19. *Theory of the Aseismic Properties of the Brace Strut (Sudikai) in a Japanese-style Building. Part I.*

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1. *Introduction.*

In the previous paper[1] we remarked that although the excellent aseismic properties of the brace struts (sudikai) are well known, no theoretical examination of these properties had yet been made, for which reason some simple cases of the problem were discussed mathematically in the same paper. However, the case shown in the previous paper was extremely simple, the building being merely one-storied with a floor that is endless, horizontally.

In the present paper we shall deal with the effect of brace struts on the vibration of a structure for the case of a finite number of spans, and also that for the case of a two-storied structure. Some special conditions of the structure shown in the previous paper are also discussed. It was found that although the aseismic properties of brace struts stand out prominently from almost every theoretical point of view, in the special condition of the struts, they are not so effective as ordinary common sense reasoning makes them to be.

2. *The case in which no elastic moment acts at the panel points of the columns of a single-storied structure.*

In the case of a Japanese-style wooden house without any bracket at the panel points, it is unlikely that the elastic moments of forces really act at the same panel points. If brace struts were added to such a structure, it would become fairly resistive to seismic forces.

For simplicity, it is assumed that the masses are concentrated at the panel points, in consequence of which the mass for horizontal movement will differ from that for vertical movement, the respective masses in question then being m_1, m_2. Let l_1, l_2, l_3 be the length of a

1) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **16** (1938), 702~713.

column, beam span, and the length of a brace strut, respectively; let also E_1I_1, E_2I_2, E_3I_3; E_1a_1, E_2a_2, E_3a_3 be the bending and longitudinal stiffnesses of the respective members just mentioned. It is also assumed that, in this case, both ends of every strut are hinged. The lateral movements y_1, y_2 and the longitudinal displacements z_1, z_3 of the respective members satisfy differential equations of types $\partial^4 y/\partial x^4=0$, $\partial^2 z/\partial x^2=0$, the solutions of which are

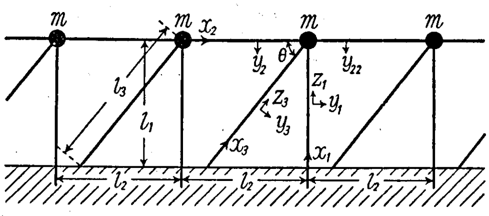

Fig. 1.

$$y_1=A_1+B_1x_1+C_1x_1^2+D_1x_1^3,\quad y_2=A_2+B_2x_2+C_2x_2^2+D_2x_2^3, \tag{1}$$

$$z_1=\alpha_1+\beta_1x_1,\quad z_3=\alpha_3+\beta_3x_3. \tag{2}$$

It is assumed that the lower end of every strut ends at the ground floor. Taking the senses of the coordinates in the respective members as shown in Fig. 1, the boundary conditions are such that

$$x_1=0;\qquad y_1=0,\quad \frac{dy_1}{dx_1}=0,\quad z_1=0, \qquad (3),\ (4),\ (5)$$

$$x_3=0;\qquad z_3=0, \qquad (6)$$

$$x_1=l_1,\ x_2=l_2,\ x_{22}=0,\ x_3=l_3;$$

$$y_2=y_{22}=-z_1,\quad z_3=z_1\sin\theta+y_1\cos\theta, \qquad (7),\ (8),\ (9)$$

$$E_1I_1\frac{d^2y_1}{dx_1^2}=0,\quad E_2I_2\frac{d^2y_2}{dx_2^2}=0,\quad E_2I_2\frac{d^2y_{22}}{dx_{22}^2}=0, \qquad (10),\ (11),\ (12)$$

$$-E_1I_1\frac{d^3y_1}{dx_1^3}+E_3a_3\frac{dz_3}{dx_3}\cos\theta=m_1p^2y_1, \tag{13}$$

$$E_2I_2\left(\frac{d^3y_{22}}{dx_{22}^3}-\frac{d^3y_2}{dx_2^3}\right)-E_3a_3\frac{dz_3}{dx_3}\sin\theta-E_1a_1\frac{dz_1}{dx_1}=m_2p^2y_2. \tag{14}$$

Substituting (1), (2) in (3)~(14), we get the following frequency equation for the free vibration of the structure:

$$\psi\gamma_1\gamma_2-\xi(\psi+\vartheta\sin^2\theta)\gamma_1-(3\psi+\vartheta\xi\cos^2\theta)\gamma_2$$
$$+\xi(3\psi+3\vartheta\sin^2\theta+\vartheta\xi\cos^2\theta)=0, \tag{15}$$

where

$$\gamma_1=\frac{m_1p^2l_1^3}{E_1I_1},\quad \gamma_2=\frac{m_2p^2l_1^3}{E_1I_1},\quad \vartheta=\frac{E_3a_3}{E_1a_1},\quad \psi=\frac{l_3}{l_1},\quad \xi=\frac{a_1l_1^2}{I_1}. \tag{16}$$

If we put $\zeta_1=0$ (and write $\vartheta_1=\vartheta$, $\xi_1=\xi$, $\alpha=1/\psi$) in equation (16) in the previous paper,[2] the same equation reduces to (15) given above.

Equation (15) gives two vibrational frequencies, one corresponding to motion of horizontal type and the other to that of vertical type.

If we assume, as is usually done; that the inertia mass of the movement in the vertical sense is the same as that in the horizontal sense, it is possible to put $\gamma_1=\gamma_2\equiv\gamma\,(m_1=m_2\equiv m)$, from which the frequency equation reduces to

$$\psi\gamma^2-\{\psi(\xi+3)+\vartheta\xi\}\gamma+\xi(3\psi+3\vartheta\sin^2\theta+\vartheta\xi\cos^2\theta)=0. \qquad (15')$$

In the present paper we shall use this equation in connexion with the numerical examples.

If we put $\vartheta=0$ in equation (15′), the same equation reduces to

$$(\gamma-3)(\gamma-\xi)=0. \qquad (15'')$$

$\gamma-3=0$, $\gamma-\xi=0$ represent equations of vibrational frequencies of a clamped-free condition for horizontal motion and vertical motion respectively. If there is no vertical movement, this is of the same form as the one that we have shown in indicating the frequency equation (20) of a braceless framed structure.[3]

With a view to ascertaining the quantitative nature of the aseismic properties of brace struts, we have selected some cases with numerical conditions.

(i) First, let $\psi(=l_3/l_1)=\sqrt{2}$, $\xi(=a_1l_1^2/I_1)=5000$, $\theta=45°$ be given. Here, as usually assumed, the mass moment of inertia for horizontal motion is to be the same as that for vertical motion. In the present particular problem, ratio l_2/l_1 is arbitrary. Since $l_3/l_1=\sqrt{2}$, and $\theta=45°$, the lower end of every strut is just at the foot of every neighbouring column. The calculation of the natural frequency for various ratios of (E_3a_3/E_1a_1) is shown in the first line in Table I and plotted in Fig. 2.

It will be seen that, since there are two freedoms of vibration, namely, horizontal and vertical, two frequencies exist for every ratio of E_3a_3/E_1a_1 as seen in Table I and in Fig. 2. The frequency of the fundamental vibration, namely, vibration mainly of horizontal type, increases enormously with small increase in the stiffness of the brace struts. With a view to comparing the panel condition of the present case with those in which the stiffness of the beam is not zero, we reproduced the results in the previous paper for the conditions $E_2I_2/E_1I_1=1, 10$, as shown in the second and third lines in Table I. This shows

2) K. SEZAWA and K. KANAI, *loc. cit.*, 1).
3) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **14** (1936), 369.

that for such ratio of E_3a_3/E_1a_1 as is greater than unity, the vibrational frequency is nearly the same for any ratio of E_2I_2/E_1I_1. This

