3. *Temperature Distribution within a Semi-gaseous Earth. Part III.*

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1. *Probable liquefaction of the earth in its early stage.*

If our earth was formed by the ejection of a gaseous filament from the central nucleus of the solar system, then, owing to heat radiation as well as condensing temperature at every point in the earth, the period of such a gaseous state could not have been very long.[1), 2)] Although the cooling of the gaseous earth in its early stage was mainly due to heat radiation, since the state of the gas was then a highly ionized one, the treatment of the problem is greatly complicated, for which reason the condition of heat radiation will not be discussed at present. However, even should the problem be restricted to the condition whether or not the temperature of any gaseous part of the earth is lower than the condensing point corresponding to that part, the change of molecular weights with increase in temperature should invariably be taken into account, in consequence of which we shall use such a formula as $\mu \propto T^{-\frac{1}{7}}$, particularly for relatively high temperature.[3)]

As to the polytropic relation of the gas, although it appears that the condition $n=3$ or $n=3/2$ is important in the case of a gaseous planet or of mono-atomic iron, since the mathematical treatment of the problem is not generally simple, the problem of a semi-gaseous earth that we gave previously was restricted to the condition that $n=1$, which case can be treated by means of simple mathematical formulae.[4)] Although the condition that $n=\infty$ corresponds to the isothermal distribution of the gas, an extremely hypothetical one, the same condition was also shown for obtaining an idea of the critical case of the problem.[5)] Emden,[6)] using the trial and error method, obtained the solutions for

1) J. H. JEANS, *The Problem of Cosmogony and Steller Dynamics* (1919).
2) H. JEFFREYS, *The Earth* (1929), 28.
3) A. S. EDDINGTON, *The Internal Constitution of the Stars* (1926), 258.
4) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 525~538.
5) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 675~684.
6) R. EMDEN, *Gaskugeln* (1907), 47~61; E. A. MILNE, *Handbuch d. Astrophysik*, **3**, 1 (1930), 186~189.

different n's in such a special case as that of a coreless gas sphere. We shall now extend his method to the condition of a semi-gaseous earth, the cases discussed being $n=2\cdot5$ and $n=4$, from which, together with the case $n=1$, the condition of the problem of a practically important case, namely, $n=3$ or $n=3/2$, can be readily ascertained by interpolation. In the present paper, some results of the previous investigation concerning $n=1$ and $n=\infty$, in improved forms, will furthermore be shown in confirmation of the conditions $n=2\cdot5$ and $n=4$.

2. *Mathematical conditions for a polytropic gas surrounding a liquid nucleus.*

Although some of the mathematical conditions were shown in the previous papers, for obtaining a complete idea of the problem, we shall rewrite the important parts in the conditions in question. Let ϕ be the gravitational potential and M_r the mass within the radius r of a spherically distributed mass, and G the gravitational constant, namely $6\cdot66.10^{-8}$ in C. G. S. Then we have

$$d\phi/dr=-GM_r/r^2, \tag{1}$$

from which the hydrostatic relation for the gaseous part is

$$dP=\rho d\phi. \tag{2}$$

The second condition satisfied by the gaseous part is Poisson's equation

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}=-4\pi G\rho. \tag{3}$$

Another condition is that for the polytropic distribution of the gas, the equation of which is

$$P=\kappa\rho^{\gamma}, \tag{4}$$

where $\gamma=1+1/n$. From (2) and (4), we get

$$\gamma\kappa\rho^{\gamma-2}d\rho=d\phi, \tag{5}$$

which, when integrated, becomes

$$\{\gamma/(\gamma-1)\}\kappa\rho^{\gamma-1}=\phi+C, \tag{6}$$

the zero of ϕ being arbitrary. Although by the usual convention, the same zero is so taken that ϕ vanishes at infinity, in the present case the potential is so adjusted that it is zero at the outer boundary of the gaseous part, from which $C=0$.[7] Remembering that $\gamma=1+1/n$, we have the relation

$$\rho=\{\phi/(n+1)\kappa\}^n, \tag{7}$$

Substituting this in (4), we have

$$P=\rho\phi/(n+1). \tag{8}$$

From (3) and (7), we get

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}+a^2\phi^n=0, \tag{9}$$

where

$$a^2=4\pi G/\{(n+1)\kappa\}^n. \tag{10}$$

Equation (9) is that of Poisson, in which the condition of polytropic gas change is involved.

