35. *Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part II.*

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1. *Introduction.*

In the previous paper[1] we dealt with the effect of brace struts on the vibration of a structure for the case of a finite number of spans, and also that for the case of a two-storied structure. It was ascertained that the frequency is considerably increased with increase in the number of spans, so that the aseismic properties are more pronounced in the case of multi-spans. In the case of a two-storied structure, the aseismic properties of brace struts are also pronounced. But, if brace struts were added to only one of the stories, the condition of the overstiff struts is rather equivalent to that of a single storied braceless structure.

The special case of the problem of a two-storied structure in which the first floor is without stiffness, corresponds to the case of a brace strut whose lower end is hinged to the intermediate point of a column of a single-storied structure; that is, the case of a partial brace (Hôdue). In the present paper, we shall discuss the aseismic properties of the partial brace just given and show the results of some cases that are supplementary to the previous paper.

2. *The stiffness of a single-storied structure with partial braces (Hôdue).*

It is assumed that there are an infinite number of spans and furthermore that the masses are concentrated at the respective panel points on the floor. Dynamically speaking, the problem can be solved by considering the equilibrium of the four parts AB, BC, CD, DB separately and applying the end conditions at A, B, C, D. There are

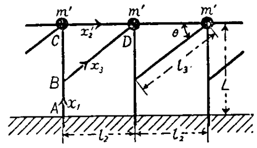

Fig. 1.

1) K. KANAI, "Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part I", *Bull. Earthq. Res. Inst.*, **17** (1939), 234~252.

two boundary conditions at A, six at B, eight at C, and also eight at D.

Since, however, the present problem is merely a special case of Section 7 of the previous paper[2] with the condition that $\zeta=E_2I_2/E_1I_1=0$, $\gamma_n=m_n p^2 l^3/E_1I_1=0$, we shall use the equations obtained in that paper. If we put

$$\zeta=0\,,\quad \gamma_n=0\,,\quad \xi'=\frac{a_1L^2}{I}(=4\xi),\quad \gamma_{\mathrm{I}}=\frac{m_1'p^2L^3}{E_1I_1}(=8\gamma_1'),$$

$$\gamma_{\mathrm{II}}=\frac{m_2'p^2L^3}{E_1I_1}(=8\gamma_2'),\quad \phi'=\frac{l_2}{L}\left(=\frac{\phi}{2}\right),\qquad (\eta'=2\phi\tan\theta) \tag{1}$$

in (100), (101), (102) of the previous paper,[3] the frequency equation then reduces to

$$\begin{aligned}\gamma_{\mathrm{I}}\gamma_{\mathrm{II}}&\Big[\vartheta\xi'\phi'^2P\cot\theta+6Q\left\{\vartheta(1-\phi'\tan\theta)+\mathrm{cosec}^3\theta\right\}\Big]\\&-\gamma_{\mathrm{I}}\xi'\left\{\vartheta\xi'\phi'^2P\cot\theta+6Q(\vartheta+\mathrm{cosec}^3\theta)\right\}\\&-6\gamma_{\mathrm{II}}\Big[\vartheta\xi'\phi'^2R\cot\theta+3S\left\{\vartheta(1-\phi'\tan\theta)+\mathrm{cosec}^3\theta\right\}\Big]\\&\qquad+6\xi'\left\{\vartheta\xi'\phi'^2R\cot\theta+3S(\vartheta+\mathrm{cosec}^3\theta)\right\}=0\,,\end{aligned} \tag{2}$$

where

$$\begin{aligned}P&=(1-\phi'\tan\theta)^2\left\{36\zeta'^2\tan^2\theta(1-\phi'\tan\theta)\right.\\&\quad-3\zeta'\tan\theta(\phi'^2\tan^2\theta+10\phi'\tan\theta-14)\\&\qquad\left.-(\phi'\tan\theta+2)(2\phi'\tan\theta-3)\right\},\\Q&=(6\zeta'\tan\theta+1)(3\zeta'+\phi'),\\R&=(6\zeta'\tan\theta+1)\left\{3\zeta'\tan\theta(4-3\phi'\tan\theta)+(3-2\phi'\tan\theta)\right\},\\S&=(6\zeta'\tan\theta+1)(12\zeta'+\phi'),\end{aligned} \tag{2}$$

in which

$$\gamma_{\mathrm{I}}=\frac{m_1'p^2L^3}{E_1I_1},\quad \gamma_{\mathrm{II}}=\frac{m_2'p^2L^3}{E_1I_1},\quad \vartheta=\frac{E_3a_3}{E_1a_1},\quad \zeta'=\frac{E_2'I_2'}{E_1I_1},\quad \phi'=\frac{l_2}{L},\quad \xi=\frac{a_1L^2}{I_1}. \tag{3}$$

