36. *Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part III.*

The Effect of Wave Scattering (Dissipation).

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1. *Structural stiffness in vibration and resonance amplitudes.*

In Parts I and II of the present series of papers[1], we discussed the effect of brace struts on the vibration of a structure, mainly with respect to its resistance to seismic forces. Thus, if the frequency of natural vibration of a structure is increased in consequence of brace struts fitted suitably to that structure, it is thought that the same structure becomes aseismic. Since, as a matter of fact, in the case of a stiff structure the vibrational deflection under seismic forces is usually small and since, besides, structural damping is always very high, it is likely that the high frequency of natural vibration implies the aseismic condition of the structure.

On the other hand, there is the well-known dynamical fact that, under resonance conditions, the vibration amplitude of any structure is pronounced, from which it is improbable that a structure with high natural frequency is always aseismic. Although the vibration amplitude of a structure under resonance is large compared with that under resonance conditions, it would be still possible for the resonance amplitude under consideration to differ greatly were its damping resistance changed.

We shall now investigate the effect of damping resistance on a structure with brace struts. There are at least two kinds of damping resistances, one originating from the inner damping force in the structure and the other resulting from the scattering (dissipation) of the vibrational energy into the ground. In the present paper the latter condition, that is, the energy scattering, alone will be studied.

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1) K. KANAI, "Theory of Aseismic Properties of the Brace Struts (Sudikai) in a Japanese-style Building. I, II," *Bull. Earthq. Res. Inst.*, **17** (1939), 234, 559.

As will be seen from the problem[2] of the scattering condition of a structure without brace struts, since the calculation of the energy dissipation is extremely complex, we shall now restrict ourselves to the case of a single-storied structure with brace struts.

2. *Equations of vibratory motion.*

For simplicity, it is assumed that the masses are concentrated at the panel points. Let l_1, l_2, l_3 be the length of the columns, beam span, and length of a brace strut, respectively; let also E_1I_1, E_1a_1; E_2I_2, 0, 0, E_3a_3 be the bending and longitudinal stiffnesses of the respective members just mentioned. The lateral displacements y_1, y_2, 0 and the longitudinal displacements z_1, 0, z_3 of the respective members satisfy differential equations of the types

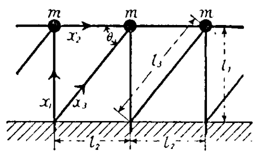

Fig. 1.

$$\frac{\partial^4 y_n}{\partial x_n^4}=0, \quad [n=1, 2] \qquad \frac{\partial^2 z_n}{\partial x_n^2}=0. \quad [n=1, 3] \tag{1}$$

The solutions of the above equations are

$$\left.\begin{aligned} y_n&=(A_n+B_nx_n+C_nx_n^2+D_nx_n^3)e^{ipt}, \qquad [n=1, 2] \\ z_n&=(\alpha_n+\beta_nx_n)e^{ipt}. \qquad [n=1, 3] \end{aligned}\right\} \tag{2}$$

The vibration of the structure occurs as the result of incident and reflected seismic waves including scattered waves. If the primary waves be transverse and incident normally on the ground surface, then the expressions of the plane incident and reflected waves are

$$u_0=e^{i(pt+kx)}, \qquad u_0'=e^{i(pt-kx)}, \tag{3}$$

where $k^2=\rho p^2/\mu$. Assuming then, for simplicity, that the scattered waves are radiated from a semi-spherical surface, $r=\varepsilon$, shown in Fig. 2, the expressions for the scattered longitudinal and transverse waves become

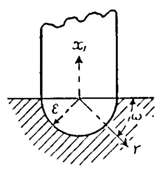

Fig. 2.

$$\left.\begin{aligned} \Delta&=\alpha_0 \cos\omega e^{i(pt-hr)}\left(\frac{1}{r}+\frac{1}{ihr^2}\right), \\ \varpi&=-\beta_0 \sin\omega e^{i(pt-kr)}\left(\frac{1}{r}+\frac{1}{ikr^2}\right), \end{aligned}\right\} \tag{4}$$

