16. *The Requisite Condition for Rayleigh-waves for Transmission through an Inner Stratum of the Earth.*

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1. *Introduction.*

Some few years back[1] we showed that it is possible for Rayleigh-type waves to be transmitted through an inner stratum of the earth at velocities ranging from those of distortional waves in the stratum to those in the outer media. The condition implied in the above case is that the velocity of the bodily waves in the stratum shall invariably be less than that in the outer media. Until recently, we had the idea that if the condition in question were reversed, the transmission would become very complex. However, on the contrary, our recent investigation[2] on the anomalous dispersion of elastic surface waves suggests that, even with the condition of the problem reversed, it is probable that transmission of waves in the manner stated does occur.

Since the type of solutions of the problem now under consideration is much the same as those for the first case just mentioned, only the mere important parts of the solutions, together with the velocity equation here simplified, will be given.

2. *Important parts of the solutions and the velocity equations.*

The dilatational and distortional components of the waves in the outer media are

$$\left.\begin{aligned} u_1 &= \frac{if}{h^2}Ae^{\mp ry+i(pt-fx)}, \quad v_1 = \pm\frac{r}{h^2}Ae^{\mp ry+i(pt-fx)}, \\ u_2 &= \mp\frac{s}{k^2}Be^{\mp sy+i(pt-fx)}, \quad v_2 = \frac{if}{k^2}Be^{\mp sy+i(pt-fx)}, \end{aligned}\right\} \tag{1}$$

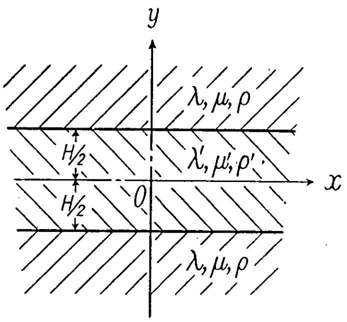

Fig. 1.

1) K. SEZAWA and G. NISHIMURA, *Bull. Earthq. Res. Inst.*, **5** (1928), 85~91.
2) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **16** (1938), 225~233.

where $r^2=f^2-h^2$, $s^2=f^2-k^2$, $h^2=\rho p^2/(\lambda+2\mu)$, $k^2=\rho p^2/\mu$, with dilatation $\Delta=\partial u_1/\partial x+\partial v_1/\partial y$. The upper and lower signs of the expressions for displacements correspond to the media for y-positive and y-negative respectively. The dilatational and distortional components of the waves in the stratum are

$$\left.\begin{aligned} u_1'&=\frac{if}{h'^2}C\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}r'y\, e^{i(pt-fx)}, & v_1'&=\pm\frac{r'}{h'^2}C\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}r'y\, e^{i(pt-fx)},\\ u_2'&=\pm\frac{s'}{k'^2}D\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}s'y\, e^{i(pt-fx)}, & v_2'&=\frac{if}{k'^2}D\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}s'y\, e^{i(pt-fx)},\end{aligned}\right\} \tag{2}$$

where $r'^2=h'^2-f^2$, $s'^2=k'^2-f^2$, $h'^2=\rho'p^2/(\lambda'+2\mu')$, $k'^2=\rho'p^2/\mu'$, with dilatation $\Delta'=\partial u_1'/\partial x+\partial v_1'/\partial y$. The upper and lower terms in all the foregoing expressions correspond to types of vibrations, both symmetrical and anti-symmetrical with respect to the middle plane of the stratum.

Using the boundary conditions

$$\left.\begin{aligned} &u_1+u_2=u_1'+u_2', \qquad v_1+v_2=v_1'+v_2',\\ &\lambda\Delta+2\mu\frac{\partial}{\partial y}(v_1+v_2)=\lambda'\Delta'+2\mu'\frac{\partial}{\partial y}(v_1'+v_2'),\\ &\mu\left\{\frac{\partial}{\partial y}(u_1+u_2)+\frac{\partial}{\partial x}(v_1+v_2)\right\}=\mu'\left\{\frac{\partial}{\partial y}(u_1'+u_2')+\frac{\partial}{\partial x}(v_1'+v_2')\right\}\end{aligned}\right\} \tag{3}$$