If the integral of the differential equation (9) can be found, it is then possible to determine the solution of the problem, using the boundary conditions at the surface of the liquid nucleus and those at the outer boundary of the gaseous part. Let R_1, R be the radii of the liquid nucleus and the outer boundary of the gaseous part respectively, and M_1, M the masses within R_1, R respectively. The conditions in question are such that

$$d\phi/dr=-GM_1/R_1^2 \tag{11}$$

at $r=R_1$ and

$$d\phi/dr=-GM/R^2, \quad \phi=0 \qquad (12), (13)$$

at $r=R$. There is an additional condition wherein

$$M-M_1=\int_{R_1}^{R}4\pi\rho r^2dr. \tag{14}$$

These conditions are sufficient for determining the problem.

With a view to ascertaining the temperature distribution, we shall consider the condition of a perfect gas, the gas equation of which is

$$\beta P=\Re\rho T/\mu \tag{15}$$

for any n, where $\Re$ is the universal gas constant $8{\cdot}26.10^7$, μ the mean molecular weight in terms of the hydrogen atom, and β the ratio of the gas pressure to the whole pressure. From (8), (15) we get

$$T=\beta\mu\phi/(n+1)\Re, \tag{16}$$

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7) A. S. Eddington, *loc. cit.*, 3), 80.

which is the relation between T and ϕ. Eddington[8] gives the relation

$$1-\beta=0\cdot 00309\ (\mathfrak{M}/\odot)^2\mu^4\beta^4, \tag{17}$$

where $\mathfrak{M}$, $\odot$ are the masses of the star (the earth in the present case) and that of the sun respectively. Since the term on the right-hand side of the above expression is very small, $\beta=1$ approximately holds. As to the value of μ, there is some uncertainty. By assuming that the gaseous part is ionized at a relatively high temperature, say, $T_0>10{,}000°K$, T_0 being the central temperature, we shall put $\mu\propto T^{-\frac{1}{7}}$. On the other hand, Hevesy shows that the mean molecular weight of the different substances, assuming the abundance of every substance that is found in the earth, is nearly 32 in the ordinary state. In our calculation here, we shall therefore assume that $\mu=32$ for $T_0=4{,}000°K$, but $\mu\propto T_0^{-\frac{1}{7}}$ for $T_0>10{,}000°K$, μ being assumed to be constant.[9]

Since the pressure distribution of the gaseous part is given by (8), it is also possible to determine the condensing temperature at every point in the gaseous part. Mercier[10] has called attention to the Clapeyron-Clausius law showing the relation between the heat of evaporation, pressure and the condensing point of a vapour, namely[11]

$$\lambda_0=4\cdot 571\frac{1}{1/T_1-1/T_2}\log_{10}(p_2/p_1), \tag{18}$$

where λ_0 is the heat of evaporation, and T_1, T_2 are the condensing points at pressures p_1, p_2 (partial pressures of the vapour for a given condensing substance), respectively. Although λ_0 diminishes with increase in temperature, since the decrement in question is not considerable, we assume that λ_0 is constant. In the present calculation, T_1 will stand for the condensing temperature of the substance (to be condensed) at atmospheric pressure p_1.

Although all the formulae just given were availed of in the present study, since the actual treatment was not simple, some further devices in calculation were resorted to, details of which will be shown in the next section.

3. *The method of numerical calculation.*

As already said, unless $n=1$, numerical treatment is difficult. When,

8) A. S. EDDINGTON, *loc. cit.* (3), 117.

9) For simplicity, we write $\mu\propto T_0^{-\frac{1}{7}}$ and not $\mu\propto T^{-\frac{1}{7}}$, the corresponding error being however rather small.

10) A. MERCIER, *Archives d. sci. e. natur.* 20 (1938), 31~58.

11) See, for example, F. H. MACDOUGALL, *Thermodynamics and Chemistry* (1926), 132; O. SACKUR CL. v. SIMSON, *Lehrbuch d. Thermochmie u. Thermodynamik.* 2-Au^a. (1928), 162.

particularly, $n=1$, the solution of (9) is of the form

$$\phi = A \sin a(r-\varepsilon)/r, \tag{19}$$

from which, with boundary conditions, (11), (12), (13), it is possible to determine fairly accurately the distribution of ϕ in the gaseous part.

