47. *Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part V. Model Experiment Confirmations.*

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1. *Introduction.*

In previous papers,[1] we ascertained mathematically that if damping inner resistance or wave scattering exists in a structure, the brace struts used in the same structure will be effective both in reducing the resonance amplitudes and in increasing the natural frequency of the structure. The present paper describes the model experiments that were made to confirm the theory just mentioned. Since, as often stated, the amplitude of seismic vibrations is particularly large in the horizontal sense, and since, moreover, the vibration of a structure is most sensible to the same horizontal movement of the ground, the experiments were made specially for the case of horizontal motion.

Although the vibrational forces from the exciter in the present experiments were all of sinusoidal type, since an oscillation of irregular type is nothing more than a combination of different sinusoidal oscillations, the present results might apply to the general case to a considerable extent.

2. *The method of experiments.*

The vibration table used in the experiments was the same as or at least similar to those used in studying the dynamic damper.[2,3] A hanging table oscillated to and fro by means of periodic forces supplied by a 1/4 H. P. motor by means of gear arrangement, oil resistance being applied to both sides of the table in order to prevent any irregular vibration of the table. The natural period of the table was about

1) K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 233～252, 559～568, 569～578, 695～712.

2) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **15** (1937), 599.

3) K. KANAI, *Bull. Earthq. Res. Inst.*, **16** (1938), 21.

1 sec. Since the periods of vibration available in our experiments were of the range 0·1 sec to 0·6 sec, resonances of such long periods as 1 sec were quite immaterial. Since, however, the motor power was not sufficiently high, it was rather difficult to avoid conspicuous changes in the vibration amplitudes of the table for different vibrational frequencies. If, on the other hand, the ratio of the amplitude of the vibration table to that of the structure were taken as we did in our analysis, the difference in the amplitudes of the table for different frequencies would not be a serious matter.

The model for our experiments was a single-storied structure of three spans with elastic floor parts, columns, and brace struts, every structural member being of sheet steel. The two ends of each brace strut were hinged. The column height (l_1) and every floor span (l_2) were 25 cm. Each column and each floor part were also of equal thickness (t) and equal width (b), that is, both members were 0·5 mm

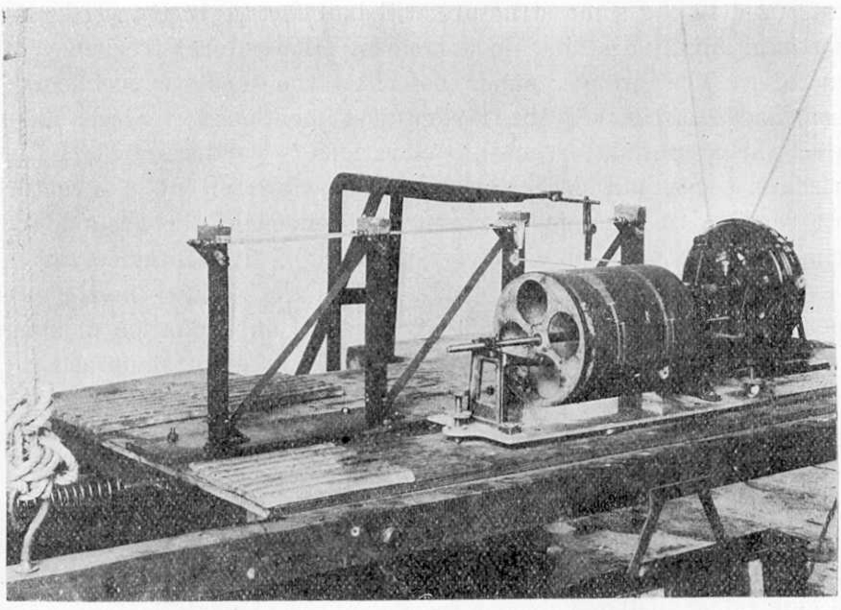

Fig. 1.

thick and 5 cm wide. The mass concentrated at each panel point was 270 gm. Since the mass of each floor span, together with that of half a column and that of half a beam span, was 60 gm, the total mass to be concentrated at each panel point in calculation was m_1=330 gm. A general view of the model, together with the vibration table, is shown in Fig. 1.

