2. *The Plasticity Conditions for the Formation of Normal and Reverse Faults. II.*

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1. *Introduction. Probable causes of the formation of normal and reverse faults.*

In a previous paper[1] we dealt with the probable causes of the formation of normal and reverse faults. When the liquid earth partly solidified, the solidfied parts changed from incompressible to compressible. If, then, there were no fault, the lateral compression at these parts would exceed the vertical, owing to the condition that under gravitational forces every point of the earth is attracted toward its centre, with its surface area tending to contract. This condition would be sufficient cause for reverse faults. On the other hand, when the solidified earth has faults, it is scarcely possible for the stress condition of a continuous solid spherical earth to be maintained. The fault surfaces, whether in reverse faults or in normal faults, mostly slide in such a direction as may be determined by the original stress conditions that give rise to those faults. Since in such a condition, the horizontal compression may be zero or at any rate less than vertical compression, the formation of normal faults is highly probable.

In the previous paper, with a view to showing the respective origins of reverse and normal faults, we dealt with a spherical earth and a plane earth. Since it is likely that the statical state of a crust bounded by faults resembles that of a plane earth without lateral stress or without lateral strain, then if the elastic or plastic condition of that plane earth were such as to give rise to normal faults, any fault that could newly form in a crust bounded by the original faults would be a normal fault. Since however the condition of the problem here given is a highly idealized one, it is impossible to ascertain the distribution of plastic stresses in a real crust bounded by faults. In these circumstances we shall now deal with the stress condition in a surface crustal block bounded by an original fault.

1) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **17** (1939), 661~674.

Although, according to our previous idea, any fault that could newly form in the crust in question must invariably be normal, the result of the present investigation shows that it is possible for the new faults to be normal, or even reverse, according to the difference in the inclination of the surface of the original fault.

2. *General mathematical solutions.*

As shown in Fig. 1, let AOB be a free surface and OC a fault surface extending to infinity. For simplicity, it is assumed that, whereas crustal block AOC is deformable, BOC is extremely rigid. The equations of equilibrium of a gravitating elastic body in cylindrical coordinates are expressed by

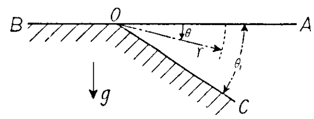

Fig. 1.

$$\left.\begin{aligned}(\lambda+2\mu)\frac{\partial\Delta}{\partial r}-\frac{2\mu}{r}\frac{\partial\varpi}{\partial\theta}&=-\rho g\sin\theta,\\ (\lambda+2\mu)\frac{1}{r}\frac{\partial\Delta}{\partial\theta}+2\mu\frac{\partial\varpi}{\partial r}&=-\rho g\cos\theta,\end{aligned}\right\} \tag{1}$$

where

$$\Delta=\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{1}{r}\frac{\partial v}{r\theta},\quad 2\varpi=\frac{1}{r}\frac{\partial(rv)}{\partial r}-\frac{1}{r}\frac{\partial u}{\partial\theta}, \tag{2}$$

in which u, v are the radial and transverse components of displacement and ρ, λ, μ the density and elastic constants, respectively. In solving (1), we shall eliminate u, v between (1), (2), which gives

$$\frac{\partial^2\Delta}{\partial r^2}+\frac{1}{r}\frac{\partial\Delta}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Delta}{\partial\theta^2}=0,\qquad \frac{\partial^2\varpi}{\partial r^2}+\frac{1}{r}\frac{\partial\varpi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varpi}{\partial\theta^2}=0. \tag{3}$$

Solving these equations and adding particular solutions of (1), we get

$$\Delta=A_0-\frac{r\rho g\sin\theta}{\lambda+\mu},\quad 2\varpi=B_0+\frac{r\rho g\cos\theta}{\lambda+\mu},\quad [n=0] \tag{4}$$

$$\left.\begin{aligned}\Delta&=\left(A_n r^n+B_n\frac{1}{r^n}\right)\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}n\theta-\frac{r\rho g\sin\theta}{\lambda+\mu},\\ 2\varpi&=\left(A_n' r^n+B_n'\frac{1}{r^n}\right)\begin{Bmatrix}\cos\\ -\sin\end{Bmatrix}n\theta+\frac{r\rho g\cos\theta}{\lambda+\mu}.\end{aligned}\right\}[n>1] \tag{5}$$

A_n, A'_n ; B_n, B'_n are not independent of each other, but in virtue of (1), such relations as

$$A'_n = -\frac{\lambda+2\mu}{\mu}A_n, \quad B'_n = \frac{\lambda+2\mu}{\mu}B_n \tag{6}$$

exist. The displacement u_1, v_1 corresponding to Δ in (4), (5) and satisfying $\varpi=0$, is expressed by

$$u_1=\frac{1}{2}A_0 r-\frac{3r^2\rho g\sin\theta}{8(\lambda+\mu)}, \quad v_1=-\frac{r^2\rho g\cos\theta}{8(\lambda+\mu)}, \quad [n=0] \tag{7}$$

$$u_1=B_1\frac{\log r+1}{2}\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}\theta, \quad v_1=B_1\frac{\log r}{2}\left.\begin{matrix}\cos\\ -\sin\end{matrix}\right\}\theta, \quad [n=1 \text{ for } B_n] \tag{8}$$

$$\left.\begin{aligned} u_1&=\left\{\frac{n+2}{4(n+1)}A_n r^{n+1}+\frac{n-2}{4(n-1)}\frac{B_n}{r^{n-1}}\right\}\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}n\theta \\ &\qquad -\frac{3r^2\rho g\sin\theta}{8(\lambda+\mu)}, \\ v_1&=\left\{\frac{n}{4(n+1)}A_n r^{n+1}-\frac{n}{4(n-1)}\frac{B_n}{r^{n-1}}\right\}\left.\begin{matrix}\cos\\ -\sin\end{matrix}\right\}n\theta \\ &\qquad -\frac{r^2\rho g\cos\theta}{8(\lambda+\mu)}. \end{aligned}\right\} \begin{bmatrix} n>1 \text{ for } A_n \\ n>2 \text{ for } B_n \end{bmatrix} \tag{9}$$

