17. *Microseisms Caused by Transmission of Atmospheric Disturbances. I.*

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1. *Causes of periodicity of microseismic oscillations.*

Although the problem of microseisms has received the attention of a number of investigators, it is not likely that their opinions with respect to their causes will perfectly agree. Gutenberg,[1] Tams,[2] Jung,[3] Wadati,[4] and the writers[5] have shown that microseims are mainly caused by the surf beating against a steep coast or by the breaking of strong waves on sea shores. On the other hand, Banerji[6] and Gherzi[7] have concluded that microseisms result from rapid changes in pressure caused either by the waves in the ocean or by the 'pumping'[7] of air pressure in cyclones, especially in tropical cyclones.

As to the periodicity of microseisms, Gutenberg has ascertained that they increase with increase in the distance of the observing station from the coast, showing that the disturbances are propagational. Kishinouye,[8] after observing microseisms at Hongo in various cases, found that, since the periods of microseims range from four to six seconds, these periods are likely to be those of the proper oscillation of the ground at Hongo. Gherzi has also shown that the prevalent periods of microseisms at Zi-Ka-Wei lie between four and eight seconds. In the case of Tokyo, it appears that, even though the cyclonic region be very far away, the prevalent periods of oscillation at an observing station is nearly the same.

Since it is improbable that the disturbances in a cyclonic region

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1) B. GUTENBERG, *ZS. f. Geophys.*, **4** (1928), 246~250; *Bull. Sesm. Soc., Amer.*, **21** (1931), 1~24.

2) E. TAMS, *ZS. f. Geophys.*, **9** (1933), 23~31.

3) K. JUNG, *ibid.*, **10** (1934), 325~329.

4) K. WADATI and K. MASUDA, *Geophys. Mag.*, **9** (1935), 299~340.

5) K. SEZAWA and G. NISHIMURA, *Bull. Earthq. Res. Inst.*, **9** (1931), 291~309.

6) S. K. BANERJI, *Phil. Trans. Roy. Soc.*, **229** (1930), 287~328.

7) E. GHERZI, *ZS. f. Geophys.*, **1** (1925), 163; *Gerl. Beitr. Geophys.*, **25** (1930), 145~147; *Notes de scism., Zi-Ka-Wei*, No. 12.

8) F. KISHINOUYE, *Bull. Earthq. Res. Inst.*, **13** (1935), 146~154; 608~615.

have always the same periods, the idea that the prevalent periods of microseisms result from proper oscillation of the ground may be a reasonable one. Since, on the other hand, the periods of microseismic oscillation at every station are of a certain range, it may be concluded that the proper oscillation of the ground at every station should be nearly the same,—a conclusion that is naturally very odd. Is it not possible to show the cause of the constancy of the periods in question in some other way?

We have the idea that there is an alternative way of explaining the prevalent periods of microseisms without invoking the proper oscillation of the ground at the station or the predominant oscillation of the original disturbances. We cannot expect any prevalent oscillation in the cyclonic disturbances. The periodicity of the vortex formation in or near a cyclone changes with difference in wind intensity. Since it is assumed that the disturbances in a cyclone consist of an aggregate of forces of various periods, if the nature of the medium were adapted to the transmission of the disturbances (in the form of waves) of a special period, it could possibly excite microseisms with a certain prevalent period.

There are two simple examples of the medium of the kind under consideration. The surface stratification of the earth's crust is an example. We have already shown that if Rayleigh-waves or Love-waves were excited in a stratified body by periodic disturbances of constant intensity at the source, the amplitudes of these waves become maximum for a certain period of the disturbance.[9] [10] It is therefore quite probable that microseisms with prevalent periods result from the movements of the ground due to the dispersive surface waves that are transmitted from the region of the cyclonic disturbances to the observing station. Should this conclusion be correct, it follows that the original forces are not restricted to atmospheric disturbances; either the beating of the surf against the coast or the pumping of air pressure could be the source. Although the microseismic periods of a certain range do not correspond to those of the origin, such periods result merely from the nature of the path of the transmitted waves. It therefore seems that microseisms, in the majority of cases, are formed in this way.

Another example of a possible medium for the transmission of

9) K. SEZAWA and K. KANAI, "Relation between the Thickness of a Surface Layer and the Amplitudes of Love-waves", *Bull. Earthq. Res. Inst.*, **15** (1937), 577~581.

10) *Ditto*, "Relation between the Thickness of a Surface Layer and the Amplitudes of Dispersive Rayleigh-waves", *Bull. Earthq. Res. Inst.*, **15** (1937), 845~859.

microseismic disturbances is atmospheric stratification, particularly that due to the polar front with a smaller kinetic energy overlying turbulent cyclonic air with greater kinetic energy. Such stratification is not necessarily in the condition of a cyclonic centre, it being rather likely to occur near lines of discontinuity that sometimes accompany the cyclone. In such a condition, it is possible for the lapse rate to be discontinuous at a certain level. The sound velocity in air below the level under consideration may be less than that above that level, which condition is the reverse of that of atmospheric stratification, including the stratosphere. It also is possible for the same condition to exist when sound waves are transmitted along layers, in either one, or each one of which, wind blows uniformly. As remarked by Gherzi, it is an observable fact that a thin layer of three or four kilometer depth often exists just above the ground, above which layer the atmospheric condition is very calm. If the disturbances from a cyclonic region were transmitted through this thin layer, it would be possible for disturbances with a certain range of periods to predominate.

