46. *Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part IV.*

The Effect of Material Inner Damping.

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1. *Wave scattering (dissipation) and material inner damping.*

In the previous paper[1] we studied wave scattering of the vibrational energy into the ground as affecting the aseismic properties of the brace strut, from which result it was ascertained that the aseismic properties of the brace strut are then much more pronounced in diminishing the resonance amplitudes than in raising the natural frequencies of the structure.

Since, however, the scattering of wave energy exists only in special kinds of structures, say, a reinforced concrete structure standing on a rather soft ground, it would be impossible to expect the existence of aseismic conditions in any structure from the mere fact of wave scattering. On the other hand, there is, in every structure, inner damping by the material regardless of the condition of the ground. It appears that the damping resistance in usual structures originates from this inner damping just mentioned.

The present investigation shows that the effect of the inner damping of the material on the aseismic properties of a braced structure is quite similar to that in the case of wave scattering.

We shall first investigate the case of a single-storied structure with an endless number of spans and next discuss the case of a single-storied structure of mono-span.

2. *Equations of vibratory motion in the case of a single-storied structure with an endless number of spans.*

For simplicity, it is assumed that the masses are concentrated at

1) K. KANAI, "Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part III. The Effect of Wave Scattering (Dissipation)", *Bull. Earthq. Res. Inst.*, **17** (1939), 569—578.

the panel points. Let l_1, l_2, l_3 be the lengths of the columns, beam span, and the length of a brace strut, respectively; let also E_1I_1, E_1a_1; E_2I_2, 0; 0, E_3a_3 be the bending and longitudinal stiffnesses of the respective members just mentioned. It is assumed that the inner damping of the material partakes of the nature of solid viscosity, when the bending and longitudinal damping resistances of the respective members will be of the forms

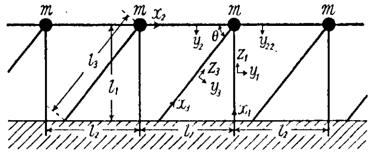

Fig. 1.

$$E_1I_1\tau_1,\ E_1a_1\kappa_1;\quad E_2I_2\tau_2,\ 0;\quad 0,\ E_3a_3\kappa_3. \tag{1}$$

The lateral displacements y_1, y_2, 0 and the longitudinal displacements z_1, 0, z_3 of the respective members satisfy differential equations of the types

$$\frac{\partial^4 y_n}{\partial x_n^4}=0,\quad [n=1,\ 2] \qquad \frac{\partial^2 z_n}{\partial x_n^2}=0.\quad [n=1,\ 3] \tag{2}$$

The solutions of the above equations are

$$\left.\begin{aligned} y_n&=(A_n+B_nx_n+C_nx_n^2+D_nx_n^3)e^{ipt}, \quad &[n=1,\ 2]\\ z_n&=(\alpha_n+\beta_nx_n)e^{ipt}. &[n=1,\ 3]\end{aligned}\right\} \tag{3}$$

If the seismic vibration be of horizontal type, the boundary conditions are such that

$$x_1=0;\qquad y_1=Ue^{ipt},\quad \frac{dy_1}{dx_1}=0,\quad z_1=0, \tag{4), (5), (6}$$

$$x_3=0;\qquad z_3=y_1\cos\theta, \tag{7}$$

$$x_1=l_1,\ x_2=l_2,\ x_{22}=0,\ x_3=l_3;\qquad y_2=y_{22}=-z_1, \tag{8), (9}$$

$$z_3=z_1\sin\theta+y_1\cos\theta,\quad \frac{dy_1}{dx_1}=\frac{dy_2}{dx_2}=\frac{dy_{22}}{dx_{22}}, \tag{10), (11), (12}$$

$$-E_1I_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{d^2y_1}{dx_1^2}-E_2I_2\left(1+\tau_2\frac{\partial}{\partial t}\right)\left(\frac{d^2y_2}{dx_2^2}-\frac{d^2y_{22}}{dx_{22}^2}\right)=0, \tag{13}$$

$$-E_1I_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{d^3y_1}{dx_1^3}+E_3a_3\left(1+\kappa_3\frac{\partial}{\partial t}\right)\frac{dz_3}{dx_3}\cos\theta=m_1p^2y_1, \tag{14}$$

$$E_2I_2\left(1+\tau_2\frac{\partial}{\partial t}\right)\left(\frac{d^3y_{22}}{dx_{22}^3}-\frac{d^3y_2}{dx_2^3}\right)-E_3a_3\left(1+\kappa_3\frac{\partial}{\partial t}\right)\frac{dz_3}{dx_3}\sin\theta$$

$$-E_1a_1\left(1+\kappa_1\frac{\partial}{\partial t}\right)\frac{dz_1}{dx_1}=0, \tag{15}$$

Substituting (3) in (4)～(15), it is possible to determine all the constants, from which the horizontal displacement of the floor is

$$y_{1x_1=l_1}=\frac{UM}{M-\gamma_1(\phi+3\zeta)(\vartheta\sin^2\theta+\psi\nu)}, \tag{16}$$

where

$$\left.\begin{aligned}
&M=3(\phi+12\zeta)(\vartheta\sin^2\theta+\psi\nu)+\vartheta\xi\nu(\phi+3\zeta)\cos^2\theta,\\
&\gamma_1=\frac{m_1p^2l_1^4}{E_1I_1(1+ip\tau_1)},\quad \vartheta=\frac{E_3a_3(1+ip\kappa_3)}{E_1a_1(1+ip\tau_1)},\quad \zeta=\frac{E_2I_2(1+ip\tau_2)}{E_1I_1(1+ip\tau_1)},\\
&\xi=\frac{a_1l_1^2}{I_1},\quad \nu=\frac{1+ip\kappa_1}{1+ip\tau_1},\quad \phi=\frac{l_2}{l_1},\quad \psi=\frac{l_3}{l_1}.
\end{aligned}\right\} \tag{17}$$

The expression (16), which is of complex form, applies to any condition in ν, ϑ, ζ, γ_1, ψ, ϕ. Since the treatment of (16) is rather complicated, we shall calculate the same expression for some special cases of τ_1, τ_2, κ_1, κ_3, by means of which it will still be possible to ascertain the general nature of the problem.

