10. *Viscosity Distribution within the Earth. Preliminary Notes.*

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(Read March 14, 1940.—Received March 20, 1940.)

1. *Introduction.*

Although the distribution of density and elasticity within the earth has been ascertained to a considerable extent by the labours of a number of authors, that of viscosity distribution has not yet been studied in any satisfactory manner. Since the viscosity of a solid or liquid diminishes greatly with increase in temperature but increases with increase in pressure, it is very difficult to determine viscosity conditions at depths within the earth. From the tables of Gutenberg[1] and Hyde[2] it appears that the viscosities of some materials change as 10^{-at^2} and as bp^2, where t and p are temperature and pressure respectively, the effect resulting from both these variables being therefore quite uncertain. Besides, since the pressure given in the data is very low compared with that at depth in the earth's crust, the data concerning the pressure cannot be applied to our problem.

The viscosity distribution could probably be known by comparing, if possible, the amplitudes of various phases of seismic waves at different stations, but since the amplitudes of waves recorded in seismograms are, quantitatively speaking, uncertain, it is necessary to devise some means of analysing seismological data quantitatively.

Judging from the investigations of Jeffreys[3], Bullen[4], and Birch[5], it is probable that the changes in density and elasticity within the earth are due mainly to pressure rather than to any difference in the physical conditions inherent in matter. Although the density and elasticities can be affected by the initial pressure, it is improbable that dynamic properties, such as viscosity, also increase so enormously with pressure increase. There is therefore some doubt regarding the increase in viscosity to an unlimited extent with increasing pressure. From our present ideas, the increment of viscosity tends to vanish beyond a cer-

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1) B. GUTENBERG, *Internal Constitution of the Earth*, 376.
2) J. H. HYDE, *Proc. Roy. Soc*, **97** (1920), 240-259.
3) H. JEFFREYS, *M. N. R. A. S. Geophys. Suppl.*, **4** (1937), 50-61.
4) K. E. BULLEN, *ibid.*, **3** (1936), 395-401.
5) F. BIRCH, *Bull. Amer. Seis. Soc.*, **29** (1939), 463-479.

tain pressure, so that the resultant viscosity within the earth's crust should actually decrease with depth, for which reason we shall first assume that the viscosity is the same everywhere in the shell outside the core of the earth, and next consider the condition of viscosity distribution for different layers.

2. *Determination of the paths of the waves.*

Although curves showing the paths of seismic waves within the earth have been drawn by Gutenberg(6) for determining the actual lengths of the curves necessary for estimating the attenuation coefficient of the waves, we shall reinvestigate the problem. It should be borne in mind that, in drawing the curves for the wave paths, we used Jeffreys's data in the International Seismological Summary etc.(7) and also the tables given by Dahm.8)

The distribution of density and that of the velocity of bodily waves within the earth are shown in Figs. 1, 2, respectively. The curve in Fig. 1 is reproduced from Bullen's paper, whereas the full lines and

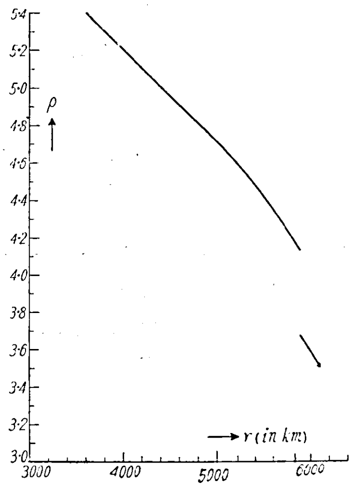

Fig. 1. Density distribution. (Reproduced from Bullen's paper)

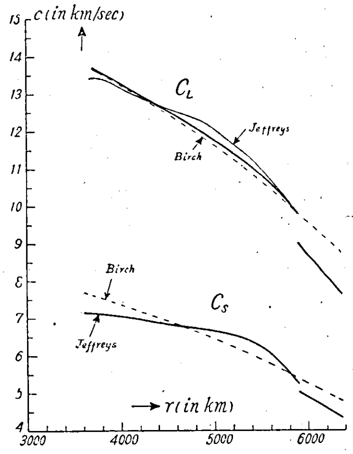

Fig. 2. Distribution of the velocities of bodily waves.

broken lines in Fig. 2 are the same as the curves obtained by Jeffreys and Birch, respectively. Since Birch's result is purely mathematical, it is not possible to avail ourselves of it much, so far as the purpose of this paper is concerned. In the present problem, as a test, we used, particularly, the mean value shown by thick lines of two different types of curves for the velocity of longitudinal waves and Jeffreys's value for

6) B. Gutenberg's earlier works.
7) H. JEFFREYS and K. E. BULLEN, *Publ. B. C. Seism. Intern.*, Fasc. 11 (1935).
8) C. G. DAHM, *Bull. Amer. Seism. Soc.*, 26 (1936), 159-171.

the velocity of transverse waves, the results of which show that Jeffreys's value naturally fits in with the various facts as revealed by actual seismological data.