The displacement corresponding to ϖ in (4), (5) and satisfying $\Delta=0$, is

$$u_2=\frac{r^2\rho g\sin\theta}{8(\lambda+\mu)}, \quad v_2=\frac{1}{2}B_0 r+\frac{3r^2\rho g\cos\theta}{8(\lambda+\mu)}, \quad [n=0] \tag{10}$$

$$u_2=B'_1\frac{\log r}{2}\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}\theta, \quad v_2=B'_1\frac{\log r+1}{2}\left.\begin{matrix}\cos\\ -\sin\end{matrix}\right\}\theta, \quad [n=1 \text{ for } B'_n] \tag{11}$$

$$\left.\begin{aligned} u_2&=\left\{\frac{n}{4(n+1)}A'_n r^{n+1}-\frac{n}{4(n-1)}\frac{B'_n}{r^{n-1}}\right\}\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}n\theta \\ &\qquad +\frac{r^2\rho g\sin\theta}{8(\lambda+\mu)}, \\ v_2&=\left\{\frac{n+2}{4(n+1)}A'_n r^{n+1}+\frac{n-2}{4(n-1)}\frac{B'_n}{r^{n-1}}\right\}\left.\begin{matrix}\cos\\ -\sin\end{matrix}\right\}n\theta \\ &\qquad +\frac{3r^2\rho g\cos\theta}{8(\lambda+\mu)}. \end{aligned}\right\} \begin{bmatrix} n>1 \text{ for } A'_n \\ n>2 \text{ for } B'_n \end{bmatrix} \tag{12}$$

There is a complementary solution that satisfies $\Delta=0$, $\varpi=0$, the expressions of which are

$$\left.\begin{aligned} u_3&=\left(A''_n r^{n-1}+\frac{B''_n}{r^{n+1}}\right)\begin{matrix}\sin\\ \cos\end{matrix}\Big\} n\theta,\\ v_3&=\left(A''_n r^{n-1}-\frac{B''_n}{r^{n+1}}\right)\begin{matrix}\cos\\ -\sin\end{matrix}\Big\} n\theta. \end{aligned}\right\} \quad [n\geqq 0]. \tag{13}$$

u_3, v_3 also satisfy equation (1) provided that they are combined with u_1, v_1 and u_2, v_2. The resultant displacement is thus expressed by

$$\left.\begin{aligned} u&=\left\{\frac{1}{2}A_0 r+\frac{1}{r}(A''_0+B''_0)\right\}+\left\{\frac{3}{8}r^2A_1+B_1\frac{\log r+1}{2}\right.\\ &\quad\left.+\frac{1}{8}A'_1r^2+B'_1\frac{\log r}{2}+A''_1+\frac{B''_1}{r^2}\right\}\begin{matrix}\sin\\ \cos\end{matrix}\Big\}\theta\\ &\quad+\sum_{n=2}^{\infty}\left\{\frac{n+2}{4(n+1)}A_n r^{n+1}+\frac{n-2}{4(n-1)}\frac{B_n}{r^{n-1}}+\frac{n}{4(n+1)}A'_n r^{n+1}\right.\\ &\quad\left.-\frac{n}{4(n-1)}\frac{B'_n}{r^{n-1}}+A''_n r^{n-1}+\frac{B''_n}{r^{n+1}}\right\}\begin{matrix}\sin\\ \cos\end{matrix}\Big\}n\theta-\frac{r^2\rho g\sin\theta}{4(\lambda+\mu)},\\ v&=\left\{\frac{1}{2}A_0 r+\frac{1}{r}(A''_0-B''_0)\right\}+\left\{\frac{1}{8}r^2A_1+B_1\frac{\log r}{2}\right.\\ &\quad\left.+\frac{3}{8}A'_1r^2+B'_1\frac{\log r+1}{2}+A''_1-\frac{B''_1}{r^2}\right\}\begin{matrix}\cos\\ -\sin\end{matrix}\Big\}\theta\\ &\quad+\sum_{n=2}^{\infty}\left\{\frac{n}{4(n+1)}A_n r^{n+1}-\frac{n}{4(n-1)}\frac{B_n}{r^{n-1}}+\frac{n+2}{4(n+1)}A'_n r^{n+1}\right.\\ &\quad\left.+\frac{n-2}{4(n-1)}\frac{B'_n}{r^{n-1}}+A''_n r^{n-1}-\frac{B''_n}{r^{n+1}}\right\}\begin{matrix}\cos\\ -\sin\end{matrix}\Big\}n\theta+\frac{r^2\rho g\cos\theta}{4(\lambda+\mu)}, \end{aligned}\right\} \tag{14}$$

where A_n, A'_n; B_n, B'_n are related to each other with the conditions shown in (6), A''_n, B''_n being arbitrary.