3. *The calculation of the special case* $\tau_1=\tau_2=\kappa_1=\kappa_3$.

If we put $\tau_1=\tau_2=\kappa_1=\kappa_3\equiv\tau$, then (16) transforms to

$$y_{1x_1=l_1}=\frac{UM'\exp.\left\{i\left(pt-\dfrac{N\sqrt{\gamma_0\tau_0}}{M'-N}\right)\right\}}{\sqrt{(M'-N)^2+N^2\gamma_0\tau_0^2}}, \tag{18}$$

where

$$\left.\begin{aligned}
&M'=3(\phi+12\zeta_0)(\vartheta_0\sin^2\theta+\psi)+\vartheta_0\xi(\phi+3\zeta_0)\cos^2\theta,\\
&N=\frac{\gamma_0}{1+\gamma_0\tau_0^2}(\phi+3\zeta_0)(\vartheta_0\sin^2\theta+\psi),\\
&\gamma_0=\frac{m_1p^2l_1^4}{E_1I_1},\quad \vartheta_0=\frac{E_3a_3}{E_1a_1},\quad \zeta_0=\frac{E_2I_2}{E_1I_1},\quad \xi=\frac{a_1l_1^2}{I_1},\\
&\phi=\frac{l_2}{l_1},\quad \psi=\frac{l_3}{l_1},\quad \tau_0=\tau\sqrt{\frac{E_1I_1}{m_1l_1^4}}.
\end{aligned}\right\} \tag{19}$$

Using these expressions it is possible to get resonance curves for the horizontal seismic vibration of the structure for any stiffness condition of the structure.

In the present examples we shall assume that the ratio of beam span to column height, that is, $\phi=l_2/l_1$, is unity, so that $\theta=45°$. Since it is further assumed that the lower end of the brace strut is exactly at the bottom of the column, the ratio of brace length to column height, that is, $\psi=l_3/l_1$, is always $\sqrt{2}$. Assuming that the structure now under consideration is similar to that shown in our previous paper,[2] we write

$$\xi\left(=\frac{a_1 l_1^2}{I_1}\right)=5000,\quad \zeta_0\left(=\frac{E_2 I_2}{E_1 I_1}\right)=10,\quad \psi\left(=\frac{l_3}{l_1}\right)=\sqrt{2},\quad \phi\left(=\frac{l_2}{l_1}\right)=1. \tag{20}$$

We then calculated the resonance curves of vibration amplitudes for various ratios of $\vartheta_0=E_3a_3/E_1a_1$, say, $\vartheta_0=0$, 0·1, 0·2, 1, 5, ∞, in three cases of $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^4})$, namely, $\tau_0=0{\cdot}0001$, 0·001, 0·01, the results being shown in Figs. 2, 3, 4. It should be borne in mind that $\sqrt{\gamma_0}(=\sqrt{m_1p^2l_1^4/E_1I_1})$ is exactly proportional to the vibrational frequency p.

It will be seen that in all cases of τ_0-value, the resonance amplitudes decrease enormously with increase in the stiffness of the brace struts, the resonance frequency increasing at the same time.

Decrease of resonance amplitudes with increase in ϑ_0 is most pronounced for $\vartheta_0=0\sim1$. For $\vartheta_0>1$, on the other hand, the decrease in resonance amplitudes with increase in ϑ_0 is rather gradual.

Comparing Figs. 2, 3, 4, it will furthermore be seen that the greater the value of τ_0, the more decreases the resonance amplitude; the same amplitude is nearly proportional to the reciprocal of damping resistance τ_0 for any ratio of ϑ_0. This shows that high damping resistance is to be recommended for diminishing the resonance amplitudes.

Another feature here is that in every case, with increase in damping resistance or with increase in stiffness of the brace strut, the resonance curve tends to have a flat form.

Although the change in resonance amplitudes with change in the natural vibration periods has been shown in Figs. 2, 3, 4, we shall show, besides, the relation between resonance amplitudes, natural vibration periods, and the stiffness of the brace struts, as given in Figs. 5, 6. It will be seen that, notwithstanding that the resonance amplitude

2) K. KANAI, *loc. cit.* 1).

diminishes enormously with increase in damping resistance, the vibration period remains nearly constant, at least with change in that damp-

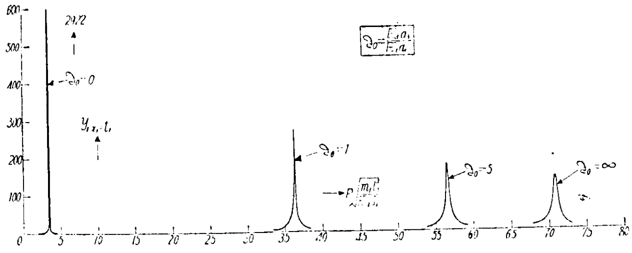

Fig. 2. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}0001$.

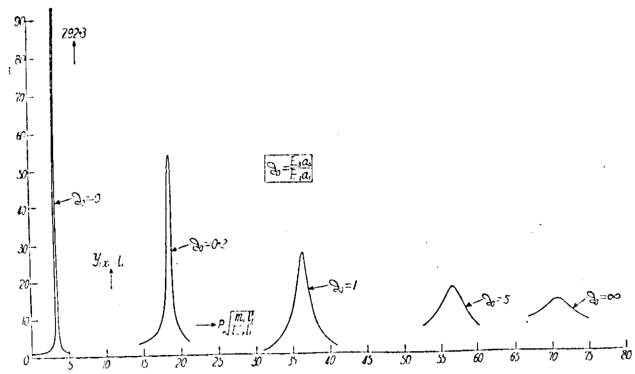

Fig. 3. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}001$.