If we put $\vartheta(=E_3a_3/E_1a_1)=0$, equation (2) reduces to

$$\gamma_{\mathrm{I}}=\frac{3(12\zeta'+\phi')}{3\zeta'+\phi'},\qquad \gamma_{\mathrm{II}}=\xi', \tag{2'}$$

2) K. KANAI, *loc. cit.* 1). pp. 247~251.
3) *ibid.*

the first being of the same form as the frequency equation of the horizontal vibration of a single-storied braceless structure and the second as the frequency equation of the vertical vibration of the same structure.

(i) As an example, we put $\phi'(l_2/L)=1$, $\vartheta(E_3a_3/E_1a_1)=1$, $\gamma_{\mathrm{I}}=\gamma_{\mathrm{II}}(m_1'=m_2')$, $\xi'(=a_1 . L^2/I)=5000$ and calculate the vibrational frequencies for various angles of θ in the three cases, $\zeta'(E_2'I_2'/E_1I_1)=1$, 10, ∞. The lower end of each brace is always at an intermediate point in the column. The results of calculation of frequencies for both horizontal and vertical vibrations are shown by full lines in Figs. 2, 3, 4.

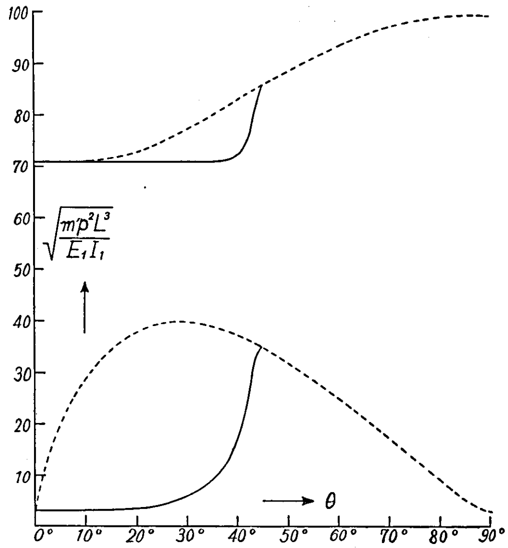

Fig. 2. Vibration frequencies (horizontal and vertical) of the case $l_2/L=1$, $E_2'I_2'/E_1I_1=1$, $E_3a_3/E_1a_1=1$. Full line: partial brace. Broken line: brace with its lower end on the ground.

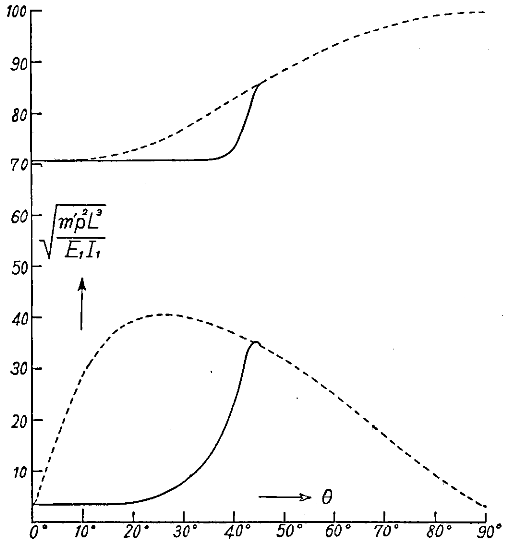

Fig. 3. Vibration frequencies (horizontal and vertical) of the case $l_2/L=1$, $E_2'l_2'/E_1I_1=10$, $E_3a_3/E_1a_1=1$. Full line: partial brace. Broken line: brace with its lower end on the ground.

The method of calculation for the case in which the lower end of the brace strut of varying inclinations is on the ground surface, was shown in another paper[4] published last year. Equation (18) of that paper represents the frequency equation. Using that equation we calculated the vibrational frequency of the case in which the lower end of each brace strut is just on the ground surface, the results of which are shown by broken lines, also

4) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, 16 (1938), 702.

in Figs. 2, 3, 4.

