# 32. *Temperature Distribution within the Earth in its Semi-gaseous State.*

By Katsutada SEZAWA and Kiyoshi KANAI,

Earthquake Research Institute.

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## 1. *The origin of the primitive earth.*

It is uncertain whether the primitive earth was formed by the ejection of a filament from the central nucleus of the solar system or by the capture of an external star by the same solar system. Since, however, the distribution of matter in the earth is somewhat similar to that of metals and slags in a smelting furnace, it is possible to assume that the present earth grew from its original liquid stage. Extending this idea to its earlier stage, it may be concluded that the earth had once passed the gaseous stage. According to Chamberlin and Moulton,[1] Jeans,[2] and Jeffreys,[3] it is highly probable that the primitive earth was formed from a gaseous filament ejected from the primitive sun. With this as a working hypothesis, we also solved the problem of the tidal deformation of a liquid sun in the event that an external massive star encounters the same sun with different velocities.[4] Although it was desirable to treat that case in which the nuclear sun and the ejected stars are all gaseous, since the problem was extremely difficult, we satisfied ourselves by discussing, particularly, the case of incompressible matter.

As pointed out by Jeffreys, although the primitive earth might have been gaseous, owing to heat radiation, the period of such a gaseous condition could not have been more than a few years.

On the other hand, Mercier,[5] in dealing with a polytropic gas sphere and the boiling temperatures of certain metals, showed that it is improbable that the primitive earth was gaseous, which result probably throws some light on the problem of the origin of the earth, but

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1), 2) J. H. JEANS, *Problems of Cosmogony and Stellar Dynamics* (1919).

3) H. JEFFREYS, *The Earth* (1929).

4) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 27.

5) A. MERCIER, "The Liquid State of the Primitive Earth" *Nature*, **141** (1938), 201; "Sur la liquéfaction du globe terrestre, dans l'hypothèse d'une sphère gazeuse initiale soumise a une loi polytropique", *Archives d. sci. phys. e. natur.*, **20** (1938), 31~58.

since Mercier's discussion is restricted to the case of the whole earth being gaseous, and does not concern itself with that of a liquid core precipitated at the centre, it is impossible to arrive at the condition of the earth at that period when the condensing metallic core was growing. In the present paper we shall deal with this condition in a way somewhat similar to that shown by Mercier.

## 2. *Mathematical equations.*

Let $R_1$ be the radius of the inner liquid core and $R$ the outer radius of the gaseous part that surrounds the core. Let $\rho$, $g$, $P$ be the density, the acceleration of gravity, and the hydrostatic pressure at any point $r$ in the gaseous part, respectively.

Since the gravitational force at $r$ is due entirely to the mass $M_r$ within $r$, we have

$$g = GM_r/r^2 , \tag{1}$$

where $G$ is the gravitational constant, namely, $6{\cdot}66 . 10^{-8}$ in C. G. S. units. If $\phi$ is the gravitational potential, we have

$$g = -d\phi/dr . \tag{2}$$

The hydrostatic relation for the gaseous part is

$$dP = -g\rho dr . \tag{3}$$

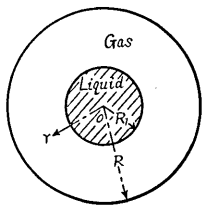

Fig. 1.

Thus, from (2), (3), we get

$$dP = \rho d\phi \tag{4}$$

in the same part. The second condition to be satisfied by the gaseous part is Poisson's equation

$$\nabla^2 \phi = -4\pi G\rho , \tag{5}$$

which for spherical symmetry reduces to

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = -\phi\pi G\rho . \tag{6}$$ [6]

Another equation to be introduced is that representing the relation between $P$ and $\rho$. The most general relation is that for the polytropic distribution of the gas, the equation of which is

$$P = \kappa\rho^\gamma , \tag{7}$$ [7]

6), 7) A. S. EDDINGTON, *The Internal Constitution of the Stars* (1926).

where $\kappa$ and $\gamma$ are constants. Although both $\kappa$ and $\gamma$ change with difference in temperature even for a given gas with certain initial conditions, in the present investigation we shall assume for simplicity that they would be constant for a given gas with given initial conditions.