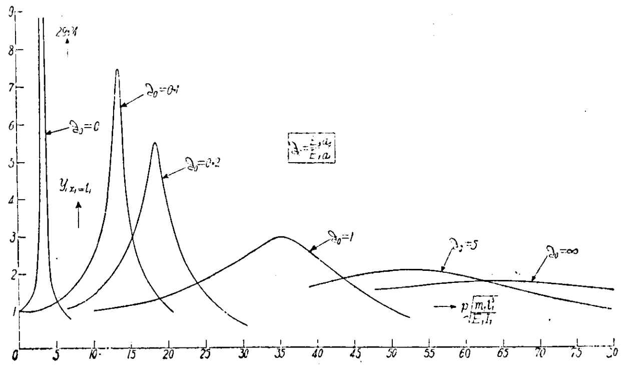

Fig. 4. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}01$.

ing resistance. On the other hand, the change of resonance amplitude and that of resonance frequency with change in the stiffness of the brace strut are of quite similar types. It holds then that if the stiffness of the brace strut were relatively small, an increase in the stiffness of the brace struts would be effective on the increase of natural vibration frequency as well as on the decrease of resonance amplitude. For a relatively large stiffness of the brace strut, the effect of the brace strut on the diminution of resonance amplitudes and also on the increase of natural vibration frequency will not be very marked. An over-strong strut is unnecessary, although it does not show a negative effect.

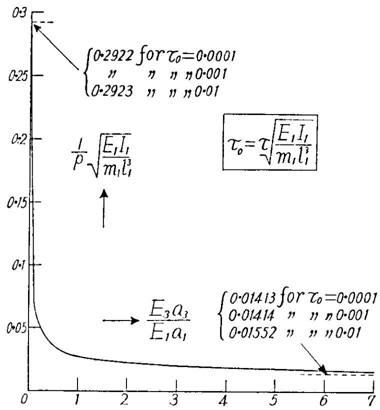

Fig. 5. Natural vibration periods.

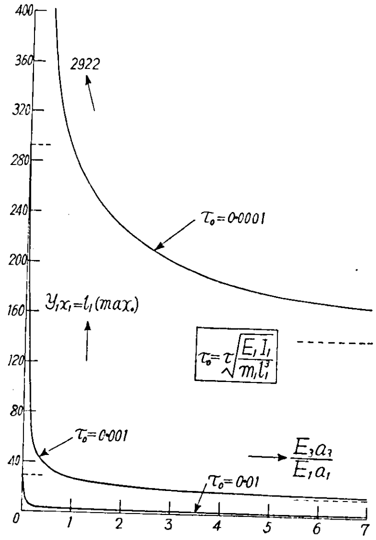

Fig. 6. Resonance amplitudes.

4. *Calculation of the special case,* $\kappa_1=\kappa_3=\tau_1$, $\tau_2=0$.

By putting $\kappa_1=\kappa_3=\tau_1\equiv\tau$, $\tau_2=0$; which represent the condition of no damping in the bending vibration of the beams, (16) transforms to

$$y_{1x_1=l_1}=U\sqrt{\frac{R^2+\gamma_0\tau_0^2S^2}{(R-P)^2+\gamma_0\tau_0^2(S-Q)^2}}\cdot\exp.\left[i\left\{pt+\tan^{-1}\frac{\sqrt{\gamma_0}\tau_0S}{R}-\tan^{-1}\frac{\sqrt{\gamma_0}\tau_0(S-Q)}{R-P}\right\}\right], \tag{20}$$

where

$$\left.\begin{aligned}R=&\left\{3(\phi+12\zeta_0)(\vartheta_0\sin^2\theta+\psi)+\vartheta_0\xi(\phi+3\zeta_0)\cos^2\theta\right\}\\&\qquad-\gamma_0\tau_0^2\phi\left\{3(\vartheta_0\sin^2\theta+\psi)+\vartheta_0\xi\cos^2\theta\right\},\\S=&6(\vartheta_0\sin^2\theta+\psi)(\phi+6\zeta_0)+\vartheta_0\xi(2\phi+3\zeta_0)\cos^2\theta,\\P=&\gamma_0(\phi+3\zeta_0)(\vartheta_0\sin^2\theta+\psi),\\Q=&\gamma_0\phi(\vartheta_0\sin^2\theta+\psi).\end{aligned}\right\}\tag{21}$$

We have calculated the resonance curves of vibration amplitudes with the same data as those of the preceding section, with the excep-

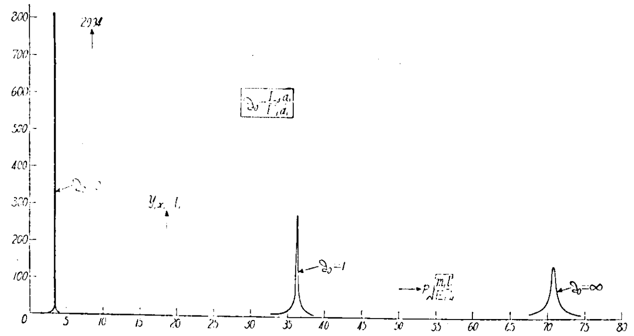

Fig. 7. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0\cdot0001$.

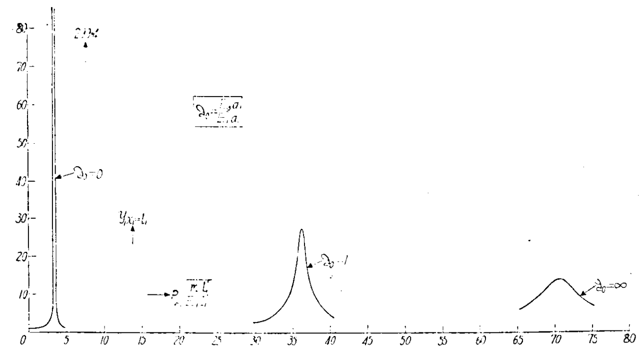

Fig. 8. Rosonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0\cdot001$.