When $n \neq 1$, that is to say, when $n=2{\cdot}5$ or $n=4$, we shall, as Emden[12] did, write $r_1 = ar$, from which (9) transforms to

$$\frac{d^2\phi}{dr_1^2} + \frac{2}{r_1}\frac{d\phi}{dr_1} + \phi^n = 0. \tag{20}$$

If furthermore we put

$$r_1 = e^{-\vartheta}, \quad \phi = e^{\lambda\vartheta} z, \quad \lambda = 2/(n-1), \quad dz/d\vartheta = y, \tag{21}$$

that is to say,

$$z = \phi r_1^\lambda, \quad y + \lambda z = -(d\phi/dr_1) r_1^{\lambda+1}, \qquad (22), (23)$$

equation (20) reduces to

$$y = -\left\{\frac{2(3-n)}{(n-1)^2} z + z^n\right\} \Big/ \left(\frac{dy}{dz} + \frac{5-n}{n-1}\right). \tag{24}$$

For solving this equation we shall draw the curves as shown in Figs. 1, 2, in which every curve represents the relation between y and z in (24)

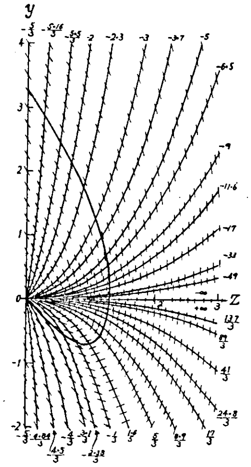

Fig. 1. $n=2{\cdot}5$.

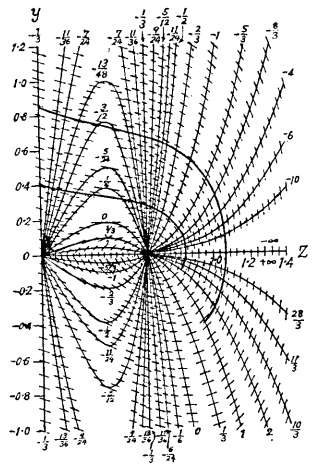

Fig. 2. $n=4$.

with the condition that dy/dz is constant in every case. Therefore, any

12) R. EMDEN, *loc. cit.* 6).

continuous track line (thick curves in Figs. 1, 2) drawn so as to intersect the above system of curves at angles whose tangents are equal to dy/dz of the respective curves just given, is the solution of equation (24).

The boundary conditions (11), (12) are now replaced by

$$d\phi/dr_1=-GM_1/\alpha R_1^2, \quad d\phi/dr_1=-GM/\alpha R^2 \qquad (25), (26)$$

at $r_1=\alpha R_1$, $r_1=\alpha R$, respectively. For getting the polytropic solutions for a coreless sphere, Eddington[13] made such substitutions as

$$\phi=\phi_0 u, \quad r=z_1/(\alpha\phi_0^{\frac{1}{2}(n-1)}), \qquad (27), (28)$$

ϕ_0 being the value of ϕ at $r=0$. The value of

$$M'=(-z_1^2du/dz_1)_{u=0}, \qquad (29)$$

which corresponds to a condition at the outer boundary of the sphere, is contained in his Table. But, since there is the relation $GM=(-r^2d\phi/dr)_{u=0}$, we get immediately

$$GM/M'=1/\{\alpha\phi_0^{\frac{1}{2}(n-3)}\}, \qquad (30)$$

from which it is possible to get the value of α for the gas sphere without nucleus, provided that the central temperature T_0 of the same sphere were given, the relation between T_0 and ϕ_0 being the same as in (16). It is assumed that if the initial central temperature were the same, the value of α just determined would apply to any stage of the liquid nucleus that is condensing. Now, if the density of the liquid nucleus be given, say, 5·525,[14] that is, the mean density of the present earth, the boundary condition (25) is known for any R_1, so that although the right-hand side of (23) is given, the right-hand side of (22) is not.

If, temporarily, we assume the value of ϕ at $r=R_1$, it is possible to get the coordinates y, z of a point in the y-z plane that corresponds to the outer boundary of the nucleus, through which coordinate point a continuous track line can be drawn. When the track line reaches the y axis, ϕ vanishes in virtue of (22), which condition represents the outer boundary of the gaseous part. If the total mass of the semi-gaseous earth is assumed to be the same as that of the present earth, namely, $M=5{\cdot}985.10^{27}$ C. G. S., the outer radius of the gaseous part is then determined by means of (23), (26). For an intermediate point on any track line in Figs. 1, 2, there are three relations to determine

13) A. S. EDDINGTON, *loc. cit.* 3), 81~83.

14) This numerical value was taken merely as a working hypothesis.