Differentiating (7), we have

$$dP=\gamma\kappa\rho^{\gamma-1}d\rho\,. \tag{8}$$

Comparing this with (8), (4),

$$\gamma\kappa\rho^{\gamma-2}d\rho=d\phi\,, \tag{9}$$

which, integrated, becomes

$$\frac{\gamma}{\gamma-1}\kappa\rho^{\gamma-1}=\phi+C\,, \tag{10}$$

where $C$ is a constant. If we put the condition that there is no temperature at such point that no mass exists, it is possible to make the constant of integration vanish, that is,

$$C=0\,. \tag{11}$$

Putting

$$\gamma=1+\frac{1}{n}\,, \tag{12}$$

we have from (10), (12)

$$\rho=\left\{\frac{\phi}{(n+1)\kappa}\right\}^{n}, \tag{13}$$

and from (13), (7)

$$P=\frac{\rho\phi}{n+1}\,, \tag{14}$$

so that from (6), (13)

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}+a^2\phi^n=0\,, \tag{15}$$

where

$$a^2=\frac{4\pi G}{\{(n+1)\kappa\}^n}\,. \tag{16}$$

(15) is Poisson's equation involving the condition of polytropic gas change. Emden,[8] using the trial and error method, obtained the solu-

8) R. EMDEN, *Gaskugeln* (1907).

tions of (15) for any $n$ in the special case that there is no liquid core at the centre of the spherical gas. Since, however, in the present case, the integrals of equation (15) for any $n$ is not simple, we shall restrict ourselves to the case $n=1$, the mathematical solutions of which are rather simple.

### 3. *Solutions for a special case; the gas equation.*

When, particularly, $n=1$, equation (15) reduces to

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}+\alpha^2\phi=0, \tag{17}$$

the solution of which is

$$\phi=\frac{A\sin\alpha(r-\varepsilon)}{r}. \tag{18}$$

The boundary conditions at the surface $r=R_1$ of the inner core and the outer surface $r=R$ of the gaseous sphere are such that

$$\frac{d\phi}{dr}=-\frac{GM_1}{R_1^2} \tag{19}$$

at $r=R_1$ and

$$\phi=0, \qquad \frac{d\phi}{dr}=-\frac{GM}{R^2} \tag{20, 21}$$

at $r = R$, where $M_1$, $M$ are masses within $R_1$, $R$, respectively.

Substituting (18) in (19), (20), (21), and eliminating $A$ and $\varepsilon$, we get

$$\alpha R+\frac{M}{M_1}\Big\{-(\alpha R_1\sin\alpha R_1+\cos\alpha R_1)\sin\alpha R$$
$$+(\sin\alpha R_1-\alpha R_1\cos\alpha R_1)\cos\alpha R\Big\}=0, \tag{22}$$

which is the equation for determining $R$ when $R_1$ is given.

The expression satisfying (20), (21) is thus

$$\phi=\frac{1}{r}\frac{GM}{\alpha R}\sin\alpha(R-r), \tag{23}$$

where $R$ can be determined by means of (22) [or by (19)].

It should be borne in mind that the boundary conditions (19), (20), (21) are approximate in the sense that the space $r>R$, where there is no gas, shall satisfy both relations $\phi=0$, $d\phi/dr=-GM/r^2$ at the same time. As a matter of fact, (19), (20), (21) should be replaced by more reasonable conditions such that, $d\phi_1/dr=-GM_1/R_1^2$ at $r=R_1$ and $\phi_1=\phi_2$, $d\phi_1/dr=d\phi_2/dr=-GM/r^2$ at $r=R$, where $\phi_1$, $\phi_2$ are potential within the gaseous sphere and that outside that sphere. In doing so, on the other hand, the distribution of matter becomes discontinuous, that is to say, from a finite density to vacuum, at the outer boundary of the gaseous sphere under consideration, which condition is rather unsatisfactory from our usual conception of the gas theory. For this reason we use the conditions shown in (19), (20), (21).