When the special point $r=0$ is not a singular point, the last solutions reduce to

$$\begin{aligned} u&=\sum_{n=0}^{\infty}\left\{\frac{-\lambda n+\mu(2-n)}{4\mu(n+1)}A_n r^{n+1}\right.\\ &\qquad\left.+A''_n r^{n-1}\right\}\begin{matrix}\sin\\ \cos\end{matrix}\Big\}n\theta-\frac{r^2\rho g\sin\theta}{4(\lambda+\mu)}, \end{aligned}$$

$$\left.\begin{aligned} v&=\sum_{n=0}^{\infty}\left\{-\frac{\lambda(n+2)+\mu(n+4)}{4\mu(n+1)}A_n r^{n+1}\right. \\ &\qquad\left.+A_n'' r^{n-1}\right\}\begin{matrix}\cos\\ -\sin\end{matrix}\Big\}n\theta+\frac{r^2\rho g\cos\theta}{4(\lambda+\mu)}, \\ \Delta&=\sum_{n=0}^{\infty}A_n r^n \begin{matrix}\sin\\ \cos\end{matrix}\Big\}n\theta-\frac{r\rho g\sin\theta}{\lambda+\mu}. \end{aligned}\right\} \quad (15)$$

In our particular problem, furthermore, the important part of the solution may be expressed as

$$\left.\begin{aligned} u&=r^2\left[\frac{-\lambda+\mu}{8\mu}\{A_1\sin\theta+C_1\cos\theta\}+\{A_3''\sin3\theta+C_3''\cos3\theta\}\right. \\ &\qquad\left.-\frac{\rho g\sin\theta}{4(\lambda+\mu)}\right]+(A_1'\sin\theta+C_1'\cos\theta), \\ v&=r^2\left[\frac{-(3\lambda+5\mu)}{8\mu}\{A_1\cos\theta-C_1\sin\theta\}+\{A_3''\cos3\theta-C_3''\sin3\theta\}\right. \\ &\qquad\left.+\frac{\rho g\cos\theta}{4(\lambda+\mu)}\right]+(A_1'\cos\theta-C_1'\sin\theta), \\ \Delta&=r\left[A_1\sin\theta+C_1\cos\theta-\frac{\rho g\sin\theta}{\lambda+\mu}\right], \end{aligned}\right\} \quad (16)$$

where symbols C_1, C_1', C_3'' are used in lieu of A_1, A_1', A_3'' for particularly showing the coefficients of $\cos\theta$ in Δ, $\sin\theta$ in ϖ, $\cos3\theta$ in u_3, respectively, shown in (5), (13).

It is well known that the stress-strain relations are

$$\left.\begin{aligned} \widehat{rr}&=\lambda\Delta+2\mu\frac{\partial u}{\partial r}, \quad \widehat{\theta\theta}=\lambda\Delta+2\mu\left(\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right), \\ \widehat{r\theta}&=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right), \end{aligned}\right\} \quad (17)$$

from which we get

$$\begin{aligned} \widehat{rr}&=\frac{r}{2}\Big\{(\lambda+\mu)(A_1\sin\theta+C_1\cos\theta) \\ &\qquad+8\mu(A_3''\sin3\theta+C_3''\cos3\theta)-2\rho g\sin\theta\Big\}, \end{aligned}$$

$$\left.\begin{aligned}\widehat{\theta\theta}&=\frac{r}{2}\Big\{3(\lambda+\mu)(A_1\sin\theta+C_1\cos\theta)\\&\qquad-8\mu(A_3''\sin3\theta+C_3''\cos3\theta)-2\rho g\sin\theta\Big\},\\\widehat{r\theta}&=\frac{r}{2}\Big\{-(\lambda+\mu)(A_1\cos\theta-C_1\sin\theta)\\&\qquad+8\mu(A_3''\cos3\theta-C_3''\sin3\theta)\Big\}.\end{aligned}\right\}\qquad(18)$$

3. *The case in which the boundary* OC *is frictionless.*

When the boundary OC is frictionless, the boundary conditions are such that

$$\widehat{\theta\theta}=0,\quad \widehat{r\theta}=0 \qquad (19)$$

at $\theta=0$ and

$$v=0,\quad \widehat{r\theta}=0 \qquad (20)$$

at $\theta=\theta_1$. The displacement of point O depends merely on the condition at a certain r which is not specified particularly in the present problem, so that the same displacement may be arbitrary. Let the distance, through which point O shifts along the surface OC, be U; then, substituting (16), (18) in conditions (19), (20), we get

$$\left.\begin{aligned}u&=U\cos(\theta_1-\theta)+\frac{r^2\rho g}{8(\lambda+\mu)}\Big[\alpha\Big\{\Big(1-\frac{\lambda}{\mu}\Big)\sin\theta+\Big(1+\frac{\lambda}{\mu}\Big)\sin3\theta\Big\}\\&\qquad+\gamma\Big\{\Big(1-\frac{\lambda}{\mu}\Big)\cos\theta+3\Big(1+\frac{\lambda}{\mu}\Big)\cos3\theta\Big\}-2\sin\theta\Big],\\v&=U\sin(\theta_1-\theta)+\frac{r^2\rho g}{8(\lambda+\mu)}\Big[\alpha\Big\{\Big(1+\frac{\lambda}{\mu}\Big)\cos3\theta-\Big(5+\frac{3\lambda}{\mu}\Big)\cos\theta\Big\}\\&\qquad+\gamma\Big\{\Big(5+\frac{3\lambda}{\mu}\Big)\sin\theta-3\Big(1+\frac{\lambda}{\mu}\Big)\sin3\theta\Big\}+2\cos\theta\Big],\end{aligned}\right\}\qquad(21)$$