ϕ, $d\phi/dr$, r, of which two are given in (22), (23), the remaining one being of the same type as (25) or (26). But since in our present calculation, the integration of a gas density whose distribution is not yet known is very troublesome, we drew, by trial and error method, a continuous curve between the two limiting points in the y-z plane, every point in the curve satisfying the relations (22), (23). There appears to be only one continuous curve that fits into these relations.

It should now be remembered that in the above treatment, the value of ϕ at the boundary of the liquid nucleus was temporarily assumed. Thus, even if the gas density that is distributed according to the law in (7) be integrated, as shown in the right-hand side in (11), the integral would not generally satisfy the relation in (11). For this reason, we shall now change the value of ϕ at the surface of the liquid nucleus, so that the position of the point of beginning of a track line in Figs. 1, 2 changes. The calculation last given is then performed again. In this way, a series of calculations are performed repeatedly, until relation (11) is almost satisfied.

4. *The numerical calculation and its results.*

We shall now consider a primitive gaseous earth for three cases of its central temperature T_0, namely, (I) $T_0=4{,}000°K$, (II) $T_0=26{,}000°K$, and (III) $T_0=48{,}000°K$. Although it is unlikely that the second or third case is obtainable in the condition of the primitive earth, it may serve to ascertain the temperature of liquid metal condensing from the gaseous part of different initial temperatures. For the reason shown in Section 2, we shall use different values of molecular weight. Thus, for (II) $T_0=2{,}600°K$, μ is nearly 16·7 and for (III) $T_0=4{,}800°K$, μ becomes 15·3, but for (I) $T_0=4{,}000°K$, it is assumed that μ remains 32 under the assumption that the gaseous part is not highly ionized.

As to the indices of the polytropy, we shall take (1) $n=1$, (2) $n=2·5$, (3) $n=4$, (4) $n=\infty$.

The radius of the liquid nucleus is generally taken either as (a) $R_1=0$[15] or (b) $R_1=3{,}000$ km, or (c) $=6{,}000$ km.

The numerical values of the constants (some of them were shown in previous sections) commonly used, are $\mathfrak{R}=8·26.10^7$, $\beta=1$, $G=6·66.10^{-8}$, $M=5·985.10^{27}$, $M_1=(4/3)\pi\,5·525\,R_1^3$, in C. G. S. units. Assuming that the iron (or nickel) is mostly condensed, we shall take $T_1=2{,}800°K$ at $p_1=1$ atm. ($=1·031.10^6$ C. G. S.) and $\lambda=9.10^4$ Cal.[16] Hevesy, however,

15) This case was treated by Mercier, *loc. cit.* 9).
16) *International Critical Tables*, **1** (1926), 102.

obtained a result that showed the atomic quantity of iron in the earth to be nearly 30 percent, with its atomic weight 55·84, the ratio of its mass to that of the whole earth being about 50 percent. The partial pressure for iron in the gaseous part could then be about 30 percent

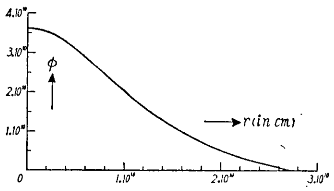

Fig. 3. Distribution of ϕ for T_0= 4,000°K; n=2·5, R_1=0.

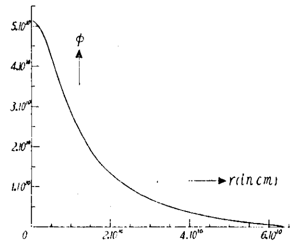

Fig. 5. Distribution of ϕ for T_0= 4,000°K; n=4, R_1=0.

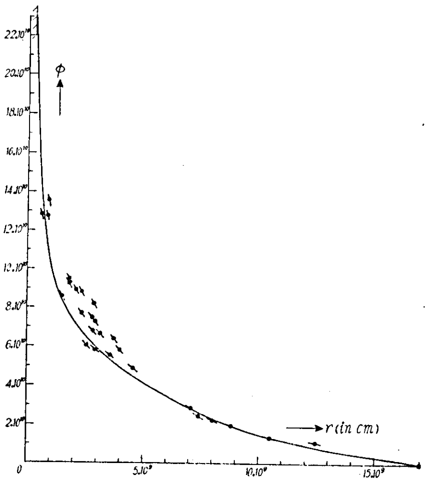

Fig. 4. Distribution of ϕ for T_0= 4,000°K; n=2·5, R_1=3,000 km.