We shall consider the condition of perfect gas. The gas equation is then

$$\beta P=\frac{\mathfrak{R}}{\mu}\rho T \tag{24}$$

for any $n$, where $\mathfrak{R}$ is the universal gas constant $8{\cdot}26 . 10^7$, $\mu$ the molecular weight in terms of the hydrogen atom, and $\beta$ the ratio of the gas pressure to the whole pressure. Hevesy shows that the mean molecular weight of the different materials, assuming the abundance of every material in the earth, is nearly 32. Since there is an additive pressure due to heat radiation in a star, $\beta$ is always less than unity. Eddington[9] gives the relation

$$1-\beta=0{\cdot}00309\left(\frac{M}{\odot}\right)^2\mu^4\beta^4\,, \tag{25}$$

where $M$, $\odot$ are the mass of the star and that of the sun respectively. Since the term on the right-hand side of (25) is very small,

$$\beta\approx 1\,. \tag{26}$$

Comparing (24) with (14), we have

$$T=\frac{\beta\mu}{(n+1)\mathfrak{R}}\phi \tag{27}$$

for any $n$.

When there is no core, that is, $M_1=0$, $R_1=0$, solution (18) becomes

$$\phi=\frac{A\sin ar}{r}\,, \tag{28}$$

so that from (20), (21)

$$\mathrm{A}=\frac{GM}{\pi}\,,\qquad aR=\pi\,, \tag{29), (30}$$

9) A. S. Eddington, *loc. cit.* 6), p. 117.

and the relation between the central temperature $T_0$ and the central potential $\phi_0$ is then of the type

$$T_0=\frac{\beta\mu}{(n+1)\Re}\phi_0 \tag{31}$$

for any $n$, $\phi_0$ being

$$\phi_0=Aa=\frac{GM}{\pi}a \tag{32}$$

from (28), (29), (30). Comparing (31) with (32), we get

$$a=\frac{T_0(n+1)\Re}{\beta\mu}\frac{\pi}{GM}. \quad (n\rightarrow 1) \tag{33}$$

If a gas sphere of such mass as equals that of the present earth has a polytropic density distribution, with its central temperature $T_0$, its central potential $\phi_0$ the constant $a$ can be determined by (33), (32). The determination of $a$ is equivalent to specifying the value of the constant $\kappa$ of the polytropic gas change. When radius $R_1$ and mass $M_1$ of the inner core are given, it is then possible to determine the outer radius $R$ by means of (22). The distributions of $\phi$, $T$ and $P$ in the gaseous sphere are then found by means of (23), (27), (24).

Although the distributions of temperature and pressure within the gaseous sphere are thus determined, there remains the question whether or not it is possible for the temperature at any point to be higher than the boiling point of the gas (metal gas in the present case) corresponding to the pressure (partial pressure for each material element[10]) at the same point. Since there is no good data for estimating the change of boiling points of metals with change in pressure, it is impossible to show accurate values of the boiling points in question. Mercier[11] used Clapeyron-Clausius empirical law showing the relation between heat of evaporation, pressure, and boiling point, namely,

$$\lambda=4{\cdot}571\frac{1}{1/T_1-1/T_2}\log\frac{p_2}{p_1}, \tag{34}$$

where $\lambda$ is the heat of evaporation, and $T_1$, $T_2$ are the boiling points at pressures $p_1$, $p_2$ (partial pressures), respectively. Mercier assumed that $\lambda$ is constant for any temperature, which, however, is rather improbable since $\lambda$ diminishes with increase in temperature. Let $T_1$ be

10) Dr. SHIRAI of our Institute kindly suggested us that the pressure should be partial.

11) A. MERCIER, *loc. cit.* 5).

the boiling point of a metal at atmospheric pressure $p_1$. From the condition that $\lambda$ diminishes with increase in $T_2$ for a given $p_2$, it holds that the rate of increase in $T_2$ with $p_2$ under the same condition of $\lambda$, is greater than that under the condition that $\lambda$ is constant.

In the present case we shall also assume, for simplicity, that $\lambda$ is constant. The error resulting from this assumption would then be that the estimated boiling point will always be unduly low. However, with this idea of the law of error in mind, it is still possible to ascertain the qualitative nature of the problem.

## 4. *Results of numerical examples and their interpretations.*

We shall consider a primitive gaseous earth for nine different cases of its central temperature $T_0$, such that (i) 2,000°K, (ii) 4,000°K, (iii) 5,000°K, (iv) 6,000°K, (v) 8,000°K, (vi) 10,000°K (vii) 50,000°K, (viii) 80,000°K, (ix) 100,000°K. Assume that $\mu=32$ (under the condition already given), $T_1=3{,}500$°K (that is for iron or nickel), $p_1=1$ atm. ($1\cdot013 . 10^6$ C. G. S.), $\lambda=9 . 10^4$ Cal.[12] (that is for iron or nickel in the usual state). The total mass of the gaseous sphere (or semi-gaseous sphere) is assumed to be the same as that of the present earth, namely, $M=5\cdot985 . 10^{27}$ C. G. S. Applying these values to equations (29) ~ (34), we get $P_0$, $\alpha$, $R$, $T_{20}$ (at the centre of the gaseous sphere) in the nine cases just mentioned, the results of which are shown below.