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2) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 682, 687, 690, 918; **14** (1936), 164, 377.

$$\left.\begin{aligned} u_1 &= -\frac{\alpha_0}{h^2}\cos\omega\frac{d}{dr}e^{i(pt-hr)}\left(\frac{1}{r}+\frac{1}{ihr^2}\right), \\ v_1 &= \frac{\alpha_0}{h^2}\sin\omega\frac{1}{r}e^{i(pt-hr)}\left(\frac{1}{r}+\frac{1}{ihr^2}\right), \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} u_2 &= -\frac{4\beta_0}{k^2}\cos\omega\frac{1}{r}e^{i(pt-kr)}\left(\frac{1}{r}+\frac{1}{ikr^2}\right), \\ v_2 &= \frac{2\beta_0}{k^2}\sin\omega\frac{1}{r}\frac{d}{dr}e^{i(pt-kr)}\left(1+\frac{1}{ikr}\right), \end{aligned}\right\} \tag{6}$$

where u_1, v_1 and u_2, v_2 satisfy exactly the equations of motion for both longitudinal and transverse bodily waves and $h^2=\rho p^2/\lambda+2\mu$; ρ, λ, μ being the density and elastic constants of the earth.

The boundary conditions are such that

$$x_1=0\,,\ x_3=0\,,\ r=\varepsilon_1; \qquad z_1=0\,,\ z_3=y_1\cos\theta\,,\ \frac{dy_1}{dx_1}=0\,, \tag{7), (8), (9}$$

$$(u_1+u_2)_{\text{max.}}=-(v_1+v_2)_{\text{max.}}, \tag{10}$$

$$y_1=u_0+u_0'+(u_1+u_2)_{\omega=0}\,, \tag{11}$$

$$\begin{aligned} &-E_1\pi\varepsilon_1^2 j_1^2\frac{d^3y_1}{dx_1^3}+E_3a_3\frac{dz_3}{dx_3}\cos\theta \\ &=\int_{\phi=0}^{\phi=\pi}\int_{\omega=0}^{\omega=\pi}\mu\left\{\frac{\partial(v_1+v_2)}{\partial r}-\frac{(v_1+v_2)}{r}+\frac{1}{r}\frac{\partial(u_1+u_2)}{\partial\omega}\right\}r^2\sin^2\omega\,d\phi\,d\omega \\ &\quad+\int_{\phi=0}^{\phi=\pi}\int_{\omega=0}^{\omega=\pi}\left\{\lambda\Delta+2\mu\frac{\partial(u_1+u_2)}{\partial r}\right\}r^2\sin\omega\cos\omega\,d\phi\,d\omega\,, \end{aligned} \tag{12}$$

$x_1=l_1$, $x_2=l_2$, $x_{22}=0$, $x_3=l_3$;

$$y_2=y_{22}=-z_1, \quad z_3=z_1\sin\theta+y_1\cos\theta, \tag{13), (14), (15}$$

$$\frac{dy_1}{dx_1}=\frac{dy_2}{dx_2}=\frac{dy_{22}}{dx_{22}}, \tag{16), (17}$$

$$-E_1I_1\frac{d^2y_1}{dx_1^2}-E_2I_2\left(\frac{d^2y_2}{dx_2^2}-\frac{d^2y_{22}}{dx_{22}^2}\right)=0\,, \tag{18}$$

$$-E_2I_2\left(\frac{d^3y_{22}}{dx_{22}^3}-\frac{d^3y_2}{dx_2^3}\right)-E_3a_3\frac{dz_3}{dx_3}\sin\theta-E_1a_1\frac{dz_1}{dx_1}=0\,, \tag{19}$$

$$-E_1I_1\frac{d^3y_1}{dx_1^3}+E_3a_3\frac{dz_3}{dx_3}\cos\theta=mp^2y_1\,, \tag{20}$$

where j is the radius of gyration of a cross section of each column. Substituting (2)$\sim$(6) in (7)$\sim$(20), it is possible to determine all the constants. For example, the values of A_1, B_1, C_1, D_1 thus determined are such that