Although the two cases above exemplified, namely, the surface stratification of the earth's crust and the turbulent or current stratification of the atmosphere, are both important in connexion with the possible existence of prevalent periods of oscillation in microseisms, since the former case has already been treated in our previous papers,[11] only the latter case will be examined in the present paper. It will finally be shown that the vibratory energy of the source of the disturbances can be transmitted more readily through atmospheric stratification than through crustal layers.

2. *The nature of atmospheric waves.*

It appears that there is an aggregate of periodic motions in microbarometric oscillations. The microbarometric waves, whose period is longer than a few minutes, are almost gravitational, and likely to be Helmholtz waves. Theoretical and experimental investigations of such waves have been made by a number of investigators, like Brunt,[12] Fujiwhara,[13] Taylor,[14] Arakawa,[15] Namekawa,[16] etc. Although the velocity of the waves under consideration increases with increase in wave length,

11) K. SEZAWA and K. KANAI, *loc. cit.*, 9), 10).
12) D. BRUNT, *Q. J. Met. Soc.*, **53** (1927), 221.
13) S. FUJIWHARA and Z. KANAGAWA, *Geophys. Mag.*, **1** (1928), 304.
14) G. I. TAYLOR, *Proc. Roy. Soc.*, **126** (1930), 169.
15) H. ARAKAWA, *Geophys. Mag.*, **5** (1932), 223; **6** (1931), 171.
16) T. NAMEKAWA, *Mem. Coll. Sci., Kyoto Imp. Univ.*, **17** (1934), 405; **18** (1935), 83, 173, 317.

the velocity, generally, is not very great; and even in such an extreme case as that in which the earth is covered by two or three wave lengths of atmospheric tides, the velocity of transmission of this tide is of the same order as that of ordinary sound waves. According to Fujiwhara[17] and Namekawa,[18] in microbarometric oscillations, periods ranging from three to eight minutes are likely to predominate, in which case the wave velocity is of the order of only ten or twenty meters per sec. If, furthermore, the period of the oscillation were as short as five or six seconds, the velocity would then be extremely small. In such a range of oscillation periods, the dynamical nature of the waves becomes almost that of compressional waves and not that of gravitational waves, in consequence of which even if the equations to be discussed concern gravitational waves as well as compressional waves, the numerical calculation here has been restricted to the case of pure compressional waves. As a matter of fact, even should the compressional force be coupled with a gravitational one, since the condition of coupling between them is very slight, the vibrations can be decomposed into almost purely compressional vibration and almost gravitational vibration.

3. *Equations of motion.*

Let the axis of z be drawn vertically downwards and the axes of x, y horizontally. The equations of motion and continuity are then[19]

$$\rho_0\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x},\quad \rho_0\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y},\quad \rho_0\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+g\rho^*, \tag{1}$$

$$\frac{D\rho}{Dt}+\rho_0\chi=0,\qquad \chi=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}, \tag{2), (3}$$

where $*$ signifies the gravitational body force. The variation in pressure is expressed by

$$\frac{\partial p}{\partial t}+\frac{\partial p_0}{\partial z}\frac{\partial z^{**}}{\partial t}=c^2\frac{D\rho}{Dt},\qquad c^2=\frac{\gamma p_0}{\rho_0}=\gamma R\theta_0, \tag{4), (5}$$

where $**$ represents the change in pressure with height. c is the sound velocity for the equilibrium temperature θ_0 (absolute) at level z, R the gas constant, and γ the ratio of the two specific heats of the gas. Writing

17) S. FUJIWHARA, *loc. cit.*, 13).
18) T. NAMEKAWA, *loc. cit.*, (16).
19) H. LAMB, *Hydrodynamics*, Chap. X.

$$p=p_0+p', \qquad \rho=\rho_0+\rho', \tag{6}$$

and neglecting small second order quantities, we get

$$\rho_0\frac{\partial u}{\partial t}=-\frac{\partial p'}{\partial x}, \quad \rho_0\frac{\partial v}{\partial t}=-\frac{\partial p'}{\partial y}, \quad \rho_0\frac{\partial w}{\partial t}=-\frac{\partial p'}{\partial z}+g\rho'^{*}, \tag{7}$$

$$\frac{\partial \rho'}{\partial t}+w\frac{\partial \rho_0}{\partial z}^{**}=-\rho_0\chi. \tag{8}$$

From (4), (5), (2), we have

$$\frac{\partial p'}{\partial t}+g\rho_0 w\left(=\frac{\partial p_0}{\partial z}\frac{\partial z}{\partial t}^{**}\right)=-\gamma p_0\chi. \tag{9}$$

From (7), (8), (9), we find that

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}(c^2\chi+gw^{**}), \qquad \frac{\partial^2 v}{\partial t^2}=\frac{\partial}{\partial y}(c^2\chi+gw^{**}),$$

$$\frac{\partial^2 w}{\partial t^2}=\frac{\partial}{\partial z}(c^2\chi+gw^{**})-\left\{\frac{dc^2}{dz}^{***}-(\gamma^{**}-1^{*})g\right\}\chi, \tag{10}$$

where $***$ denotes the change of c^2 with height.