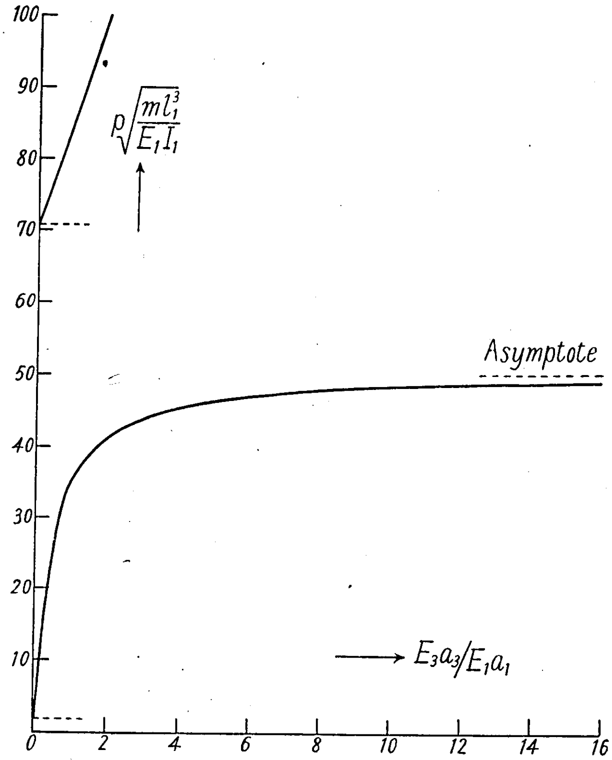

Fig. 2. $l_3/l_1=\sqrt{2}$, l_2/l_1=arbitrary, $a_1l_1^2/I_1=5000$, $\theta=45°$, $m_1=m_2\equiv m$.

feature has already been shown in the previous paper.[4] Furthermore, beyond a certain ratio of E_3a_3/E_1a_1, say, $E_3a_3/E_1a_1=5$, the frequency of the horizontal vibration assumes an asymptotic value.

From the above conditions of the problem, it holds that for the horizontal type of vibration, as in the case of the natural frequency of such a condition of the braced strut as $E_3a_3/E_1a_1>5$, even should there be no bending stiffness in the panels, the vibrational frequency of the structure is nearly equal to that with infinitely stiff beams and struts.

In Fig. 2 and Table I, the vibrational frequencies for the vertical type of vibration are also added. The frequency of this type of vibration increases very rapidly with increase in E_3a_3/E_1a_1. Here, in the nature of things, the increase in the stiffness of the beam, that is, E_2I_2, has almost no effect on the change in natural frequency of the structure.

4) K. SEZAWA and K. KANAI, *loc. cit.*, 1). p. 706, (ii) and Fig. 3 of that paper.

Table I. The value of $p\sqrt{ml_1^3/E_1I_1}$ for horizontal (and vertical) type of vibration.

E_2I_2/E_1I_1 \ E_3a_3/E_1a_1	1	2	10	∞
0	1·732 (70·71)	34·8 (85·6)	48·2 (195)	50·01 (∞)
1	3·125 (70·71)	34·9 (85·6)	48·2 (195)	50·05 (∞)
10	3·420 (70·71)	34·9 (85·6)	48·2 (195)	50·06 (∞)

(ii) We shall next consider the effect of change in beam span, the bottom hinge of the strut being always at the foot of the column. The ratios $\vartheta(=E_3a_3/E_1a_1)=1$, $\xi(=a_1l_1^2/I_1)=5000$, $\psi(=l_3/l_1)=\operatorname{cosec}\theta$, $l_3^2=l_1^2+l_2^2$ are given. The calculation is shown in Fig. 3.

It will be seen that for the horizontal type of vibration, there is

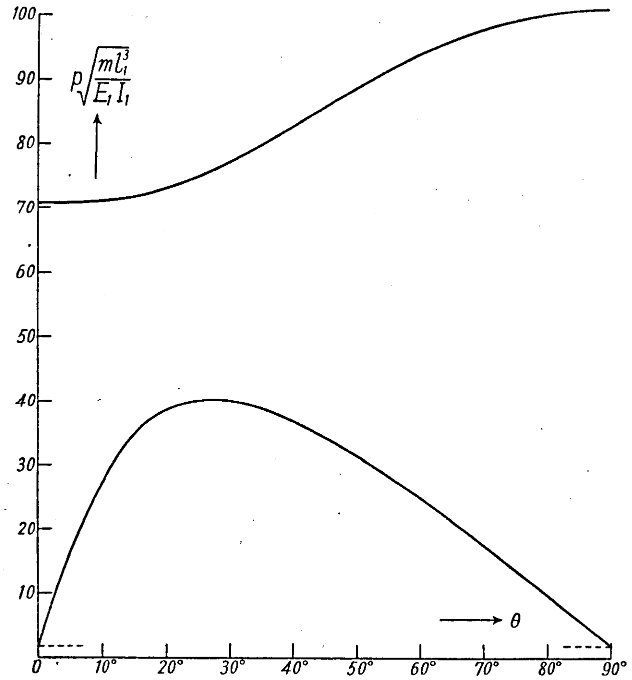

Fig. 3. $E_3a_3/E_1a_1=1$, $a_1l_1^2/I_1=5000$, $l_1/l_3=\sin\theta$, $l_3^2=l_1^2+l_2^2$, $m_1=m_2\equiv m$.

a maximum of vibrational frequency at that angle near $\theta=30°$. For the vertical type of vibration, on the other hand, the frequency assumes its greatest value at $\theta=90°$,—an obvious fact from the nature of the problem.

Since in a Japanese-style building, it is very difficult to make the panel points resistive against moments of forces, it is advisable to use brace struts, by means of which the building could be made so stiff that beams of infinite stiffness may be used.

3. *A single-storied structure of mono-span with brace strut.*

For simplicity, it is assumed that although the inertia mass is ineffective for vertical motion, the elastic stiffness of every member is operative for such motion. Using notations similar to those in the preceding section, the solutions of the vibratory motion of the columns and the beam in both lateral and longitudinal senses are

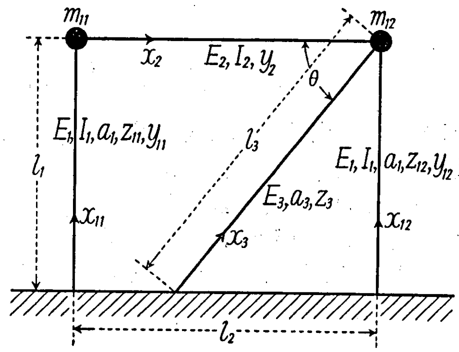

Fig. 4.

$$\left.\begin{aligned} y_{1n}&=A_{1n}+B_{1n}x_{1n}+C_{1n}x_{1n}^2+D_{1n}x_{1n}^3, \quad [n=1,\ 2] \\ y_2&=A_2+B_2x_2+C_2x_2^2+D_2x_2^3, \end{aligned}\right\} \tag{17}$$

$$z_{1n}=\alpha_{1n}+\beta_{1n}x_{1n}, \quad [n=1,\ 2] \qquad z_3=\alpha_3+\beta_3x_3. \tag{18}$$

Although the panel points are rigid, elastically, both ends of the strut are hinged. The boundary conditions are such that

$$x_{1n}=0; \qquad y_{1n}=0, \quad \frac{dy_{1n}}{dx_{1n}}=0, \quad z_{1n}=0, \quad [n=1,\ 2] \qquad (19),\ (20),\ (21)$$

$$x_3=0; \qquad z_3=0, \tag{22}$$

$$x_{11}=l_1,\ x_2=0; \qquad y_2=-z_{11}, \tag{23}$$

$$\frac{dy_{11}}{dx_{11}}=\frac{dy_2}{dx_2}, \quad -E_1I_1\frac{d^2y_{11}}{dx_{11}^2}+E_2I_2\frac{d^2y_2}{dx_2^2}=0, \qquad (24),\ (25)$$

$$E_2I_2\frac{d^3y_2}{dx_2^3}-E_1a_1\frac{dz_{11}}{dx_{11}}=0, \quad -E_1I_1\frac{d^3y_{11}}{dx_{11}^3}=m_{11}p^2y_{11}+T, \qquad (26),\ (27)$$

$$x_{12}=l_1,\ x_2=l_2,\ x_3=l_3; \qquad y_2=-z_{12}, \tag{28}$$

$$z_3=z_{12}\sin\theta+y_{12}\cos\theta, \tag{29}$$

$$\frac{dy_{12}}{dx_{12}}=\frac{dy_2}{dx_2}, \quad -E_1I_1\frac{d^2y_{12}}{dx_{12}^2}-E_2I_2\frac{d^2y_2}{dx_2^2}=0, \qquad (30),\ (31)$$

$$-E_2I_2\frac{d^3y_2}{dx_2^3}-E_3a_3\frac{dz_3}{dx_3}\sin\theta-E_1a_1\frac{dz_{12}}{dx_{12}}=0, \tag{32}$$

$$-E_1I_1\frac{d^3y_{12}}{dx_{12}^3}+E_3a_3\frac{dz_3}{dx_3}\cos\theta=m_{12}p^2y_{12}-T, \tag{33}$$

$$x_{11}=x_{12}=l_1; \qquad y_{11}=y_{12}, \tag{34}$$

where T is the thrust in every beam.

