizontal motion of Rayleigh-type waves and

$$M=\rho H, \quad c=\pi^2(\lambda+2\mu)/4H, \quad \tau=(\lambda'+2\mu')\pi^2/4H \tag{53}$$

for the vertical motion of Rayleigh-type waves. By assuming, for simplicity, that the relations in (52) apply even to the vertical motion of Rayleigh-type waves, we get, in the case of Love-type waves,

$$\left.\begin{array}{l} c=1\cdot23.\ 10^6\ \text{C. G. S.}, \quad \tau=7\cdot4.\ 10^4\ \text{C. G. S.}, \quad \dfrac{M\alpha^2}{c}=0\cdot5, \\ f_1/k=1\cdot0245, \quad f_2=2\cdot22.\ 10^{-7}(\text{cm}^{-1})=2\cdot22.\ 10^{-2}(\text{km}^{-1})\,; \end{array}\right\} \tag{54}$$

and in the case of Rayleigh-type waves,

$$\left.\begin{array}{l} c=1\cdot23.\ 10^6\ \text{C. G. S.}, \quad \tau=7\cdot4.\ 10^4\ \text{C. G. S.}, \quad \dfrac{M\alpha^2}{c}=0\cdot5, \\ f_2=1\cdot08.\ 10^{-7}(\text{cm}^{-1})=1\cdot08.\ 10^{-2}(\text{km}^{-1}), \end{array}\right\} \tag{55}$$

the results in (54), (55) being obtained by means of (9) and (25), respectively.

When the viscosities of a semi-infinite body without surface layer are the same as those in the soil layer in the present problem, the attenuation coefficient of Rayleigh-waves transmitted along that semi-infinite body (in which $\lambda=\mu, \lambda'=\mu'$), is expressed by[5)]

$$f_2=\frac{f_1}{2}\frac{\mu'}{\mu}, \tag{56}$$

from which if $\rho=2\cdot5$, $\sqrt{\mu/\rho}=4$ km/sec, $\alpha=5\cdot55\ \text{sec}^{-1}$, as in the preceding cases, we obtain

$$f_2=\frac{\alpha^2/2}{0\cdot9192\sqrt{\mu/\rho}}\frac{\mu'}{\mu}=1\cdot05.\ 10^{-3}(\text{km}^{-1}). \tag{57}$$

Comparing the results for the three cases above given, namely,

5) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 12, equation (13').

$f_2 = 2 \cdot 22.\ 10^{-2}(\text{km}^{-1})$ for Love-type waves in this paper,
$f_2 = 1 \cdot 08.\ 10^{-2}(\text{km}^{-1})$ for Rayleigh-type waves in this paper,
$f_2 = 1 \cdot 05.\ 10^{-3}(\text{km}^{-1})$ for Rayleigh-waves in a semi-infinite body,

the attenuation coefficient in the cases of the soil layer serving as a dynamic damper is considerably larger than the coefficient in the case of surface waves transmitted through a visco-elastic body in the usual sense. In the present case we assumed that $H=100$ m and $\sqrt{\mu/\rho}=500$ m/sec. for the soil layer. If a greater value of H and a smaller value of $\sqrt{\mu/\rho}$ were used, the attenuation coefficient in the case of the soil layer, acting as a dynamic damper, would become much greater than the coefficient of the usual surface waves.

6. *General summary and concluding remarks.*

By mathematical investigation it was ascertained that a surface soil layer serves as a dynamic damper to seismic surface waves provided that the layer is of fairly large thickness and of small elasticities, with certain viscosities. The attenuation coefficient of the boundary waves in the soil layer with a given viscosity that rests on a semi-infinite subjacent elastic medium without viscosity, is much greater than the coefficient of Rayleigh-waves transmitted through a semi-infinite body with the same elasticities as those of the subjacent medium and with the same viscosities as those of the soil layer. Although the coefficient of the waves in the case of the surface layer and the subjacent medium, both being viscous, is naturally greater than that of the case of the surface layer alone being viscous, the increment of the coefficient in question would not be very marked.

Let M be the mass of the layer per unit surface area, c the modulus of elasticity of the layer, and α the frequency of the waves. When the surface layer acts as a dynamic damper, the attenuation coefficient generally increases with increase in $M\alpha^2/c$, tending to infinity at $M\alpha^2/c=1$. Further, when M is given, the coefficient diminishes with increase in c and, when c is given, the same coefficient increases with increase in M. On the other hand, the velocity of transmission diminishes with $M\alpha^2/c$ and also with M alone, but increases with c alone. It holds then that in accord with increase or decrease in the velocity of transmission due to change in either M or c, the attenuation coefficient invariably diminishes or increases. In any case, the attenuation coefficient is always proportional to the coefficient of viscosity and the square of the vibrational frequency of the waves.

As to the effect of the water of the sea on the seismic boundary waves, it is possible to say that it can scarcely contribute to the damping of the waves, which is also obvious from the condition that the viscosity of the water is very small compared with that of solid matter.