$$\left.\begin{aligned} &A_1=\frac{2QR}{QR-\gamma\tau PS}, & &B_1=0, \\ &C_1=\frac{3\gamma R(\phi+6\zeta)(\psi+\vartheta\sin^2\theta)}{QR-\gamma\tau PS}, & &D_1=\frac{-\gamma R(\phi+12\zeta)(\psi+\vartheta\sin^2\theta)}{QR-\gamma\tau PS}, \end{aligned}\right\} \tag{21}$$

where

$$P=3(\phi+12\zeta)(\psi+\vartheta\sin^2\theta)+(\phi+3\zeta)\vartheta\xi\cos^2\theta,$$

$$Q=\{3(\phi+12\zeta)(\psi+\vartheta\sin^2\theta)+(\phi+3\zeta)\vartheta\xi\cos^2\theta\}-\gamma(\phi+3\zeta)(\psi+\vartheta\sin^2\theta),$$

$$R=R_1+iR_2, \qquad S=S_1+iS_2,$$

$$R_1=\gamma\nu\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda}{\mu}+2}\right)+3\left(\frac{\lambda}{\mu}+2\right),$$

$$R_2=\sqrt{\nu\gamma}\left\{3\left(\frac{\lambda}{\mu}+2+\sqrt{\frac{\lambda}{\mu}+2}\right)+\nu\gamma\left(\sqrt{\frac{\lambda}{\mu}+2}-2\right)\right\},$$

$$S_1=5+\frac{2\lambda}{\mu}-\gamma\nu,$$

$$S_2=\sqrt{\gamma\nu}\left(1+2\sqrt{\frac{\lambda}{\mu}+2}\right),$$

$$\gamma=\frac{mp^2l_1^3}{E_1I_1}, \quad \nu=\frac{E_1\rho I_1\varepsilon_1^2}{\mu ml_1^3}, \quad \tau=\frac{3E_1I_1}{2\pi\mu\varepsilon l_1^3}, \quad \vartheta=\frac{E_3a_3}{E_1a_1}, \quad \zeta=\frac{E_2I_2}{E_1I_1},$$

$$\phi=\frac{l_2}{l_1}, \quad \psi=\frac{l_3}{l_1}, \quad \xi=\frac{a_1l_1^2}{I_1}. \tag{22}$$

We shall now calculate the horizontal displacements of the floor, the results of which are

$$y_{1\,x_1=l_1}=2\{Q+\gamma(\phi+3\zeta)(\psi+\vartheta\sin^2\theta)\}\sqrt{\frac{R_1^2+R_2^2}{(QR_1-\gamma\tau PS_1)^2+(QR_2-\gamma\tau PS_2)^2}}$$

$$\cdot\exp.\, i\left[pt+\tan^{-1}\frac{R_2}{R_1}-\tan^{-1}\frac{QR_2-\gamma\tau PS_2}{QR_1-\gamma\tau PS_1}\right]. \tag{23}$$

Using these expressions it is possible to get the resonance curves for the horizontal seismic vibration of the structure for any stiffness of the structure and the ground. It should be borne in mind that no inner damping force of the structure is present, in which condition it is possible to ascertain the effect of wave scattering on the vibrational nature of the same structure.

3. *Numerical examples and their interpretation.*

In the present examples, we shall assume that the ratio of beam span to column height, that is, $\phi = l_2/l_1$, is unity, so that $\theta = 45°$. Since it is further assumed that the lower end of the brace strut is exactly at the bottom of the column, the ratio of brace length to column height, that is, $\psi = l_3/l_1$, is always $\sqrt{2}$. We shall also assume that the Poisson ratio of the ground is 1/4, so that $\lambda = \mu$.