Table I. The values of $P_0$, $\alpha$, $R$, $T_{20}$ for the primitive gaseous earth for the nine different cases of $T_0$.

| $T_0$ (K) | 2,000° | 4,000° | 5,000° | 6,000° | 8,000° |
|---|---|---|---|---|---|
| $P_0$ (C.G.S.) | $4\cdot215.10^5$ | $6\cdot75 . 10^6$ | $1\cdot650.10^7$ | $3\cdot415.10^7$ | $1\cdot080.10^8$ |
| $\alpha$ (C.G.S) | $8\cdot136.10^{-11}$ | $1\cdot628.10^{-10}$ | $2\cdot034.10^{-10}$ | $2\cdot441.10^{-10}$ | $3\cdot254.10^{-10}$ |
| $R$ (cm) | $3\cdot862.10^{10}$ | $1\cdot929.10^{10}$ | $1\cdot545.10^{10}$ | $1\cdot287.10^{10}$ | $9\cdot652.10^9$ |
| $T_{20}$ (K) | $3\cdot767.10^3$ | $4\cdot121.10^3$ | $4.487.10^3$ | $4\ 853.10^3$ | $5\cdot888.10^3$ |

(*continued.*)

| $T_0$ (K) | 10,000° | 50,000° | 80,000° | 100,000° |
|---|---|---|---|---|
| $P_0$ (C.G.S.) | $2\cdot635.10^8$ | $1\cdot649.10^{11}$ | $1\cdot080.10^{12}$ | $2\cdot637.10^{12}$ |
| $\alpha$ (C.G.S.) | $4\cdot068.10^{-10}$ | $2\cdot034.10^{-9}$ | $3\cdot254.10^{-9}$ | $4\cdot07 . 10^{-9}$ |
| $R$ (cm) | $7\cdot722.10^9$ | $1\cdot544.10^9$ | $9\cdot65 . 10^8$ | $7\cdot72 . 10^8$ |
| $T_{20}$ (K) | $6\cdot209.10^3$ | $3\cdot855.10^4$ | $10\cdot12 . 10^4$ | $16\cdot22 . 10^4$ |

12) *International Critical Tables*, I (1926), p. 102; $\lambda=9.10^4$ Cal=380 Kilo-joules.

Although $T_{20}$ (or even $T_2$) for a given material, say, iron or nickel, should be partial pressure that corresponds to that material, since the ratio of $p_2$ to $p_1$ in (34) is only effective, the usual pressure was taken in calculation.

Since $T_0$, $T_{20}$ are the respective actual temperature and boiling point at the centre of the gaseous sphere, if $T_0$ is higher than $T_{20}$, it is possible for the central part, at least, of the earth to remain in a gaseous state, whereas if $T_0$ is lower than $T_{20}$, the part of the gas at the centre will be liquefied. The higher temperature between the $T_0$, $T_{20}$ in each column of Table I is underlined.

It will be seen from Table I if $T_0$ is relatively low, say, $T_0 <$ 4,000°K, or very high, say, $T_0 > 80{,}000$°K, the central part of the primitive earth will be liquid if, on the other hand, $T_0$ is medium, say, 5,000°K $< T_0 <$ 50,000°K, the central part under consideration can remain in a gaseous condition.

The above consideration, however, does not concern with the condition of the gas for any radius of the sphere. Even should the centre of the sphere be gaseous, it does not follow that the matter at any radius is also gaseous. It is also desirable to know the condition for the case in which a liquid core exists at the centre of the sphere. For ascertaining these conditions, we calculated the distribution of $T$ and $T_2$ together with $\phi$, within the whole gaseous part of the sphere, the equations used being (23), (27), (24), (34). For determining the mass $M_1$ of the central core of radius $R_1$, we assumed that the density of the core is equal to the mean density 5·525 of the present earth. The results of calculations for three characteristic cases, namely, (i) $T_0=4{,}000$°K, (ii) $T_0=50{,}000$°K, (iii) $T_0=100{,}000$°K, are shown in Figs. 2∼13. The distributions of $\phi$ are plotted in Figs. 2, 6, 9 and those of $T$, $T_2$ in the remaining figures (actual temperature $T$ in full lines and boiling point $T_2$ in broken lines). The radius $R_1$ of any liquid core in every case is shown hatched.