at $y=\pm H/2$, we get the velocity equation, the simplified form of which is

$$\left(\frac{\mu'}{\mu}\right)^2\left(1-\frac{rs}{f^2}\right)\left\{\frac{4r's'}{f^2}X+\left(2-\frac{k'^2}{f^2}\right)^2Y\right\}+\left\{\frac{r's'}{f^2}X+Y\right\}\left\{\left(2-\frac{k^2}{f^2}\right)^2-\frac{4rs}{f^2}\right\}$$
$$+\frac{\mu'k^2k'^2}{\mu f^4}\left\{\frac{r's}{f^2}XY-\frac{rs'}{f^2}\right\}+\frac{2\mu'}{\mu}\left\{\frac{2r's'}{f^2}X+\left(2-\frac{k'^2}{f^2}\right)Y\right\}\left\{\frac{2rs}{f^2}-\left(2-\frac{k^2}{f^2}\right)\right\}=0, \tag{4}$$

where $X=\tan(r'H/2)$, $Y=\tan(s'H/2)$ for the symmetrical type of vibration and $X=-\cot(r'H/2)$, $Y=-\cot(s'H/2)$ for the anti-symmetrical type of vibration.

3. *The condition for existence of the waves.*

Even should the densities and the elastic constants of both outer media be assumed to be the same, there are four typical kinds of stratification. As is well known, let p^2/h'^2, p^2/k'^2, p^2/h^2, p^2/k^2 be the velocities of the dilatational and distortional waves in the inner stratum and those in the outer media, the four kinds of stratification just mentioned are then indicated by the following relations:

$$\text{(I)} \quad k'^2 > \begin{Bmatrix} h'^2 > k^2 \\ k^2 > h'^2 \end{Bmatrix} > h^2, \tag{5}$$

$$\text{(II)} \quad k'^2 > k^2 > h^2 > h'^2, \tag{6}$$

$$\text{(III)} \quad k^2 > \begin{Bmatrix} h^2 > k'^2 \\ k'^2 > h^2 \end{Bmatrix} > h'^2, \tag{7}$$

$$\text{(IV)} \quad k^2 > k'^2 > h'^2 > h^2. \tag{8}$$

(I) When relation (5) exists, there are six cases to be discussed, namely,

(i)	$1 > k'^2/f^2$;	$r^2 > 0$,	$s^2 > 0$,	$r'^2 < 0$,	$s'^2 < 0$,
(ii)	$k'^2/f^2 > 1 > (h'^2/f^2,\ k^2/f^2)$;	$r^2 > 0$,	$s^2 > 0$,	$r'^2 < 0$,	$s'^2 > 0$,
(iii)	$h'^2/f^2 > 1 > k^2/f^2$;	$r^2 > 0$,	$s^2 > 0$,	$r'^2 > 0$,	$s'^2 > 0$,
(iv)	$k^2/f^2 > 1 > h'^2/f^2$;	$r^2 > 0$,	$s^2 < 0$,	$r'^2 < 0$,	$s'^2 > 0$,
(v)	$(h'^2/f^2,\ k^2/f^2) > 1 > h^2/f^2$;	$r^2 > 0$,	$s^2 < 0$,	$r'^2 > 0$,	$s'^2 > 0$,
(vi)	$h^2/f^2 > 1$;	$r^2 < 0$,	$s^2 < 0$,	$r'^2 > 0$,	$s'^2 > 0$.

Whereas in cases (i), (ii), (iii) equation (4) is solvable without using complex values for the quantities given in the same equation, and furthermore, owing to the conditions $r^2 > 0$, $s^2 > 0$, the energy of the waves is accumulated near the stratum. In cases (iv), (v), (vi), on the other hand, equation (4) becomes complex, and owing to the condition $r^2 < 0$ or $s^2 < 0$, the energy of the waves is not accumulated near the stratum. Since cases (i), (ii), (iii) come within the condition that $1 > k^2/f^2$, it follows that the velocity of the waves transmitted along the stratum is less than that of the distortional waves in the outer media, but is possibly greater than zero.