If we write

$$\Gamma_1^2\chi=\frac{\partial^2\chi}{\partial x^2}+\frac{\partial^2\chi}{\partial y^2}, \tag{11}$$

we get from (10)

$$\frac{\partial^4\chi}{\partial t^4}=c^2\Gamma^2\frac{\partial^2\chi}{\partial t^2}+\left(\frac{dc^2}{dz}^{***}+\gamma g^{**}\right)\frac{\partial^3\chi}{\partial t^2\partial z}$$

$$+g\left\{\frac{dc^2}{dz}^{***}-(\gamma^{**}-1^{*})g\right\}\Gamma_1^2\chi. \tag{12}$$

Since in the case of periodic waves transmitted along the surface of the earth, it is possible to put $\chi\propto e^{-i\sigma t}C(kr)$, (12) transforms to

$$\frac{1}{k^2}\frac{\partial^2\chi}{\partial z^2}+\frac{1}{c^2k^2}\left(\frac{dc^2}{dz}^{***}+\gamma g^{**}\right)\frac{\partial\chi}{\partial z}$$

$$+\left\{\left(\frac{\sigma^2}{k^2c^2}-1\right)+\frac{g^{2**}}{c^2\sigma^2}\left(\gamma^{**}-1^{*})-\frac{1}{g}\frac{dc^2}{dz}^{***}\right)\right\}\chi=0. \tag{13}$$

As will be seen later, the length of prevalent periodic waves is of the order of six kilometers with a velocity of 330 meters per second, in which case, although the term σ^2/k^2c^2-1 in (13) is 0·04, the term $(g^2/c^2\sigma^2)(\gamma-1)$ is 0·0026. It is then possible for the term $\{(\sigma^2/k^2c^2-1)+(g^2/c^2\sigma^2)(\gamma-1)-(1/g)(dc^2/dz)\}$ in (13) to be approximated to σ^2/k^2-1, the term $(1/g)(dc^2/dz)$, owing to its nature, being neglected. In the same way, it is fairly possible to neglect the term $dc^2/dz+\gamma g$ in (13). Equation (13) may therefore be replaced by

$$c^2\frac{\partial^2\chi}{\partial z^2}+(\sigma^2-k^2c^2)\chi=0. \tag{13'}$$

The condition implied in the above equation is virtually the same as neglecting all the terms marked *, **, *** in the early stage of using the equations.

Dynamically, even should gravity and compressibility be both taken into consideration, since the coupling condition for the two elements is very slight, the solutions are virtually the same as that of treating the problem of gravity and that of compressibility separately.

It is now possible for differential equation (12) to be replaced by the simple expression

$$\frac{\partial^2\chi}{\partial t^2}=c^2\nabla^2\chi, \tag{12'}$$

and equation (13) by (13′) as has already been mentioned.

From (9) the rate of pressure change is

$$\frac{\partial p'}{\partial t}=-g\rho_0 w^{**}-\gamma p_0\chi, \tag{14}$$

and from (10) the vertical velocity is expressed by

$$w=-\frac{c^2}{\sigma^2}\frac{\partial\chi}{\partial z}. \tag{15}$$

Since on the ground surface, w is assumed to be zero, the rate of pressure change on the ground becomes

$$\frac{\partial p'}{\partial t}=-\gamma p_0\chi \tag{16}$$

even in the case of accurate calculation.

Although microseisms are, as a matter of fact, movements of the ground, since in the present condition of the problem the same movements are results merely of pressure change in the atmosphere, the hydrodynamical equations alone will here be dealt with, under the assumption that microseisms are stress changes on the ground surface.

4. *The solutions of the problem for the case of the original disturbance being in the layer.*

For the reason given in the preceding section, we shall solve equation (12′) with relation (15).

Let the thickness of the atmospheric layer be H. Taking the origin of coordinates at a point on the boundary between the layer and the upper atmosphere, the elementary solutions of equation (12′) for both media take the forms

$$\left.\begin{array}{ll}\chi_{01}=A_0e^{ikx-\alpha z-i\sigma t}, & \chi_{02}=A_0e^{ikx+\alpha z-i\sigma t}, \\ \chi_1=Ce^{ikx+\alpha z-i\sigma t}, & \chi_2=Be^{ikx-\alpha z-i\sigma t},\end{array}\right\} \qquad (17)$$

$$\chi_1'=De^{ikx-\beta z-i\sigma t}, \qquad (18)$$

in which χ_{01}, χ_{02} represent the initial disturbances in the layers that are directed upward and downward respectively, and χ_1, χ_2 correspond to the reflected parts of disturbances at the upper and lower boundaries of the layer respectively. It is assumed that the sound velocities c, c' in the layer and the upper atmosphere are constant, α, β being thus written

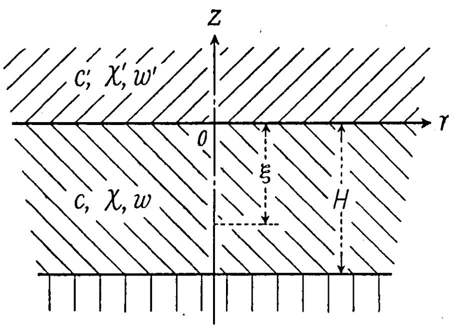

Fig. 1.

$$\alpha=\sqrt{k^2-\frac{\sigma^2}{c^2}}, \quad \beta=\sqrt{k^2-\frac{\sigma^2}{c'^2}}. \qquad (19)$$

Applying the boundary conditions that $w_{02}+w_2=0$ at $z=-H$ and $w_{01}+w_1=w_1'$, $\chi_{01}+\chi_1=\chi_1'$ at $z=0$, it is possible to determine the constants B, C, D in terms of A_0.