It will be seen from these figures that the partial brace here considered is not so effective on the aseismic properties of the structure as the brace strut that ends on the ground surface. In the present examples, the vibrational frequency of the structure with partial braces is nearly equal to that without the same braces unless the lower ends of the braces are quite near the ground surface. This feature does not change much even should the ratio of ζ', that is, E_2I_2/E_1I_1, be very small or very large.

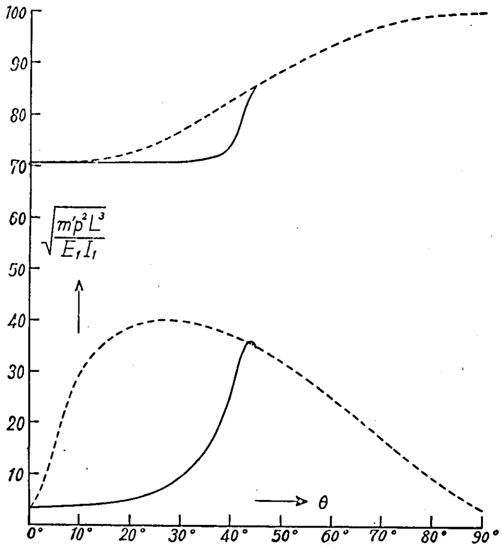

Fig. 4. Vibration frequencies (horizontal and vertical) of the case $l_2/L=1$, $E_2'I_2'/E_1I_1=\infty$, $E_3a_3/E_1a_1=1$. Full line: partial brace. Broken line: brace with its lower end on the ground.

The fact that the values of the vibrational frequency are nearly the same for any stiffness of the beam, indicates that the frame part which consists of BC, CD, DB in Fig. 1, remains as a rigid body. Since both ends of the brace are in a hinge condition, the braces with horizontal inclination never contribute to the change of frequency. Thus, in case, $\zeta'=\infty$, for example, the vibrational frequency is given by $p\sqrt{m_1L^3/E_1I_1}=\sqrt{12}$, which is the same as that for a single-storied structure with an extremely rigid floor.

The above consideration concerns cases of both horizontal and vertical vibrations. If we consider both cases of vibrations separately, there are certain special conditions for the respective cases of vibrations under consideration. Since the vibrational frequency of the vertical vibration is very high compared with that for the horizontal vibration, whereas the brace added in a suitably selected position is greatly effective on the increase in frequency of the vibration of horizontal type, the addition of the brace in any position does not appreciably change the frequency of vibrations of vertical type.

It should be borne in mind that, although the most effective condition of the partial brace is that the lower end of that brace shall be near the ground surface, if there be a slight inclination θ in the partial brace, the vibrational frequency of the structure then becomes even higher than that of the case of a braceless structure with an infinitely rigid floor.

(ii) We shall next consider the effect of the change in the stiffness of the partial brace on the vibrational frequency of the structure. As numerical examples, we put $\phi'(=l_2/L)=1$, $\zeta'(=E_2'I_2'/E_1I_1)=1$, $\gamma_{\text{I}}=\gamma_{\text{II}}(m_1'=m_2')$, $\xi'(a_1L^2/I)=5000$ and calculate the vibrational frequencies for various angles of θ in two cases $E_3a_3/E_1a_1=1$, 10. The results of calculation are shown in Fig. 5.

It will be seen that if the lower end of every brace were near the ground surface, the vibrational frequency of the structure would increase with increase in the sectional area of the same brace.

(iii) Let us finally consider the effect of partial braces in the case in which the ratio of beam span to column height varies. For this purpose, we take $\zeta'(=E_2'I_2'/E_1I_1)=1$, $\vartheta(=E_3a_3/E_1a_1)=1$, $\gamma_{\text{I}}=\gamma_{\text{II}}$ $(m_1'=m_2')$, $\xi'(=a_1L^2/I)=5000$ and calculate the vibrational frequencies for various angles of θ in the two cases, $\phi'(=l_2/L)=1/2$, 2, the lower end of each brace being always at an intermediate point in the column. The results of calculation of frequencies for both horizontal and vertical vibrations are shown by full lines in Figs. 6,