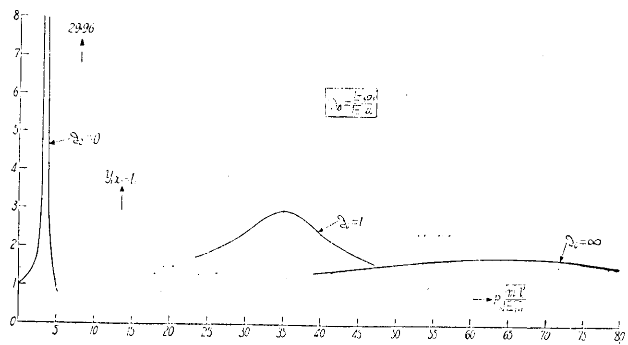

Fig. 9. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/_1ml_1^3})=0\cdot01$.

tion of the conditions $\kappa_1=\kappa_3=\tau_1$, $\tau_2=0$. The results of calculation are shown in Figs. 7, 8, 9. It will be seen that the resonance curves here obtained are almost of the same forms as those given in the preceding section. Although the resonance amplitudes for a smaller ratio of $\vartheta_0(=E_3a_3/E_1a_1)$ are, in the present case, somewhat less than those in the previous case, say, less than by two or three percent, the same amplitudes for a larger ratio of ϑ_0 are nearly the same for both cases.

The relations between resonance amplitudes, natural vibration periods, and the stiffness of the brace struts are also similar to those in the preceding section, as will be seen in Figs. 10, 11. The conclusion given in the preceding section therefore applies directly to the present case.

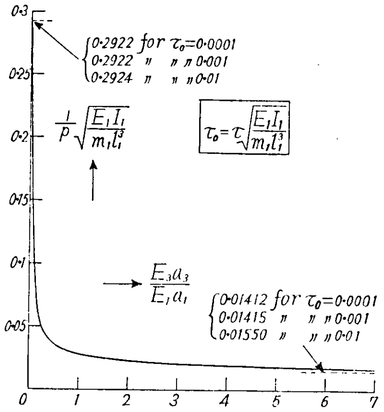

Fig. 10. Natural vibration periods.

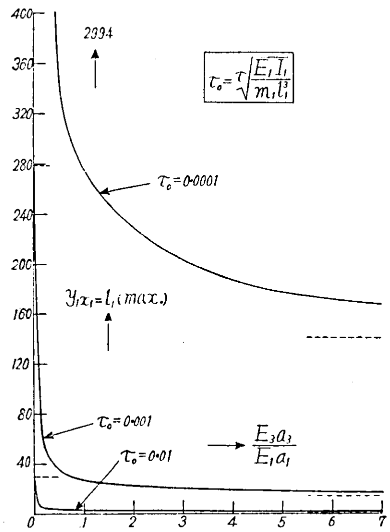

Fig. 11. Resonance amplitudes.

We have now the idea that the damping inner resistance of a braced structure arises mainly from the columns and the brace struts. It is, however, uncertain whether it is the resistance in the columns or the resistance in the brace struts that plays the important part in the damping under consideration.

5. *Calculation of the special case,* $\kappa_1=\kappa_3=\tau_2\equiv\tau,\ \tau_1=0$.

When there is no damping inner resistance in the bending vibration of the columns, equation (16) reduces to

$$y_{[x_1=l_1]}=U\sqrt{\frac{R^2+S^2}{P^2+Q^2}}\,\text{exp.}\left\{i\left(pt+\tan^{-1}\frac{S}{R}-\tan^{-1}\frac{Q}{P}\right)\right\}, \tag{22}$$

where

$$R=3(\phi+12\zeta_0-12\gamma_0\zeta_0\tau_0^2)(\vartheta_0\sin^2\theta+\phi) + \vartheta_0\xi\cos^2\theta\left\{(\phi+3\zeta_0)-\gamma_0\tau_0^2(\phi+9\zeta_0)\right\},$$

$$
\left.\begin{aligned}
S &= \sqrt{\gamma_0}\,\tau_0\Big\{3(\phi+24\zeta_0)(\vartheta_0\sin^2\theta+\psi) \\
&\qquad\qquad + \vartheta_0\xi\cos^2\theta\,(2\phi+9\zeta_0-3\gamma_0\zeta_0\tau_0^2)\Big\}, \\
P &= R+\gamma_0(\phi+3\zeta_0-3\gamma_0\zeta_0\tau_0^2)(\vartheta_0\sin^2\theta+\psi), \\
Q &= S+\sqrt{\gamma_0^3}\,\tau_0(\phi+6\zeta_0)(\vartheta_0\sin^2\theta+\psi).
\end{aligned}\right\} \quad (23)
$$

We have also calculated the resonance curves of the vibration amplitudes with the same data as those in Section 2, excepting the conditions $\kappa_1=\kappa_3=\tau_2$, $\tau_1=0$.

In the present condition, the intermediate case of τ_0, namely, the case $\tau_0=0{\cdot}001$, was omitted. The results of calculation are shown in Figs. 12, 13.

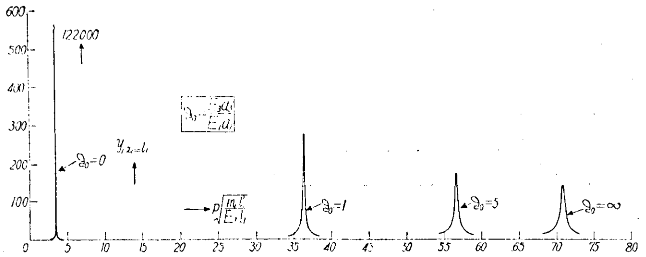

Fig. 12. Resonance curves for $\tau_0(=\tau_1\sqrt{E_1I_1/m_1l_1^4})=0{\cdot}0001$.

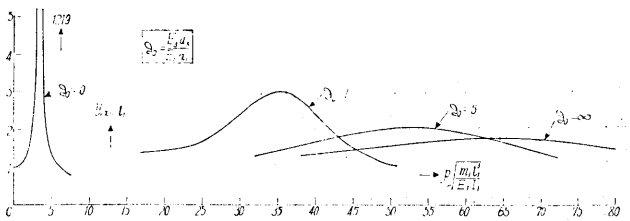

Fig. 13. Resonance curves for $\tau_0(=\tau_1\sqrt{E_1I_1/m_1l_1^4})=0{\cdot}01$.

In the present condition, the resonance amplitudes for the case $\vartheta_0(=E_3a_3/E_1a_1)=0$ are nearly forty times those in the previous cases. The other features of the problem are also very similar to those in the preceding sections. Since the case $\tau_1=0$ corresponds to the condition that no damping resistance exists in the bending vibration of the columns in the braceless structure, it is quite probable that the resonance amplitudes for $\vartheta_0=0$ become fairly large. The resonance amplitudes for the cases $\vartheta_0>1$ are almost equal to those in the previous cases, indicating that the damping action of the inner resistance in the brace struts is much more effective than that in the columns.