There were five cases of brace struts, as shown in Table I. Since the column height and the span were the same, that is, 25 cm, the

Table I. Five cases of brace struts.

Case	A	B	C	D	E
Thickness (mm)	0	0·5	0·5	0·5	1·0
Width (mm)	·0	5	10	20	20
$\vartheta_0 = \dfrac{E_3 a_3}{E_1 a_1}$	0	0·1	0·2	0·4	0·8

length of the brace was $\sqrt{2}.25$ cm. It was furthermore assumed that the Young's modulus is always constant $E_1 = E_2 = E_3 = 2{\cdot}1 . 10^{12}$ in C.G.S. The moment of inertia of every member was calculated from the usual formula $I = bt^3/12$, where t, b are the thickness and width of a section of that member.

The amplitudes and vibrational frequencies were observed with the aid of a smoke cylinder, driven by a synchronous motor that was calibrated separately. Since the whole contrivance was firmly fixed to the vibration table, the disturbing effect was found to be negligible. In the experiments, the displacements of the floor relative to the vibration table and those of the table relative to the room floor were recorded on the revolving cylinder without magnification.

3. *Results of experiments.*

Let the ratio of the vibration amplitude of the structure (at its floor) $(y_t - U)$ to that of the vibration table (U), both recorded on the smoke cylinder, be $(y_t - U)/U$, where y_t is the displacement of the floor relative to the room space. The values of $(y_t - U)/U$ obtained in the experiments for cases A, B, C, D, E are shown in Figs. 2—6.

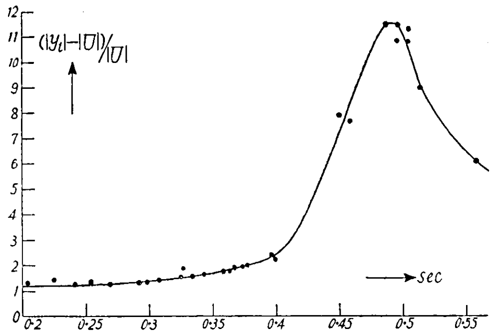

Fig. 2. Case A, $\vartheta_0 (= E_3 a_3 / E_1 a_1) = 0$.

It will be seen that the greater the ratio of $\vartheta_0 = E_3 a_3 / E_1 a_1$, the more the diminution in resonance amplitude. Since there was no special damper fitted to any structural part and since, furthermore, it is unlikely that wave scattering existed in the present condition of the model, the vibration damping would probably have come about from inner damping of the material. It is then possible for the damping resistance to increase with mere increase in the sectional area of the brace struts.

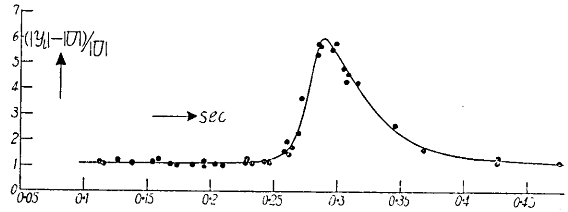

Fig. 3. Case B, $\vartheta_0(=E_3a_3/E_1a_1)=0{\cdot}1$.

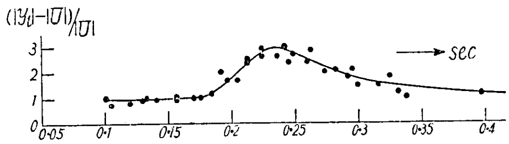

Fig. 4. Case C, $\vartheta_0(=E_3a_3/E_1a_1)=0{\cdot}2$.

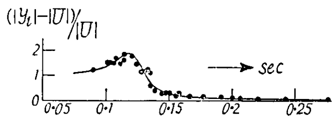

Fig. 5. Case D, $\vartheta_0(=E_3a_3/E_1a_1)=0{\cdot}4$.