Let the source of the disturbance in the layer be, ideally, a point at the level $H-\xi$ above the ground surface, from which point periodic pressure waves of constant amplitude with various frequencies are generated. Then, the same disturbance is expressed by

$$\left.\begin{aligned}\chi_0&=\frac{e^{-i(\sigma t-hR)}}{hR}=\frac{e^{-i\sigma t}}{2}\int_{-\infty}^{\infty}\frac{e^{\alpha(z+\xi)}}{\alpha}H_0^{(1)}(kr)\,kdk \qquad [z<-\xi]\\&=\frac{e^{-i\sigma t}}{2}\int_{-\infty}^{\infty}\frac{e^{-\alpha(z+\xi)}}{\alpha}H_0^{(1)}(kr)\,kdk, \qquad [z>-\xi]\end{aligned}\right\}\tag{20}$$

where $h^2=\sigma^2/c^2$, $h'^2=\sigma^2/c'^2$. The waves χ_0 are successively reflected and refracted at the boundaries $z=0$ and $z=-H$ in agreement with the boundary conditions just given. The resultant χ, including χ_0, is then expressed by[20]

$$\left.\begin{aligned}\chi&=\frac{e^{-i\sigma t}}{2}\int_{-\infty}^{\infty}\frac{\left\{\left(\alpha+\beta\frac{c'^2}{c^2}\right)e^{\alpha\xi}+\left(\alpha-\beta\frac{c'^2}{c^2}\right)e^{-\alpha\xi}\right\}\left\{e^{\alpha(z+H)}+e^{-\alpha(z+H)}\right\}}{\alpha\left\{\left(\alpha+\beta\frac{c'^2}{c^2}\right)e^{\alpha H}-\left(\alpha-\beta\frac{c'^2}{c^2}\right)e^{-\alpha H}\right\}}\\&\qquad\qquad\cdot H_0^{(1)}(kr)\,kdk, \qquad [(z+\xi)<0]\\\chi&=\frac{e^{-i\sigma t}}{2}\int_{-\infty}^{\infty}\frac{\left\{\left(\alpha+\beta\frac{c'^2}{c^2}\right)e^{-\alpha z}+\left(\alpha-\beta\frac{c'^2}{c^2}\right)e^{\alpha z}\right\}\left\{e^{\alpha(H-\xi)}+e^{-\alpha(H-\xi)}\right\}}{\alpha\left\{\left(\alpha+\beta\frac{c'^2}{c^2}\right)e^{\alpha H}-\left(\alpha-\beta\frac{c'^2}{c^2}\right)e^{-\alpha H}\right\}}\\&\qquad\qquad\cdot H_0^{(1)}(kr)\,kdk, \qquad [(z+\xi)>0]\end{aligned}\right\}\tag{21}$$

and the resultant χ' is written

$$\chi'=\frac{e^{-i\sigma t}}{2}\int_{-\infty}^{\infty}\frac{e^{-\beta z}\left\{e^{\alpha(H-\xi)}+e^{-\alpha(H-\xi)}\right\}}{\left\{\left(\alpha+\beta\frac{c'^2}{c^2}\right)e^{\alpha H}-\left(\alpha-\beta\frac{c'^2}{c^2}\right)e^{-\alpha H}\right\}}H_0^{(1)}(kr)\,kdk. \tag{22}$$

On the ground surface, χ becomes

$$\begin{aligned}\chi_{z=-H}&=\frac{2\pi H_0^{(1)}(\kappa r)e^{-i\left(\sigma t-\frac{\pi}{2}\right)}\frac{\kappa}{h}}{\sqrt{h^2-\kappa^2}F'(\kappa)}\Big[c^2\sqrt{h^2-\kappa^2}\cos\sqrt{h^2-\kappa^2}\,\xi\\&\quad+c'^2\sqrt{\kappa^2-h'^2}\sin\sqrt{h^2-\kappa^2}\,\xi\Big]\\&\quad+\frac{\kappa}{h}e^{i\left(h'r-\sigma t+\frac{\pi}{2}\right)}\frac{c'^2}{c^2}\frac{\cos\sqrt{h^2-h'^2}(H-\xi)}{(h^2-h'^2)\sin^2\sqrt{h^2-h'^2}H}\frac{1}{r^2}+\cdots,\end{aligned}\tag{23}$$

where κ is the root of

$$F(k)=\left(\alpha+\beta\frac{c'^2}{c^2}\right)e^{\alpha H}-\left(\alpha-\beta\frac{c'^2}{c^2}\right)e^{-\alpha H}=0, \tag{24}$$

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20) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **13** (1935), 1.

and

$$F'(\kappa) = \left(c^2\kappa H + \frac{c'^2\kappa}{\sqrt{\kappa^2 - h'^2}}\right)\cos\sqrt{h^2-\kappa^2}H$$

$$+ (c^2\kappa + c'^2\kappa H\sqrt{\kappa^2 - h'^2})\sin\sqrt{h^2-\kappa^2}H/\sqrt{h^2-\kappa^2}. \qquad (25)$$

The first term in the expression in (23) represents the pressure waves transmitted through the layer with a velocity that is determined by solving (24) and the second term the waves whose velocity is equal to the sound velocity in the upper atmosphere. The velocity obtained from (24) is intermediate between the sound velocity c in the layer and c' in the upper atmosphere. Since the pressure of the waves corresponding to the second term in question varies as r^{-2}, the pressure of the waves represented by the first term fulfils an important part in the microseismic disturbance.