As is well known, the relation between the velocity of bodily waves c and the coordinates r, θ, for the respective paths of the waves, is

$$\frac{r^2}{c^2}=p\left\{\left(\frac{dr}{d\theta}\right)^2+r^2\right\}^{\frac{1}{2}}, \tag{1}$$

where p is a constant that can be determined by

$$p=dT/d\Delta, \tag{2}$$

in which T, Δ are the time and the epicentral distance, respectively, of the travel-time curves. The use of Jeffreys's table and Dahm's, just given, enabled us to determine the value of p. From (1), the coordinate r_1 of the point nearest to the centre of the earth for every path of the waves is $r=r_1=cp$; three special points in every ray being thus determined rather readily. For any other point, the inclination of the curve and the radial distance from the centre of the earth are ascertained from equation (1). From these conditions, the curve of every path of the waves can be drawn fairly approximately. The curves of the wave paths determined in this way are shown in Fig. 3; the full lines and broken lines indicating the paths for longitudinal waves and those for transverse waves, respectively.

It will be seen that the types of the respective curves are quite similar to those given by Gutenberg, etc. Since our present aim is to

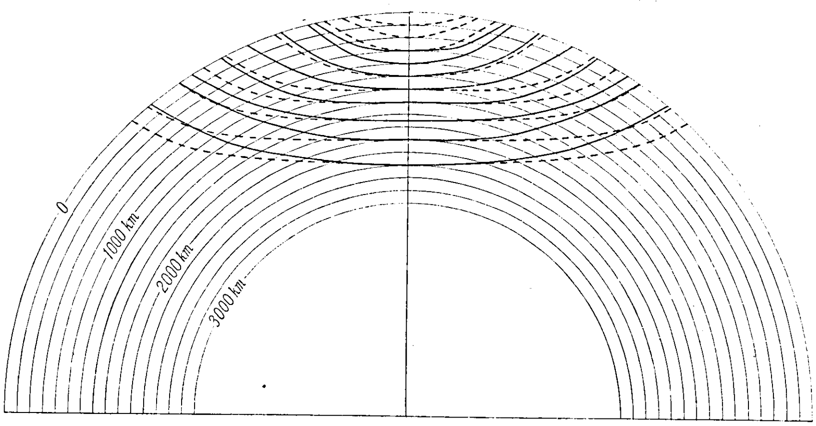

Fig. 3. Curves showing wave paths in rocky shell, full lines and broken lines representing P-waves and S-waves, respectively.

get the attenuation coefficient of the waves for every path, a small deviation in the curve, does not matter.

3. *Determination of the attenuation coefficient of waves belonging to every path.*

It has often been shown that when the waves do not diverge three-dimensionally, the attenuation factors of longitudinal and transverse waves transmitted through a uniform medium are expressed by

$$e^{-\frac{\alpha^2}{2C_L}\frac{\lambda'+2\mu'}{\lambda+2\mu}x}, \quad e^{-\frac{\alpha^2}{2C_S}\frac{\mu'}{\mu}x}, \tag{3}$$

respectively, where $C_L(=\sqrt{(\lambda+2\mu)/\rho})$, $C_s(=\sqrt{\mu/\rho})$ are the velocities of the two kinds of waves, in which λ, μ, λ', μ' are the elastic and viscous coefficients, x the focal distance, and α the vibrational frequency of the waves.