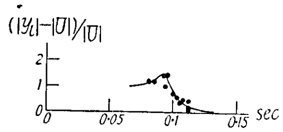

Fig. 6. Case E, $\vartheta_0(=E_3a_3/E_1a_1)=0{\cdot}8$.

In the present experiments, with a view to getting the resonance period of the model within the range of vibration periods such that 0·1 sec∼0·6 sec, the thickness of every member was made as small as possible, in consequence of which the thrust in every brace strut often exceeded the critical for the buckling of that brace strut. Notwithstanding that no bending moment was transmitted through any brace strut, the strut itself vibrated transversely in every vibration of the model, as the result of which the effective stiffness of every member against the longitudinal force in that member would have become very small compared with that calculated from Table I. We shall first examine whether or not the longitudinal force in any member exceeds Euler's critical thrust for buckling. Since, however, the thrusts in the brace struts are particularly large, the condition in those struts alone will be considered. Let U, y_t be the amplitudes of the table and that of the model floor, both relative to the room space. The horizontal inertia force for the mass m_1 at each panel point is then $P'=y_t(2\pi/T)^2m_1$, its component along the brace strut being $P''=y_t(2\pi/T)^2m_1\cdot\cos\theta$. On the other hand, Euler's critical thrust for buckling is $P=$

$E_3 I_3 \pi^2 / l_3^2$, where $E_3 I_3$ is the bending stiffness of the strut. The results of calculation for both these thrusts for the three cases, B, C, D are shown in Table II.

When the horizontal displacement of the panel point relative to the vibration table is $y_l - U$, the effective strain of the brace strut along its length is $(y_l - U)\cos\theta / l_3$. Equating P'' to $E_3' a_3'$.(strain), namely, to $E'_3 a_3' (y_l - U)\cos\theta / l_3$, it is possible to get the effective longitudinal stiffness of that brace strut in the form

$$E_3' a_3' = \frac{y_l (2\pi/T)^2 m_1}{(y_l - U)/l_3}. \tag{1}$$

If it is defined that the effective Young's modulus E_3' is equal to the actual one E_3, say, $2{\cdot}1 .\ 10^{12}$ in C. G. S., then the effective area of the strut a_3' will be determined. The results of calculation for the cases B, C, D are shown in Table II, the ratio of a_3'/a_3 being given also in the same table. On the other hand, the quantity $\xi = a_1 l_1^2 / I_1$ is important

Table II.

Case	B	C	D
$P = E_3 I_3 \pi^2 / l_3^2$ (C.G.S.)	$1{\cdot}05 .\ 10^5$	$2{\cdot}11 .\ 10^5$	$4{\cdot}22 .\ 10^5$
$P'' = y_l (2\pi/T)^2 m_1 \cos\theta$ (C.G.S.)	$2{\cdot}46 .\ 10^5$	$1{\cdot}49 .\ 10^5$	$1{\cdot}14 .\ 10^5$
$a_3' = \dfrac{y_l (2\pi/T)^2 m_1}{E_3 (y_l - U)/l_3}$ (cm^2)	$2{\cdot}65 .\ 10^{-6}$	$4{\cdot}78 .\ 10^{-6}$	$3{\cdot}77 .\ 10^{-5}$
a_3'/a_3	$1{\cdot}06 .\ 10^{-4}$	$0{\cdot}96 .\ 10^{-4}$	$3{\cdot}77 .\ 10^{-4}$
$\xi' = \dfrac{a_1' l_1^2}{I_1}$ (estimation from experiments)	265	239	944
ξ'' (used in calculation)	500	500	500

in the calculation that comes in the next section. If we assume that the effective area of every column for longitudinal thrust is diminished by the same law as that for the brace strut, then it is possible to write $a_1'/a_1 = a_3'/a_3$, from which condition ξ is also replaced by $\xi' = a_1' l_1^2 / I_1$. Since $\xi = a_1 l_1^2 / I_1 = 2{,}450{,}000$ in our case, the values of ξ' thus determined are shown in Table II. The value of ξ' actually used in calculation was determined from an alternative condition. ξ'' in Table II represents that value.