5. *The solution of the problem for the case of the original disturbance being in the upper atmosphere.*

In the present case, the original disturbance is assumed to be at level $H+\xi$ above the ground surface. Let the original disturbance be expressed by

$$\chi_0 = \frac{e^{-i(\sigma t - h'R)}}{h'R}. \qquad (26)$$

Proceeding in the same way as in the preceding section, we get

$$\chi_{z=-H} = 2\pi\frac{h}{h'}e^{-i\left(\sigma t-\frac{\pi}{2}\right)}H_0^{(1)}(\kappa r)$$

$$\cdot\frac{c'^2e^{-\sqrt{\kappa^2-h'^2}\xi}}{\left(c^2\kappa H+\frac{c'^2\kappa H}{\sqrt{\kappa^2-h'^2}}\right)\cos\sqrt{h^2-\kappa^2}H+\left(c'^2\kappa H\sqrt{\frac{\kappa^2-h'^2}{h^2-\kappa^2}}+\frac{c^2\kappa}{\sqrt{h^2-\kappa^2}}\right)\sin\sqrt{h^2-\kappa^2}H}$$

$$+e^{-\left(h'r-\sigma t+\frac{\pi}{2}\right)}\frac{\kappa}{h'}\frac{c'^2}{c^2}$$

$$\cdot\left\{\frac{c'^2\cos\sqrt{h^2-h'^2}H-c^2\sqrt{h^2-h'^2}\xi\sin\sqrt{h^2-h'^2}H}{c^2(h^2-h'^2)\sin^2\sqrt{h^2-h'^2}H}\right\}\frac{1}{r^2}+\cdots, \qquad (27)$$

the meaning of every term being the same as before.

6. *Calculation of the amplitudes of the pressure waves generated from the original disturbance of the same intensity.*

We shall write

$$2\pi H_0^{(1)}(\kappa r)e^{-i\left(\sigma t-\frac{\pi}{2}\right)}=K \tag{28}$$

in expression (23). In order that the intensity of the original disturbance shown in (27) may equal that in (23), we should assume, in virtue of the denominators in (20), (26), that

$$2\pi\frac{h'}{h}H_0^{(1)}(\kappa r)e^{-i\left(\sigma t-\frac{\pi}{2}\right)}=K \tag{29}$$

in expression (27).

Using the results in (23), (27), it is possible to get the amplitudes of the pressure waves that are generated from the original disturbance of the same intensity.

We shall now take two examples, namely, (i) $c=327\,m/s$, $c'=337\,m/s$; (ii) $c=330{\cdot}34\,m/s$, $c'=333{\cdot}66\,m/s$. As already remarked in the opening section of the present paper, the sound velocity in the layer is rather less than that in the upper atmosphere, which condition is possible of existing in the case of the lower layer being turbulent or in the case of uniform current in either one of the layers. Gravitational waves in such a stratification is however rather unstable. Although the case in which the original disturbance lies in the upper atmosphere is also calculated, it is nothing more than a model to idealize the condition of the original disturbance applied at some area of the boundary between the two media. It should be borne in mind, however, that if the sound velocity in the upper atmosphere exceeds that in the lower layer, the periodic waves through the strata are quickly damped. Thus, in the case of wind blowing uniformly in the layer, although the disturbance transmitted in the direction opposite to that of the wind is maintained in the form of two-dimensional waves, that transmitted in the direction of the wind is dispersed. In the case of wind blowing uniformly in the upper atmosphere, the condition of the problem is exactly reversed.[21]

At all events, if the conditions in examples (i) and (ii) were replaced by the temperature of a gas in statical equilibrium, we have (i) $(c'/c)^2=\theta'/\theta\,(\text{abs.})=1{\cdot}061$, (ii) $(c'/c)^2=\theta'/\theta\,(\text{abs.})=1{\cdot}020$, for the respective cases.

21) The problem is somewhat similar to the case of gravitational waves in an atmospheric layer, outside or within which wind is blowing uniformly.

The velocities of wave transmission through the layer should be calculated beforehand, the result of which is shown in Fig. 2.

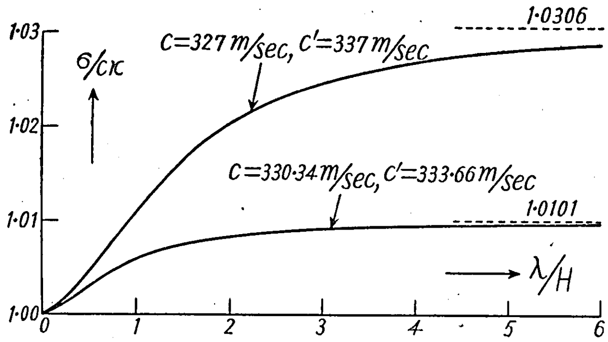

Fig. 2. Dispersion curves.

With a view to calculating the amplitudes of the waves, we shall use equations (23), (27), and the results in Fig. 2. The amplitudes of the waves in cases (i) and (ii) are shows in Figs. 3, 4. In every case the level of the original source is varied as (a) $\xi/H=1$, (b) $\xi/H=0{\cdot}5$, (c) $\xi/H=1$, (d) $\xi/H=-0{\cdot}5$, (e) $\xi/H=-1$. Since the ordinates χ corresponding to the pressure change are represented in terms of K, it is possible to conclude that the higher the ordinate, the greater the amplitude of the waves.