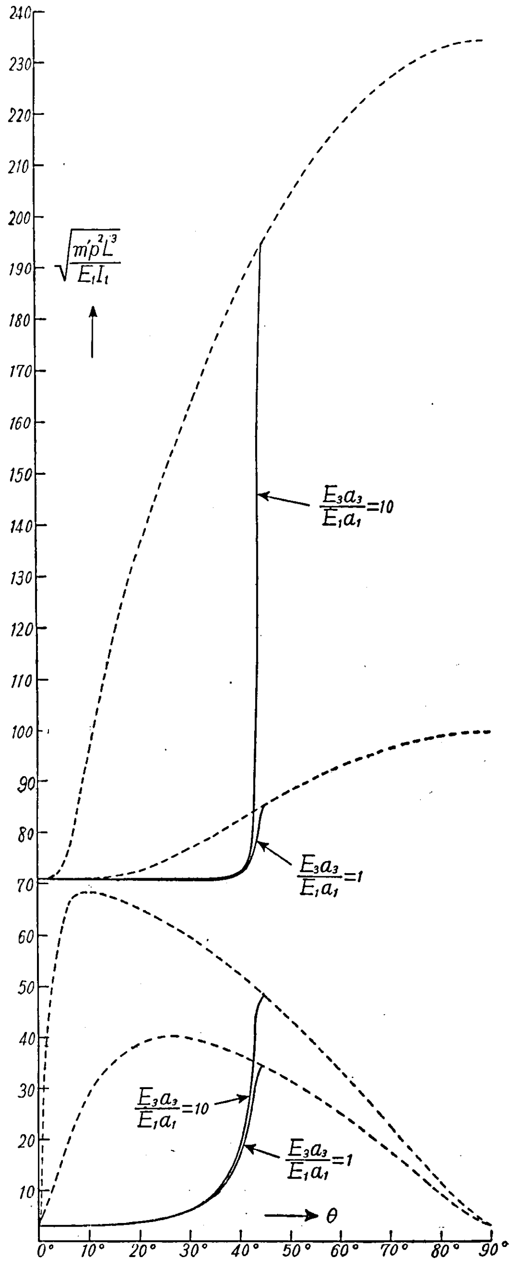

Fig. 5. Vibration frequencies (horizontal and vertical) of the case $l_2/L=1$, $E_2'I_2'/E_1I_1=1$, Full line: partial brace. Broken line: brace with its lower end on the ground.

7. The frequencies for the case in which the lower end of each column is always on the ground surface are also calculated in the same way as in (i), the results of which are shown by broken lines in the same figures.

It will be seen that the nature of the problem is quite similar to that shown in (i). In the present case, since the ratio of beam span to column height is not unity, the abscissa of the point in each curve at which full line and broken line meet is not necessarily near $\theta=45°$. Notwithstanding this fact, the nature of the vibrational frequency resulting from the addition of the braces is almost similar to that shown in (i), the reason for which is probably that the condition of the structure with partial braces is rather equivalent to that of a single-storied braceless structure (with a rigid floor) of column height that is equal to the structural height below the lower ends of the partial braces. The above feature holds, as a matter of fact, for horizontal vibration as well as for vertical vibration.

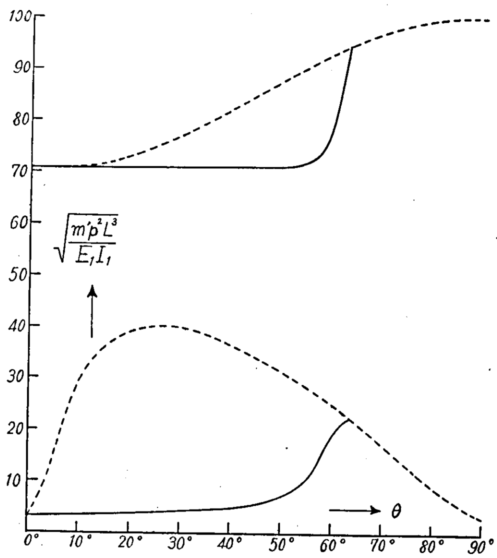

Fig. 6. Vibration frequencies (horizontal and vertical) of the case $l_2/L=1/2$, $E_2'I_2'/E_1I_1=1$, $E_3a_3/E_1a_1=1$. Full line: partial brace. Broken line: brace with its lower end on the ground.