(II) When relation (6) exists, there are five cases to be discussed, namely,

(i)	$1 > k'^2/f^2$;	$r^2 > 0$,	$s^2 > 0$,	$r'^2 < 0$,	$s'^2 < 0$,
(ii)	$k'^2/f^2 > 1 > k^2/f^2$;	$r^2 > 0$,	$s^2 > 0$,	$r'^2 < 0$,	$s'^2 > 0$,
(iii)	$k^2/f^2 > 1 > h^2/f^2$;	$r^2 > 0$,	$s^2 < 0$,	$r'^2 < 0$,	$s'^2 > 0$,
(iv)	$h^2/f^2 > 1 > h'^2/f^2$;	$r^2 < 0$,	$s^2 < 0$,	$r'^2 < 0$,	$s'^2 > 0$,
(v)	$h'^2/f^2 > 1$;	$r^2 < 0$,	$s^2 < 0$,	$r'^2 > 0$,	$s'^2 > 0$.

Although, in cases (i), (ii), equation (4) is solvable without using a complex value for any quantity in the same equation and, owing to the conditions $r^2>0$, $s^2>0$, the energy of the waves is accumulated near the stratum. In the remaining cases, equation (4) becomes complex and, owing to the condition $r^2<0$ or $s^2<0$, the waves are not of boundary type. It follows then that the velocity of the waves is less than that of the distortional waves in the outer media, and that it is possibly greater than zero.

(III) When relation (7) exists, there are six cases to be discussed, namely,

(i)	$1>k^2/f^2$:	$r^2>0$,	$s^2>0$,	$r'^2<0$,	$s'^2<0$,
(ii)	$k^2/f^2>1>(h^2/f^2,\ k'^2/f^2)$;	$r^2>0$,	$s^2<0$,	$r'^2<0$,	$s'^2<0$,
(iii)	$k'^2/f^2>1>h^2/f^2$;	$r^2>0$,	$s^2<0$,	$r'^2<0$,	$s'^2>0$,
(iv)	$h^2/f^2>1>k'^2/f^2$;	$r^2<0$,	$s^2<0$,	$r'^2<0$,	$s'^2<0$,
(v)	$(h^2/f^2,\ k'^2/f^2)>1>h'^2/f^2$;	$r^2<0$,	$s^2<0$,	$r'^2<0$,	$s'^2>0$,
(vi)	$h'^2/f^2>1$;	$r^2<0$,	$s^2<0$,	$r'^2>0$,	$s'^2>0$.

In case (i), both r^2 and s^2 are positive, whereas in all the remaining cases, either one or both of r^2 and s^2 are negative, the waves corresponding to case (i) alone thus being of boundary type and transmitted with a velocity that is less than that of the distortional waves in the outer media.

(IV) Finally, when relation (8) exists, there are five cases to be discussed, namely,

(i)	$1>k^2/f^2$;	$r^2>0$,	$s^2>0$,	$r'^2<0$,	$s'^2<0$,
(ii)	$k^2/f^2>1>k'^2/f^2$;	$r^2>0$,	$s^2<0$,	$r'^2<0$,	$s'^2<0$,
(iii)	$k'^2/f^2>1>h'^2/f^2$;	$r^2>0$,	$s^2<0$,	$r'^2<0$,	$s'^2>0$,
(iv)	$h'^2/f^2>1>h^2/f^2$;	$r^2>0$,	$s^2<0$,	$r'^2>0$,	$s'^2>0$,
(v)	$h^2/f^2>1$;	$r^2<0$,	$s^2<0$,	$r'^2>0$,	$s'^2>0$.

In the present condition, too, from the signs of r^2, s^2, the waves corresponding to case (i) alone are of boundary type, the velocity of which is less than that of the distortional waves in the outer media.

It is now possible to conclude that even should the density or the elastic constant of the stratum be either greater or less than that in the outer media, the velocity of the waves under consideration would invariably be less than that of the distortional waves in the outer media.

From numerical calculations with equation (4), it could furthermore be shown that although the waves of cases (I)-(i) and (II)-(i) may possibly exist under the foregoing criterion of the problem, they do not really exist. Interpreted physically, since (I) and (II) represent the condition that the velocity of distortional waves in the stratum is less than that in the outer media, if relations (I)-(i) and (II)-(i) held, then the velocity of the boundary waves would be less than that of every bodily wave in the media of transmission, which is rather improbable in the present particular condition of the problem.