$$\left.\begin{aligned}\widehat{rr}&=\frac{r\rho g}{2}\Big\{\alpha(\sin\theta+\sin3\theta)+\gamma(\cos\theta+\cos3\theta)-2\sin\theta\Big\},\\\widehat{\theta\theta}&=\frac{r\rho g}{2}\Big\{\alpha(3\sin\theta-\sin3\theta)+3\gamma(\cos\theta-\cos3\theta)-2\sin\theta\Big\},\\\widehat{r\theta}&=\frac{r\rho g}{2}\Big\{\alpha(\cos3\theta-\cos\theta)+\gamma(\sin\theta-3\sin3\theta)\Big\},\end{aligned}\right\}\qquad(22)$$

where

$$\left.\begin{aligned} \alpha &= \frac{\cos\theta_1(3\sin 3\theta_1-\sin\theta_1)}{\left(2+\frac{\lambda}{\mu}\right)(3\cos\theta_1\sin 3\theta_1-\sin\theta_1\cos 3\theta_1)}, \\ \gamma &= \frac{\cos\theta_1(\cos 3\theta_1-\cos\theta_1)}{\left(2+\frac{\lambda}{\mu}\right)(3\cos\theta_1\sin 3\theta_1-\sin\theta_1\cos 3\theta_1)}. \end{aligned}\right\} \tag{23}$$

The maximum shear stress, namely, the plastic stress, is represented by

$$\tau=\sqrt{(\widehat{rr}-\widehat{\theta\theta})^2/4+\widehat{r\theta}^2}, \tag{24}$$

its direction β relative to that of r-θ being

$$\tan 2\beta=(\widehat{rr}-\widehat{\theta\theta})/2\widehat{r\theta}. \tag{25}$$

(22), (23) show that when the material is incompressible, that is to say, $\lambda/\mu=\infty$, then

$$\widehat{rr}=\widehat{\theta\theta}=-r\rho g\sin\theta, \quad \widehat{r\theta}=0, \tag{26}$$

the plastic stress therefore being zero. Since, furthermore, $\widehat{r\theta}$ is zero at the surfaces $\theta=0$ and $\theta=\theta_1$, the planes of the maximum shear stress intersect with these surfaces at 45° or 135°. The condition whether the angle in question is 45° or 135° depends on the difference in θ_1.

Using (22), (23), (24), (25), we calculated the values of τ, β, $\widehat{rr}$ $\widehat{\theta\theta}$, $\widehat{r\theta}$ for seven different cases, namely, $\theta_1=15°$, 30°, 45°, 60°, 120°,

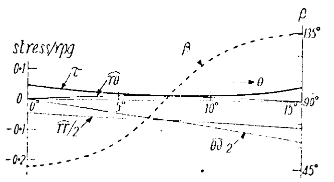

Fig. 2. $f=0$, $\theta_1=15°$; τ, β being the plastic stress and its direction relative to r-θ. f is frictional coefficient.

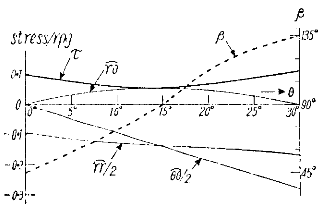

Fig. 3. $f=0$, $\theta_1=30°$; τ, β being the plastic stress and its direction relative to r-θ.

135°, 150° (the Poisson's ratio being 1/4), the results of which are plotted in Figs. 2∼8 by thick full lines for τ, thick broken lines for β, and thin full lines for $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{r\theta}$, respectively. These figures show that the nature of the change in the quantities τ, β, $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{r\theta}$ with θ for the case $\theta_1<90°$ differs entirely from that in the same quantities with θ for the case $\theta_1>90°$.

The value of $\widehat{rr}$ at the free surface is in the sense of compression for $\theta_1 < 90^\circ$ and in that of tension for $\theta_1 > 90^\circ$. Both $\widehat{rr}$ and $\widehat{\theta\theta}$ are compressive at the surface $\theta = \theta_1$, the value of $\widehat{\theta\theta}$ at that surface invariably exceeding that of $\widehat{rr}$. From these conditions, the curve for $\widehat{rr}$

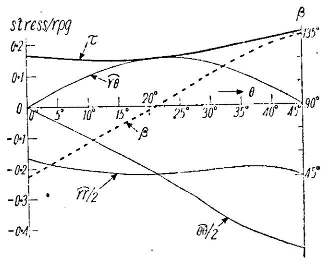

Fig. 4. $f=0$, $\theta_1=45^\circ$; τ, β being the plastic stress and its direction relative to r-θ.

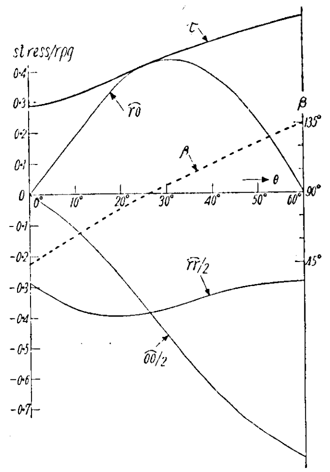

Fig. 5. $f=0$, $\theta_1=60^\circ$; τ, β being the plastic stress and its direction relative to r-θ.