Substituting (17) and (18) in (19)$\sim$(34), we get the following frequency equations for the structure:

$$\left.\begin{aligned}\gamma\Phi=12\psi(6\xi\zeta\phi^2+\xi\phi^3+24\zeta)+\vartheta\{\xi(3\xi\zeta\phi^2+2\xi\phi^3+48\zeta)\cos^2\theta\\ -72\xi\zeta\phi\cos\theta\sin\theta+12(6\xi\zeta\phi^2+\xi\phi^3+12\zeta)\sin^2\theta\},\\ \Phi=\psi(3\xi\zeta\phi^2+2\xi\phi^3+48\zeta)+\vartheta(3\xi\zeta\phi^2+2\xi\phi^3+24\zeta)\sin^2\theta,\end{aligned}\right\} \tag{35}$$

where

$$\left.\begin{aligned}\gamma=\frac{(m_{11}+m_{12})p^2l_1^3}{E_1I_1},\quad \vartheta=\frac{E_3a_3}{E_1a_1},\quad \zeta=\frac{E_2I_2}{E_1I_1},\quad \psi=\frac{l_3}{l_1}\\ \phi=\frac{l_2}{l_1},\quad \xi=\frac{a_1l_1^2}{I_1}.\end{aligned}\right\} \tag{36}$$

By putting, specially, $\vartheta(=E_3a_3/E_1a_1)=0$ in (35), we get

$$\gamma=\frac{12(6\xi\zeta\phi^2+\xi\phi^3+24\zeta)}{3\xi\zeta\phi^2+2\xi\phi^3+48\zeta}, \tag{35'}$$

which is the frequency equation of a braceless structure of mono-span.

If we put $\theta=0$, (35) again reduces to (35′), which means that although the bottom hinge is on the ground surface, if the length of the brace strut were infinitely long, horizontally, the same strut would become ineffective even in resistance to horizontal motion.

(i) As a numerical example, we put $\zeta(=E_2I_2/E_1I_1)=1$, $\psi(=l_3/l_1)=\sqrt{2}$, $\phi(=l_2/l_1)=1$, $\xi(=a_1l_1^2/I_1)5000$, $\theta=45°$, $m_{11}=m_{12}\equiv m$. The condition of

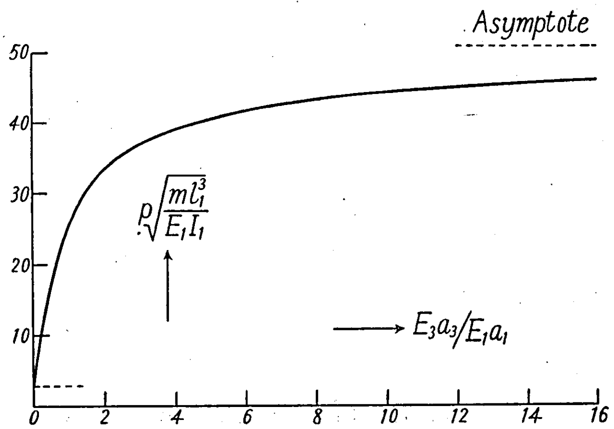

Fig. 5. $E_2I_2/E_1I_1=1$, $l_2/l_1=1$, $l_3/l_1=\sqrt{2}$, $a_1l_1^2/I_1=5000$, $\theta=45°$, $m_{11}=m_{12}\equiv m$.

the structure is then such that the brace is a diagonal of a square-shaped structure. The calculation is shown in Fig. 5. It will be seen that if the ratio of E_3a_3/E_1a_1 be greater than 5, the vibrational frequency is virtually the same as that for the condition $E_3a_3/E_1a_1=\infty$. Even in the condition $E_3a_3/E_1a_1=2$, the frequency is six or seven times that in the condition of a braceless structure.

(ii) We shall next assume that $\vartheta(=E_3a_3/E_1a_1)=1$, $\zeta=1$, $\psi=\operatorname{cosec}\theta$, $\phi=1$, $\xi=5000$, $m_{11}=m_{12}\equiv m$ and vary the inclination θ of the strut, the bottom hinge being always on the ground surface. The result of calculation is shown in Fig. 6. It will be seen that in this case, too,

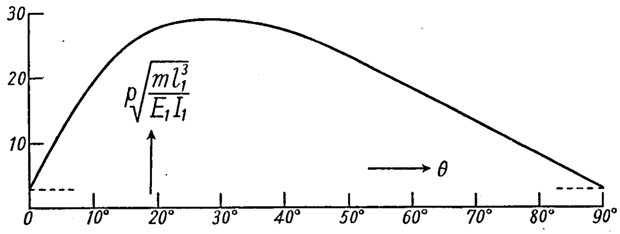

Fig. 6. $E_3a_3/E_1a_1=1$, $E_2I_2/E_1I_1=1$, $l_2/l_1=1$, $a_1l_1^2/I_1=5000$, $l_1/l_3=\sin\theta$, $m_{11}=m_{12}\equiv m$.

the natural frequency becomes maximum at an angle near $\theta=30°$. For $\theta=0$, namely, for the condition of the strut being horizontal, the effect of the strut disappears, as has been shown in equation (35′).

4. *A single-storied structure of two spans with brace struts.*

In this case, too, it is assumed that the inertia mass is ineffective for vertical motion, but in such a sense the elastic resistance of every member is effective. Since the large motion of any great earthquake at a certain epicentral distance is mainly horizontal, the condition just mentioned fits the practical problem. The solutions of the vibratory motions of every member in both lateral and longitudinal senses are

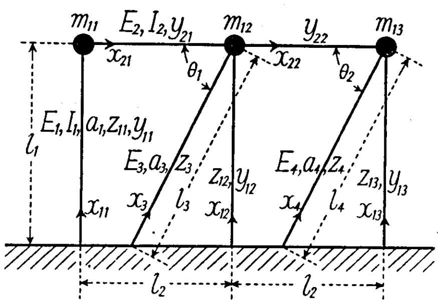

Fig. 7.

$$\left.\begin{aligned} y_{1n}&=A_{1n}+B_{1n}x_{1n}+C_{1n}x_{1n}^2+D_{1n}x_{1n}^3, \qquad [n=1,\ 2,\ 3] \\ y_{2n}&=A_{2n}+B_{2n}x_{2n}+C_{2n}x_{2n}^2+D_{2n}x_{2n}^3, \qquad [n=1,\ 2] \end{aligned}\right\} \tag{37}$$

$$\left.\begin{aligned} z_{1n} &= \alpha_{1n} + \beta_{1n} x_{1n}, \qquad [n=1,\ 2,\ 3] \\ z_n &= \alpha_n + \beta_n x_n. \qquad [n=3,\ 4] \end{aligned}\right\} \tag{38}$$