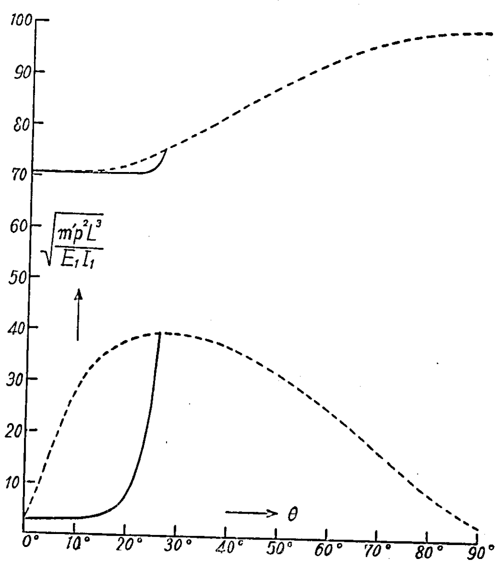

Fig. 7. Vibration frequencies (horizontal and vertical) of the case $l_2/L=2$, $E_2'I_2'/E_1I_1=1$, $E_3a_3/E_1a_1=1$. Full line: partial brace. Broken line: brace with its lower end on the ground.

3. *The stiffness of a single-storied structure of mono-span with brace strut.*

Although the problem of

this case has been shown in a previous paper,[5] owing to the mathematical difficulty, we have calculated only a few simple cases. It was so adjusted that no vertical displacement of the structure exists. We shall here discuss more extended cases by calculating various conditions of the problem, particularly in the stiffness ratios E_2I_2/E_1I_1, E_3a_3/E_1a_1, and the ratio of beam span to column height l_2/l_1.

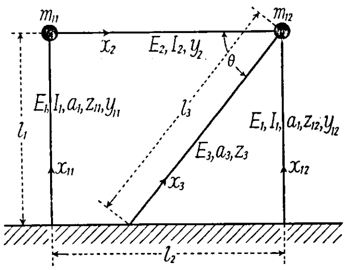

Fig. 8.

The equations used in calculation are the same as (35), (36), (37) in the previous paper.[6]

(i) As numerical examples, we first put $\phi(=l_2/l_1)=1$, $\psi(=l_3/l_1)=\sqrt{2}$, $\xi(=a_1l_1^2/I_1)=5000$, $\theta=45°$ and calculate the vibrational frequencies in the cases $\zeta(=E_2I_2/E_1I_1)=0$, 1, 10. The results are shown in Figs. 9, 10, 11. It will be seen from these figures that the effect of the change

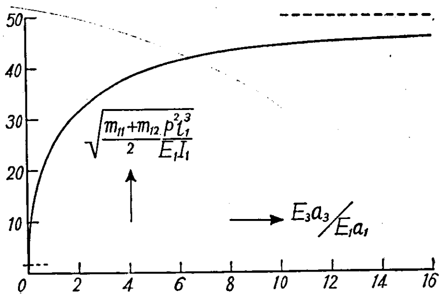

Fig. 9. Vibration frequencies of the case, $E_2I_2/E_1I_1=0$, $l_2/l_1=1$, $l_3/l_1=\sqrt{2}$, $a_1l_1^2/I_1=5000$, $\theta=45°$.

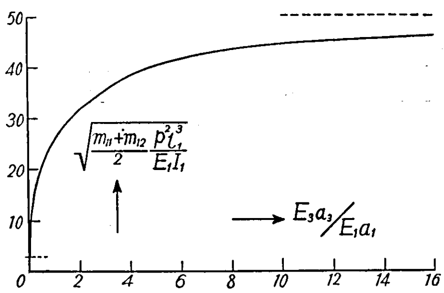

Fig. 10. Vibration frequencies of the case, $E_2I_2/E_1I_1=1$, $l_2/l_1=1$, $l_3/l_1=\sqrt{2}$, $a_1l_1^2/I_1=5000$, $\theta=45°$.