7. *The amplitude of waves, whose length is twice the thickness of the layar, is maximum.*

It will be seen from Figs. 3, 4 that in every amplitude curve, the oridinate is maximum at abscissa near $\lambda/H=2$. In other words, the amplitudes of the waves become maximum when the wave length λ is twice the thickness H of the layer, whence it follows that in microseisms, or at least in microbarometric vibrations, waves whose length is twice the thickness of the layer, would predominate. If the thickness of the layer be three kilometers as stated by Gherzi, the length of the predominant waves in the disturbances would be six kilometers. Assuming that $c\approx c'\approx 330$ meters/sec, the predominant period of the waves is 18 seconds. If we assume that the thickness of the layer is a kilometer, the prevalent period of the disturbance is then six seconds. Thus, even from the present idea, the existence of prevalent periods of oscillation, pointed out by Gherzi and by Kishinouye, is quite probable.

The condition that the amplitude of waves whose length is twice the thickness of the boundary layer is maximum, is not restricted to sound waves. It has already been shown that in the case of Rayleigh-waves or in the case of Love-waves,[22] the wave amplitude becomes maximum when their length is nearly twice the thickness of the layer. Even

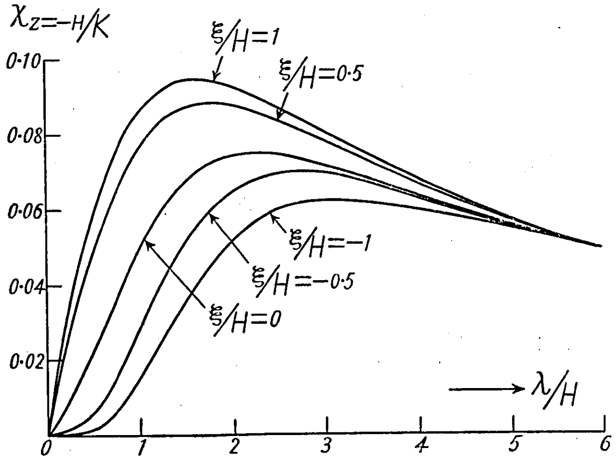

Fig. 3. Amplitude of pressure change in case $c'=337$ m/sec, $c=327$ m/sec.

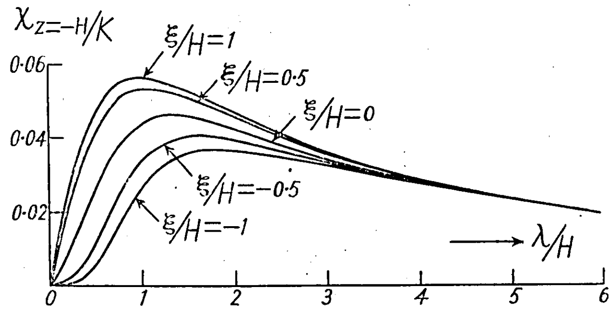

Fig. 4. Amplitude of pressure change in case $c=330{\cdot}34$ m/sec, $c'=333{\cdot}66$m/sec.

in the case of gravitational waves (sea waves) the same condition holds as shown in Fig. 5, the data of which have been reproduced from our previous investigation.[23]

The reason that the amplitude of waves whose length is twice the thickness of the layer is maximum in any case, is relatively simple. If the standing natural vibration of a layer, of which the lower boundary is fixed and the upper free, were specially considered, the wave length of that vibration is, obviously, twice the thickness of the same layer, from which it follows that in the case of atmospheric waves or elastic surface waves or even sea waves, the period of oscillation whose

22) K. SEZAWA and K. KANAI, *loc. cit.*, 9), 10).

23) *Ditto*, "Prevalent Periods of Oscillations in Tidal Waves", *Bull. Earthq. Res. Inst.*, **15** (1937), 888.

amplitude is maximum corresponds to that of the standing vibration of the layer that is concerned in this problem. This shows that the specially large amplitude of the waves for a certain period of oscillation

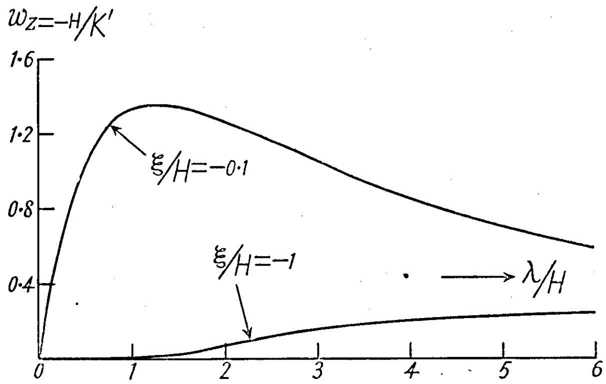

Fig. 5. Amplitude of gravitational waves.

results either from the condition that the oscillation of the layer near the origin is likely to be resonant with periodic forces or from the condition that the layer in the path of the waves sympathizes with waves of special periods.

In treating the present problem, it has been assumed that the effect of gravity is negligible. Since, however, even in the case of pure gravitational waves, the amplitude is maximum for waves whose length is twice the thickness of the layer, it is likely that the amplitude of waves that are affected by the compressibility of the air as well as by gravitational forces, will also be maximum when the wave length is twice the thickness of the layer.