The boundary conditions are such that

$$x_{1n}=0; \qquad y_{1n}=0, \quad \frac{dy_{1n}}{dx_{1n}}=0, \quad z_{1n}=0, \quad [n=1,\ 2,\ 3] \qquad (39),\ (40),\ (41)$$

$$x_n=0; \qquad z_n=0, \qquad [n=3,\ 4] \qquad (42)$$

$$x_{11}=l_1,\ x_{21}=0; \qquad y_{21}=-z_{11}, \quad \frac{dy_{11}}{dx_{11}}=\frac{dy_{21}}{dx_{21}}, \qquad (43),\ (44)$$

$$-E_1I_1\frac{d^2y_{11}}{dx_{11}^2}+E_2I_2\frac{d^2y_{21}}{dx_{21}^2}=0, \quad E_2I_2\frac{d^3y_{21}}{dx_{21}^3}-E_1a_1\frac{dz_{11}}{dx_{11}}=0, \qquad (45),\ (46)$$

$$x_{12}=l_1,\ x_{21}=l_2,\ x_{22}=0,\ x_3=l_3;$$

$$y_{21}=y_{22}=-z_{12}, \quad z_3=z_{12}\sin\theta_1+y_{12}\cos\theta_1, \qquad (47),\ (48)$$

$$\frac{dy_{12}}{dx_{12}}=\frac{dy_{21}}{dx_{21}}=\frac{dy_{22}}{dx_{22}}, \tag{49}$$

$$-E_1I_1\frac{d^2y_{12}}{dx_{12}^2}-E_2I_2\frac{d^2y_{21}}{dx_{21}^2}+E_2I_2\frac{d^2y_{22}}{dx_{22}^2}=0, \tag{50}$$

$$-E_2I_2\frac{d^3y_{21}}{dx_{21}^3}+E_2I_2\frac{d^3y_{22}}{dx_{22}^3}-E_3a_3\frac{dz_3}{dx_3}\sin\theta_1-E_1a_1\frac{dz_{12}}{dx_{12}}=0, \tag{51}$$

$$x_{13}=l_1,\ x_{22}=l_2,\ x_4=l_4;$$

$$y_{22}=-z_{13}, \quad z_4=z_{13}\sin\theta_2+y_{13}\cos\theta_2, \qquad (52),\ (53)$$

$$\frac{dy_{13}}{dx_{13}}=\frac{dy_{22}}{dx_{22}}, \qquad -E_1I_1\frac{d^2y_{13}}{dx_{13}^2}-E_2I_2\frac{d^2y_{22}}{dx_{22}^2}=0, \qquad (54),\ (55)$$

$$-E_2I_2\frac{d^3y_{22}}{dx_{22}^3}-E_4a_4\frac{dz_4}{dx_4}\sin\theta_2-E_1a_1\frac{dz_{13}}{dx_{13}}=0, \tag{56}$$

$$x_{11}=x_{12}=x_{13}=l_1; \qquad y_{11}=y_{12}=y_{13}, \tag{57}$$

$$m_{11}p^2y_{11}+m_{12}p^2y_{12}+m_{13}p^2y_{13}=-E_1I_1\left(\frac{d^3y_{11}}{dx_{11}^3}+\frac{d^3y_{12}}{dx_{12}^3}+\frac{d^3y_{13}}{dx_{13}^3}\right)$$
$$+E_3a_3\frac{dz_3}{dx_3}\cos\theta_1+E_4a_4\frac{dz_4}{dx_4}\cos\theta_2=0. \tag{58}$$

Substituting (37), (38) in (39)$\sim$(58), we get the frequency equation

$$P_2(R_1Q_3-Q_1R_3)+Q_2(P_1R_3-R_1P_3)+R_2(Q_1P_3-P_1Q_3)=0, \tag{59}$$

where

$$
\begin{aligned}
P_1 &= 2\gamma\zeta(\psi_2+\vartheta_2\sin^2\theta_2)\sin\theta_1-(\psi_2+\vartheta_2\sin^2\theta_2)(3\xi\zeta\phi\cos\theta_1+18\zeta\sin\theta_1)\\
&\quad -2\vartheta_2\xi\zeta\cos^2\theta_2\sin\theta_1,\\
Q_1 &= 3\gamma\zeta^2\phi(\zeta+2\phi)(\psi_2+\vartheta_2\sin^2\theta_2)\sin\theta_1-3\vartheta_2\xi\zeta^2\phi(\zeta+2\phi)\cos^2\theta_2\sin\theta_1\\
&\quad -(\psi_2+\vartheta_2\sin^2\theta_2)\{\xi\phi^2(6\zeta^3+21\zeta^2\phi+20\zeta\phi^2+8\phi^3)\cos\theta_1\\
&\quad +6\zeta(3\zeta^2+16\zeta\phi+16\phi^2)\cos\theta_1-18\zeta\phi^2(5\zeta+8\phi)\sin\theta_1\},\\
R_1 &= 3\gamma\zeta\phi(\zeta^2+8\zeta\phi+8\phi^2)(\psi_2+\vartheta_2\sin^2\theta_2)\sin\theta_1\\
&\quad -3\vartheta_2\zeta\{\xi\phi(\zeta^2+8\zeta\phi+8\phi^2)\cos\theta_2+2(3\zeta^2+16\zeta\phi+16\phi^2)\sin\theta_2\}\cos\theta_2\sin\theta_1\\
&\quad -6\zeta(\psi_2+\vartheta_2\sin^2\theta_2)\{\xi\phi^2(\zeta^2+7\zeta\phi+7\phi^2)\cos\theta_1\\
&\quad -(3\zeta^2+16\zeta\phi+16\phi^2)\cos\theta_1+12\phi^2(\zeta+\phi)\sin\theta_1\},\\
P_2 &= 3\xi\zeta\phi(\psi_1+\vartheta_1\sin^2\theta_1)-2\vartheta_1\xi\zeta\cos\theta_1\sin\theta_1,\\
Q_2 &= \xi\phi^2(6\zeta^3+21\zeta^2\phi+20\zeta\phi^2+8\phi^3)(\psi_1+\vartheta_1\sin^2\theta_1)\\
&\quad -3\vartheta_1\xi\zeta^2\phi(\zeta+2\phi)\cos\theta_1\sin\theta_1+6\psi_1\zeta(3\zeta^2+16\zeta\phi+16\phi^2),\\
R_2 &= 6\xi\zeta\phi(\zeta^2+7\zeta\phi+7\phi^2)(\psi_1+\vartheta_1\sin^2\theta_1)-3\vartheta_1\xi\zeta\phi(\zeta^2+8\zeta\phi+8\phi^2)\cos\theta_1\sin\theta_1\\
&\quad -6\psi_1\zeta(3\zeta^2+16\zeta\phi+16\phi^2),\\
P_3 &= 12\xi\phi\zeta(\psi_2+\vartheta_2\sin^2\theta_2)-4\vartheta_2\xi\zeta\cos\theta_2\sin\theta_2,\\
Q_3 &= 4\xi\phi^2(\psi_2+\vartheta_2\sin^2\theta_2)(6\zeta^3+18\zeta^2\phi+13\zeta\phi^2+4\phi^3)\\
&\quad -6\vartheta_2\xi\zeta^2\phi(\zeta+2\phi)\cos\theta_2\sin\theta_2+12\zeta(\psi_2+\vartheta_2\sin^2\theta_2)(3\zeta^2+16\zeta\phi+16\phi^2),\\
R_3 &= 4\xi\phi^2(\psi_2+\vartheta_2\sin^2\theta_2)(6\zeta^3+45\zeta^2\phi+49\zeta\phi^2+4\phi^3)\\
&\quad -6\vartheta_2\xi\zeta\phi(\zeta^2+8\zeta\phi+8\phi^2)\cos\theta_2\sin\theta_2+12\psi_2\zeta(3\zeta^2+16\zeta\phi+16\phi^2), \qquad (60)
\end{aligned}
$$

in which

$$
\left.\begin{aligned}
\gamma=\frac{(m_{11}+m_{12}+m_{13})p^2l_1^3}{E_1I_1},\quad \vartheta_1=\frac{E_3a_3}{E_1a_1},\quad \vartheta_2=\frac{E_4a_4}{E_1a_1},\quad \zeta=\frac{E_2I_2}{E_1I_1}\\
\psi_1=\frac{l_3}{l_1},\quad \psi_2=\frac{l_4}{l_1},\quad \phi=\frac{l_2}{l_1},\quad \xi=\frac{a_1l_1^2}{I_1}.
\end{aligned}\right\} \qquad (61)
$$

Since, in this case, we neglected the vertical component of the inertia mass, there is only one vibrational frequency.