The relation between the resonance amplitudes, natural vibration periods, and the stiffness of the brace struts is shown in Figs. 14, 15, these curves being also similar to those in the previous sections.

At all events, from the results in Section 4 and the present section it is possible to conclude that the inner dampings of the material of the columns as well as of the beams do not participate much in the decrease in resonance amplitudes. Brace struts of moderate stiffness and of the greatest possible damping resistance are likely to be those most adapted for contributing to the aseismic properties of a structure.

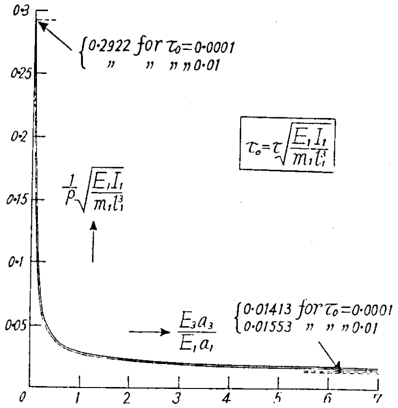

Fig. 14. Natural vibration periods.

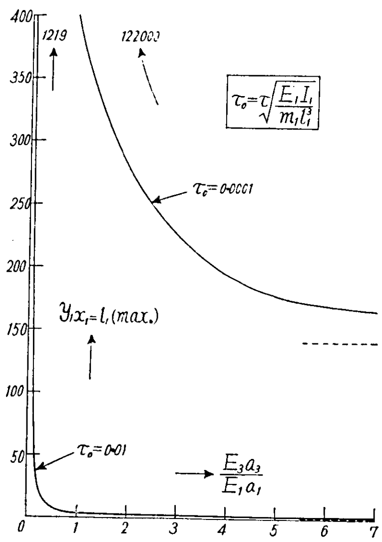

Fig. 15. Resonance amplitudes.

6. *Calculation of the speical case* $\kappa_1=\kappa_3=0$, $\tau_1=\tau_2$.

In this case no damping resistance exists in the longitudinal vibrations of the columns as well as in the brace struts. Since damping does not exist also in the bending vibration of the brace struts, the brace struts have no damping.

The conditions $\kappa_1=\kappa_3=0$, $\tau_1=\tau_2\equiv\tau$ reduce equation (16) to

$$y_{1x_1=l_1}=U\sqrt{\frac{R^2+S^2}{P^2+S^2}}\exp.\left\{i\left(pt+\tan^{-1}\frac{S}{R}-\tan^{-1}\frac{S}{P}\right)\right\}, \tag{24}$$

where

$$\left.\begin{aligned} R&=3(\phi+12\zeta_0)(\vartheta_0\sin^2\theta+\psi)+\vartheta_0\xi(\phi+3\zeta_0)\cos^2\theta, \\ S&=3\sqrt{\gamma_0}\,\tau_0(\phi+12\zeta_0)(\vartheta_0\sin^2\theta+\psi), \\ P&=R-\gamma_0(\phi+3\zeta_0)(\vartheta_0\sin^2\theta+\psi). \end{aligned}\right\} \quad (25)$$

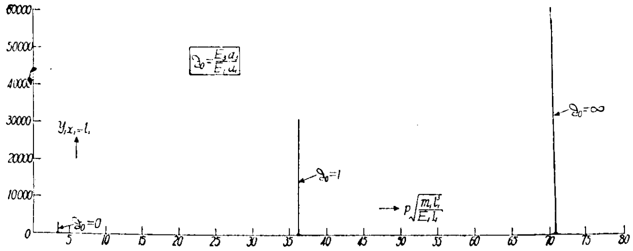

Fig. 16. Resonance curves for $\tau_0(=\tau_1\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}0001$.

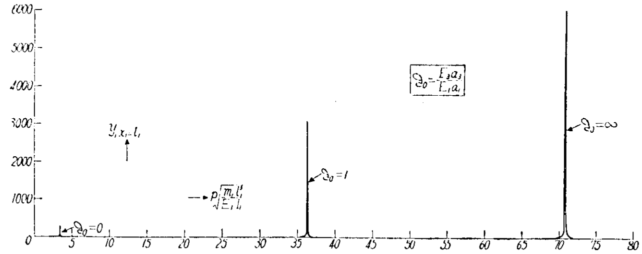

Fig. 17. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}001$.

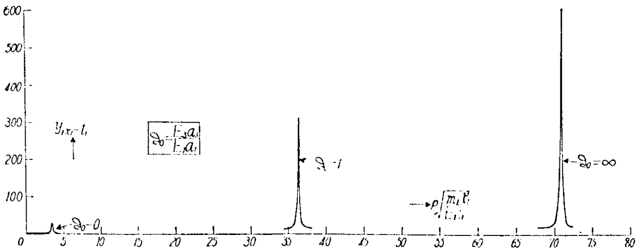

Fig. 18. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}01$.

We have calculated the resonance curves of the vibration amplitudes with the same data as those in Sections 3, 4, 5, with the exception of conditions $\kappa_1=\kappa_3=0$, $\tau_1=\tau_2$, the results of which are shown in Figs. 16, 17, 18. It will be seen that although the resonance amplitudes for the case $\vartheta_0=0$, that is, the case of the braceless structure, are nearly the same as those in Sections 3, 4, they are much less than those in

Section 5, which fact shows that, although the damping in the bending vibration of the columns is effective in diminishing the resonance amplitude, the damping in the longitudinal vibration in the same columns is not.

The resonance curves for the cases $\vartheta_0 > 1$ are very odd. The resonance amplitudes increase with increase in the ratio of ϑ_0. Although the resonance frequency tends to increase with increase in ϑ_0, the condition is quite reversed in the case of resonance amplitudes.