In cases (III)-(i) and (IV)-(i), boundary waves can exist, but, since from the conditions $r'^2<0$, $s'^2<0$, the distribution of displacements of the waves in the stratum is hyperbolic, the waves possible in this case are merely of the fundamental type, waves of higher order types, that is, waves of displacements with nodal planes parallel to the stratum being inconsistent. It may be added that the higher the order of the waves, the greater the number of the nodal planes under consideration.

In cases (I)-(ii), (I)-(iii), (II)-(ii), owing to the positive signs of r'^2, s'^2, the distribution of the displacements of the waves in the stratum is sinusoidal, waves of higher order types also existing.

As already remarked, in the remaining cases, namely, in cases (I)-(iv)—(vi), (II)-(iii)—(v), (III)-(ii)—(vi), (IV)—(ii)—(v) no boundary wave could possibly exist.

As already said, the condition for existence of waves of higher order types results from the signs of r'^2, s'^2. If, furthermore, the condition of symmetry and anti-symmetry of the amplitude distribution of the waves were considered, another restriction would be demanded for the possible existence of the waves. Since, for waves of symmetrical vibration, the forms of X, Y in equation (4) are hyperbolic, while for waves of anti-symmetrical vibration, any one or every one of X, Y remains in a trigonometrical form, the result is that the last conclusion regarding the existence of waves of higher order type holds only for anti-symmetrical vibration, no wave of higher order type existing for the symmetrical vibration.

4. *The nature of the wave dispersion in various possible cases.*

The condition of the possible range of the velocity of the waves has already been ascertained. Using equation (4), we shall calculate the dispersion curves for various possible cases. The cases considered are such that (a) $\mu'/\mu=0{\cdot}47$, (b) $\mu'/\mu=0{\cdot}4927$, (c) $\mu'/\mu=0{\cdot}5$, (d) $\mu'/\mu=0{\cdot}54$, (e) $\mu'/\mu=0{\cdot}5625$, (f) $\mu'/\mu=0{\cdot}6$. In every one of these cases it is assumed that $\rho'/\rho=1/2$, $\lambda=\mu$, $\lambda'=\mu'$. The calculation for all the cases is shown in Tables I$\sim$VI and Fig. 2.

The full lines, broken lines, and chain lines in Fig. 2 respectively

Table I. (a) $\mu'/\mu=0{\cdot}47$. i: fundamental, ii: 1st harmonics.

	$V=k/f$		1	0·998	0·996	0·995	0·990	0·9798	0·9695
$\frac{L}{H}$	Symm.		2·484	1·567	1·129	0·968	0·4087	0·1591	0
	Anti-symm.	i	∞	19·00	9·97	7·748	2·038	0·3499	0
		ii	0·4198	0·3719	—	—	0·1975	—	0

Table II. (b) $\mu'/\mu=0{\cdot}4927$. i: fundamental, ii: 1st harmonics.

	$V=k/f$		1	0·998	0·995	0·9927
$\frac{L}{H}$	Symm.		2·063	1·104	0·4445	0
	Anti-symm.	i	∞	14·76	4·983	0
		ii	0·2179	0·1745	—	0

Table III. (c) $\mu'/\mu=0{\cdot}5$.

	$V=k/f$	1	0·9995	0·998	0·9965	0·9952
$\frac{L}{H}$	Symm.	1·924	1·4547	0·940	0·5592	0
	Anti-symm.	∞	38·2	13·43	7·255	0

Table IV. (d) $\mu'/\mu=0{\cdot}54$.

	$V=k/f$	1	0·9997	0·99943
$\frac{L}{H}$	Symm.	1·133	0·6575	0
	Anti-symm.	∞	34·45	0

Table V. (e) $\mu'/\mu=0{\cdot}5627$.

	$V=k/f$	1		0·99995	
$\frac{L}{H}$	Symm.	0			
	Anti-symm.	0	∞	0·780	78·06

Table VI. (f) $\mu'/\mu=0{\cdot}6$.

	$V=k/f$	1		0·99995	
$\frac{L}{H}$	Anti-symm.	2·397	∞	3·208	40·9

represent dispersion curves of the waves for symmetrical vibration, for anti-symmetrical (asymmetrical) vibration of the fundamental type, and for the same vibration of the first order type.