As an example, we shall assume that $E_2I_2/E_1I_1=1$, $l_3/l_1=l_4/l_1=\sqrt{2}$, $l_2/l_1=1$, $a_1l_1^2/I_1=5000$, $\theta_1=\theta_2=45°$, $m_{11}=m_{12}=m_{13}\equiv m$ and vary the ratio of $E_3a_3/E_1a_1(=E_4a_4/E_1a_1)$. The brace strut is evidently a diagonal of a square structure. The calculation is shown in Fig. 8. It will be seen from this figure, too, that if the ratio of $E_3a_3/E_1a_1(=E_4a_4/E_1a_1)$ were greater than 5, the condition of the structure would then be virtually the same as that in the case of infinitely stiff struts.

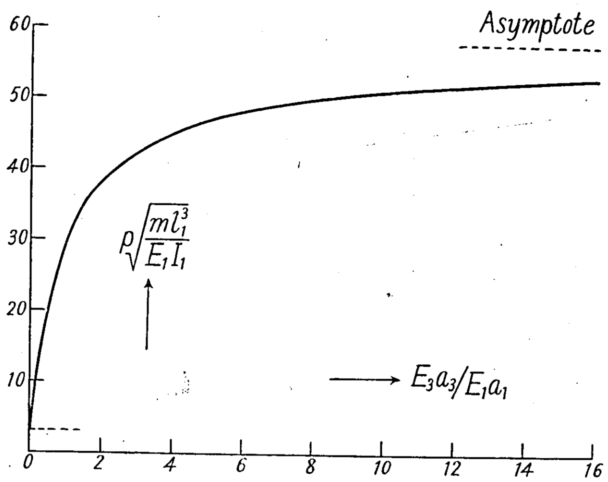

Fig. 8. $l_3/l_1=l_4/l_1=\sqrt{2}$, $l_2/l_1=1$, $E_2I_2/E_1I_1=1$, $a_1l_1^2/I_1=5000$, $\theta_1=\theta_2=45°$, $m_{11}=m_{12}=m_{13}\equiv m$.

5. *Comparison of cases of different number of spans.*

In Sections 2, 3, 4, we obtained the results for different numbers of spans, all of which correspond to a single-storied framed structure. We shall now compare these cases for various conditions of E_3a_3/E_1a_1, the values of $E_2I_2/E_1I_1=1$, $l_3/l_1(=l_4/l_1)=\sqrt{2}$, $l_2/l_1=1$, $a_1l_1^2/I_1=5000$, $\theta(=\theta_1=\theta_2)=45°$ being kept constant. As will be seen from Table II, the ratio of E_3a_3/E_1a_1 varies as 0, 1, 10, ∞, whereas the number of spans taken is 1, 2, ∞.

Table II. The values of $p\sqrt{ml_1^3/E_1I_1}$ for different numbers of spans and the ratio of E_3a_3/E_1a_1.

E_3a_3/E_1a_1 / No. of spans	0	1	10	∞
1	2·896	25·96	44·67	50·57
2	2·954	29·64	51·05	57·80
∞	3·124	36·29	62·49	70·80

It will be seen from this table that in every number of spans, a small increase in the ratio of E_3a_3/E_1a_1 is effective for stiffening the structure. As to the relation between the ratio of E_3a_3/E_1a_1 and the number of spans that would affect the vibrational frequency of the structure, there is a special feature to be pointed out. When there is no brace strut, that is to say, when $E_3a_3/E_1a_1=0$, the increase in vibrational frequency with increase in the number of spans, is very slight, whereas when the brace struts are very stiff, the same vibrational frequency increases enormously with small increase in the number of spans.

It may therefore be concluded that the efficiency of the brace struts is specially marked when there is a sufficient number of spans, say, three or four. This does not include that condition in which the brace strut is added to a structure with such a small number of spans as only, say, one or two. As already shown, even in the case of a monospan, the vibrational frequency is increased considerably by the addition of a brace strut.

6. *The case of a two-storied structure with brace struts in the first floor.*

The solutions of this problem is quite similar to those shown in previous sections. Referring to Fig. 9, the lateral and longitudinal displacements of every member are expressed by

$$\left.\begin{aligned} y_1&=A_1+B_1x_1+C_1x_1^2+D_1x_1^3,\\ y_1'&=A_1'+B_1'x_1'+C_1'x_1'^2+D_1'x_1'^3,\\ y_2&=A_2+B_2x_2+C_2x_2^2+D_2x_2^3,\\ y_2'&=A_2'+B_2'x_2'+C_2'x_2'^2+D_2'x_2'^3, \end{aligned}\right\}\quad(62)$$

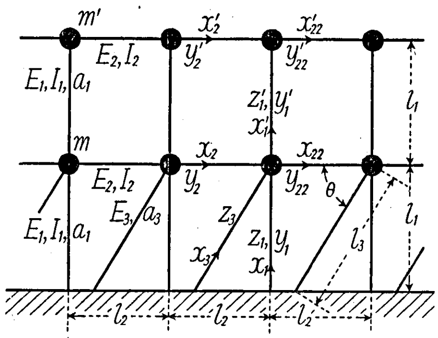

Fig. 9.

$$z_1=\alpha_1+\beta_1x_1,\quad z_1'=\alpha_1'+\beta_1'x_1',\quad z_3=\alpha_3+\beta_3x_3. \tag{63}$$

In the present case, the vertical component of every inertia mass is not neglected. The bending moments at every panel point are taken as usual, so that, naturally, the stiffness of the beam is effective.

The boundary conditions are now such that

$$x_1=0;\qquad y_1=0,\ \frac{dy_1}{dx_1}=0.\quad z_1=0, \qquad (64),\ (65),\ (66)$$

$$x_3=0;\qquad z_3=0, \qquad (67)$$

$$x_1=l_1,\ x_1'=0,\ x_2=l_2,\ x_{22}=0,\ x_3=l_3;$$

$$y_1=y_1',\ y_2=y_{22}=-z_1=-z_1',\ z_3=z_1\sin\theta+y_1\cos\theta, \qquad (68),\ (69),\ (70)$$

$$\frac{dy_1}{dx_1}=\frac{dy_1'}{dx_1'}=\frac{dy_2}{dx_2}=\frac{dy_{22}}{dx_{22}}, \tag{71}$$

$$-E_1I_1\frac{d^2y_1}{dx_1^2}+E_1I_1\frac{d^2y_1'}{dx_1'^2}-E_2I_2\left(\frac{d^2y_2}{dx_2^2}-\frac{d^2y_{22}}{dx_{22}^2}\right)=0, \tag{72}$$

$$-E_1I_1\frac{d^3y_1}{dx_1^3}+E_1I_1\frac{d^3y_1'}{dx_1'^3}+E_3a_3\frac{dz_3}{dx_3}\cos\theta=m_1p^2y_1, \tag{73}$$

$$E_2I_2\left(\frac{d^3y_{22}}{dx_{22}^3}-\frac{d^3y_2}{dx_2^3}\right)-E_3a_3\frac{dz_3}{dx_3}\sin\theta-E_1a_1\frac{dz_1}{dx_1}+E_1a_1\frac{dz_1'}{dx_1'}=m_2p^2y_2, \tag{74}$$

$$x_1'=l_1,\ x_2'=l_2,\ x_{22}'=0; \qquad y_2'=y_{22}'=-z_1', \tag{75}$$

$$\frac{dy_1'}{dx_1'}=\frac{dy_2'}{dx_2'}=\frac{dy_{22}'}{dx_{22}'}, \quad -E_1I_1\frac{d^2y_1'}{dx_1'^2}-E_2I_2\left(\frac{d^2y_2'}{dx_2'^2}-\frac{d^2y_{22}'}{dx_{22}'^2}\right)=0, \quad (76),\ (77)$$

$$-E_1I_1\frac{d^3y_1'}{dx_1'^3}=m_1'p^2y_1', \quad E_2I_2\left(\frac{d^3y_{22}'}{dx_{22}'^3}-\frac{d^3y_2'}{dx_2'^3}\right)-E_1a_1\frac{dz_1'}{dx_1'}=m_2'p^2y_2'. \quad (78),\ (79)$$