Since we now deal with a special case of the polytropic distribution of the gas, namely, $n=1$, equation (13) shows that the distribution of mass density $\rho$ is quite similar to that of $\phi$. It is therefore possible for $\phi$-distributions in Figs. 2, 6, 9 to be those of mass density merely with a scale change in the ordinate.

It will be seen from these figures that if $T_0$ is as low as 4,000°K, the distribution of $\phi$ or $\rho$ is changed greatly with increase in the radius $R_1$ of the liquid core. In the case of $T_0$ being as high as 50,000°K or 100,000°K the distributions under consideration are nearly the same for any radius $R_1$ of the liquid core.

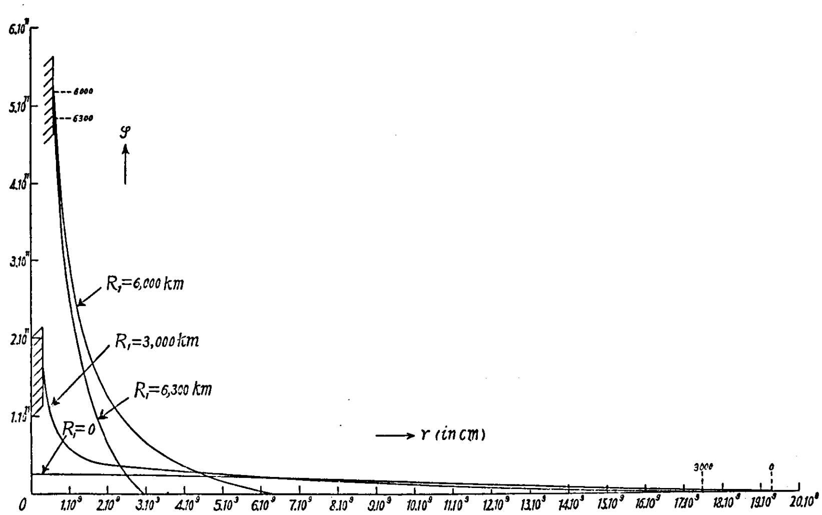


Fig. 2. Distribution of $\phi$ for (i) $T_0$=4,000°K.

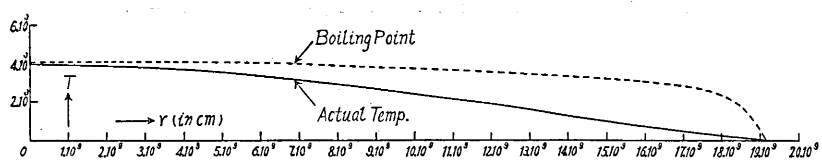


Fig. 3. $T$ and $T_2$ for case (i) $T_0$=4,000°K; $R_1$=0.

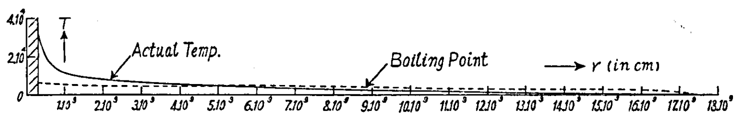


Fig. 4. $T$ and $T_2$ for case (i) $T_0$=4,000°K; $R_1$=3,000 km.

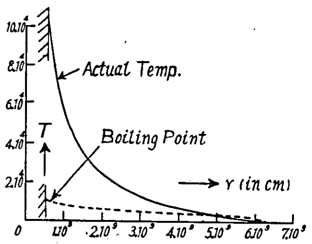


Fig. 5. $T$ and $T_2$ for case (i) $T_0$=4,000°K; $R_1$=6,000 km.