From the above relations, we may write the attenuation factors of both waves in a heterogeneous medium in the forms,

$$\Pi\left(e^{-\frac{\alpha^2}{2C_L}\frac{\lambda'+2\mu'}{\lambda+2\mu}x_n}\right), \quad \Pi\left(e^{-\frac{\alpha^2}{2C_S}\frac{\mu'}{\mu}x_n}\right), \tag{4}$$

respectively, where x_n is an elementary part of the wave path and C_L, C_s, λ, μ, λ', μ' are functions of x_n. It holds, then, that the attenuation coefficients multiplied by the focal distances for both waves are expressed by

$$\sum_n \frac{\alpha^2}{2C_L}\frac{\lambda'+2\mu'}{\lambda+2\mu}x_n, \quad \sum_n \frac{\alpha^2}{2C_s}\frac{\mu'}{\mu}x_n, \tag{5}$$

respectively. With the aid of these relations, the attenuation coefficients

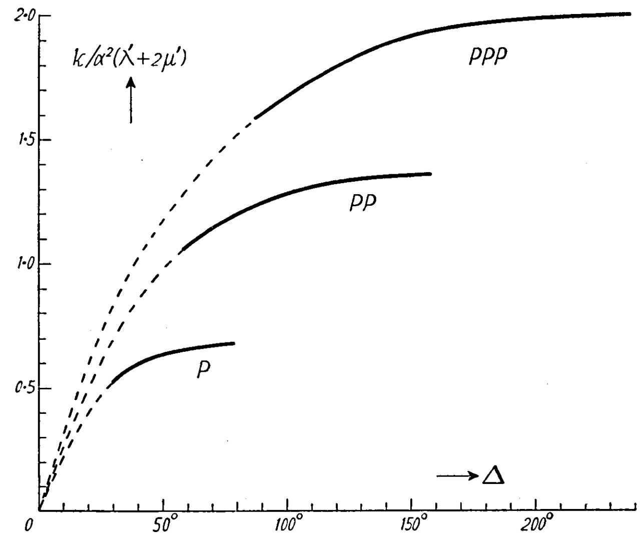

Fig. 4. Attenuation coefficient for longitudinal waves, the effect of three-dimensional transmission being omitted.

of various phases of waves trasmitted along the paths shown in Fig. 3

can be determined. In the present case, for convenience, let the attenuation factor of the waves at every epicentral distance be

$$e^{-k}; \tag{6}$$

then coefficient k is a quantity that can be interpreted very simply.

4. *The case of uniform viscosities.*

We shall now assume that C_L, C_S, λ, μ are the same as those in the actual earth, but λ', μ' are constant throughout the earth's crust. The values of k (the effect of distance being involved) for P, PP, PPP, S, SS, SSS for different values of Δ, calculated with the data in Fig. 3 and relations in (5), are shown in Figs. 4, 5.

From these figures it will be seen that in every case of P, PP, PPP, S, SS, SSS, constant k does not sensibly increase with Δ beyond a certain value of Δ. This arises from the condition that, owing to relations (3), the effective attenuation coefficient decreases with depth in

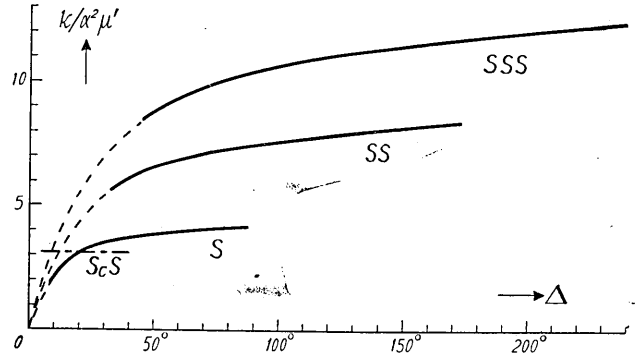

Fig. 5. Attenuation coefficient for transverse waves, the effect of three-dimensional transmission being omitted.

the earth's crust. The curves PP, PPP, SS, SSS were drawn under the assumption that no other wave is formed when the primary waves are reflected on the free surface of the earth. But the coefficients k for these waves are very large, that is to say, the amplitudes of the reflected waves ought to become very small—a result from the condition that the effective damping of waves near the free surface is much greater than that at depth.

In Fig. 5, the constant k for S_cS waves that are transmitted in a vertical direction, is also shown. It will be seen that notwithstanding that the length of the path for these waves is greater than that of the arc corresponding to 50°, their coefficient k is nearly equal to that for S at 20°, the reason for which is the same as that for S, SS, SSS just mentioned.

The ordinates in Figs. 4, 5 indicate that the coefficient in any case is proportional to the value of $\lambda'+2\mu'$ or μ' and also to the square of the vibrational frequency of the waves.

It should now be remembered that in the present problem, it is assumed that the waves do not diverge three-dimensionally. Were such three-dimensional propagation taken into account, then since the wave amplitude diminishes with a factor that is inversely proportional to the actual path, the value of k would be additionally increased with increase in Δ.