Substituting (62), (63) in (64)~(79), we get the following frequency equation:

$$\Big[\gamma_1'\big\{(\gamma_1\psi-\vartheta\xi\cos^2\theta)(36\zeta^2+36\zeta\phi+7\phi^2)-12\psi(72\zeta^2+63\zeta\phi+8\phi^2)\big\}$$
$$-12\big\{(\gamma_1\psi-\vartheta\xi\cos^2\theta)(36\zeta^2+18\zeta\phi+\phi^2)-3\psi(144\zeta^2+36\zeta\phi+\phi^2)\big\}\Big]$$
$$\cdot\big\{\gamma_2\gamma_2'\psi-\gamma_2\psi\xi-\gamma_2'\xi(\vartheta\sin^2\theta+2\psi)+\xi^2(\vartheta\sin^2\theta+\psi)\big\}$$
$$-\vartheta^2\xi^2\cos^2\theta\sin^2\theta(\gamma_2'-\xi)\big\{\gamma_1'(36\zeta^2+36\zeta\phi+7\phi^2)$$
$$-12(36\zeta^2+18\zeta\phi+\phi^2)\big\}=0, \tag{80}$$

in which

$$\left.\begin{aligned}&\gamma_1=\frac{m_1p^2l_1^3}{E_1I_1},\quad \gamma_2=\frac{m_2p^2l_1^3}{E_1I_1},\quad \gamma_1'=\frac{m_1'p^2l_1^3}{E_1I_1},\quad \gamma_2'=\frac{m_2'p^2l_1^3}{E_1I_1},\quad \vartheta=\frac{E_3a_3}{E_1a_1},\\ &\zeta=\frac{E_2I_2}{E_1I_1},\quad \psi=\frac{l_3}{l_1},\quad \phi=\frac{l_2}{l_1},\quad \xi=\frac{a_1l_1^2}{I_1}.\end{aligned}\right\} \tag{81}$$

With a view to confirming equation (80), we put $\vartheta=0$ in the same equation, from which we get a pair of frequency equations of the forms:

$$\gamma_1'\{\gamma_1(36\zeta^2+36\zeta\phi+7\phi^2)-12(72\zeta^2+63\zeta\phi+8\phi^2)\}$$
$$-12\{\gamma_1(36\zeta^2+18\zeta\phi+\phi^2)-3(144\zeta^2+36\zeta\phi+\phi^2)\}=0, \tag{80'a}$$

$$\gamma_2\gamma_2'-\gamma_2\xi-2\gamma_2'\xi+\xi^2=0. \tag{80'b}$$

The first one is of the same form as that of a braceless structure, which we obtained previously.[5] The latter is the frequency equation

5) K. SEZAWA and K. KANAI, *loc. cit.*, 3). Writing $\gamma_2=\gamma_1'$, $\eta_1=\eta_2=\zeta/\phi$, $\nu=\xi=1$ in equation (21) of that paper, the same equation becomes identical with (80'a) in the present case.

of the vertical motion of a two-storied braceless structure.

Since in the present case there are four freedoms of motion, namely, two in vertical motion and two in horizontal motion, there are, as will be seen from equation (80), four vibrational frequencies.

We shall now take two cases in the value of ζ, namely, $E_2I_2/E_1I_1=1$ and 10 and assume that $\gamma_1=\gamma_2=\gamma_1'=\gamma_2'(m_1=m_2=m_1'=m_2')$, $\psi(=l_3/l_1) =\sqrt{2}$, $\phi(=l_2/l_1)=1$, $\xi(=a_1l_1^2/I_1)=5000$, $\theta=45°$.

Using equation (80), we obtained the results as shown in Table III and Fig. 10. In every case, the values of the lowest two frequen-

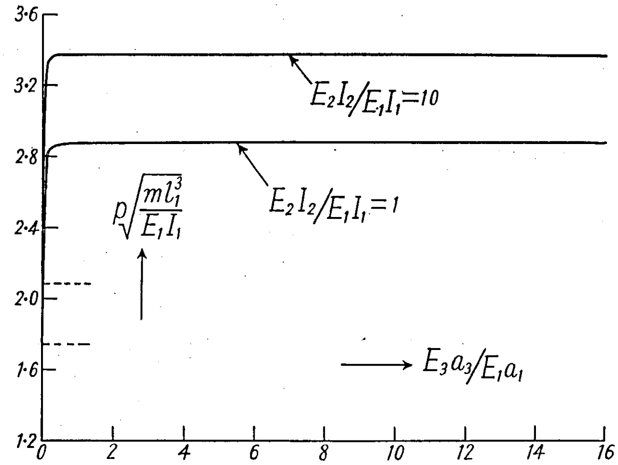

Fig. 10. $l_3/l_1=\sqrt{2}$, $l_2/l_1=1$, $a_1l_1^2/I_1=5000$, $\theta=45°$, $m_1=m_2=m_1'=m_2'\equiv m$.

cies for $\vartheta=0$ become identical with those of corresponding cases in Table II of the previous paper.[6] This is obvious from the fact that equation (80) can be decomposed into the two equations in (80′). The

Table III. The values of $p\sqrt{ml_1^3/E_1I_1}$ for various E_3a_3/E_1a_1 in cases $E_2I_2/E_1I_1=1$ and 10.

E_2I_2/E_1I_1 \ E_3a_3/E_1a_1	0	1	10	∞
1	1·747 5·199 43·70 114·41	2·875 32·73 53·62 120·9	2·883 37·93 86·27 203·7	2·887 38·34 92·38 ∞
10	2·086 5·550 43·70 114·4	3·366 32·73 53·61 120·9	3·377 37·94 86·27 203·7	3·379 38·34 92·38 ∞

6) K. SEZAWA and K. KANAI, *loc. cit.*, 3). p. 370. Table II, $\eta=1$, 10.

remaining two frequencies last given, correspond to those for the vertical motion.

The last column in the above table represents that case in which the stiffness of the brace struts becomes infinitely large. In such a condition, the two lowest vibrational frequencies are respectively nearly equal to those of a braceless single-storied structure, as might be seen by comparing these frequencies with those of the corresponding cases in the previous paper.[7] The larger the ratio of E_2I_2/E_1I_1, the closer the resemblance between the values of the frequencies of the special case in the present condition and those of corresponding ones in the previous paper.

It will be seen, at any rate, that in the case of a two-storied structure with brace struts in either one of the storeys, an unlimited increase in the stiffness of the brace struts does not necessarily increase the vibrational frequency of that structure, but results in the appearance of a natural vibration of a single-storied braceless structure corresponding to the other floor of the two-storied one just mentioned.

7. *The case of a two-storied structure with brace struts in the second floor.*

Although in this case, there is no brace strut in the first floor, there is one in every span of the second floor, both ends of the brace strut in question always reaching the panel points of the two floors.

The solutions of the problem, which are similar to those in the preceding section, are

$$\left.\begin{aligned} y_n &= A_n + B_n x_n + C_n x_n^2 + D_n x_n^3, \\ y'_n &= A'_n + B'_n x'_n + C'_n x'^2_n + D'_n x'^3_n, \end{aligned}\right\} \quad [n=1,\ 2] \tag{82}$$

$$z_1 = \alpha_1 + \beta_1 x_1, \quad z'_1 = \alpha'_1 + \beta'_1 x'_1, \quad z'_3 = \alpha'_3 + \beta'_3 x'_3. \tag{83}$$

The boundary conditions are written

$$x_1 = 0; \qquad y_1 = 0, \quad \frac{dy_1}{dx_1} = 0, \quad z_1 = 0, \qquad (84),\ (85),\ (86)$$

$$x_1 = l_1,\ x'_1 = 0,\ x_2 = l_2,\ x_{22} = 0,\ x'_3 = 0;$$

$$y_1 = y'_1,\ y_2 = y_{22} = -z_1 = -z'_1,\ z'_3 = z_1 \sin\theta + y_1 \cos\theta, \qquad (87),\ (88),\ (89)$$

$$\frac{dy_1}{dx_1} = \frac{dy'_1}{dx'_1} = \frac{dy_2}{dx_2} = \frac{dy_{22}}{dx_{22}}, \tag{90}$$

7) K. SEZAWA and K. KANAI, *loc. cit.*, 3). p. 369. Table I. $\eta=1,\ 10$.