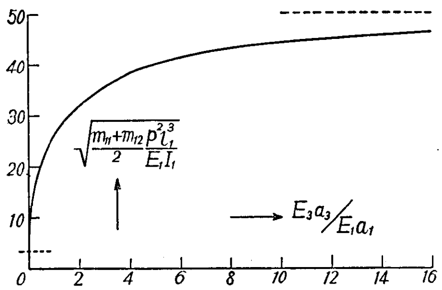

Fig. 11. Vibration frequencies of the case, $E_2I_2/E_1I_1=10$, $l_2/l_1=1$, $l_3/l_1=\sqrt{2}$, $a_1l_1^2/I_1=5000$, $\theta=45°$.

5), 6) K. KANAI, *loc. cit.* 1). p. 238.

in the stiffness of the beam on the vibrational frequency of the braced framed structure is not appreciable, which feature is quite similar to that in the case of a structure with an infinite number of spans. At all events, if the lower end of each brace strut is on the ground surface, the vibrational frequency of the structure is not seriously affected by the stiffness of the beam in any case.

(ii) We shall next consider the effect of the change in the ratio of $l_2\, l_1$, that is to say, the ratio of the beam span to the column height. Keeping the ratio of E_2I_2/E_1I_1 as unity, the ratio l_2/l_1 is changed to 1/2, 2. The results of calculation for both cases are shown in Figs. 12, 13.

It will be seen that, whereas in the case of a narrow span, the vibrational frequency assumes an asymptotic value for a certain ratio of E_3a_3/E_1a_1, in the case of a wide span the frequency is augmented increasingly with increase in the ratio of E_3a_3/E_1a_1, no asymptotic value of the frequency existing for a moderate value of E_3a_3/E_1a_1. It is therefore possible for the case of a relatively large span to be stiffened without limit by the increase in the cross section of each strut.

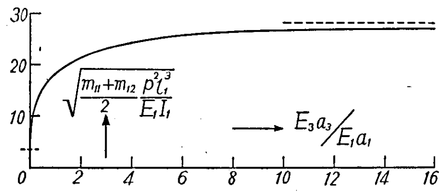

Fig. 12. Vibration frequencies of the case, $E_2I_2/E_1I_1=1$, $l_2/l_1=1/2$, $l_3/l_1=\sqrt{5}$, $a_1l_1^2/I_1=5000$, $\theta=\tan^{-1}2$.

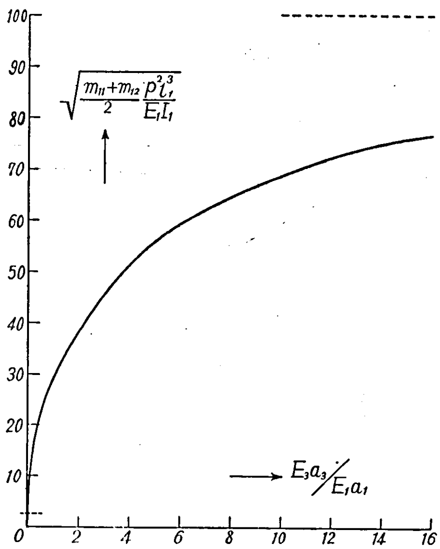

Fig. 13. Vibration frequencies of the case, $E_2I_2/E_1I_1=1$, $l_2/l_1=2$, $l_3/l_1=\sqrt{5}$, $a_1l_1^2/I_1=5000$, $\theta=\tan^{-1}1/2$.

4. *The stiffness of a single-storied structure of two spans with brace struts.*

The mathematical solution of the present case was shown also in the previous paper,[7] the condition of no vertical displacement of the vibration being replaced by suitable stresses as shown in the same paper.

In the previous paper we considered that the ratio of the stiffness

7) K. KANAI, *loc. cit.* 1), p. 240.

of the beam to that of the column is unity. In the present problem, on the other hand, we shall take the case in which no stiffness exists in the beams and also the one in which the stiffness of the beams is extremely large. Using equations (59), (60), (61) in the previous paper, we have calculated the case just mentioned, the results of which are shown in Figs. 15, 16. Comparing these with the result in Fig. 8 of the previous paper, it will be seen that in the present cases too the beam stiffness, even should it be very small or very large, is not important in the change of vibrational frequency of the braced structure.

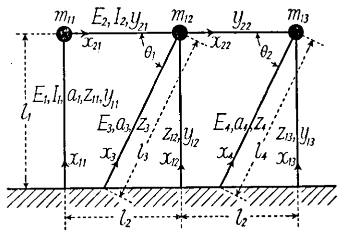

Fig. 14.