In the present paper, we assumed, for convenience, the existence of vibrators of relatively simple type instead of considering the actual medium in the soil layer. The treatment of that case in which the soil layer is really visco-elastic and is a continuous medium, being hopeful, a discussion of it will be attempted in the near future.

In conclusion, we wish to express our thanks to Messrs. Watanabe and Kodaira who assisted us greatly in the mathematical calculations.

We also wish to express our warmest thanks to the Officials of the Division of Scientific Research in the Department of Education for financial aid (Funds for Scientific Research) granted for a series of our investigations, of which this study is a part.

Notes added to the paper—"Damping of Periodic Visco-elastic Waves with Increase in Focal Distance, I, II," by K. SEZAWA and K. KANAI in *Bull. Earthq. Res. Inst.*, **16** (1938), 491-503; **17** (1939), 9-26.

Although in the second of these papers, we concluded that the attenuation coefficient of Rayleigh-waves as referred to increase in focal distance is always intermediate between those of the two bodily waves, and also that the damping coefficient of Rayleigh-waves as referred to time increase is always intermediate between those of the two bodily waves, further investigation has shown that the condition, particularly for the attenuation coefficient, is not exact. Since the attenuation coefficient for Rayleigh-waves is

$$f_2=\frac{pf_1}{2}\frac{\left\{\dfrac{\lambda'+2\mu'}{\lambda+2\mu}h_1^2\sqrt{\dfrac{1-k_1^2}{1-h_1^2}}+\dfrac{\mu'}{\mu}k_1^2\left(\sqrt{\dfrac{1-h_1^2}{1-k_1^2}}-2+k_1^2\right)\right\}}{\left\{h_1^2\sqrt{\dfrac{1-k_1^2}{1-h_1^2}}+k_1^2\left(\sqrt{\dfrac{1-h_1^2}{1-k_1^2}}-2+k_1^2\right)\right\}},$$

where $pf_1=p^2/V_R$ for a given p, and the coefficients for longitudinal and transverse waves are

$$\frac{pf_1}{2}\frac{\lambda'+2\mu'}{\lambda+2\mu},\quad \frac{pf_1'}{2}\frac{\mu'}{\mu},$$

respectively, where $pf_1=p^2/V_L, pf_1'=p^2/V_S$, our previous conclusion is not correct unless $V_R=V_L=V_S$. Since, however, it is plausible that the difference between $(\lambda'+2\mu')/(\lambda+2\mu)$ and μ'/μ would generally be greater than that between V_R, V_L, V_S, the conclusion is practically correct. As to the damping coefficient of Rayleigh-waves, we obtained the relation

$$p_2=\frac{p_1^2}{2}\frac{\left[\dfrac{\lambda'+2\mu'}{\lambda+2\mu}+\dfrac{\mu'}{\mu}\dfrac{k_1^2}{h_1^2}\left\{\dfrac{1-h_1^2}{1-k_1^2}-(2-k_1^2)\sqrt{\dfrac{1-h_1^2}{1-k_1^2}}\right\}\right]}{1+\dfrac{k_1^2}{h_1^2}\left\{\dfrac{1-h_1^2}{1-k_1^2}-(2-k_1^2)\sqrt{\dfrac{1-h_1^2}{1-k_1^2}}\right\}},$$

where $p_1^2=(fV_R)^2$ for a given f, and the coefficients for longitudinal and transverse waves are

$$p_1^2(\lambda'+2\mu')/2(\lambda+2\mu), \quad p_2^2\mu'/2\mu,$$

respectively, and where $p_1^2=(fV_L)^2, p_2^2=(fV_S)^2$, so that the previous conclusion is fairly approximate, though not exact. Thus, it is likely that our result shown in the first of our previous papers gives rather the real condition. Since Rayleigh-waves are in the condition of dilatational waves coupled with distortional waves, we trust we are justified in feeling that the object of our paper was fulfilled, at any rate, in the numerical calculations.

Notes added to other papers—"A kind of waves transmitted over a Semi-infin'te Solid Body of Varying Elasticity," by K. SEZAWA in *Bull. Earthq. Res. Inst.*, **9** (1931) 310; and "On Shallow Water Waves...., with Special Reference to Love-waves in Heterogeneous Media," by K. SEZAWA and K. KANAI in *Bull. Earthq. Res. Inst.*, **17** (1939), 685 694.