In the case of symmetrical vibration, only one dispersion curve

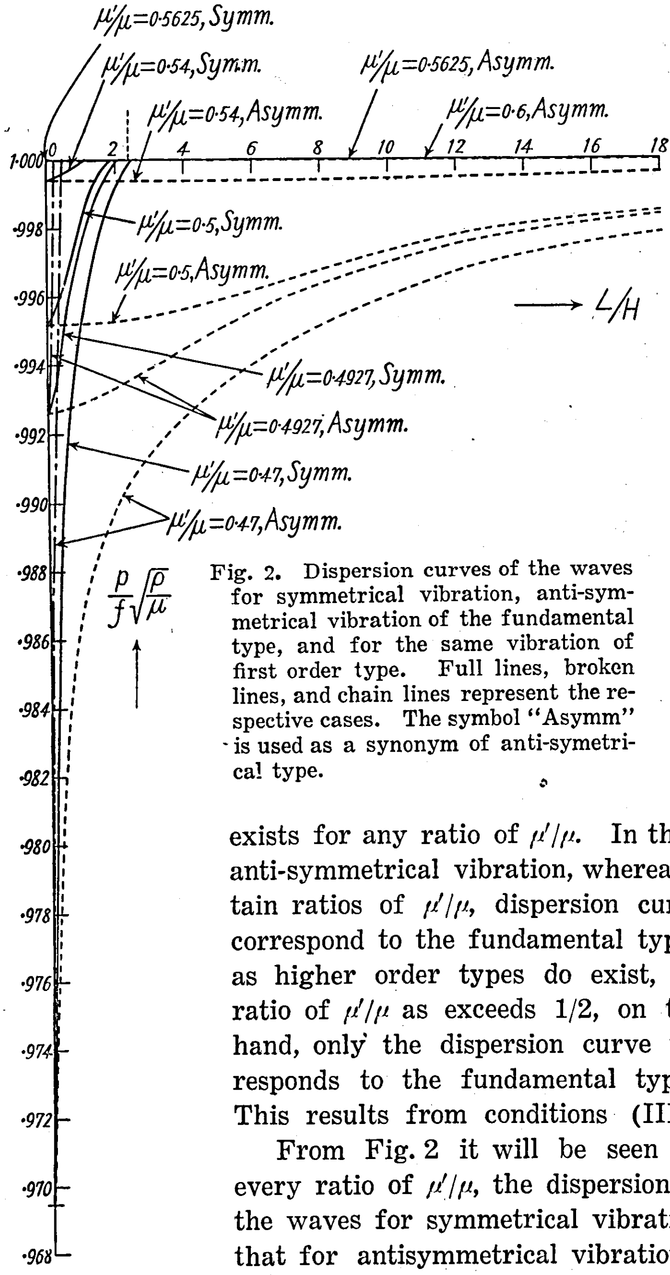

Fig. 2. Dispersion curves of the waves for symmetrical vibration, anti-symmetrical vibration of the fundamental type, and for the same vibration of first order type. Full lines, broken lines, and chain lines represent the respective cases. The symbol "Asymm" is used as a synonym of anti-symetrical type.

exists for any ratio of μ'/μ. In the case of anti-symmetrical vibration, whereas for certain ratios of μ'/μ, dispersion curves that correspond to the fundamental type as well as higher order types do exist, for such ratio of μ'/μ as exceeds 1/2, on the other hand, only the dispersion curve that corresponds to the fundamental type exists. This results from conditions (III), (IV).

From Fig. 2 it will be seen that for every ratio of μ'/μ, the dispersion curve of the waves for symmetrical vibration meets that for antisymmetrical vibration of any order type at a point with abscissa $L/H=0$. Although the dispersion curves of the waves for anti-symmetrical vibration of the fundamental type tend to approach the asymptotic line $k/f=1$ for $L/H=\infty$, the similar curves for the symmetrical vibration and anti-symmetrical vibration of higher order types extend through relatively narrow ranges of L/H, and tend to end discontinuously at certain ratios of L/H. Furthermore, it is likely that the higher the order of anti-symmetrical vibration, the greater the steepness of the inclination of the corrsepond-ing dispersion curve, as a result of which all dispersion curves of waves for anti-symmetrical vibration of such orders as are higher than the second, virtually crowd together quite near the abscissa $L/H=0$.