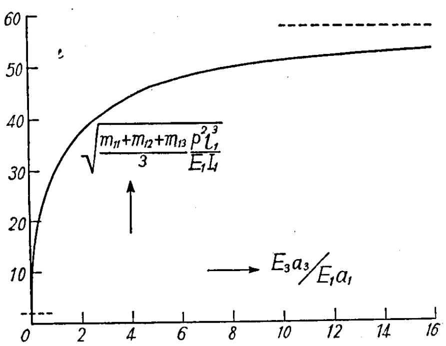

Fig. 15. Vibration frequencies of the case, $E_2I_2/E_1I_1=0$, $l_3/l_1=\sqrt{2}$, $l_2/l_1=1$, $a_1l_1^2/I_1=5000$, $\theta=45°$.

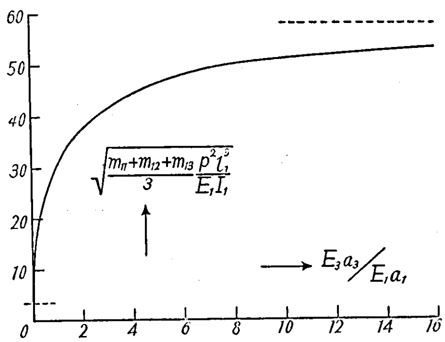

Fig. 16. Vibration frequencies of the case, $E_2I_2/E_1I_1=\infty$, $l_3/l_1=\sqrt{2}$, $l_2/l_1=1$, $a_1l_1^2/I_1=5000$, $\theta=45°$.

5. *General summary and concluding remarks.*

By mathematical investigation, we have ascertained the dynamical theory of the aseismic properties of partial braces (Hôdue) and also such parts of the theory of the usual brace struts that were omitted in the previous paper.

It has been ascertained that the frequency of the horizontal vibration of the structure with partial braces does not greatly differ from that without the same braces unless the lower ends of the braces are quite near the ground surface. Since the frequency of the vertical vibration of the structure is inherently very high, the addition of a brace in any possible position does not affect the change in the frequency of the same vibration. At all events, the condition of the structure with partial braces is rather equivalent to that of a single storied braceless structure with a rigid floor of column height equal to that of the part of the original column below the lower end of each strut.

It should be borne in mind that, although the most effective condition of the brace is that the lower end of that brace shall be near the ground surface, if there be a slight inclination even in the partial brace, the vibrational frequency of the structure then becomes higher even than that of the case of a braceless structure with an infinitely rigid floor.

With regard to the additional problems of the single-storied structure with usual braces, it is remarkable that in the case of a narrow span, a small increase in the cross section of the brace is equivalent to making that section infinitely great, whereas in the case of a wide span, the stiffness of the structure is augmented increasingly with increase in the cross section of the brace.

In conclusion, I wish to express my thanks to Messrs. Unoki and Watanabe for valuable assistance in the present series of investigation. The present investigation was made at Professor Sezawa's suggestion in connection with his research work as member of the Investigation Committee for Earthquake-proof Construction, of the Japan Society for the Promotion of Scientific Research. I wish also to express my sincerest thanks to Professor Sezawa for valuable aid given to me.

35. 筋違の耐震効果の理論 (本論其 2)

地震研究所　金　井　清

數個の張間のある1階建や無限數の張間のある建物に筋違があるときの耐震性を前回の報告で述べたが，今回は建物に部分的の筋違，卽ち極端にいへば方杖のある場合のそれをしらべた．

研究の結果によれば部分的筋違は餘り効果がない事がわかつた．筋違の下端が床又は地面まで來てをるものがよいのである．部分的方杖をつける事は，その方杖から下に當る柱の部分を高さとする筋違無しの柱がある構造（床の剛なる）と同じになるからである．

玆に注意すべき事は，筋違の下端が地面迄來てをれば最も理想的であるとはいへ，たとへ方杖型であつてもそれに幾分かの傾斜がありさへすれば，筋違がなくて床が極端に剛い場合の振動數より高い振動數を持つやうになるのである．

尙，この研究中には第1報で述べなかつた事柄を附加へてあるがその中でも，張間の狹い構造に限り筋違の剛度を少しく增すと筋違の剛度が無限に大きくなつたのと同じ効果のあるといふ事などは特記すべき點である．