4. *Comparison of the experimental results with the mathematical calculation.*

We shall now assume that the damping of the vibration originated only from the inner damping by the material and, furthermore, that the condition of three spans is nearly the same as that of an endless number of spans. If, for simplicity, the damping coefficients satisfy the conditions, $\tau_1=\tau_2=\kappa_1=\kappa_3$,[4] equation (18) of the previous paper,[5] that is,

$$y_{1\,x_1=l_1}=\frac{UM'\exp.\left\{i\left(pt-\dfrac{N\sqrt{\gamma_0}\,\tau_0}{M'-N}\right)\right\}}{\sqrt{(M'-N)^2+N^2\gamma_0\tau_0^2}} \tag{2}$$

is available in the present case. It should be remembered that the case of three spans is very similar to that of an endless number of spans. In the present case

$$\left.\begin{aligned} M'&=3(\phi+12\zeta_0)(\vartheta_0\sin^2\theta+\psi)+\vartheta_0\xi(\phi+3\zeta_0)\cos^2\theta,\\ N&=\frac{\gamma_0}{1+\gamma_0\tau_0^2}(\phi+3\zeta_0)(\vartheta_0\sin^2\theta+\psi),\end{aligned}\right\} \tag{3}$$

where

$$\left.\begin{aligned} &\gamma_0=\frac{m_1p^2l_1^3}{E_1I_1}, \quad \tau_0=\tau\sqrt{\frac{E_1I_1}{m_1l_1^3}}, \quad \vartheta_0=\frac{E_3a_3}{E_1a_1}, \quad \zeta_0=\frac{E_2I_2}{E_1I_1}=1,\\ &\phi=\frac{l_2}{l_1}=1, \quad \psi=\frac{l_3}{l_1}=\sqrt{2}, \quad \xi=\frac{a_1l_1^2}{I_1}.\end{aligned}\right\} \tag{4}$$

Furthermore, from the reasoning at the end of the previous section, it is possible to write

$$\vartheta_0=\frac{E_3'a_3'}{E_1'a_1'}=\frac{E_3a_3}{E_1a_1}. \tag{5}$$

If it is so adjusted that the calculated resonance period for case C is just equal to the experimental one for the same case, it is possible to determine ξ' (independent of the value of ξ' shown in the preceding section), which we shall represent by ξ''.

We have thus

$$\xi'\left(=\frac{a_1'l_1^2}{I_1}\right)=500\equiv\xi'',$$

which is shown in the final line in Table II. It will be seen that the value of ξ'' thus determined is eventually intermediate between the greatest and the smallest values of ξ' shown in the same table.

4) Comparison of the calculation with experimental results shows that the condition $\tau_1=\tau_2=\kappa_1=\kappa_3$ is not satisfactory. A preferable condition is likely to be $\kappa_3\gg\tau_1$ (τ_2, κ_1 being unimportant). Equation (2) is used merely for simplicity.

5) K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 695~712.

Now, if we put $\tau_0=\tau\sqrt{E_1I_1/m_1l_1^3}=0{\cdot}055$ in the case $\vartheta_0=0{\cdot}2$, equation (2) shows that the mathematical resonance amplitude is exactly equal to the experimental in Fig. 4. Using this value of τ_0, we calculated the resonance curves for various ratios of $\vartheta_0=E_3a_3'/E_1a_1'$, namely, $\vartheta_0=$ 0, 0·1, 0·2, 0·4, 0·8, the results of which are plotted in Fig. 7. It should

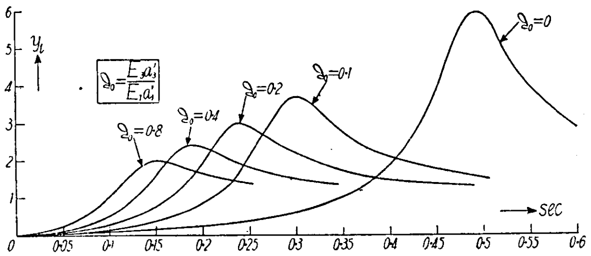

Fig. 7. Resonance curves for various ratios of $\vartheta_0(=E_3a_3'/E_1a_1')$ under the condition $\xi''(=a_1'l_1^2/I_1)=500$, $\tau_0=(\tau\sqrt{E_1I_1/m_1l_1^3})=0{\cdot}055$.