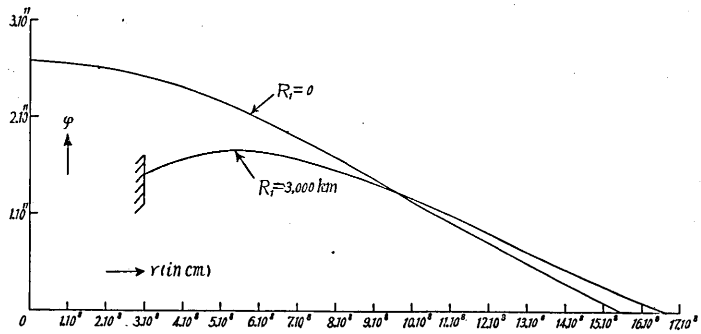


Fig. 6. Distribution of $\phi$ for (ii) $T_0=50{,}000°K$.

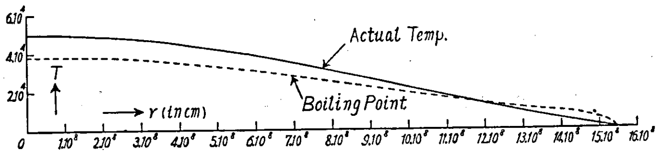


Fig. 7. $T$ and $T_2$ for case (ii) $T_0=50{,}000°K$; $R_1=0$.

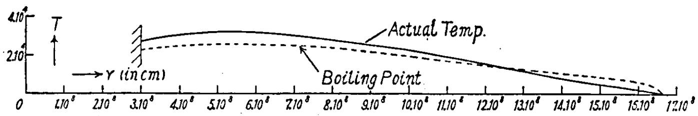


Fig. 8. $T$ and $T_2$ for case (ii) $T_0=50{,}000°K$; $R_1=3{,}000$ km.

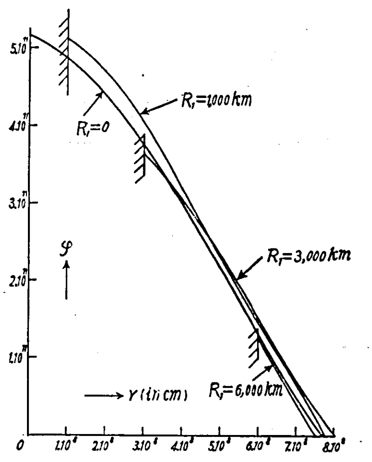


Fig. 9. Distribution of $\phi$ for (iii) $T_0=100{,}000°K$.

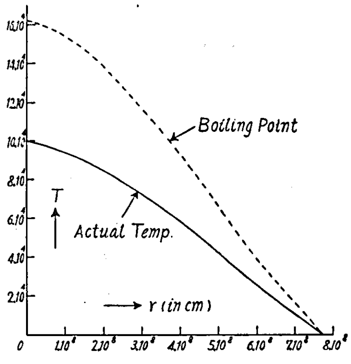


Fig. 10. $T$ and $T_2$ for case (iii) $T_0=100{,}000°K$; $R_1=0$.

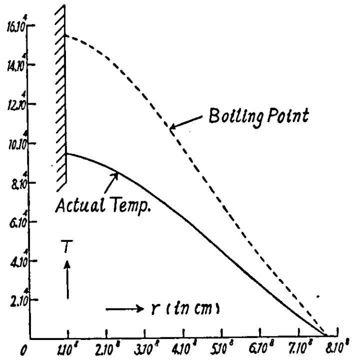


Fig. 11. $T$ and $T_2$ for case (iii) $T_0$=100,000°K; $R_1$=1,000 km.

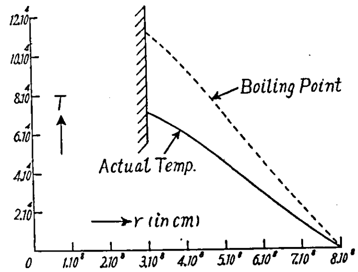


Fig. 12. $T$ and $T_2$ for case (iii) $T_0$=100,000°K; $R_1$=3,000 km.

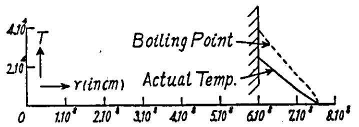


Fig 13. $T$ and $T_2$ for case (iii) $T_0$=100,000°K; $R_1$=6,000 km.

The above condition is likely to be in agreement with the change of the outer radius of the gaseous part surrounding the core. Whereas in case, $T_0$=4,000°K, the outer radius of the gaseous part diminishes rapidly with increase in the radius of the core, in case $T_0$=50,000°K or 100,000°K, the core density being the same as the mean density of the present earth, the outer radius last mentioned remains nearly the same for any radius of the core.