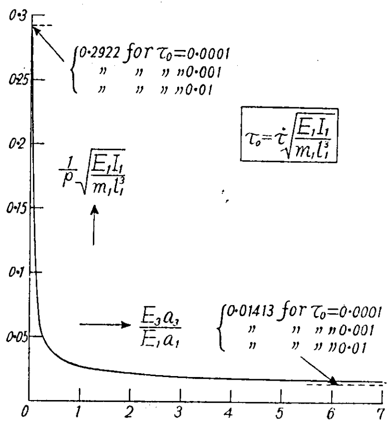

Fig. 19. Natural vibration periods.

The above fact may be interpreted as follows. The elastically important part in the structure is, as a matter of fact, the brace struts. The important part in the damping of the structural vibration would also be the brace struts. With increase in ϑ_0, the brace struts become more important in the vibration of the structure. Since, in the present case, on the other hand, no damping inner resistance exists in the same brace struts, effective damping of the vibration would then decrease, from which condition it is likely that the resonance amplitudes increase without limit with increase in the ratio of ϑ_0.

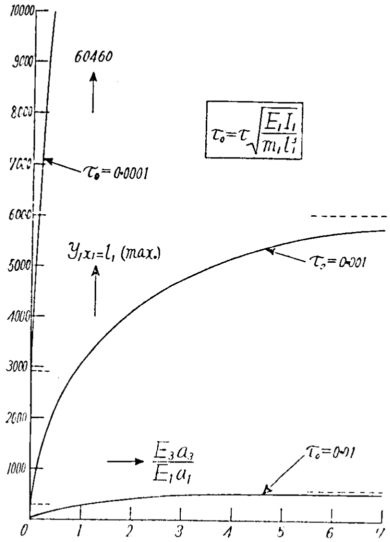

Fig. 20. Resonance amplitudes.

The relation between the resonance amplitudes, natural vibration periods, and the stiffness of the brace struts is shown in Figs. 19, 20. From the condition of the problem just mentioned, the feature of the vibrational periods is exactly the reverse of that of the resonance amplitudes for any value of τ_0.

7. *The equations of vibratory motion for the case of a single-storied structure of mono-span.*

As in the case of an endless number of spans, we shall use the quantities $l_1, l_2, l_3, E_1I_1, E_1a_1, E_2I_2, E_3a_3, E_1I_1\tau_1, E_1a_1\kappa_1, E_2I_2\tau_2, E_3a_3\kappa_3$, the meanings of which are the same as those in the last case. The lateral displacements y_{11}, y_{12}, y_2, 0 and the longitudinal displacements z_{11}, z_{12}, 0, z_3 of the four members show in Fig. 21 are

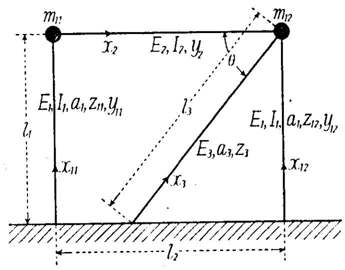

Fig. 21.

$$\left.\begin{aligned} y_{1n} &= (A_{1n}+B_{1n}x_{1n}+C_{1n}x_{1n}^2+D_{1n}x_{1n}^3)e^{ipt}, \quad [n=1,\ 2] \\ y_2 &= (A_2+B_2x_2+C_2x_2^2+D_2x_2^3)e^{ipt}, \end{aligned}\right\} \tag{26}$$

$$z_{1n}=(\alpha_{1n}+\beta_{1n}x_{1n})e^{ipt}, \quad [n=1,\ 2] \qquad z_3=\alpha_3+\beta_3x_3. \tag{27}$$

If the vibration of the structure is caused by horizontal seismic displacement of the ground, the boundary conditions are such that

$$x_{1n}=0; \qquad y_{1n}=Ue^{ipt}, \quad \frac{dy_{1n}}{dx_{1n}}=0, \quad z_{1n}=0, \quad [n=1,\ 2] \quad (28),\ (29),\ (30)$$

$$x_3=0; \qquad z_3=Ue^{ipt}\cos\theta, \tag{31}$$

$$x_{11}=l_1,\ x_2=0; \qquad y_2=-z_{11}, \quad \frac{dy_{11}}{dx_{11}}=\frac{dy_2}{dx_2}, \quad (32),\ (33)$$

$$-E_1I_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{d^2y_{11}}{dx_{11}^2}+E_2I_2\left(1+\tau_2\frac{\partial}{\partial t}\right)\frac{d^2y_2}{dx_2^2}=0, \tag{34}$$

$$E_2I_2\left(1+\tau_2\frac{\partial}{\partial t}\right)\frac{d^3y_2}{dx_2^3}-E_1a_1\left(1+\kappa_1\frac{\partial}{\partial t}\right)\frac{dz_{11}}{dx_{11}}=0, \tag{35}$$

$$-E_1I_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{d^3y_{11}}{dx_{11}^3}=m_{11}p^2y_{11}+T, \tag{36}$$

$$x_{12}=l_1,\ x_2=l_2,\ x_3=l_3; \qquad y_2=-z_{12}, \quad z_3=z_{12}\sin\theta+y_{12}\cos\theta, \quad (37),\ (38)$$

$$\frac{dy_{12}}{dx_{12}}=\frac{dy_2}{dx_2}, \tag{39}$$

$$-E_1I_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{d^2y_{12}}{dx_{12}^2}-E_2I_2\left(1+\tau_2\frac{\partial}{\partial t}\right)\frac{d^2y_2}{dx_2^2}=0, \tag{40}$$

$$-E_2I_2\left(1+\tau_2\frac{\partial}{\partial t}\right)\frac{d^3y_2}{dx_2^3}-E_3a_3\left(1+\kappa_3\frac{\partial}{\partial t}\right)\frac{dz_3}{dx_3}\sin\theta-E_1a_1\left(1+\kappa_1\frac{\partial}{\partial t}\right)\frac{dz_{12}}{dx_{12}}=0, \tag{41}$$

$$-E_1I_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{d^3y_{12}}{dx_{12}^3}+E_3a_3\left(1+\kappa_3\frac{\partial}{\partial t}\right)\frac{dz_3}{dx_3}\cos\theta=m_{12}p^2y_{12}-T, \tag{42}$$

$$x_{11}=l_1,\ x_{12}=l_1; \qquad y_{11}=y_{12}. \tag{43}$$

Substituting (26), (27) in (28)∼(43), we get for the vibration amplitude (horizontal) of the floor