be borne in mind that in the calculation of $(y_t-U)/U$, the phase for U in the denominator was 90° in lead of that for U in the numerator. The resonance amplitudes thus obtained and those observed experimentally are also plotted by broken and full lines in Fig. 8. Comparing the two curves in Fig. 8, it will be seen that, although for an intermediate value of ϑ_0, the resonance amplitudes obtained mathematically fairly agree with the experimental, for a greater and smaller value of ϑ_0, they do not.

The resonance periods obtained experimentally and those calculated mathematically are plotted in Fig. 9. Both results again fairly agree.

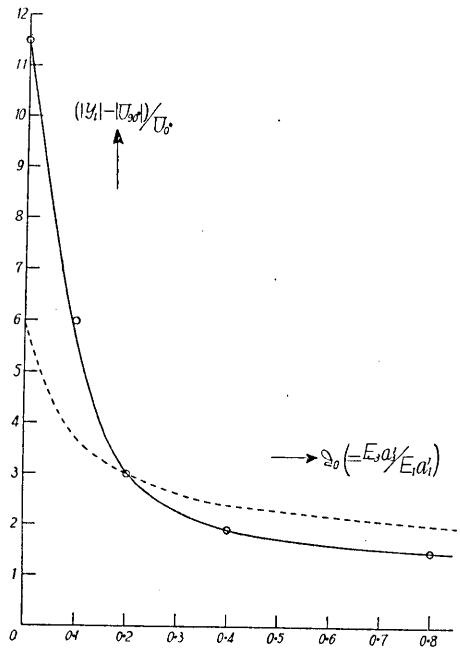

Fig. 8. Resonance amplitudes. For avoiding complexity, it is assumed that $\tau_1=\tau_2=\kappa_1=\kappa_3$ in calculation. Full line, experimental; broken line, calculation.

We have now ascertained that if the effective areas of the columns and the brace struts were used, the experimental results are in agreement with the theoretical results. Furthermore, since in the present case, the value of τE is approximately

2·826. 10^{10}(C. G. S.), it is likely that the inner damping by the material is, somewhat greater than that originating from solid viscosity. Thus, the damping may partly be due to wave scattering.

Since, as a matter of fact, we used the same ξ'' for any case and the same relation, $\tau_1=\tau_2=\kappa_1=\kappa_3$, the deviations of both kinds of curves in Figs. 8, 9 are inevitable. If, on the other hand, we used different ξ'''s for the different cases shown in Table II and another relation between τ_1, τ_2, κ_1, κ_3, the two kinds of curves in Figs. 8, 9 would agree to a considerable extent. If we specially select such a condition that κ_3 is much larger than the remaining damping coefficients, there would be better agreement between both curves in Fig. 8.

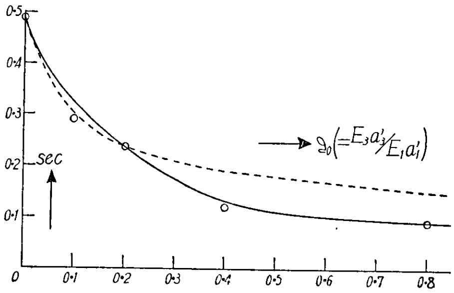

Fig. 9. Resonance periods. For simplicity, it is assumed that $\xi''=500$ for any case in calculation. Full line, experimental; broken line, calculated.

As in the case of mathematical calculation, the experimental relation between the resonance amplitude and the stiffness of the brace strut is quite similar to that between the natural vibration period and the same stiffness. It is thus confirmed experimentally that the struts are effective both in reducing resonance amplitude and in increasing resonance frequency.