8. *Microbarometric oscillations of longer and smaller periods.*

It may now be stated that, even should the amplitude of pure gravitational waves be maximum in the case of their length being twice the thickness of the layer, the period of such waves is extremely long. Let the densities of the lower and the upper layers be ρ, ρ' respectively. Then, the treatment of hydrodynamical equations with boundary conditions as indicated in Section 3, shows that the velocity of gravitational waves is $\sqrt{\{g(1-\rho'/\rho)k\}/\{\coth kH+\rho'/\rho\}}$, where $2\pi/k$ is the wave length. In this case, the effective pressure in the upper layer should be less than that in the lower layer; otherwise the vibration would become unstable. If we take $H=3$ km, $\rho'/\rho=19/20$, the velocity V becomes 15·48 m/sec. Since the amplitude of the waves is maximum when $L/H=2$, the period of the waves T is then 387·5 sec. If we take $H=1$ km, $L=2$ km, the velocity is 8·933 m/sec and the prevalent period of oscillation is 224 sec, but since the effect of compressibility of the air on the atmospheric oscillation of a period as long as 224 seconds, is rather negligible, the existence of pure gravitational waves in the air strata also becomes possible. It seems that the microbarometric oscillations of periods ranging from three to eight minutes are of this kind, as may be observed in barographs. If the velocity of long waves whose length is half the circumference of the earth, be 330 m/sec, the ratio of ρ'/ρ for stratosphere ($H=13$ km) is 0·145, so that for $L=26$ km, we get $V=173{\cdot}8$ m/sec and $T=149{\cdot}6$ sec. Oscillation of the eight minute period is also likely to be that of the layer extending to the stratosphere. Although it is probable that microseismic oscillations of periods ranging from four to six seconds are not observable in meteorological instruments, it is likely that the same oscillation has still energy enough to induce microseisms of the ground, which condition may be attributed to their extremely short periods. It is obvious that if the vibration of an atmospheric layer results from gravitational force coupled with compressional force, since two vibration periods of the coupled condition differ enormously from each other, the solutions of the problem are virtually the same as those for pure gravitational and pure compressonal problems.[24]

It is a well-known fact that seismographs of longer periods often register not only microseisms with a period of a few seconds, but also slow movement of the ground of periods as long as a few minutes in the case of no earthquake.[25] In our opinion, it may be concluded that oscillations of smaller periods result from microbarometric waves mainly of a compressional nature in a majority of cases, and also from the elastic surface waves in a rather less number of cases. Oscillation of longer periods, on the other hand, is caused by microbarometric oscillations, mainly of a gravitational nature.

9. *The possible existence of microseisms transmitted through a layer, outside or within which there is a steady air current.*

In Section 3 the transmission of disturbance through a turbulent layer is studied. If the conception implied in that section be valid, it is also possible for the disturbance to be transmitted through an atmospheric layer outside or within which a steady air current is blowing. The condition for the atmospheric disturbance being transmitted through

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24) A well-established dynamical principle for coupled vibrations under different elastic resistances.

25) The records usually assumed as tilting motion of long periods.

a layer in the form of compressional waves is that the velocity of the sound in the layer shall invariably be less than that outside the same layer. For the case in which the disturbance is transmitted through the layer in the form of gravitational waves, the condition is rather reversed, particularly in density distribution. In any case, there should be some source of disturbance, for example, action resembling that of pumping.

Let us now assume that the source of disturbance is near the centre of a cyclone and that the seismological station is outside of that centre. Then, if there be a steady air flow in the lower atmosphere in a certain direction, it would be possible for the disturbance near the cyclonic centre to be transmitted through the lower atmosphere to the station in the form of compressional waves, microseisms of short periods thus being possible[26] The transmission of the disturbance through the layer is not restricted to waves of a compressional nature. It is possible for the same condition to exist also in the case of gravitational waves.

The fact that, as remarked by Gherzi and Kishinouye, microseisms are unlikely when the cyclonic centre is right above the observing station, but occur when the same centre approaches or recedes from the station, may be interpreted from the present explanation.

10. *The probability of elastic surface waves and that of microbarometric waves causing microseismic oscillation.*

Although we have shown in the beginning of this paper that elastic surface waves and microbarometic waves are both important in connexion with microseismic oscillation, for a more comprehensive idea of the problem, we shall now examine the sort of waves that have largely to do with the problem.

We shall leave aside such a cause as the surf beating against the coast, assuming all disturbances to be of atmospheric origin. If the disturbance were transmitted through the earth's crust, the vibrational energy in the air should first be transmitted into the crust. Since the transmissibility in such a case is very low, only a small percentage of the energy is transmitted into the ground. The disturbance that is absorbed into the ground is then propagated outwards through the crustal layers. Furthermore, damping of the elastic waves through the layers is very great in the case of rocks.

If, on the other hand, the vibrational energy were immediately

26) The possibility of compressional boundary waves transmitted through a layer, outside or within which there is a steady air current, is now being studied, the result of which will be published in Part II of the present investigation.

transmitted through atmospheric strata, there is no such reduction of energy in passing from the air into the solid body. Moreover, damping of the waves in air is extremely small. From these considerations, it is very probable that the vibrational energy in the atmosphere is more readily transmitted through air strata than through the crustal layer of the earth. Even should the energy of the disturbance transmitted only through the air be reduced, at the stage of its transfer to the ground at the observing station, at the same rate that corresponds to the transmissibility of the energy between the air and the solid as last given, the small extent of the damping of air waves still has an important part in the problem.