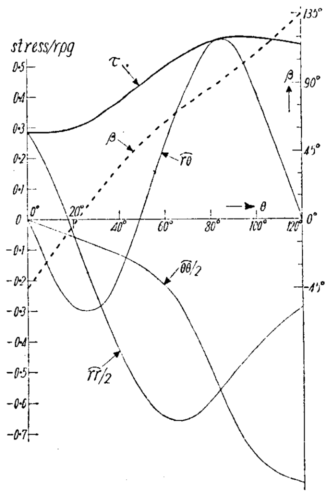

Fig. 6. $f=0$, $\theta_1=120^\circ$; τ, β being the plastic stress and its direction relative to r-θ.

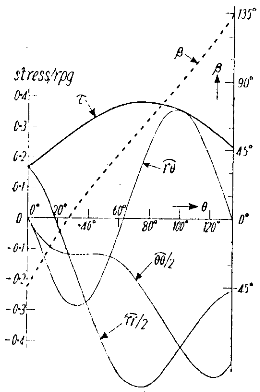

Fig. 7. $f=0$, $\theta_1=135^\circ$; τ, β being the plastic stress and its direction relative to r-θ.

and that for $\widehat{\theta\theta}$, both with abscissa θ, intersect once for $\theta_1 < 90^\circ$ and twice for $\theta_1 > 90^\circ$. The value of $\widehat{r\theta}$ is of the same sign for any θ in the case $\theta_1 < 90°$, but changes its sign for an intermediate θ in the case $\theta_1 > 90°$.

The type of curve for the maximum shear stress τ also changes with change in θ_1. When θ_1 is small, say, less than 45°, there is a minimum of τ for an intermediate value of θ and, when θ_1 is large, say, greater than 135°, a maximum of τ occurs also for an intermediate value of θ. When θ_1 is medium, say, $120° > \theta_1 > 60°$, there is a point that satisfies $d^2\tau/d\theta^2 = 0$ for a certain value of θ.

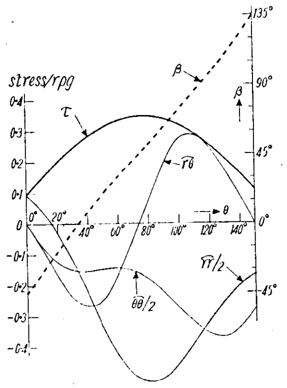

Fig. 8. $f=0$, $\theta_1=150°$; τ, β being the plastic stress and its direction relative to r-ϑ.

There is also a certain regularity in the variation of β. For $\theta_1 < 90°$, the angle β changes from 45° to 135°, while for $\theta_1 > 90°$, the same angle changes from $-45°$ to 135°, (extension of $\beta = -45°$ in opposite sense), from which it follows that for $\theta_1 > 90°$, the sign of the maximum shear stress at the free surface is the same as that at the surface $\theta = \theta_1$, whereas for $\theta_1 < 90°$, the condition is reversed.

In order to verify the foregoing results, the values of τ at both surfaces $\theta = 0$, $\theta = \theta_1$, that is to say, the values of $(\widehat{rr} - \widehat{\theta\theta})/2$, particularly, at these surfaces, are plotted in Figs. 9, 10. The same values for $f \neq 0$ are also shown in these figures. It will be seen that if θ_1

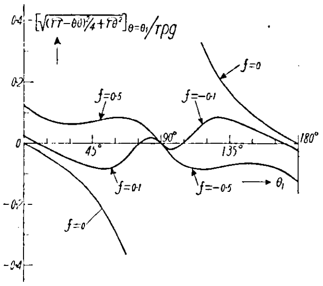

Fig. 9. The values of τ at the free surface for different f and θ_1.

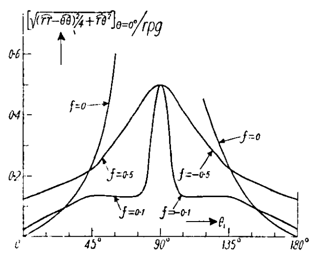

Fig. 10. The value of τ at the sliding surface for different f and θ_1.

changes from $\theta_1 < 90°$ to $\theta_1 > 90°$, the stress on the free surface changes from compressive to tractive, whereas the value of $(\widehat{rr} - \widehat{\theta\theta})/2$ at the surface $\theta = \theta_1$ retains the same sign for any case in θ_1. If the geological condition of a fault block were $\theta_1 > 90°$ in the present study, then such faults as can newly form on the surface of the same block would be of normal type, whereas if the condition were $\theta_1 < 90°$, the new faults on the same free surface would be of reverse type, although the new faults on the surface $\theta=\theta_1$ would invariably be of normal type, regardless of whether $\theta_1<90°$ or $\theta_1>90°$. Our previous idea that new faults that could form in a crust bounded by original faults will invariably be normal, has thus been modified.

The deformation and stress of the free surface (or even the sliding surface) vary in parabolic law (excepting bodily deformation) and linear law, respectively, both along that surface.

When $f=0$, every quantity shown in (21), (22), (24) tends to infinity for $\theta_1\to 90°$, but when $f\neq 0$, it does not. Since, however, the boundary conditions in (17) can scarcely be satisfied if $f=0$ and θ_1 is exactly 90°, our solutions in such a special case do not apply to the case $\theta_1=90°$. This results from the treatment that, for fulfilling the conditions (20), both sine terms and cosine terms were used. For the solutions being determinate, some small frictional resistance should exist at the surface $\theta=\theta_1$. At all events, that the stresses and strains shall tend to increase for $\theta_1\to 90°$ is probable, physically speaking.