Although it may seem that the dispersion curves of waves of anti-symmetrical vibration of the fundamental type tend to increase, even if gradually, with increase in L/H, this is only apparent. Strictly speaking, the fundamental dispersion curves lying quite near the line $k/f=1$ deflect somewhat downward for a medium value of L/H, and then tend to approach the line $k/f=1$ asymptotically. Thus, as will be seen from Table V, the dispersion curve for $\mu'/\mu=0{\cdot}5627$, which passes once through such a special point with zero abscissa and zero ordinate, is deflected to $k/f=0{\cdot}99995$ at $L/H=0{\cdot}780$ and $78{\cdot}06$, and again tends to approach the line $k/f=1$ for $L/H=\infty$. In the same way, the dispersion curve for the case $\mu'/\mu=0{\cdot}6$ begins at $L/H=2{\cdot}397$ discontinuously, and after being deflected downward, even if very slightly, tends to approach the line $k/f=1$ at $L/H=\infty$.

The above features are likely to concern the condition that, in the case of dispersion curves of waves of anti-symmetrical vibration of the fundamental type, the greater the ratio of μ'/μ, the smaller the steepness of the inclinations of those dispersion curves. Since, on the other hand, the steepness of inclination of dispersion curves of waves of symmetrical vibration and the anti-symmetrical vibration of the higher order types generally increase with increase in μ'/μ, it is improbable that the dispersion curves of these waves deflect downward for a medium value of L/H, in consequence of which for $\mu'/\mu>0{\cdot}5627$, no dispersion curve of waves of the kinds last mentioned can exist.

Dispersion curves do not exist below such an ordinate as $k/f<k'/f'$, which coresponds to conditions (I)-(i) and (II)-(i).

The ordinates of certain dispersion curves at abscissa L/H represent the velocity of the Stoneley-waves that are transmitted along any boundary between the stratum and the outer media. The condition for Stoneley-waves to exist is that $r^2>0$, $s^2>0$, $r'^2<0$, $s'^2<0$, which correspond to conditions (III)-(i) and (IV)-(i) at $L/H=0$. It is possible to show that for the range $\mu'/\mu > 0{\cdot}4927$ at $L/H=0$, Stoneley-waves exist. This has been shown also in our previous paper.[3] The fact that the dispersion curve of waves for symmetrical vibration, always meets that for anti-symmetrical vibration at a point with abscissa $L/H=0$, is related to the condition for the existence of Stoneley-waves.

5. *Summary and Concluding remarks.*

From mathematical calculation, we found various forms of Rayleigh-type waves that are transmitted through an inner stratum of the earth, which we shall now summarize as follows.

(1) The velocity of the boundary waves, in any case, ranges between the velocity of distortional waves in the stratum and that in the outer media.

(2) In the case of the displacement distribution of the waves that is symmetrical with respect to the middle plane of the stratum, there is only one dispersion curve for any ratio of μ'/μ. In the case of the distribution that is anti-symmetrical with respect to that middle plane, there are a number of dispersion curves for certain ratios of μ'/μ. These dispersion curves correspond to the waves of the fundamental and higher order types, namely, waves with several nodal planes of vibration parallel to the stratum.

(3) Even in the case of waves of anti-symmetrical vibration, if the velocity of distortional waves in the stratum is higher than that in the outer media, there is only one dispersion curve that corresponds to waves of fundamental vibration.

(4) The dispersion curve for waves of symmetrical vibration meets that for anti-symmetrical vibration at a point with abscissa $L/H=0$ for any ratio of μ'/μ.

(5) Whereas in the case of anti-symmetrical vibration of the fundamental type, the steepness of the dispersion curve decreases with increase in μ'/μ, in the cases of symmetrical vibration and anti-symmetrical vibration of the higher order types, the steepness of the curves under consideration increases with increase in μ'/μ.