It should however be borne in mind that the amplitude decrease in waves is not the immediate consequence of the small damping coefficient of the medium of transmission. From our investigation, the amplitude of waves of an elastic nature decreases with the damping coefficient (as referred to time)[27]

$$\frac{2\pi^2\gamma}{T^2\rho V^3}, \tag{30}$$

where T, V, γ, ρ are the vibration period, wave velocity, viscous coefficient, and density, respectively. The viscous coefficent of any rock, according to Iida,[28] is greater than 10^5 (C. G. S.) and that of any metal greater than 10^6 (C. G. S.).[29] By assuming that $\gamma=10^5$ for rock and $\gamma=0{\cdot}17$ for air, the densities of both the media being $2{\cdot}7$ and 10^{-3} respectively, and the velocities of waves of elastic nature in the rock and in the air are roughly 3300 meters/sec and 330 meters/sec, respectively, the damping coefficients of the waves (with distance) in rock and in air are then

$$\frac{2\pi^2}{T^2}\frac{10^5}{2{\cdot}7.3{\cdot}3^3.10^{15}}, \qquad \frac{2\pi^2}{T^2}\frac{0{\cdot}17}{3{\cdot}3^3.10^{15}} \tag{31}$$

in C. G. S. respectively, the ratio of the coefficients for both cases being as great as 10^5.

It will be seen that even if the effective damping coefficient of the waves be taken, the coefficient for waves through the rock is extremely large compared with that for waves through the atmosphere for any period of vibration.

It is now possible to conclude that if the source of microseisms were

27) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **16** (193?), 491.
28) K. IIDA, *Bull. Earthq. Res. Inst.*, **15** (1937), 828.
29) K. SEZAWA and K. KUBO, *Rep. Aeron. Res. Inst.*, *No.* 89 (1932).

atmospheric disturbance, movements of the ground would result from the microbarometic waves in the air. If, on the other hand, the source is of the kind like the surf beating against a coast, it is possible for microseisms to result from the elastic surface waves.

11. *Concluding remarks.*

From mathematical investigation, it has been ascertained that the prevalent periods of vibration in microseisms result from the nature of the stratification in the path of transmission of the disturbances. Since the damping coefficient for waves through rocks is extremely large compared with that for waves through the atmosphere for any period of vibration, if the source of microseisms were atmospheric disturbance, the disturbance would be transmitted through the air strata in the form of atmospheric waves, while, if, on the other hand, the same microseisms were from any other origin, such as surf beating against the coast, the disturbances would possibly be transmitted through the crustal layer in the form of elastic surface waves. In any case, the amplitude of waves whose length is twice the thickness of the layer of transmission, is maximum, the origin of the prevalent periods of the microseisms being thus ascertained. A stratified condition of the atmosphere possibly exists when there is a turbulent thin layer on the ground or, at any rate, when there is a steady air current outside or within the lower part of the atmosphere.

It has also been ascertained that in the case of atmospheric waves, there is the possibility of two different prevalent periods of the vibrations, one corresponding to waves of chiefly a gravitational nature and the other to waves of mainly a compressional nature. In either case, the layer never corresponds to the condition of the stratosphere, and is much thinner than the thickness of the layer below that stratosphere. Whereas (stable) waves of a gravitational nature have prevalent periods as long as a few minutes, those of a compressional nature have the same of only a few seconds. Even should compressional force in air be coupled with gravitational force, owing to the great difference in nature between these forces, the coupled condition is virtually the same as the resultant of the pure compressional and the pure gravitational conditions. It is also added that even in the case of the stratosphere being considered with data given by Taylor,[30] the period of the maximum amplitude of the gravitational waves is also 150 sec.

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30) G. I. TAYLOR *loc. cit.*, 14).

17. 大氣波の傳播に伴ひ得る脈動，第1報

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脈動の原因について多くの人の議論があるが未だに一致しないやうに見える．海岸を打つ波の衝撃が彈性波として傳播するといふ說と，大氣中の振動が壓力波又は彈性波として傳播するといふ說とは互に相讓らぬやうである．

只今の硏究では，海岸を打つ波の場合には勿論彈性波として傳播してよいけれども，大氣中の振動はやはり大氣波として傳播しなくてはならぬといふ結論に達するのである．尤も何れの場合にも波動が2次元的でないと充分な振動勢力が傳はらない．波が海岸を打つて出る波については既に多くの硏究が試みられてあるから玆では取扱はない．

大氣中に極く僅かの不連續をもつ層があると，低氣壓の附近から發した振動中の特定の振動周期の成分の振動のみがよく傳播することがわかる．不連續層の厚さの2倍程度の波長をもつ成分の振幅が特に大きい．

このやうな波動が一端地中の層に入つて傳播するとすれば，大氣中の層の中を傳はるときよりも 10^5 倍程度の効果的減衰係數を以て減衰するから，どうしても大氣の層中で傳播する方の可能性が遙かに高い．

大氣層の存在は必しも靜力學的でなくてよろしく，一方の層又は兩方の層に一定の氣流があるといふ力學的の層があつても同じ結果が得られるのである．この委しい議論は第2報で述べる．

數秒程度の周期の波が氣象計器に必しも明瞭に現れないのはむしろこのやうな短周期の波に對する感度の相違とした方がよいかも知れぬ．

大氣の層を考へるときに，重力と壓縮力との兩方の影響があるけれども脈動程度の場合には兩方の影響が餘りに違ひ過ぎるために，聯成振動が純粹の重力波と純粹の壓縮波とに分離する．大氣層の厚さの2倍の波長の振幅が卓越することは兩方の波について何れにもいひ得るのであつて，實際の場合に重力波は數分の周期となり，壓縮波は數秒の周期となるのである．

層の厚さの2倍位の波長のものの振幅が大きくなる事はレーリー波やラブ波の場合でも同樣であつて，之は彈性波として傳はる場合にもいひ得るのである．