$$y_{11x_{11}=l_1}=\frac{UQe^{ipt}}{Q-\gamma P}, \tag{44}$$

where

$$\left.\begin{aligned}
P&=\psi\left\{48\zeta+\xi\nu\phi^2(3\zeta+2\phi)\right\}+\vartheta\sin^2\theta\left\{24\zeta+\xi\nu\phi^2(3\zeta+2\phi)\right\},\\
Q&=12\psi\left\{24\zeta+\xi\nu\phi^2(6\zeta+\phi)\right\}+\vartheta\Big[\xi\nu\cos^2\theta\left\{48\zeta+\xi\nu\phi^2(3\zeta+2\phi)\right\}\\
&\quad-72\xi\nu\phi\zeta\cos\theta\sin\theta+12\sin^2\theta\left\{\xi\nu\phi^2(6\zeta+\phi)+12\zeta\right\}\Big],\\
\gamma&=\frac{m_1p^2l_1^3}{E_1I_1(1+ip\tau_1)},\quad \vartheta=\frac{E_3a_3(1+ip\kappa_3)}{E_1a_1(1+ip\tau_1)},\quad \zeta=\frac{E_2I_2(1+ip\tau_2)}{E_1I_1(1+ip\tau_1)},\\
\xi&=\frac{a_1l_1^2}{I_1},\quad \nu=\frac{1+ip\kappa_1}{1+ip\tau_1},\quad \phi=\frac{l_2}{l_1},\quad \psi=\frac{l_3}{l_1}.
\end{aligned}\right\} \tag{45}$$

The expression in (44) is of complex form. By putting $\kappa_1=\kappa_3=\tau_1=\tau_2\equiv\tau$, equation (44) becomes real as shown in the next section.

8. *Calculation for the special case* $\kappa_1=\kappa_3=\tau_1=\tau_2$.

If we put $\kappa_1=\kappa_3=\tau_1=\tau_2\equiv\tau$ in expressions (44), (45), the same expressions become real, assuming the forms

$$y_{11x_{11}=l_1}=\frac{UQ_1(1+\gamma_0\tau_0^2)}{\sqrt{\left\{Q_1(1+\gamma_0\tau_0^2)-2P_1\gamma_0\right\}^2+4P_1^2\gamma_0^3\tau_0^2}}\cdot\exp.\left[i\left\{pt-\tan^{-1}\frac{2P_1\sqrt{\gamma_0^3}\,\tau_0}{Q_1(1+\gamma_0\tau_0^2)-2P_1\gamma_0}\right\}\right], \tag{46}$$

$$\left.\begin{aligned}
P_1&=\psi\left\{48\zeta_0+\xi\phi^2(3\zeta_0+2\phi)\right\}+\vartheta_0\sin^2\theta\left\{24\zeta_0+\xi\phi^2(3\zeta_0+2\phi)\right\},\\
Q_1&=12\psi\left\{24\zeta_0+\xi\phi^2(6\zeta_0+\phi)\right\}+\vartheta_0\Big[\xi\cos^2\theta\left\{48\zeta_0+\xi\phi^2(3\zeta_0+2\phi)\right\}\\
&\quad-72\xi\phi\zeta_0\cos\theta\sin\theta+12\sin^2\theta\left\{\xi\phi^2(6\zeta_0+\phi)+12\zeta_0\right\}\Big],\\
\gamma_0&=\frac{m_1p^2l_1^3}{E_1I_1},\quad \tau_0=\tau\sqrt{\frac{m_1l_1^3}{E_1I_1}},\quad \vartheta_0=\frac{E_3a_3}{E_1a_1},\quad \zeta_0=\frac{E_2I_2}{E_1I_1},\\
\xi&=\frac{a_1l_1^2}{I_1},\quad \phi=\frac{l_2}{l_1},\quad \psi=\frac{l_3}{l_1}.
\end{aligned}\right\} \tag{47}$$

Using the above expressions, we calculated the resonance curves of the vibration amplitudes for various ratios of ϑ_0, namely, $\vartheta_0=0$, 0·1 0·2, 1, 2, 5, ∞ in the three cases of τ_0, namely, $\tau_0=0\cdot0001$, 0·001, 0·01. The conditions are therefore quite similar to those shown in Section 3. The results of calculation are shown in Figs. 22, 23, 24. It will be

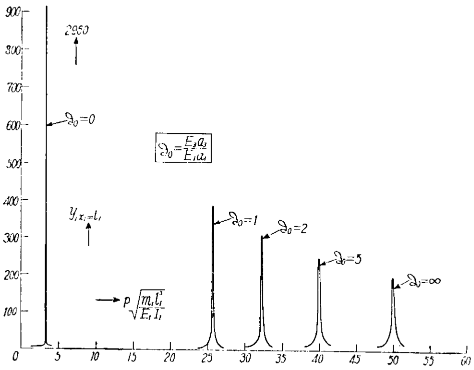

Fig. 22. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0\cdot0001$.

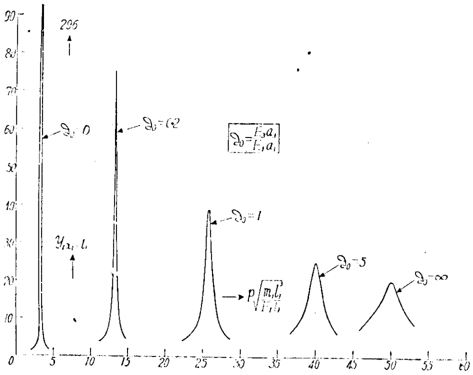

Fig. 23. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0\cdot001$.

seen that the general features of the resonance curves are almost similar to those in Section 3, the case of an endless number of spans. A closer examination of these curves shows, however, that, whereas for a smaller value of ϑ_0, say $\vartheta_0=0$, the resonance curves are nearly the same for the case of a mono-span and for the case of an endless number of spans, for a greater ratio of ϑ_0, say, $\vartheta_0=\infty$, the resonance amplitude

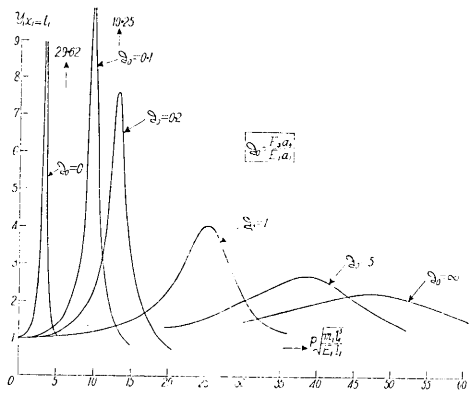

Fig. 24. Resonance curves for $\tau_0(=\tau\sqrt{E_1I_1/m_1l_1^3})=0\cdot01$.

in the former case is greater than that in the latter case by about 50 percent. For an intermediate ratio of ϑ_0, the difference in resonance amplitudes for both cases is also intermediate.