The first and second papers here given will be called Paper I and Paper II, respectively. Although, as far back as 1931, one of us published Paper I, our principal aim was merely to show the possible solution of that case in using hypergeometric series, the treatment being therefore fairly approximate as will be seen from the expression (35) in that paper. Since, furthermore, even if the type of the solution indicated that the amplitude of the waves at great depth tends to be infinitesimal, it was uncertain whether or not the total energy integrated through the whole depth is finite. For this reason, we gave the solution for the waves transmitted through stratified heterogeneous layers, the total energy of which waves can be finite (*Bull. Earthq. Res. Inst.*, **17** (1939), 12–25). As the solutions of that case were rather abstract, we furthermore solved the same case using confluent hypergeometric functions, the results of which were published in Paper II. As stated in Paper II, although the title of the paper concerned shallow water waves, the problem gave rise to the solution of Love-type waves by merely replacing parameters for shallow water waves by those for elastic waves. This condition, as well as the inexactness of Paper I were pointed out in our lecture in July, last year, and also in Paper II. Since, however, it is likely that our recently improved results in Paper II are not yet well understood, we shall now show the important parts concerning Love-waves that were contained in Paper II, using parameters corresponding to Love-waves in heterogeneous media, which may serve as a reply to the questions raised by some of our readers.

Let l be thickness of the surface layer and ρ', ρ, μ', μ the densities and the rigidities of the surface layer and the subjacent medium, respectively. It is assumed that, besides $\rho'=\rho$, the upper layer is uniform and the rigidity of the lower layer varies as $\mu=\mu_1 y^2/l^2$. From the equations of motion for both media, namely,

$$\frac{\partial^2\zeta'}{\partial t^2}=\frac{\mu'}{\rho}\left(\frac{\partial^2\zeta'}{\partial x^2}+\frac{\partial^2\zeta'}{\partial y^2}\right), \quad \frac{\partial^2\zeta}{\partial t^2}=\frac{\mu}{\rho}\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial}{\partial y}\left(\frac{\mu}{\rho}\frac{\partial\zeta}{\partial y}\right),$$

ζ', ζ being horizontal displacements, and the boundary conditions

$$\frac{\partial\zeta'}{\partial y}=0\,; \quad \zeta'=\zeta, \quad \mu'\frac{\partial\zeta'}{\partial y}=\mu\frac{\partial\zeta}{\partial y}$$

at $y=0$ and $y=l$, we finally obtain the relations

$$\frac{1}{\nu}\sqrt{\left(\frac{1}{4}-m^2\right)\nu-\frac{\rho_1^2}{4}}\tan\sqrt{\left(\frac{1}{4}-m^2\right)\nu-\frac{\rho_1^2}{4}}$$
$$=\frac{(1+k-\rho_1/2)W_{k,m}(\rho_1)+W_{k+1,m}(\rho_1)}{W_{k,m}(\rho_1)},$$

where $\nu=\mu_1/\mu'$, $k=0$, $\rho_1=4\pi l/L$, from which the velocity $pL/2\pi$ of the waves can be determined. The results of calculations for three cases, $\mu_1/\mu'=1, 5, 30$, are shown in Fig. 18, L, l being the wave length and the thickness of the layer, respectively.

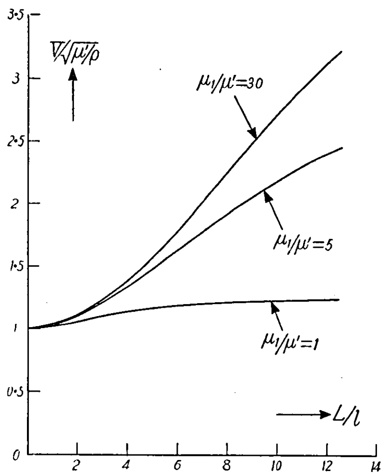

Fig. 18. Dispersion curves for Love-waves in the case of the lower layer being of varying rigidities.

It will be seen that the dispersion is quite normal and also that the velocity generally increases with increase in the ratio of rigidity of the lower layer to that of the upper layer.

Although we assumed that, for simplicity, the rigidity of the lower layer varies as $\mu=\mu_1 y^2/l^2$, the nature of the problem would not change much even should a different law in regard to variation in rigidity be assumed. When there is a surface layer, a possible existence of Love-type waves is certain, but if there is no surface layer, the possible condition for Love-waves being existent, particularly with the conception of finite energy of the waves, differs with difference in the rigidity distribution or density distribution in the medium.

9. 地表土及大洋が地震表面波に及ぼす力學的制振作用；附報，二三の前報告の補遺

地震研究所 {妹澤克惟 / 金井清}

數學的研究により，地表土の層ができるだけ厚く、その彈性ができるたけ小さく，粘性ができるだけ大きければ，地震表面波に力學的制振器として働く事がわかつた．粘性のない半無限彈性體の上に粘性のある地表土層があるとき，之に傳はる地震表面波の減衰性は，地表土の無い半無限體の全體が地表土に相當する粘性を持つ場合の地震表面波の減衰性よりも遙かに高い事がわかる．之がレーリー型の波について行はれる事は勿論であるが，ラブ型波についても同樣な性質がある．地震波の波長が短くなれば波の速度も遲くなるが，力學的制振器としての作用は一層よくなる．

大洋の水が同樣に力學的制振器として作用するかどうかをしらべて見ると，あらゆる場合に殆ど其効果の無い事がわかる．之は水の粘性が餘りにも少くて振動力の吸收ができぬからである．