We shall next consider the distributions of $T_0$ and $T$.

Case (i) $T_0$<4,000°K. Although in condition $R_1=0$, the actual temperature $T$ is lower than the boiling point $T_2$ for any radius, in condition $R_1$=3,000 km or 6,000 km, $T$ is higher than $T_2$ for smaller radius, but $T$ is still lower than $T_2$ for a larger radius. Since the distributions of $T$ and $T_2$ for $R_1=0$ is possible only when the radius of the earth is as large as $19.10^9$ cm, no sooner than a shower of liquid metals pours out from the whole gaseous sphere toward the earth's centre, than the temperature distribution of the sphere changes to such a condition that whereas $T$ is higher than $T_2$ within the gaseous part near the liquid core, $T$ is lower than $T_2$ outside that part, the metal shower then falling only from the part of large radius. Since $T_2$ for iron or nickel is higher than that for almost any other material, the metals falling in showers would be iron or nickel.

Case (ii) 5,000°K$<T_0<$50,000°K. In any condition of $R_1$, $T$ is higher than $T_2$ for smaller radius, while the reverse holds for larger radius. In this condition, therefore, showers of liquid metal fall only from the part of large radius. It should be borne in mind that in the present condition, for an intermediate value of $R_1$, the maxima of $\phi$, $\rho$, $T$, and $T_2$ do not necessarily lie just on the boundary of the liquid core. They become maximum rather at radii intermediate between the surface, $R$, of the gaseous part and the surface, $R_1$, of the liquid core.

Case (iii) $T_0>$50,000°K. In any condition of $R_1$, $T$ is lower than $T_2$ for any radius. This means that the feature of a metal shower pouring from the whole of the earth is continued or rather accelerated until the whole earth becomes liquid. Since, in this case, the outer radius $R$ is nearly the same for any $R_1$, it is possible for the medium of the gas to liquefy at its original position without any shower of liquid metal falling toward the centre of the earth.

From the distribution of materials in the present earth, we may say that the metal showers fell from the upper gaseous sphere or, at least, convection movements of metal liquid occurred at the very early stage of the earth's formation. It follows then that the state corresponding to case (iii) was unlikely unless an extremely perfect convection of liquid occurred. Thus, cases (i) and (ii) support the formation of the present earth. It is, however, impossible to conclude whether it is case (i) or case (ii) that corresponds to the origin of the present earth.

Finally, it should be remarked that in the present calculation we assumed that $\lambda$ is constant. If $\lambda$ diminishes with temperature rise, the ordinates of curves corresponding to boiling points in Figs. 3, 4, 5, 7, 8, 10, 11, 12, 13 are likely to be higher than those shown in the respective figures, in agreement with which the underlined values in the last line of Table I also increase, while those in the first line of the same table diminish. Thus, if $\lambda$ diminishes enormously with temperature rise, the result would be that the conditions will be classified merely into (i) and (iii). The condition would then change immediately from (i) to (iii) with increase in $T_0$. From this idea, it holds that case (i) is the only probable one for the origin of the primitive earth.

## 5. *Summary and concluding remarks.*

From mathematical investigation it has been ascertained that if the central temperature of the primitive earth is relatively low, say, $T_0<$4,000°K, or very high, say, $T_0>$80,000°K, the central part of the primitive earth will be liquid, and if, on the other hand, $T_0$ is medium, say, $5{,}000°K < T_0 < 50{,}000°K$, the central part under consideration could remain in a gaseous condition.

If $T_0 < 4{,}000°K$, although in a coreless condition the actual temperature would be lower than boiling point for any radius, in the condition with a core of a certain size the temperature will be higher than the boiling point for a smaller radius, but lower than boiling point for a larger radius. In this case, as soon as the liquid shower tends to pour from the whole gas sphere toward the earth's centre, the temperature distribution of the sphere changes to such a condition that the liquid shower falls only from the part of large radius. In the present case the outer radius of the gaseous part diminishes enormously with increase in the core radius.

If $5{,}000°K < T_0 < 50{,}000°K$, in the conditionwith a core or without it, the temperature will be higher than boiling point for a smaller radius, while the reverse holds for a larger radius. In this case, the shower will fall always from the part of larger radius.