The relation between the resonance amplitudes, natural vibration periods, and the stiffness of the brace strut is shown in Figs. 25, 26. It will be seen that the two kinds of curves are quite similar. The brace strut is effective both in diminution of resonance amplitude and in increase of resonance frequency.

We have thus ascertained that the aseismic properties of the brace struts with damping inner resistance are nearly the same, regardless of whether the structure is of an endless number of spans or a mono-span, whence it follows that the aseismic properties of a braced structure with a few spans would not differ much from those shown in the present paper.

It is also possible to assume that in the case of no damping inner resistance within the brace strut, the resonance amplitude would increase with increase in the ratio of ϑ_0 even were the structure of mono-span.

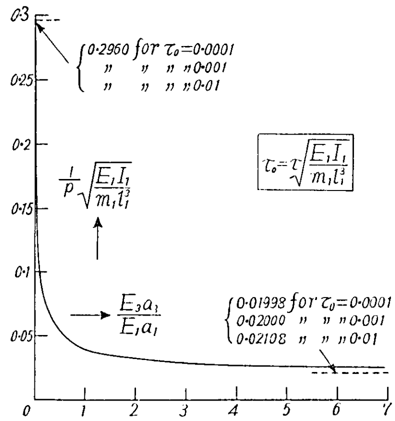

Fig. 25. Natural vibration periods.

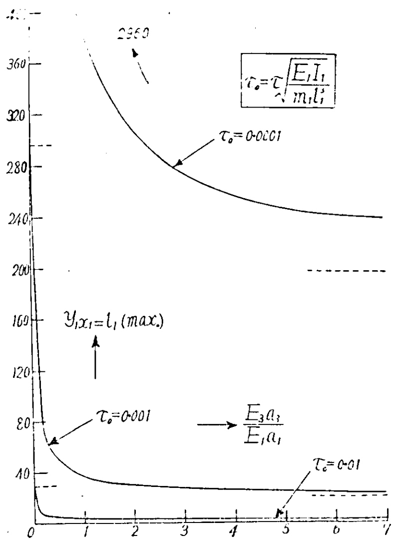

Fig. 26. Resonance amplitudes.

9. *General summary and concluding remarks.*

From mathematical investigation we have ascertained that if a damping inner resistance exists in a structure, the brace struts are effective both in reducing the resonance amplitudes and in increasing the natural frequency of the structure, the nature of these effects being thus quite similar to that if wave scattering existed.

Although the resonance amplitude diminishes enormously with increase in damping resistance, the vibration period remains nearly constant, at least with the change in damping resistance. On the other hand, the change of resonance amplitude and that of resonance frequency with change in the stiffness of the brace strut are of quite similar types.

The damping resistance in the beam (of moderate size) is not likely to be effective in diminishing the resonance amplitudes. Although the damping resistance in the longitudinal vibration of the columns is also ineffective on the aseismic properties of the structure, that in the bending vibration of the same columns, however, somewhat affects these properties.

The damping inner resistance in the brace struts, on the other hand, is extremely effective on the damping action of the vibration of the structure. If, however, there is no damping inner resistance in a brace strut, the resonance amplitude will tend to increase with increase in the stiffness of the same strut—a result that seems very odd. Our investigation therefore makes it clear that an over-strong strut is unnecessary. Thus, a brace strut of moderate size with the greatest possible damping inner resistance is recommended for obtaining aseismicity in a structure.

It should be borne in mind, however, that the aseismic properties of the brace struts with damping inner resistance are nearly the same, whether the structure be of an endless number of spans, a few spans, or mono-span.

In conclusion, I wish to record here my thanks to Messrs. Unoki and Watanabe for valuable assistance rendered in the present series of investigation. The present investigation was made at Professor Sezawa's suggestion in connection with his research work as member of the Investigation Committee for Earthquake-proof Construction, of the Japan Society for the Promotion of Scientific Research. I wish also to express my sincerest thanks to Professor Sezawa for valuable aid given to me.

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46. 筋違の耐震効果の理論 (本論其 4)

構造內部の減衰力の影響

地震研究所 金 井 清

前回の報告では構造に震動逸散性のある場合の筋違の耐震効果の理論を述べたが，今回は構造內部の減衰力例へば固體粘性のある場合のそれを研究した．

研究の結果によれば，構造內部の減衰力があるときには筋違は構造物の振動周期を短くするこ

同時にその共振振幅を少くすることに役立つ事がわかつた．即ち震動逸散のある場合と非常によく似てをる譯である．但し構造内部の減衰力のみを極端に大きくすると共振振幅はいくらでも小さくなるが，振動周期はそれによつて餘り變化しない．

筋違の大さを變化した場合の振動周期を表す曲線と共振振幅を表す曲線とは非常に似てをる．

床の中の減衰力は床が餘り厚くないときには構造全體の振動減衰に餘り役に立たない．柱の中の減衰力は柱がそれ程大きくなくても構造全體の振動減衰に相當役立つ．

筋違の中の減衰力は構造全體の振動減衰に非常によくきくことがわかる．而して若し筋違の中に減衰力が全然無い場合には，筋違を強くすると，共振の周期だけは短くなるけれども，構造全體の減衰が反て少なくなることがわかつた．之によつて適當な大さの筋違の材料的減衰力をできるだけ大きくしたものが最も耐震力をよくする事がわかつた．