We shall now refer the data of the deep-focus earthquake (0·06 R) shown in Scrase's paper[9], namely, the earthquake that occurred on February 20, 1931 at 44·0° N, 138·0° E near the Japan Sea; the seismic records used being those at the station $\Delta=17{\cdot}1°$, 75·8°. In these data, the phases for P, S, SS, S_cS are fully recorded. Although the amplitudes shown in seismograms usually differ with differences in the sensibilities of the respective seismographs and also with the character of the respective stations, it is likely that a comparison of the amplitudes of different phases will offer some suggestion, at least qualitatively.

Comparing the phases for S, SS, S_cS in the data of Scrase's paper[10], it is possible to conclude that the relative amplitudes of these phases are merely the results of the three-dimensional transmission of the waves, provided the viscous constant $\mu'\ll 1$ in km. gr. sec. units (when $\lambda=\mu$, $\lambda'=\mu'$), that is to say, $\mu'\ll 10^{10}$ in C. G. S. units, which value is rather less than the coefficient of viscosity of surface rocks. This suggests that since the above value of μ' is the mean of the viscosities at different depths, the viscosity at depth will be less than that at the surface.

5. *Viscosities of the earth's crust near the surface, estimated from the data of surface waves.*

A few years back,[11] we estimated the viscosity of the earth's crust by investigating the change in the ratio of amplitude of Rayleigh-waves to that of Love-waves, together with the change in the ratio of vibration periods of the two waves with increasing epicentral distance. Since the transmission of these waves is over the same path at the earth's surface, no correction for the diverging nature of waves was required. Since our result, shown in the paper just mentioned, was

$$\mu'/2\rho=c/4\pi^2,$$

where $c=5{\cdot}90\ \mathrm{km}^2/\mathrm{sec}$, if $\rho=2{\cdot}5$, we get

9) F. J. SCRASE, *Phil. Trans. Roy. Soc.*, **231** (1933), 234.

10) For comparing the data of Scrase's paper, the effect of three-dimensional transmission was also taken into account, although approximately.

11) K. SEZAWA and K. KANAI, "Periods and Amplitudes of Oscillations in *L*- and *M*-phases," *Bull. Earthq. Res. Inst.*, **13** (1935), 26.

$$\mu'=3.10^9\ \rho \text{ in C.G.S. units}=7\cdot5,\ 10^9 \text{ in C.G.S. units.} \quad (8)$$

The relation $\mu'/\rho=0\cdot3$ in C. G. S. units, shown in that paper, was incorrect, due to Sezawa's error in the units taken.

Since the layer, through which the disturbances of Rayleigh-waves or Love-waves are transmitted, is of a few tens of kilometers in thickness, that is, probably less than 40 km or 50 km, it follows that the viscosities of rocks within the layers near the surface are of the order of $7\cdot5.\ 10^9 \approx 10^{10}$ in C. G. S. units. This value is somewhat greater than the mean viscosity of the earth's crust as given in the last section.

6. *Viscosities of the earth's crust at depth, estimated from the data of two kinds of bodily waves.*

The result in Section 4 shows that the mean viscosity of the earth's crust ought to be much less than 10^{10} in C. G. S. units. The result in Section 5, on the other hand, gives rise to the condition that the viscosity of rocks in the surface layer of thickness less than 40 km or 50 km, cannot exceed 10^{10} in C. G. S. units, whence it holds that the viscosities of the crust at depth would be very much less than 10^{10} in C. G. S. units.

It is impossible to use different phases of only one kind of wave, whether longitudinal or transverse, to the determination of the viscosities of the crust at depth. If, on the other hand, the two kinds of waves, namely, P-type and S-type, were used simultaneously, it would be possible to estimate, qualitatively, the values of the viscosities under consideration, in the same way as in the case of both Rayleigh-waves and Love-waves being applied in determining the nature of the earth's surface. It should be borne in mind that in treating the problem in the present case, it is assumed that the ratio of the amplitudes of P-waves to that of S-waves is finite even at the wave source.

We shall again use Scrase's data. The ratio of amplitude of P-type waves to that of S-type waves is about 1/2 for $\Delta=17\cdot1°$ and 1/3 for $\Delta=75\cdot8°$, the corresponding vibrations having periods of 6 sec at $17\cdot1°$ and 10 sec at $75\cdot8°$. We shall assume, for simplicity, that there are two layers concerned in this question of viscosity, namely, the surface layer of 50 km thickness and the inner crust. From damping of the surface waves, the coefficient of viscosity in the surface layer is 10^{10} C. G. S. units.