If $T_0 > 50{,}000°K$, the temperature will always be lower than the boiling point and, furthermore, the radius of the outer boundary of the gaseous sphere will never remain nearly the same for any size of the core, from which it is possible to conclude that the medium of the gas liquefies at its original position without any shower of metal liquid. Thus, unless an extremely perfect convection movement of the liquid occurred in the past stage of the earth, the liquefied condition of the last case is not in agreement with the configuration of the stratification of the present earth.

It holds then that the present earth did not result immediately from the high temperature condition $T_0 > 50{,}000°K$, but was formed from the primitive gaseous earth with relatively low temperature or from the one that had been greatly cooled from the high temperature condition just given.

In the present calculation, the heat of evaporation is assumed to be constant for every material. If that heat diminishes with temperature rise, it is likely that the conditions to be classified are merely two, namely, the condition corresponding to the relatively low central temperature of the primitive gaseous earth and the condition corresponding to the extremely high central temperature of the same earth. Of these conditions, the former is naturally more plausible.

In conclusion, we wish to express our sincerest thanks to Dr. Shirai for his valuable suggestions in the present study.

# 32. 半瓦斯狀地球に於ける溫度分布

地震研究所 {妹澤克惟
{金井清

地球が太陽系中で獨立したものか，外部から捕獲されたかについてはあまりはつきりした事がいへないが，液狀から固體狀の方へ向つたものである事はいへると思ふ．この考方を延長すると極めて短時間ではあるけれども瓦斯狀の時代があつたかも知れぬ譯である．地球全體が瓦斯であつたとした場合に於ける溫度の分布をその壓力に相當する沸點と比較した研究に Mercier のものがある．只今の研究では氣球の中央に凝固した液の球がある場合に Mercier の考方に如何なる變化があるかをしらべたのである．

問題を解くには，液の球と外部の眞空部との間の瓦斯の部分に Poisson の數式と瓦斯の Polytropic distribution の關係式を用ひ，內部及び外部の境界條件にあはせるのである．尙，溫度の分布を出すために瓦斯の式を必要とする．

地球の場合にその最初の瓦斯狀の球の中心の溫度を與へると Polytropic distribution の常數の一部が定まる．又，中心にある液狀球の大さと質量も種々に變化できるが，それに相當する氣球の半徑も定まる譯である．その結果氣球部の壓力及び溫度の分布が定まる．

一方に於て各氣壓に相當する沸點の決定が必要であるが，之は非常に厄介である．Clapyron の氣化熱，沸點，沸點壓間の關係を利用して，之から逆に沸點の變化を定める．但し氣化熱が沸點の上昇と共に下降するから，その考慮も必要である．

さて，半瓦斯狀地球全體の質量を現在の地球の質量と同じにし，中心にある液狀核の密度も便宜上現在の地球の平均密度と同じに取る．このやうにしてその最初の狀態卽ち地球全體が瓦斯のときの中心の溫度（絕對溫度）が 2,000°，4,000°，5,000°，6,000°，8,000°，10,000°，50,000°，80,000°，100,000° の各場合を計算して見たのである．

(i) 中心の最初の溫度が 4,000° よりも低い場合には，中心核がなければ如何なる半徑でも溫度が沸點よりも低いけれども，中心核ができると半徑の小さな所では溫度が沸點よりも高いのに半徑の大きな所では溫度が沸點よりも低い．故に中心核がない間は如何なる部分からも物質の雨（特に鐵かニッケルの雨）が中心へ向けて降るけれども，一旦中心核ができ出すと，高い氣界からのみ雨が降る事になる譯である．

(ii) 中心の最初の溫度が 5,000° 乃至 50,000° になると中心核があつてもなくても，小さな半徑では溫度が沸點よりも高く，大きな半徑では溫度が沸點よりも低い．故に如何なる中心核のの場合でも鐵の雨が高い氣界のみから降る．

(iii) 中心の最初の溫度が 50,000° 以上になると溫度が常に沸點よりも低い．而も球が氣體であつても液體であつてもその大さが大して變らない．之では金屬の雨が降る機會がない譯である．在現の地球はこの狀態から直接液化したものでない事は明かである．卽ち溫度が (iii) から (ii) 乃至 (i) に下降して氣界が出來てから雨が降り出したものである事がいはれる．