4. *The case in which there is friction on the boundary* OC.

When a frictional force that is proportional to the normal pressure resists the movement of the crustal block AOC, the boundary conditions are such that

$$\widehat{\theta\theta}=0, \quad \widehat{r\theta}=0 \tag{27}$$

at $\theta=0$ and

$$v=0, \quad \widehat{r\theta}=f\,\widehat{\theta\theta} \tag{28}$$

at $\theta=\theta_1$. Substituting (16), (18) in these conditions, we get the expressions for displacement and stresses, the forms of which are the same as those in (21), (22), with the conditions that

$$\left.\begin{aligned} \alpha\Phi &= \cos\theta_1(3\sin 3\theta_1-\sin\theta_1)\\ &\quad +f\left\{(3+2\sin^2\theta_1-3\cos 2\theta_1)+\frac{3\lambda}{\mu}\sin\theta_1(\sin\theta_1-\sin 3\theta_1)\right\},\\ \gamma\Phi &= \cos\theta_1(\cos 3\theta_1-\cos\theta_1)\\ &\quad +f\left\{(2\cos\theta_1\sin\theta_1+\sin 2\theta_1)+\frac{\lambda}{\mu}\sin\theta_1(3\cos\theta_1-\cos 3\theta_1)\right\}, \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned}\Phi=\left(2+\frac{\lambda}{\mu}\right)(3\cos\theta_1\sin3\theta_1-\sin\theta_1\cos3\theta_1)\\+f\left\{(9-7\cos2\theta_1-2\cos\theta_1\cos3\theta_1)+\frac{6\lambda}{\mu}(1-\cos2\theta_1)\right\},\end{aligned}\right)$$

the expressions for the maximum shear stress and its direction being the same as those in (24) and (25).

In the present case, even should the material be incompressible, the plastic stress is not zero, the same stress at any point being proportional to the coefficient f.

From the numerical calculation to be shown later or as will immediately be seen from the nature of the expressions for the stresses, it has been ascertained that f should be taken positive for $\theta_1<90°$ and negative for $\theta_1>90°$, the reason for which is that the condition of sliding of the surface for $\theta_1>90°$ is the reverse of that for $\theta_1<90°$.

In the present case, too, the deformation and stress of the free surface (or even the sliding surface) vary according to the parabolic law (excepting bodily deformation) and linear law, respectively, both along that surface.

Using (22), (29), (24), (25), we calculated the values of τ, β, $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{r\theta}$ for six different cases, namely, (1) $f=0{\cdot}1$, $\theta_1=45°$, (2) $f=0{\cdot}2$, $\theta_1=45°$, (3) $f=0{\cdot}4$, $\theta_1=45°$, (4) $f=0{\cdot}5$, $\theta_1=45°$, (5) $f=-0{\cdot}1$, $\theta_1=135°$, (6) $f=-0{\cdot}5$, $\theta_1=135°$; the results of which are shown in Figs. 11∼16.

From these figures it will be seen that, whereas the change in

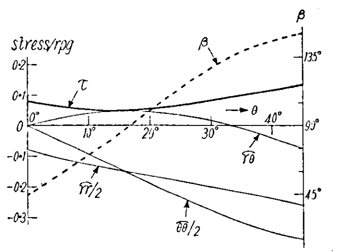

Fig. 11. $f=0{\cdot}1$, $\theta_1=45°$; τ, β being the plastic stress and its direction relative to r-θ.

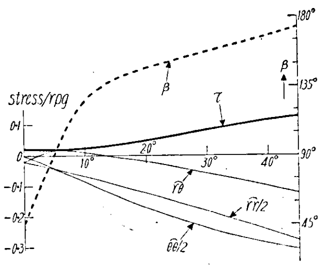

Fig. 12. $f=0{\cdot}2$, $\theta_1=45°$; τ, β being the plastic stress and its direction relative to r-θ.

every quantity for $f=0{\cdot}1$ or $f=0{\cdot}2$ or for $f=-0{\cdot}1$ resembles that for $f=0$, the change in the same quantity for $f=0{\cdot}5$ or $f=-0{\cdot}5$ or for $f=0{\cdot}4$ differs entirely from that for $f=0$. In the case of $f=0{\cdot}5$ or $f=-0{\cdot}5$ or $f=0{\cdot}4$, the surface stress is tractive for $\theta_1<90°$ and compressive for $\theta_1 > 90°$, the nature of the problem being the reverse of the case $f=0$. The condition of the plastic stress on the surface $\theta=\theta_1$ is also greatly modified. Since in this case there is tangential friction on the surface, the angle of the plane of the maximum shear stress is

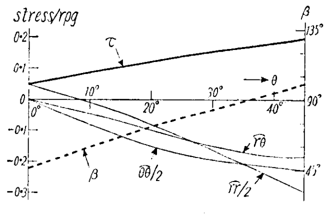

Fig. 13. $f=0{\cdot}4$, $\theta_1=45°$; τ, β being the plastic stress and its direction relative to r-ϑ.