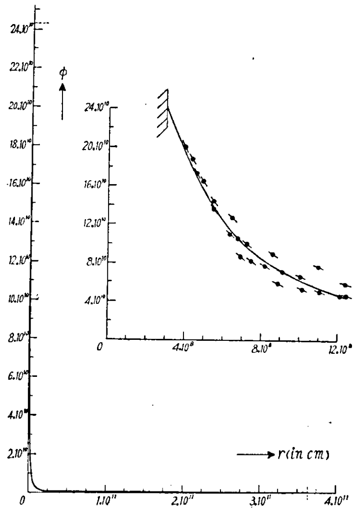

Fig. 6. Distribution of ϕ for T_0= 4,000°K; n=4, R_1=3,000 km.

of the gas pressure. Thus, even if the pressure calculated from ϕ be 1 atm., p_2 in the expression (18) would be 0·3 atm.

The calculations for $n=1$ and $n=\infty$ were made with the aid of mathematical formulae and that for $n=2·5$ and $n=4$ by the trial and error method shown in the preceding section; the results of calculation for the distributions of ϕ in cases $n=2·5$ and $n=4$ are shown in Figs. 3∼8, and those in cases $n=1$ and $n=\infty$ in the previous papers.[17] The

17) *loc. cit.* 4), 5).

parts shown with hatches in these figures correspond to the liquid nucleus.

The temperature distributions for cases $n=1, 2{\cdot}5, 4, \infty$ were determined in every case, with the aid of equation (16), the results of which

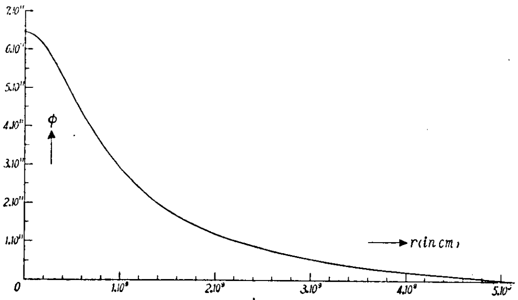

Fig. 7. Distribution of ϕ for $T_0=26{,}000°K$; $n=4$, $R_1=0$.

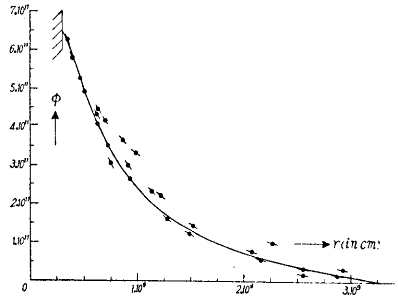

Fig. 8. Distribution of ϕ for $T_0=26{,}000°K$; $n=4$, $R_1=3{,}000$ km.

are shown by full lines in Figs. 9∼26 (cases for $T_0=4{,}000°K$ in Figs. 9∼16, those for $T_0=26{,}000°K$ in Figs. 17∼21, and those for $T_0=48{,}000°K$ in Figs. 22∼26). The part shown with hatches in every figure represents the radius of the liquid nucleus. As just said, the distribution

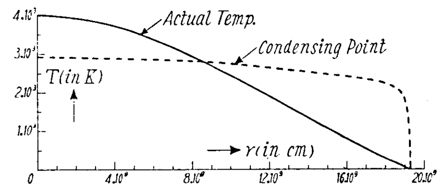

Fig. 9. Distributions of temperatures for $T_0=4{,}000°K$; $n=1$, $R_1=0$.

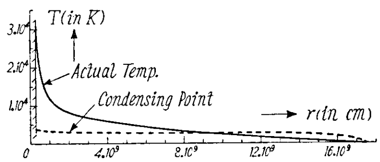

Fig. 10. Distributions of temperatures for $T_0=4{,}000°K$; $n=1$, $R_1=3{,}000$ km.

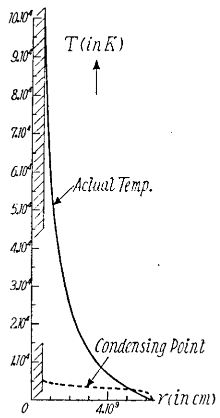

Fig. 11. Distributions of temperatures for $T_0=4{,}000°K$; $n=1$, $R_1=6{,}000$ km.

of the condensing points for iron corresponding to the gas pressure was obtained by means of (18), the results of which are shown by broken lines in the same figures.

As shown in (7), since the density distribution is proportional to ϕ^n, the decrease in density with radius in the gaseous part is more pronounced for a larger n. The density near the outer boundary of the gaseous part tends to be zero in every case.