(i) As a first example, we shall take $E_1 = 5 \cdot 10^8$ dyne/cm², $\mu = 5 \cdot 10^7$ dyne/cm², $\rho = 2$, $l_1 = 400$ cm, $\varepsilon_1 = 25$ cm, $m = 7 \cdot 10^7$ gr. mass, from which we have $\xi = 1004$, $\tau = 0{\cdot}000915$, $\nu = 0{\cdot}0000103$. For approximation, we shall write

$$\xi\left(=\frac{a_1 l_1^2}{I_1}\right)=1000\,, \quad \tau=\frac{3E_1 I_1}{2\pi\mu\varepsilon_1 l_1^3}=0{\cdot}001\,, \quad \nu=\frac{E_1\rho_1 I_1 \varepsilon_1^2}{\mu m l_1^3}=0{\cdot}00001\,.$$

The values of E_1, l_1, ε_1, m here assumed represent the condition that a 70 ton mass (column mass being included) rests on the top of a wooden circular column of 50 cm dia. and 4 meters long. The mass here assumed is fairly heavy from the point of view of material resistance. Even should the mass be reduced to a certain fraction of that given above, the nature of scattering would not change much. The values of ρ, μ here given are likely to correspond to those for dense soil.

We have calculated the resonance curves of vibration amplitudes for various ratios of $\vartheta(=E_3 a_3/E_1 a_1)$, say, $\vartheta = 0, 1, 2, 5, \infty$, in two cases of $\zeta(=E_2 I_2/E_1 I_1)$, namely, $\zeta = 1, 10$, the results of which are shown in Figs. 3, 4. It should be remembered that $\sqrt{\gamma}\,(=\sqrt{mp^2 l_1^3/E_1 I_1})$ is exactly proportional to the vibrational frequency p.

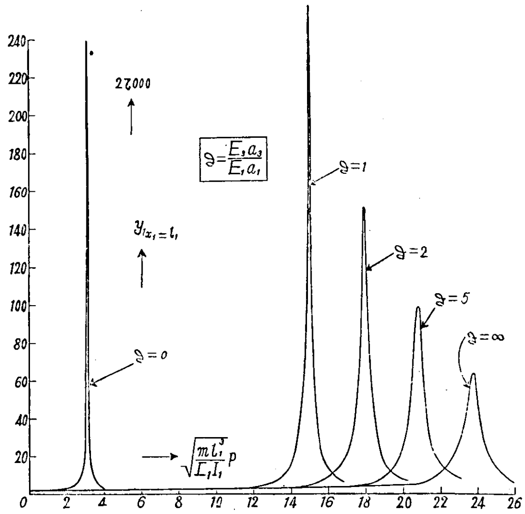

Fig. 3. Resonance curves for $\zeta(=E_2 I_2/E_1 I_1)=1$ in case (1).

It will be seen that the resonance amplitudes decrease enormously with increase in the stiffness of the brace struts in both cases of ζ-value, the resonance frequency increasing at the same time, though gradually. The curves show that the increase in the stiffness of the brace struts is much more effective on the decrease of resonance amplitudes than on the increase of natural frequencies of vibration.

Decrease of resonance amplitudes with increase in ϑ, is most pronounced for $\vartheta=0\sim1$. For $\vartheta>1$, on the other hand, the decrease in resonance amplitudes with increase in ϑ is rather gradual.

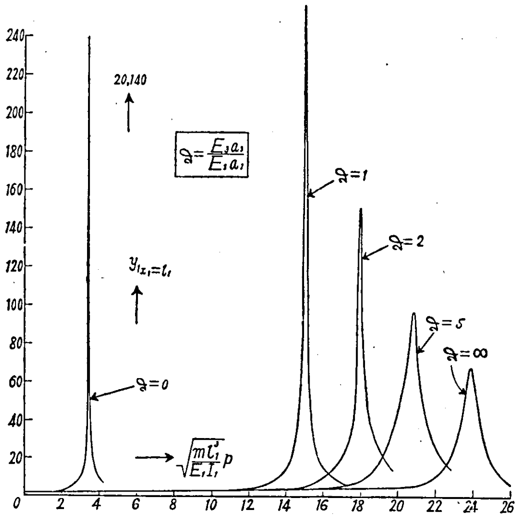

Fig. 4. Resonance curves for $\zeta(=E_2I_2/E_1I_1)=10$ in case (i).