(6) If the dispersion curve of waves of anti-symmetrical vibration of the fundamental type lies near the line $k/f=1$, the same curve is likely to deflect downward for a medium value of L/H. Such feature does not exist in the dispersion curves of waves of the remaining types, from which it is possible to conclude that $\mu'/\mu < 0{\cdot}5625$ in the present example is the critical condition for the existence of waves of the re-

3) K. SEZAWA and K. KANAI, "The Range of Possible Existence of Stoneley Waves, and Some Related Problems", *Bull. Earthq. Res. Inst.*, **17** (1939), 1~8.

maining types in question.

(7) It is possible for waves of anti-symmetrical vibration of the fundamental type to exist for any ratio of L/H. It is also possible for waves of symmetrical vibration and those of anti-symmetrical vibration of higher order types to exist for relatively smaller ratios of L/H. The higher the order of the anti-symmetrical vibration, the smaller the range of L/H within which the dispersion curves rest.

(8) The ordinates of the dispersion curves at abscissa $L/H=0$ represent the velocities of Stoneley-waves that are transmitted along any one boundary between the inner layer and the outer media. In the present example, namely, in case $\rho'/\rho=1/2$, the velocity of Stoneley-waves lies between $k/f=0{\cdot}5625\sim0{\cdot}4927$.

Although there are a number of other features to be discussed, since their essential parts have been referred to in the respective sections, no further explanation of them will be given here.

Gutenberg[4] found G-waves that are transmitted parallel to the surface of the earth with velocities exceeding that of Love-waves. According to him, besides the waves being distinguishable from Love-waves, the seismic records of the same waves at some stations appear to have also the component of the displacement that is parallel to the direction of propagation of the waves. Although in the present paper, we have not discussed the case of the waves being orientated horizontally in sense normal to the direction of propagation, to which G-waves are likely to correspond, it is possible that in some cases, the G-waves may be of the kind that we have examined in the present paper.

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16. レーレー型波が地殼の内部層に傳播する條件

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十年程前にレーレー型波が地殼の内部層に傳播し得る事を述べた．今回はその傳播についての正確なる條件を研究して見たのである．研究結果の要點は次の如くである．

（1） 波動の速度は内部層の横波と外部層の横波との間にある．

（2） 内部層の中間面について對稱なる振幅分布をなす波動の場合には各 μ'/μ に對して１つ

4) B. GUTENBERG and C. F. RICHTER, *Gerl. Beitr. z. Geophys.*, **43** (1934), 56～133.

の分散曲線しかないが，斜對稱の振幅分布をなす場合には數多くの分散曲線がある．之等の曲線は互に異なる數の節平面が內部層中に存在する場合に相當するのである．換言すれば零次，一次，二次，……の振動を示す譯である．

（3） 斜對稱の場合でも，內部層中での橫波速度が外部のそれよりも高いときには，零次の振動の分散曲線しか存在しない．

（4） 各 μ'/μ について對稱の場合の分散曲線も斜對稱の場合の分散曲線も $L/H=0$ なる線上の一點に集まる．

（5） 零次の斜對稱の振動の場合には分散曲線の L/H についての傾斜が μ'/μ の增加とともに減少するけれども，高次の斜對稱や，對稱の振動の場合にはその傾斜が μ'/μ の增加とともに增加する．

（6）零次の斜對稱の振動には分散曲線があらゆる L/H に涉つてをるけれども，他の場合には分散曲線が極く小なる L/H の範圍にしか存在しない．

（7） 零次の斜對稱の振動の分散曲線が波動速度 $k/f=1$ の附近にあるものは，中位の L/H について中だるみの傾向をなす．他の種類の振動の分散曲線にはこのやうな事がない．

（8） 分數曲線の $L/H=0$ の附近の値は2種の層の境界を傳播する Stoneley 波の速度を與へる．この論文の例の場合には $\rho'/\rho=1/2$ であり，Stoneley 波の速度は $k/f=0{\cdot}5625\sim0{\cdot}4927$ の間にある．

Gutenberg は G 波といふものを考へた．之は大體に於て Love 波の場合と似た振動をなすが，速度は Love 波のそれよりも高く，且つ地球の表面の弧に沿うて傳播する．この論文で述べたものはむしろ Rayleigh 波の型に近いが，G 波も場合によつては只今の場合に含まれてをるものがあるかも知れぬ．