$$-E_1I_1\frac{d^2y_1}{dx_1^2}+E_1I_1\frac{d^2y_1'}{dx_1'^2}-E_2I_2\left(\frac{d^2y_2}{dx_2^2}-\frac{d^2y_{22}}{dx_{22}^2}\right)=0, \tag{91}$$

$$-E_1I_1\frac{d^3y_1}{dx_1^3}+E_1I_1\frac{d^3y_1'}{dx_1'^3}-E_3a_3\frac{dz_3'}{dx_3'}\cos\theta=m_1p^2y_1, \tag{92}$$

$$E_2I_2\left(\frac{d^3y_{22}}{dx_{22}^3}-\frac{d^3y_2}{dx_2^3}\right)+E_3a_3\frac{dz_3'}{dx_3'}\sin\theta-E_1a_1\frac{dz_1}{dx_1}+E_1a_1\frac{dz_1'}{dx_1'}=m_2p^2y_2, \tag{93}$$

$x_1'=l_1'$, $x_2'=l_2$, $x_{22}'=0$, $x_3'=l_3$;

$$y_2'=y_{22}'=-z_1', \quad z_3'=z_1'\sin\theta+y_1'\cos\theta, \tag{94), (95}$$

$$\frac{dy_1'}{dx_1'}=\frac{dy_2'}{dx_2'}=\frac{dy_{22}'}{dx_{22}'}, \tag{96}$$

$$-E_1I_1\frac{d^2y_1'}{dx_1'^2}-E_2'I_2'\left(\frac{d^2y_2'}{dx_2'^2}-\frac{d^2y_{22}'}{dx_{22}'^2}\right)=0, \tag{97}$$

$$-E_1I_1\frac{d^3y_1'}{dx_1'^3}+E_3a_3\frac{dz_3'}{dx_3'}\cos\theta=m_1'p^2y_1', \tag{98}$$

$$E_2'I_2'\left(\frac{d^3y_{22}'}{dx_{22}'^3}-\frac{d^3y_2'}{dx_2'^3}\right)-E_3a_3\frac{dz_3'}{dx_3'}\sin\theta-E_1a_1\frac{dz_1'}{dx_1'}=m_2'p^2y_2'. \tag{99}$$

Substituting (82), (83) in (84)$\sim$(99), we get the frequency equation

$$\begin{aligned}&\{\gamma_2\gamma_2'\psi\eta\eta'-\gamma_2\xi\eta(\vartheta\eta'\sin^2\theta+\psi)-\gamma_2'\xi(\vartheta\eta\eta'\sin^2\theta+\psi\eta+\psi\eta')\\&\quad+\xi^2(\vartheta\eta'\sin^2\theta+\psi)\}(\eta P_1Q_3-P_3Q_1)\\&\qquad+2\psi(\eta'^2\vartheta\xi\cos\theta\sin\theta)^2(\gamma_2\eta+\gamma_2'\eta-\xi)(P_2X_2-\eta Q_2X_1)=0,\end{aligned} \tag{100}$$

where

$$\begin{aligned}P_1&=3\gamma_1\psi\eta^2\eta'\phi(6\zeta'\eta'+\phi)+2\vartheta\xi\eta'^2\cos^2\theta\{3\eta(12\zeta\zeta'\eta'^2+4\zeta\eta'\phi+4\zeta'\eta'\phi+\phi^2)\\&\quad+\eta'\phi(3\zeta'\eta'+\phi)\}+6\psi\{12\eta(12\zeta\zeta'\eta'+\zeta\phi+\zeta'\phi)+\phi(12\zeta'\eta'+\phi)\},\\Q_1&=\gamma_1\psi\eta^3\eta'\phi(6\zeta'\eta'+\phi)+2\vartheta\xi\eta\eta'^2\cos^2\theta\{3\eta(6\zeta\zeta'\eta'^2+2\zeta\eta'\phi+2\zeta'\eta'\phi+\phi^2)\\&\quad+\eta'\phi(3\zeta'\eta'+\phi)\}+6\psi\{6\eta^2(12\zeta\zeta'\eta'+\zeta\phi+\zeta'\phi)\\&\quad+\eta\phi(12\zeta'\eta'+\phi)+\eta'\phi(6\zeta'\eta'+\phi)\},\\P_2&=3\eta(12\zeta\zeta'\eta'^2+4\zeta\eta'\phi+4\zeta'\eta'\phi+\phi^2)+\eta'\phi(3\zeta'\eta'+\phi),\\Q_2&=3\eta(6\zeta\zeta'\eta'^2+2\zeta\eta'\phi+2\zeta'\eta'\phi+\phi^2)+\eta'\phi(3\zeta'\eta'+\phi),\\P_3&=\gamma_1'\psi\eta'\Big[3(6\zeta'\eta'+\phi)\{\eta^2\phi+2\eta\eta'(6\zeta\eta'+\phi)+\eta'^2\phi\}\\&\quad-\eta'^2\{12\eta(12\zeta\zeta'\eta'+\zeta\phi+\zeta'\phi)+\phi(12\zeta'\eta'+\phi)\}\Big]\\&\quad-2\vartheta\xi\eta'^2\cos^2\theta\{3\eta(12\zeta\zeta'\eta'^2+4\zeta\eta'\phi+4\zeta'\eta'\phi+\phi^2)+\eta'\phi(3\zeta'\eta'+\phi)\}\\&\quad-6\psi\{12\eta(12\zeta\zeta'\eta'+\zeta\phi+\zeta'\phi)+\phi(12\zeta'\eta'+\phi)\},\end{aligned}$$

$$Q_3=\gamma_1'\psi\eta'\Big[(6\zeta'\eta'+\phi)\{\eta^2\phi+3\eta\eta'(6\zeta\eta'+\phi)+3\eta'^2\phi\}$$
$$-\eta'^2\{6\eta(12\zeta\zeta'\eta'+\zeta\phi+\zeta'\phi)+\phi(12\zeta'\eta'+\phi)\}\Big]$$
$$-2\vartheta\xi\eta'^2\cos^2\theta\{3\eta(6\zeta\zeta'\eta'^2+2\zeta\eta'\phi+2\zeta'\eta'\phi+\phi^2)+\eta'\phi(3\zeta'\eta'+\phi)\}$$
$$-6\psi\{6\eta(12\zeta\zeta'\eta'+\zeta\phi+\zeta'\phi)+\phi(12\zeta'\eta'+\phi)\},$$
$$X_1=3\gamma_1\eta^2\phi(6\zeta'\eta'+\phi)+\gamma_1'\{3\phi\eta^2(6\zeta'\eta'+\phi)+6\eta\eta'(12\zeta\zeta'\eta'^2+4\zeta\eta'\phi$$
$$+4\zeta'\eta'\phi+\phi^2)+2\eta'^2\phi(3\zeta'\eta'+\phi)\},$$
$$X_2=\phi(6\zeta'\eta'+\phi)(\gamma_1\eta^3+6)+\gamma_1'\eta\{\phi\eta^2(6\zeta'\eta'+\phi)+3\eta\eta'(12\zeta\zeta'\eta'^2+4\zeta\eta'\phi$$
$$+4\zeta'\eta'\phi+\phi_2)+2\eta'^2\phi(3\zeta'\eta'+\phi)\}, \qquad (101)$$

in which

$$\left.\begin{aligned}&\gamma_n=\frac{m_n p^2 l^3}{E_1 I_1},\quad \gamma_n'=\frac{m_n' p^2 l^3}{E_1 I_1},\quad [n=1,\ 2]\quad \vartheta=\frac{E_3 a_3}{E_1 a_1},\quad \zeta=\frac{E_2 I_2}{E_1 I_1},\\ &\zeta'=\frac{E_2' I_2'}{E_1 I_1},\quad \psi=\frac{l_3}{l},\quad \phi=\frac{l_2}{l},\quad \eta=\frac{l_1}{l},\quad \eta'=\frac{l_1'}{l},\quad \xi=\frac{a_1 l^2}{I_1}.\end{aligned}\right\} \qquad (102)$$

If we put, specially, $\vartheta=0$, $\zeta=\zeta'$, $\eta=\eta'=1$ equation (100) reduces to

$$\left.\begin{aligned}&\gamma_1\gamma_1'(36\zeta^2+36\zeta\phi+7\phi^2)-12\gamma_1(36\zeta^2+18\zeta\phi+\phi^2)\\ &\quad-12\gamma_1'(72\zeta^2+63\zeta\phi+8\phi^2)+36(144\zeta^2+36\zeta\phi+\phi^2)=0,\\ &\gamma_2\gamma_2'-\gamma_2\xi-2\gamma_2'\xi+\xi^2=0,\end{aligned}\right\} \qquad (100')$$

the first one being of the same form as the frequency equation of a two-storied braceless structure, as has been shown in a previous paper[8] and also in equation (80′) in the preceding section of this paper. The vibrations in such a special case are of four kinds, namely, two of purely horizontal motion and two of purely vertical motion.