Although, in the present experiments, owing to the restricted range in the frequency of forced vibration, we used a model whose members are so thin that the longitudinal stiffness of every member is the effective one, if it is possible to use a large model in which the buckling condition of the members can scarcely occur, it would then hold that the actual longitudinal stiffness operates.

5. *General summary and concluding remarks.*

Our previous mathematical results with respect to the aseismic properties of brace struts were confirmed by means of model experiments. From the requirement that our forced vibration periods shall be within a certain restricted range, each member of the model structure was so thin that the effective longitudinal stiffness (calculated separately) of that member should be used. If it were possible to use a large model, the longitudinal stiffness would then be the actual stiffness.

With the above condition of the problem in mind, we compared the experimental result with the mathematical calculation, from which it was ascertained that both results fairly agree in resonance amplitude as well as in resonance period.

In the mathematical calculation, the reduction of the effective areas of the members is the same for any ϑ_0, and the damping coefficients of different kinds are equal. If it be possible to make different reduction for different ϑ_0 and furthermore the damping coefficient for the brace strut is specially large, the agreement between the mathematical and experimental results would be more considerable.

As in the case of the mathematical calculation, the experimental relation between the resonance amplitude and the stiffness of the brace strut is quite similar to that between the natural vibration period and the same stiffness. Thus, experimental results also show that the brace struts fitted to a structure are effective both in reducing the resonance amplitude and in increasing the resonance frequency.

In conclusion, I wish to express my thanks to Messrs. Teizi Watanabe and Yosikazu Kodaira, who assisted me in the present investigation. The present investigation was made at Professor Sezawa's suggestion in connection with his research work as member of the Investigation Committee for Earthquake-proof Construction, of the Japan Society for the Promotion of Scientific Research. I wish also to express my sincerest thanks to Professor Sezawa for valuable aid given to me.

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47. 筋違の耐震効果の理論 (其 5)
模型實驗的確め

地震研究所 金 井 清

筋違の耐震効果の理論を確めるために，簡單な模型を振動臺の上に載せて振動試驗を試みた．模型の性質上，減衰力は構造內部のそれのやうである．實驗から出した共振曲線と數學的に出した共振曲線とを比較したのであるが，構造各部分の板が餘り薄いために，之等の板の長さの方向の强さについては相當の減少があるから，斷面の効果的面積を振動部分の變位から算定し，之から比較したのである．その結果，實驗的の共振振動振幅と振動周期とは計算のそれと可なり一致する事がわかつた．但し只今の研究では斷面の效果面積が筋違の大小に關せず一定としたし，又，振動の減衰係數が如何な部分でも同じであるとしたから，實驗と計算との間に多少の相違も出たが，若しその効果面積を場合場合について調整し且つ筋違の減衰係數のみが特に大きいとすれば兩方の結果は一層よく一致するのである．後者の方が特に影響するけれどもその計算が非常に複雜になる．振動臺の性質がもつとよければ，模型を大きくして材料の効果的斷面などを取る

必要がないのである．

以上の點に關係なく，筋違の效果としては構造物の振動周期を短くするだけでなく共振振幅を少くするのに一層効果のある事がわかつた．又，筋違の斷面を變化してできる周期變化の曲線と共振振幅の曲線とは殆ど相似である事もわかつた．即ち定性的にはこれ迄の理論による筋違の耐震性が相當よく確められた譯である．

尙，上述の實驗結果から逆に構造材料の振動の減衰係數を出すと $2{\cdot}826\,.\,10^{10}$ (C. G. S.) 位になり，之は材料の固體粘性のそれよりも少し大きな値である．即ち振動逸散の影響も多少含まれてをつたものと思はれる．

又，只今の實驗でよくわかつた點は筋違の長さの方向の減衰係數が柱其他の部分の減衰係數よりも相當大きいとして計算すれば，計算結果が實驗値に合ひ得るといふ事である．この事柄は筋違の減衰力をできるだけ大きくすれば耐震效果が上るといふ理論を一層强める譯である．