We shall next consider the temperature distributions of the gaseous part and the corresponding condensing points. In every case (excepting that for (II 4) $n=\infty$, $T_0=26{,}000°K$ and (III 4) $n=\infty$, $T_0=48{,}000°K$) the curve for the temperature distribution and that for the

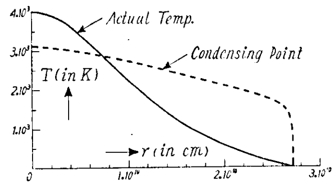

Fig. 12. Temperature distribution for $T_0=4{,}000°K$; $n=2{\cdot}5$, $R_1=0$.

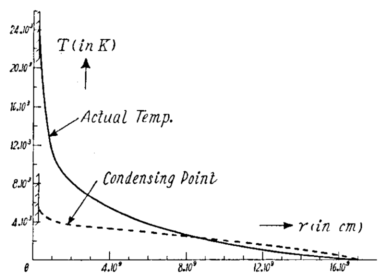

Fig. 13. Temperature distribution for $T_0=4\ 000°K$; $n=2{\cdot}5$, $R_1=3{,}000$ km.

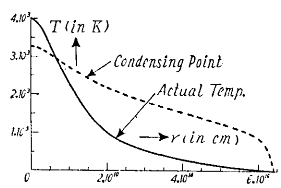

Fig. 14. Temperature distribution for $T_0=4{,}000°K$; $n=4$, $R_1=0$.

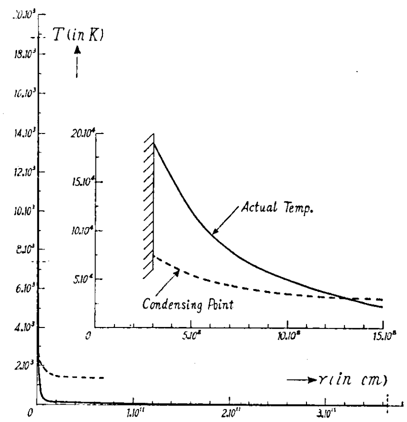

Fig. 15. Temperature distribution for $T_0=4{,}000°K$; $n=4$, $R_1=3{,}000$ km.

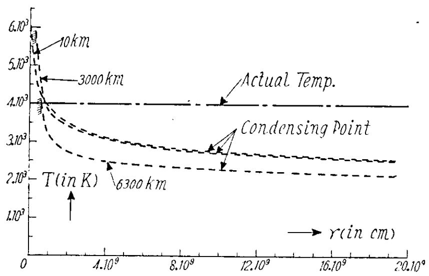

Fig. 16. Temperature distribution for $T_0=4{,}000°K$; $n=\infty$; $R_1=10$, 3000, 6300 kms.

condensing point intersect at a certain radius. Within the spherical surface corresponding to this intersection, the actual temperature is always higher than the condensing point, whereas, outside the same surface, the reverse condition holds, showers of metals being likely to fall from that part of large radius.[18] It will furthermore be seen that the tem-

18) A. MERCIER, *loc. cit.* 10). In Mercier's case, the condition of the problem is very similar to the present one.

perature of the gas (and also the condensing point) at the spherical surface corresponding to the intersection of both curves just given is in the range between 3,000°K and 4,000°K in the case of $T_0=4{,}000°K$ and between 5,000°K and 7,000°K in the case of $T_0>26{,}000°K$, regardless of whether the temperature of the primitive gas sphere is high

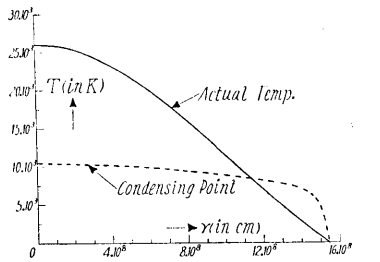

Fig. 17. Temperature distribution for $T_0=26{,}000°K$; $n=1$, $R_1=0$.

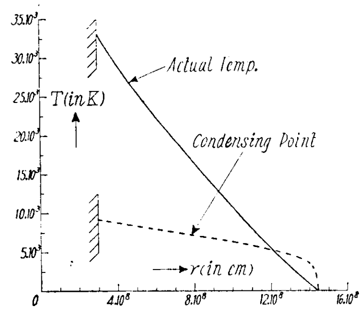

Fig. 18. Temperature distribution for $T_0=26{,}000°K$; $n=1$, $R_1=3{,}000$ km.