We shall put $\lambda=\mu$, $\lambda'=\mu'$ for any layer. The distribution of densities and the velocities of waves within the surface layer are the same as the mean values of those given by Imamura[12] and Matuzawa[13],

12) A. Imamura, K. Kishinouye and T. Kodaira, *Bull. Earthq. Res. Inst.*, **7** (1929), 471-487.

13) T. Matuzawa, K. Yamada and T. Suzuki, *Bull. Earthq. Res. Inst.*, **7** (1929), 241-260.

as follows:

	Density	C_L	C_S
The upper layer of 20 km thickness	2·7	5·0 km/s	3·15 km/s
The second layer of 30 km thickness	3·0	6·1 km/s	3·70 km/s

The distributions of densities and velocities of bodily waves within the inner crust are the same as those determined by Bullen[14] and Jeffreys[15].

Applying these data to equation (4) and using Figs. 4, 5, it is possible to form the simultaneous relations:

$$\begin{array}{l} S\text{-waves:} \\ P\text{-waves:} \end{array} \quad \frac{Ae^{-0\cdot434}e^{-2\cdot54\mu'}}{Be^{-0\cdot310}e^{-1\cdot22\mu'}}=2, \quad \frac{Ae^{-0\cdot174}e^{-1\cdot56\mu'}}{Be^{-0\cdot124}e^{-0\cdot786\mu'}}=3, \tag{9}$$

for $\Delta=17\cdot1^\circ$ and $\Delta=75\cdot8^\circ$, respectively, A, B being the amplitudes of the S-waves and P-waves at the wave source. The first exponential in each line represents the damping factor for the surface layer and the second that for the inner crust. Solving these equations, we find that the coefficient of viscosity of the inner crust should be $\mu'=5.10^9$ in C. G. S., which value is rather greater than that expected in the preceding sections. The reason for the coefficient of viscosity in the inner crust being not very small compared with that in the surface layer, is that, owing to the increase in elasticity at depth, the effective viscosity is greatly reduced, any decrease in the coefficient of viscosity without limit being thus improbable.

From our present investigation, it has been ascertained that the coefficient of viscosity in the earth's rocky shell lies between 10^{10} and 10^9.

7. *General summary and concluding remarks.*

From analysis of seismic records and from the theory of surface waves as well as bodily waves, we ascertained that the coefficient of viscosity of the rocky shell of the earth lies between 10^{10} and 10^9 in C. G. S. units. The coefficient near the surface rather exceeds that at depth. Since, moreover, damping action is reduced with increase in elasticity, the effective viscosity at depth is much less than the viscosity as actually estimated from the analysis.

From Jeffrey's investigation on the persistence in variation in latitude and the damping of seismic waves, the viscosity of the earth's crust is about 5.10^{20} in C. G S. units. The difference is probably owing to the condition that, in our case, we assumed only one kind of vis-

14) K. E. BULLEN, *loc. cit.* 4).
15) H. JEFFREYS, *lcc. cit.* 3).

cosity, whereas Jeffreys assumed two, namely, elastico-viscosity and firmo-viscosity.

In conclusion, we wish to express our thanks to Messrs. Watanabe and Kodaira who assisted us greatly in our calculations. We also wish to express our warmest thanks to the Officials of the Division of Scientific Research in the Department of Education for financial aid (Funds for Scientific Research) granted for a series of our investigation, of which this study is a part.

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10. 地球の內部に於ける粘性の分布（序報）

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地震記象の分析と表面波及固體波の理論とから地殼の粘性係數が 10^{10} と 10^{9} (C.G.S.) との間にある事がわかつた．地表の粘性係數は深層部のそれよりも寧ろ高い事もわかつた．即ち，地表近くで 10^{10} であり，數十粁から數千粁までは 10^{9} であるといふ意味である．又，同じ粘性係數であつても，地中深い所では彈性の增加のためにその効果的粘性は著しく減ずる譯である．Jeffreys が緯度變化の問題と地震波のそれとの兩方から出した地殼の粘性係數は 5.10^{20} である．この差は Jeffreys の場合には 2 種の粘性を態々考へたのに對し，我々のは一種しか考へない事から起つたのかも知れぬ．