Comparing Fig. 3 and Fig. 4 it will moreover be seen that notwithstanding that the ζ-values of both cases differ enormously, the curves in Fig. 4 are virtually the same as those in Fig. 3. This tells us that the change in beam (floor) stiffness is not very effective on the aseismic properties of a structure with braces.

(ii) We shall next consider another example in which the values of τ, ν are fairly large compared with those in the preceding example. Write

$$\xi\left(=\frac{a_1l_1^2}{I_1}\right)=1000\,,\qquad \tau\left(=\frac{3E_1I_1}{2\pi\mu\varepsilon_1l_1^3}\right)=0{\cdot}1\,,\qquad \nu\left(=\frac{E_1\rho_1I_1\varepsilon_1^2}{\mu ml_1^3}\right)=0{\cdot}0005\,,$$

which values are scarcely realized in the actual condition. Thus, in this case the resonance amplitudes even in the braceless condition of the structure are not too marked.

The calculation was made for various values of $\vartheta(=E_3a_3/E_1a_1)$, say, $\vartheta=0$, 0·1, 1, 5, ∞, in the two cases of $\zeta(=E_2I_2/E_1I_1)$, namely, $\zeta=1$, 10, the results of which are shown in Figs. 5, 6.

It will be seen that in the present case, too, the resonance amplitudes decrease with increase in the stiffness of the brace struts, the natural frequency of vibration increasing at the same time.

In the present case, however, the decrease in resonance amplitudes with increasing natural frequency, particularly near $\vartheta=0\sim1$, is not so marked as in the preceding example. The resonance amplitudes for a structure without brace strut, on the other hand, are still very small.

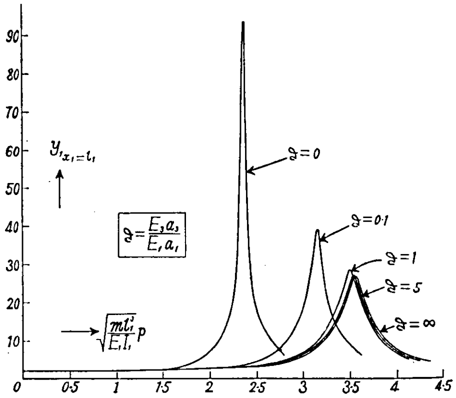

Fig. 5. Resonance curves for $\zeta(=E_2I_2/E_1I_1)=1$ in case (ii).

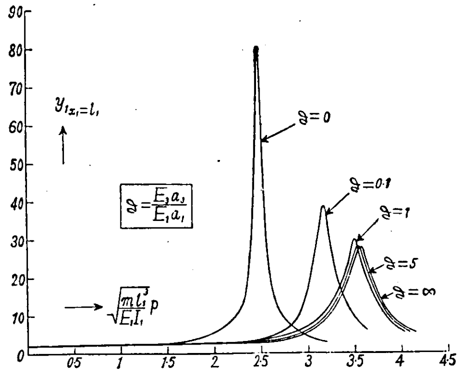

Fig. 6. Resonance curves for $\zeta(=E_2I_2/E_1I_1)=10$ in case (ii).

In this case, comparing the result in Fig. 5 with that in Fig. 6, it will be seen that the effect of the change in the beam (floor) stiffness on the aseismic properties of a structure with brace struts is practically negligible.

It may be concluded, at all events, that if the wave scattering nature of the structure and the ground is marked, the effect of the brace struts on diminution of resonance amplitudes will not be very pronounced.

4. *Comparison of natural frequencies of vibration with resonance amplitudes.*

Although the change in resonance amplitudes with change in the natural vibration periods has been investigated in two preceding sections, yet for ascertaining the same condition more clearly, we shall now show the relation between resonance amplitudes, natural vibration periods, and the stiffness of the brace struts, as given in Figs. 7, 8 and Tables I, II.