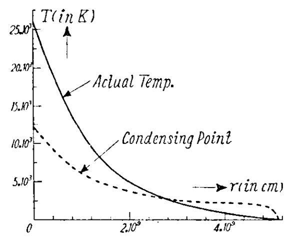

Fig. 19. Temperature distribution for $T_0=26{,}000°K$; $n=4$, $R_1=0$.

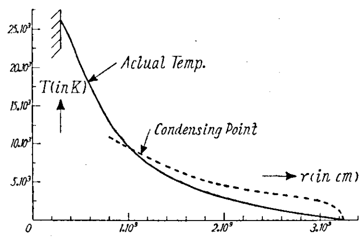

Fig. 20. Temperature distribution for $T_0=26{,}000°K$; $n=4$, $R_1=3{,}000$ km.

or low. Besides, with increase in the size of the liquid nucleus, the temperature in the gaseous part near the same nucleus tends to increase, whereas that near the spherical surface corresponding to the intersection of the two kinds of curves in Figs. 9∼26 (excepting those in Figs. 21, 26) is still from 3,000°K to 4,000°K in the case of T_0 being 4,000°K and from 5,000°K to 7,000°K in the case of T_0 being greater than 26,000°K. It is then likely that, unless the condition is isothermal, the temperature in the shower when it is formed will invariably be of a certain range, and not very high.

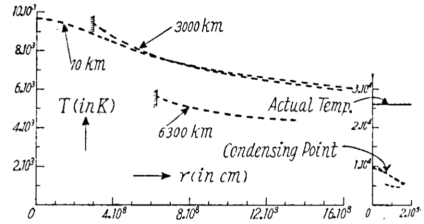

Fig. 21. Temperature distribution for $T_0=26{,}000°K$; $n=\infty$; $R_1=10$, 3000, 6300 kms.

Since, on the other hand, the temperature in the gaseous part near the liquid nucleus is higher than the condensing point, it is possible for the liquid metal of the shower or in the nucleus to evaporate. The temperature of the liquid metal of the shower is raised by direct heating as well as by the resistance of the gaseous part near the nucleus, whereas that in the nucleus rises with its surface being always exposed to the high temperature of the gaseous part, from which circumstances the problem of liquefaction becomes complex.

5. *Conditions for the liquefaction of the gaseous earth, and concluding remarks.*

From the statements made at the end of the last section, it would be difficult to show whether or not the liquid nucleus grows under both influences of the shower fall of and the evaporation of the liquid metal. As to the vaporization of the liquid metal in the shower, since the time required for its traverse through the gaseous part is not very long, and since its traversing velocity is not very great, it is likely that the shower remains in a liquid condition until it reaches the nucleus, even without a great rise in its temperature. As to the evaporation of the liquid metal in the nucleus, since fresh liquid metal in the shower drops upon the surface of that nucleus in continuous succession, and since, moreover, convection movements of the metal in the same nucleus is not likery to occur, it would be impossible to say that the effect of evaporation will exceed that of the shower.

The conditions given above are merely probable and does not generally hold. If, eventually, the rate of fall of the shower of liquid

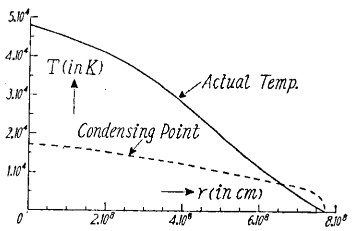

Fig. 22. Temperature distribution for $T_0=48,000°K$; $n=1$, $R_1=0$.

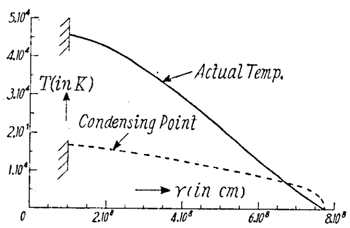

Fig. 23. Temperature distribution for $T_0=48,000°K$; $n=1$, $R_1=1,000$ km.

metal is below a certain critical point, the liquid nucleus cannot grow, whereas if that rate exceeds this point, growth of the nucleus is then possible. For the possible formation of a liquid state of the earth from its gaseous state even in the former condition of rate of shower fall it is requisite that heat shall radiate from the gaseous part. From the investigations of many authors, it appears that radiation of heat is

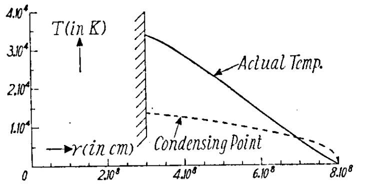

Fig. 24. Temperature distribution for $T_0=48{,}000°K$; $n=1$, $R_1=3{,}000$ km.