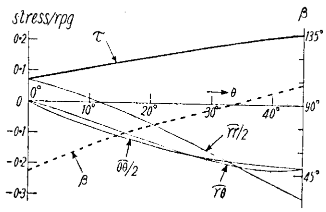

Fig. 14. $f=0{\cdot}5$, $\theta_1=45°$; τ, β being the plastic stress and its direction relative to r-θ.

also modified. Whereas in the case $f=0$, the variations of β with θ are in given ranges for any θ_1 that is less than 90° and also for any θ_1 that is greater than 90°, namely, between 45° and 135° for $\theta_1 < 90°$ and between $-45°$ and 135° for $\theta_1 > 90°$, respectively; in the case $f \neq 0$, on the other hand, the value of β varies from 45° (and $-45°$) to an

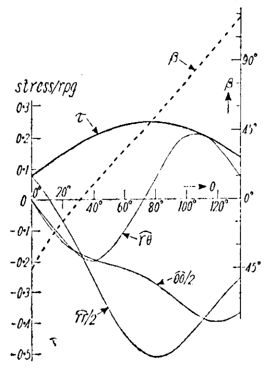

Fig. 15. $f=-0{\cdot}1$, $\theta_1=135°$; τ, β being the plastic stress and its direction relative to r-θ.

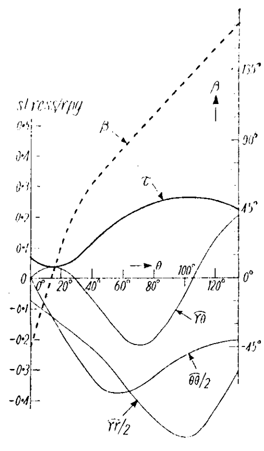

Fig. 16. $f=-0{\cdot}5$, $\theta_1=135°$; τ, β being the plastic stress and its direction relative to r-θ.

angle that depends on the difference in θ_1. Furthermore, the law of variation of the upper critical in β is not simple. For instance, the range in question is between 45° and 150° for (1) $f=0{\cdot}1$, $\theta_1=45°$, between 45° and 173° for (2) $f=0{\cdot}2$, $\theta_1=45°$, between 45° and 100° for (3) $f=0{\cdot}4$, $\theta_1=45°$, between 45° and 103° for (4) $f=0{\cdot}5$, $\theta_1=45°$, between $-45°$ and $118°$ for (5) $f=-0{\cdot}1$, $\theta_1=135°$, and between $-45°$ and $167°$ for (6) $f=-0{\cdot}5$, $\theta_1=135°$. It is however possible to conclude that in the case $\theta_1=45°$, the β-value (excepting the value at the free surface) for any $|f|$ that is less than 0·25 and that for any $|f|$ that is greater than 0·25 are, respectively, greater and less than their values for $f=0$; and in the case $\theta_1=135°$, reverse condition holds.

In the previous section it has been shown that in the case $f=0$, if $\theta_1 \to 90°$, every stress tends to increase; and if θ_1 is exactly 90°, the stress becomes infinitely great, although such condition is mathematically inconsistent. In the case $f \neq 0$, on the other hand, the condition is greatly modified, any stress being then never infinitely great. For example, in the case $\theta_1=90°$, even if the frictional coefficient f be infinitesimal, the surface stress vanishes and the normal pressure as well as the tangential stress at the sliding surface also vanishes, the vertical compression at the surface $\theta=\theta_1$ being ρgr for any r. Furthermore, if $f \neq 0$, the stress $\widehat{rr}$ on the surface is not zero even in the case $\theta_1=0$. The results of calculation for the cases $|f|=0{\cdot}1$ and $|f|=0{\cdot}5$ are shown also in Figs. 9, 10.

With a view to ascertaining the effect of the change in f on the general feature of the problem, we calculated the horizontal and vertical displacements (excepting the bodily displacement of the crust) and stresses on the free surface for a wide range of f, the results for $\theta_1=45°$ and 135° being shown in Figs. 17, 18. It will be seen from these figures that with increase in f, the horizontal stress changes from

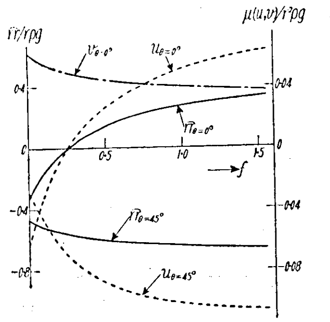

Fig. 17. Variations in deformations and stresses with f for $\theta_1=45°$.
The part corresponding to bodily displacement is excluded from u and v.

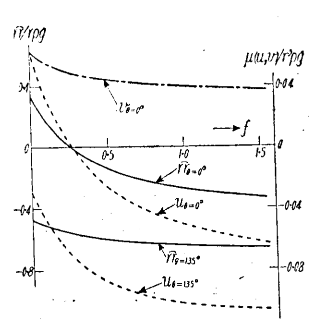

Fig. 18. Variations in deformations and stresses with f for $\theta_1=135°$.
The part corresponding to bodily displacement is excluded from u and v.

compressive to tractive (at $f=0{\cdot}25$) in the case $\theta_1=45°$ and from tractive to compressive (also at $f=0{\cdot}25$) in the case $\theta_1=135°$. It holds then that with frictional resistance acting on the sliding surface, the stress in the crust assumes a condition that is the reverse of that in the case without frictional force on the sliding surface under consideration. As to the vertical and horizontal displacements, if the bodily displacement that corresponds to the movement of the point O be omitted, the variation in the same displacement is parabolic.

We now arrive at the conclusion that if the frictional resistance on the sliding surface is great, such faults as could form in the crustal block would be of normal type for $\theta_1 < 90°$ and of reverse type for $\theta_1 > 90°$. It thus appears that the frictional force in the sliding plane plays an important part in the problem.