It will be seen that, in case (i), notwithstanding that the vibration period remains nearly constant beyond $\vartheta=1$, the resonance amplitude diminishes enormously for the range between $\vartheta=1$ and 5. In case (ii), on the other hand, both the vibration periods and the resonance amplitudes are nearly constant for ϑ even beyond $\vartheta=0{\cdot}5$. Now, it will be seen that the aseismic properties of the structure is pronounced in the case in which the scattering nature is medium (even if rather greater than the condition of the actual structure and the ground), the stiffness of brace struts being, besides, greater than the lengthwise stiffness of the columns. An over-strong brace strut is unnecessary, although it does not show a negative effect.

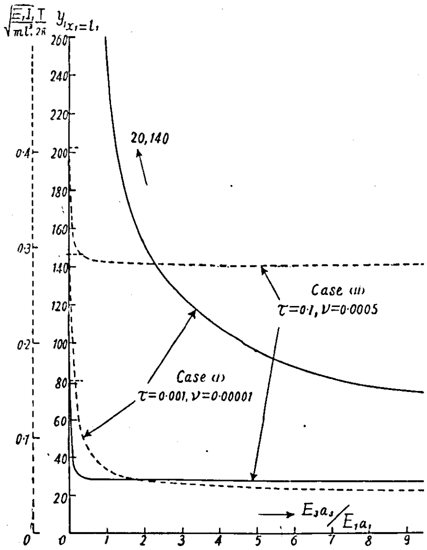

Fig. 7. Resonance amplitudes (full lines) and natural vibration periods (broken lines) in the case $\zeta=(E_2I_2/E_1I_1)=1$.

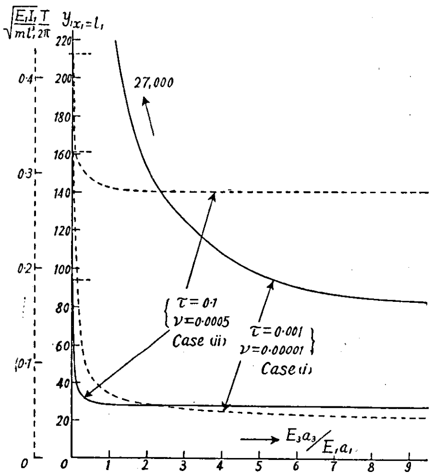

Fig. 8. Resonance amplitudes (full lines) and natural vibration periods (broken lines) in the case $\zeta(=E_2I_2/E_1I_1)=10$.

Table I. $y_{\max.}$ and $\sqrt{\gamma}$ in resonance in Case (i), $\tau=0{\cdot}001$, $\nu=0{\cdot}00001$.

	E_3a_3/E_1a_1	0	1	2	5	∞
$E_2I_2/E_1I_1=1$	$y_{\max.}$	27000	258	152	97·08	64·62
	$\sqrt{mp^2l_1^3/E_1I_1}$	3·11	14·96	17·85	20·78	23·79
$E_2I_2/E_1I_1=10$	$y_{\max.}$	20140	257	152	96·9	64·6
	$\sqrt{mp^2l_1^3/E_1I_1}$	3·41	15·01	17·9	20·8	23·8

Table II. $y_{\max.}$ and $\sqrt{\gamma}$ in resonance in Case (ii), $\tau=0{\cdot}1$, $\nu=0{\cdot}0005$.

	E_3a_3/E_1a_1	0	0·1	1	5	∞
$E_2I_2/E_1I_1=1$	$y_{\max.}$	93·9	39·2	28·6	27·5	27·2
	$\sqrt{mp^2l_1^3/E_1I_1}$	2·36	3·15	3·50	3·55	3·56
$E_2I_2/F_1I_1=10$	$y_{\max.}$	80·2	38·5	28·6	27·5	27·2
	$\sqrt{mp^2l_1^3/F_1I_1}$	2·47	3·17	3·51	3·55	3·56

5. *Summary and concluding remarks.*

From mathematical investigation it was ascertained that increase in the stiffness of brace struts fitted to a structure is invariably accompanied by increased natural frequency of the same structure as well as decreased resonance amplitudes.