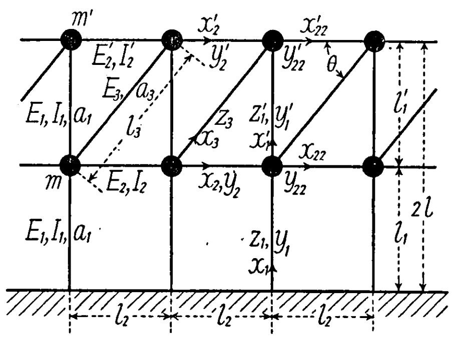

Fig. 11.

As a special case of equation (100), we put $\eta=0$, $\eta'=1$, which represents the condition that the floor height of the first floor is zero; then equation (100) reduces to

8) *loc. cit.*, 3).

$$\gamma_1'\gamma_2'\phi^2(3\zeta'+\phi)-\gamma_2'\psi\{\vartheta\xi(3\zeta'+\phi)\cos^2\theta+3\psi(12\zeta'+\phi)\}$$
$$-\gamma_1'\xi\psi(3\zeta'+\phi)(\vartheta\sin^2\theta+\psi)+\xi\{3\psi(12\zeta'+\phi)(\vartheta\sin^2\theta+\psi)$$
$$+\vartheta\xi\psi(3\zeta'+\phi)\cos^2\theta\}=0. \quad (103)$$

If we write $\gamma_1'=\gamma_1$, $\gamma_2'=\gamma_2$, $\vartheta=\vartheta_1$, $\zeta'=\zeta_1$, $\xi=\xi_1$, $\phi=1/\beta$, $\psi=1/\alpha$ (103) becomes of the same form as equation (16) of the previous paper,[9] namely, the frequency equation of a single-storied braced structure. This confirms the accuracy of equation (100).

As a numerical example, we put $\gamma_1=\gamma_2=\gamma_1'=\gamma_2'(m_1=m_2=m_1'=m_2')$, $\psi=\sqrt{2}$, $\phi=1$, $\eta=1$, $\eta'=1$, $\xi=5000$, $\theta=45°$, and calculate the vibrational frequencies for various ratios of E_3a_3/E_1a_1 in the two cases, $E_2I_2/E_1I_1=1$ and 10. These ratios and cases correspond well to those in the previous sections. The results of calculation are shown in Table IV and Fig. 12.

The values of the lowest two frequencies in the first column in this

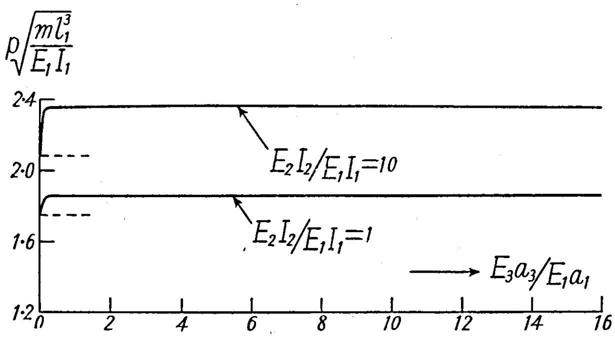

Fig. 12. $l_3/l_1=\sqrt{2}$, $l_1/l=l_1'/l=l_2/l=1$, $a_1l^2/I_1=5000$, $\theta=45°$, $m_1=m_2=m_1'=m_2'\equiv m$.

table are the same as those in Table III in the preceding section, so that they correspond to the frequencies in the case of a two-storied braceless structure, as has been shown in a previous paper. The motions corresponding to these cases are, naturally, of the horizontal type. The above result can be expected from the first equation in (100').

The first frequency, in every case of E_2I_2, in the last column in Table IV represents that for a single-storied braceless structure; for the squares of the respective numerical value in the column just given, are nearly twice the squares of the values of frequencies for $\eta=1$ and 10, shown in Table I in a previous paper.[10] This condition would be

9) *loc. cit.*, 1).
10) *loc. cit.*, 3). Table I, $\eta=1$, 10.

Table IV. The values of $p\sqrt{ml^3/E_1I_1}$ in the cases $E_2I_2/E_1I_1=1$, 10.

E_2I_2/E_1I_1 \ E_3a_3/E_1a_1	0	1	10	∞
1	1·747 5·199 43·70 114·41	1·856 42·40 63·93 133·8	1·857 42·84 87·50 305·3	1·857 42·87 90·12 ∞
10	2·086 5·550 43·70 114·4	2·359 41·32 56·17 132·0	2·362 42·43 80·84 281·5	2·362 42·50 84·33 ∞

more pronounced if the ratio of E_2I_2/E_1I_1 were further increased.

The above feature of the problem shows that if the brace struts in the second floor were fairly stiffened, that part between the first and second floors would vibrate as a single solid body, in consequence of which the condition of the problem becomes one in which the mass on the top of the columns in the first floor were doubled.

8. *General summary and concluding remarks.*

By mathematical investigation, we have ascertained the dynamical theory of the aseismic properties of some complex cases of brace struts. It has been shown that, even where there is no moment of force at the panel points, the addition of brace struts is just as effective on the aseismic properties of the structure as if beams of infinite stiffness were added to that structure.

Although in the case of a braceless structure, the vibrational frequency does not change much with increase in the number of spans, in the case of a braced structure, on the other hand, the frequency is fairly increased with increase in the number of spans, so that the aseismic properties are more pronounced in the case of multi-spans.

In the case of a two-storied structure, the aseismic properties of brace struts are also pronounced. But, if the brace struts were added to only one of the storeys, the condition of the over-stiff struts is rather equivalent to that of a single-storied braceless structure.

We are now examining such a special case of the problem of a two-storied structure in which the floor is without stiffness, which exactly corresponds to the case of a brace strut whose lower end is hinged to the intermediate point of a column of a single-storied structure, the result of which will be published in the near future.

In conclusion, we wish to express our thanks to Mr. Unoki, whose aid in the present investigation was of great value. It should be added that the present investigation was made at Professor Sezawa's suggestion in connection with his research work as member of the Investigation Committee for Earthquake-proof Construction, of the Japan Society for the Promotion of Scientific Research. I wish also to express my sincerest thanks to Professor Sezawa for valuable aid given me.

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19. 筋違の耐震効果の理論 (本論其 1)

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筋違が耐震上に効果があるのに拘らず，その正しい理論がない以上は筋違の寸法が決定できにくい事をこの前の序報で述べ，ついでにその簡單な場合の理論を示して置いた．しかしそれは極く特殊の場合だけであつたから，今少しく一般的に且つ具體的にこの理論を研究することにしたのである．

この論文では1階建で張間が1個か2個ある場合に筋違の及ぼす影響と，2階建で無限數の張間がある場合に筋違が何れかの床にあるときの影響をしらべた．尙，柱と梁との間にモーメントが働かぬ場合の筋違の影響をも確めて置いた．この場合には適當の筋違を附加へさへすれば，無限に剛い梁のある構造物と同じ耐震性を與へることがわかつた．

研究の結果によれば，筋違のない場合には構造物の振動數が張間の數によつて大して影響を受けぬに拘らず，筋違のある場合には張間が增加するにつれて振動數が著しく增加し，耐震性がよくなる事がわかつた．

2階建の場合にも筋違が耐震性に及ぼす影響が著しいが，筋違を單に何れかの階に附した場合には單に筋違の剛度を增す事は，何れか殘りの階に筋違がない場合の耐震性を與へるのに過ぎぬ事がわかつた．

尙筋違の傾斜その他の部分的事項は勿論それぞれ詳しく計算して，その有効度の理論を示して置いた．