5. *Summary and concluding remarks.*

From mathematical investigation we ascertained the stress and strain distributions in a surface crustal block bounded by one original fault extending to infinity, the plastic condition being taken only in the sense of the maximum shear stress.

The surface displacement and surface stress vary according to parabolic law and linear law, respectively, both along that surface. If there were no frictional resistance on the sliding surface, the stress and strain on the free surface would be compressive for $\theta_1 < 90°$ and tractive for $\theta_1 > 90°$. The faults that could form on the surface of the crustal block are thus of reverse type for $\theta_1 < 90°$ and of normal type for $\theta_1 > 90°$. If, on the other hand, the frictional resistance on the sliding surface is great, the condition of the problem is quite reversed, namely, the strain and stress on the free surface is tractive for $\theta_1 < 90°$ and compressive for $\theta_1 > 90°$. In this case, the faults that can form on the surface of the crustal block would be of normal type for $\theta_1 < 90°$ and of reverse type for $\theta_1 > 90°$.

The above statement explains the only condition of the free surface. If the inner condition of the crustal block were considered, the problem would be fairly complicated. However, one simple feature to be mentioned is that, when $f = 0$, the stress condition on the surface $\theta = \theta_1$ is such that new faults that form on the surface $\theta = \theta_1$ would invariably be of normal type regardless of whether $\theta_1 < 90°$ or $\theta_1 > 90°$.

Since the elastic or plastic state of a solid earth without fault has already been treated in the previous paper, no reference to it has been made in the present paper. However, from the results shown in the previous and the present papers, it holds that although a normal fault is formed in a crustal block that is bounded by original faults, the origin of the reverse fault, on the other hand, is not restricted to local conditions, but is also related to the deformation of the entire earth, particularly in its early historical stage.

In the present paper, we considered a crustal block bounded by an original fault. This condition of the problem was given merely to serve as a working hypothesis. The treatment of a block that is bounded by many original faults may be a more general one. Furthermore, if we could solve the problem of a crust in a folded state, it would also be possible for us to get a result that is somewhat similar to that in the present paper.

Although our knowledge of geology is meagre, it is possible for us to imagine from the present results that the faults found in the Japanese Islands would be of reverse or normal type, whereas those (imaginable) that neighbour the Japan Trench, or large faults in the Continent of Asia, would probably be of reverse type.

In conclusion, we wish to express our thanks to Messrs. Watanabe and Kodaira who assisted us considerably in our calculations.

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2. 正斷層及逆斷層の生成に關する プラスチック論的條件（第2報）

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金井清 }

地球の重力作用によつて正斷層のできる場合と逆斷層のできる場合のある事をこの前の報告で示して置いた．但し正斷層の生成を説明するに當つて，既成斷層（逆斷層又は正斷層の）で圍まれた地帶の存在を必要とするけれども，之を具體的に解くのは數學的困難を伴ふから，無垢の半無限固體の場合のうちで水平方向に横歪み又は横應力のないやうな狀態のものを以て置換へて考へたのである．

上の考へ方では問題が相當に抽象化されてをるから，プラスチック應力の分布を確める事ができ難い．之を明瞭にするために，この論文では任意の傾斜をなす一つの既成平面斷層が存在する場合に，プラスチック應力が如何に分布するかを計算して見たのである．その結果，新たに生ずる筈の斷層は種々の條件によつて正斷層のみでなく逆斷層にもなり得る事がわかつた．

唯今考へるやうな既成斷層があるときに，地表の變位及び應力は地表に沿うて夫々二次曲線的（一體的變位を除き）及び直線的に變化する．考へんとする地帶の境界をなす既成斷層に摩擦力がないときにもその傾きが 90 度よりも小なるか又は大なるかに從て，地表に生ずる應力及び歪みは壓し又は延びになる．卽ち，この傾きが 90 度よりも小なるか大なるかに從て，地表に新に

ずべき斷層が逆斷層か正斷層かになる．若し既成斷層面に相當の摩擦力があるこ問題が恰度逆になり，その傾斜が 90 度よりも小なるか大なるかに從て，地表に生ずる應力及び歪みが延びになるか壓しになる．この場合には傾きが 90 度よりも小なるか大なるかに從て，地表に新に生ずべき斷層が正斷層か逆斷層になる．

上に述べた事柄は地表についてのみであるが，地帶內部に關しては問題が相當複雜になる．しかし最も簡單な事柄は，斷層面に摩擦力がないときに，この斷層面に生ずる新斷層は常に逆斷層になるこいふ事である．

前回の論文及びこの論文に照して，正斷層の生成は地殼の局部的條件にのみ關係するけれごも逆斷層のそれは地殼の局部的條件及び地球全體の條件の兩方に關係する事がわかる．

この論文では既成斷層が唯一個ある場合を考へたが，之は計算の便宜上のためである．既成斷層が數多くある場合の方がより一般的である．又，既成斷層でなく，既成の皺曲の場合を考へても唯今の結果こ似たものができるやうに考へられる．

地質に關する我々の知識は甚だ貧しいけれごも，我々の計算結果から次の事だけはいはれる．內地に見出だされる斷層は正斷層であつたり，逆斷層であつたりしてよいが，日本海溝に接續するこ考へられる深い大斷層面や，亞細亞大陸內の大皺曲は恐らく殆ご逆斷層及びそれに相應すべき地殼變形のみであうらこいふ事である．