In the case in which the wave scattering nature of the structure and the ground is medium (even if somewhat greater than that under actual conditions), the resonance amplitude diminishes enormously with increase in the stiffness of the brace struts, the resonance frequency also increasing at the same time, though gradually. In the case in which the wave scattering nature of the structure and the ground is marked, the decrease in resonance amplitudes with increasing natural frequency, particularly near $\vartheta=0\sim 1$, is not pronounced. In the latter case, the resonance amplitudes for a structure without brace strut are still very small.

In every case, even should the beam stiffness be changed to any considerable extent, no appreciable change in resonance amplitudes nor in natural vibration periods would occur.

At all events, the aseismic properties of brace struts are much more pronounced in diminishing the resonance amplitudes than in raising the natural frequencies of the structure. We are now studying the effect of material inner damping on the aseismic properties of a structure with brace struts, in which case it is likely that the resonance amplitudes will also greatly decrease as the result of brace struts having been fitted to that structure. Therefore, the addition of brace struts to any structure is recommendable in the sense of diminishing the resonance amplitudes rather than raising the natural vibration frequencies.

In conclusion, I wish to express my hearty thanks to Mr. Unoki, with whose aid the numerical calculations were successfully made. It should be added that the present investigation was made at Professor Sezawa's suggestion in connection with his research work as member of the Investigation Committee for Earthquake-proof Construction, of the Japan Society for the Promotion of Scientific Research. I wish also to express my sincerest thanks to Professor Sezawa.

36. 筋違の耐震効果の理論

(本論其3, 振動散逸の影響)

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筋違の耐震効果について述べた之迄の議論は單に耐震の剛度に關してであつた．たとひ如何に高い自己振動數を持つ構造でも共振狀態になると單に剛度といふ考方では不滿足になる．卽ち共振に於ける振動振幅が如何になるものかといふ事を說明しなければならないのである．

共振に於ける振幅を左右するものとして，構造自身中で勢力が熱に變化する減衰力と振動勢力が地中へ散逸する波動的のものとが重要である．只今は振動が波動として散逸する場合を研究する．

問題を簡單にして筋違のある1階建のものが水平震動をなす場合を取る．構造部の振動式及び地中での波動式とを境界條件が滿足するやうに組合して問題を解くのである．

研究の結果，筋違を設けた結果として自己振動數の高くなつてるものは同時に共振に於ける振動振幅も低い事がわかる．卽ち剛度を高くする事は共振振幅を少くするといふ事と同じになる譯である．

筋違のある構造では梁の剛度を如何に高くしても構造の振動數が大して變らず，筋違の性質のみがその問題を左右したが，共振の振幅をしらべても，やはり併行的の性質がある，卽ち適當の筋違があれば梁を如何に丈夫にしても共振振幅が餘り變らない．

振動數の變化を考へたとき筋違の剛度が柱の剛度と同程度になると振動數が餘り變化しなくなつたが，共振振幅を考へても筋違の剛度が柱の剛度と同じ位になると筋違の剛度が無限大になつたのと同じ程度の振幅になる．卽ち筋違の大さは，振動數の變化と共振振幅の變化とに比例的性質を與へる．

このやうにして見ると筋違の効果は單に構造物の耐震的剛度を與へるだけでなく，その共振振幅を少くする性質がある譯である．只今は振動の散逸性のみを考慮に入れて計算を試みたが，粘性其他の減衰を考へても大體に於て同じ傾向になるのである．之は次回に述べる事にする．

Corrigenda to paper 19, "The Theory of the Aseismic Properties of the Brace Strut. Part I", *Bull. Earthq. Res. Inst.*, **17** (1939), Part 2. Page 242, line 12.

Read

$$+6\zeta(2\psi_1+\vartheta_1 \sin^2\theta_1)(3\zeta^2+16\zeta\phi+16\phi^2)$$

for

$$6\psi_1\zeta(3\zeta^2+16\zeta\phi+16\phi^2).$$