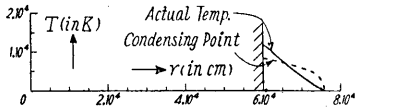

Fig. 25. Temperature distribution for $T_0=48{,}000°K$; $n=1$, $R_1=6{,}000$ km.

sufficient cause for the formation of a liquid earth, so that the condition shown in this paper may be an additional cause in the liquefaction of a gaseous earth. It should be remembered that in the gaseous state of the earth, atomic disintegration and a radioactive state of nearly all substances are possible, resulting in evolution of heat. But this heat would be much less than that in radiation, and it would not even be enough to overcome the liquid metal's falling as a shower.

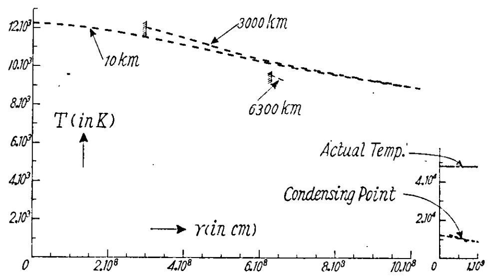

Fig. 26. Temperature distribution for $T_0=48{,}000°K$; $n=\infty$; $R_1=10$, 3000, 6300 kms.

In the present paper, no clear explanation of the rate of shower fall has been given. But, seeing that the temperature and the condensing point in the gaseous part are distributed almost in the same manner for any stage of growth of the liquid nucleus (Figs. 9∼26), it is possible to imagine that the condensation of the liquid metal in the manner of the present anaylsis will proceed much more quickly than in the case of cooling of a gaseous sphere with gradual radiations of heat, in consequence of which there is still the possibility of liquefaction of the gaseous earth, even without regard to heat radiation.

It should be borne in mind that in the calculation of temperature distribution, we have taken into account the ionisation merely in such a sense as will affect the values of the molecular weights. If every effect of that ionisation were considered, the problem would be concerned with a number of recently developed concepts of physics, the treatment being then far beyond this classical theory. Another condition to be noted is that, when iron (or nickel) is almost liquefied, the partial pressures of the various substances still remaining in the gaseous part should be relatively low, the condensing points for these substances being then very low, in consequence of which condensation of the gaseous part is not likely to occur.[19]

In conclusion, we wish to express our thanks to Messrs. Watanabe and Kodaira, who assisted us much in the numerical calculation. In the course of the present investigation, Professor Nagaoka and Dr. Shirai gave us many valuable suggestions, and to these gentlemen we wish also to express our hearty thanks.

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3. 半瓦斯狀地球に於ける溫度分布 (第3報)

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地球が太陽系の中から瓦斯狀のままでとび出たとしても，その瓦斯狀各部の溫度と各部の壓力に相當する液化點との關係によつて瓦斯狀のままで長く留り難い事は前2回の報告に示した通りである．實際問題としては輻射によつて溫度が急激に下り，氣體の狀態が長く續かない譯であるが，我々の研究では，それを別にしても液化し勝ちである事を示したのである．

この前の研究は數式をもつて容易に計算し得る場合のみであつたが，今回は Emden の試みた圖式計算法を液狀核がある場合に適當せしめる事によつて稍一般的な場合を取扱つたのである．又，瓦斯が高溫の場合にイオン化する結果として物質の分子量が減少する事をも計算に入れて見たのである．

計算の結果，液狀核の大さ如何に拘らず半徑の大なる氣界では瓦斯の溫度が液化點よりも低く且つ常に一定以下の溫度になり，半徑の小なる氣界では瓦斯の儘に留り得る事がわかつた．從て高い氣界から金屬の雨が降り，液狀核面からは金屬が蒸發する事になる．しかし種々の事を考に入れると，雨の降る速度の方が速く，液狀核は次第に大きくなる傾向がある．

何れにしても輻射以外にも地球液化の可能性のある事が確められた譯である．尙，瓦斯の高溫に於けるイオン化によつて分子量が變化する事以外に瓦斯體に種々の現象があり得るが，之等をすべて考慮に入れる事はこの古典的方法の範圍外である．

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19) H. LORENZ recently indicated that in the present state of the earth, its core can remain gaseous with high temperature and great density; *Z. f. Geophys.*, **15** (1939), 371～379. We shall study this condition in the not distant future.