22. *Stationary Vibrations due to an Obliquely Incident Transversal Wave of Harmonic Type of the Surface-Layer of an Elastic Earth's Crust.* (*First Paper.*)

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1. The writers recently studied the forced oscillation of the surface-layer of an elastic earth when an infinite train of harmonic dilatational waves is obliquely incident on the bottom surface of it, and obtained a number of theoretical facts helpful in studying the seismic oscillation of a surface-layer of the earth's crust, especially its resonance conditions[1]. To study, seismometrically, the seismic vibrations of the surface-layer of the earth's crust, a theoretical knowledge is necessary of the properties of the stationary oscillation of the surface-layer, excited by a distortional wave that is obliquely incident on the bottom surface of that layer. The present paper is one of the results of the writers' attempts to study this problem.

We shall begin with the general expressions of the vibration of the surface-layer when an infinite train of harmonic distortional waves is obliquely incident on the bottom surface of the surface-layer. Next, the special case is discussed, in which the incidence angle of this wave is 45°, and the ratio of the rigidity of the surface-layer to that of the subjacent infinite medium is 1/10, and the densities of the materials are exactly the same for both media[2]. These conditions, assumed in the present paper, are exactly the same as in the previous paper[3], in which the primary incident wave is a dilatational wave of infinite harmonic train.

2. The horizontal boundary of a surface-layer of thickness H and a subjacent infinite medium is $y=0$, and the axes of x and y are drawn on and perpendicular to this common boundary respectively, the axis of

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1) G. NISHIMURA and T. TAKAYAMA, *B. E. R. I.*, **15** (1937), 394~440; **17** (1939), 30~45.

2) Both media, the surface layer and the subjacent medium, are also assumed to satisfy Cauchy's elasticity condition.

3) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

y being downward positive, as shown in Fig. 1. The density and Lamé's elastic constants of the subjacent medium $y \geq 0$ are ρ, and λ, μ, and those of the surface-layer $-H \leq y \leq 0$ are ρ', and λ', μ' respectively.

Let the primary incident distortional wave of propagational velocity $V_2(=\sqrt{\mu/\rho})$ in the subjacent medium $y \geq 0$ be of the type

$$\psi_0 = \mathfrak{A}\exp\{i(fx-sy-pt)\}, \tag{1}$$

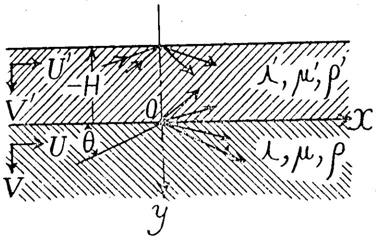


Fig. 1.

where

$$p = 2\pi/T,\ s^2 = (p/V_2)^2 - f^2,\ f = (2\pi/L)\cos\theta, \tag{2}$$

and T, L, θ are respectively the period, the wave-length, and the incidence angle of this wave. Then, when this wave is incident on the bottom surface of the surface-layer, the following six kinds of waves are necessary for satisfying the boundary-conditions on the top and the bottom surfaces of the surface-layer:

$$\phi = A\exp\{i(fx+ry-pt)\}, \tag{3}$$

$$\psi = B\exp\{i(fx+sy-pt)\}, \tag{4}$$

$$\phi_1' = C\exp\{i(fx-r'y-pt)\}, \tag{5}$$

$$\phi_2' = D\exp\{i(fx+r'y-pt)\}, \tag{6}$$

$$\psi_1' = E\exp\{i(fx-s'y-pt)\}, \tag{7}$$

$$\psi_2' = F\exp\{i(fx+s'y-pt)\}, \tag{8}$$

where

$$\left.\begin{aligned} r^2 &= \rho p^2/(\lambda+2\mu) - f^2, \quad s^2 = (\rho p^2/\mu) - f^2, \\ r'^2 &= \rho' p^2/(\lambda'+2\mu') - f^2, \quad s'^2 = (\rho' p^2/\mu') - f^2, \end{aligned}\right\} \tag{9}$$

and A, B, C, D, E, F are any constants to be determined from the boundary-conditions at $y=0$ and $y=-H$. In expression (3)∼(8), ϕ and ψ are the dilatational and distortional waves in the subjacent medium reflected at the bottom surface of the surface-layer, ϕ_1' and ψ_1' are the refracted dilatational and distortional waves in the surface-layer due to the incidence of the primary wave (1), and ϕ_2' and ψ_2' are the reflected dilatational and distortional waves in the same layer. These wave-expressions (1)∼(8) satisfy their respective wave-equations[4].

4) The dilatational waves satisfy the equation $\frac{\partial^2\phi}{\partial t^2} = v^2\nabla^2\phi$, where v is the propagation velocity of that wave; and the distortional waves with propagation velocity V_2 satisfy, generally, the equation $\frac{\partial^2\psi}{\partial t^2} = V_2^2\nabla^2\psi$.

The displacement of particles in the surface-layer may easily be obtained from expressions (5) ~ (8), as in the following forms:

$$U'=if[C\exp\{i(fx-r'y-pt)\}+D\exp\{i(fx+r'y-pt)\}]$$
$$+is'[-E\exp\{i(fx-s'y-pt)\}+F\exp\{i(fx+s'y-pt)\}], \qquad (10)$$

$$V'=ir'[-C\exp\{i(fx-r'y-pt)\}+D\exp\{i(fx+r'y-pt)\}]$$
$$+if[-E\exp\{i(fx-s'y-pt)\}-F\exp\{i(fx+s'y-pt)\}], \qquad (11)$$

where U' is the horizontal component of displacement of the particle, and V' is the vertical component.

The displacement of the particles in the subjacent semi-infinite medium is usually derived from expressions (1), (3), (4), as follows:

$$U=is[-\mathfrak{A}\exp\{i(fx-sy-pt)\}+B\exp\{i(fx+sy-pt)\}]$$
$$+ifA\exp\{i(fx+ry-pt)\}, \qquad (12)$$

$$V=-if[\mathfrak{A}\exp\{i(fx-sy-pt)\}+B\exp\{i(fx+sy-pt)\}]$$
$$+irA\exp\{i(fx+ry-pt)\}, \qquad (13)$$

where U and V are the horizontal and vertical components of the movement of the particle respectively.

Now, the displacements (vertical and horizontal components), and the stresses (normal and shear components) of both media should be continuous at the common boundary, and the top surface of the surface-layer must be a free surface. The conditions at $y=0$ are analytically expressed as follows:

$$\left.\begin{aligned}&U=U',\ V=V',\\&\mu\left(\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}\right)=\mu'\left(\frac{\partial V'}{\partial x}+\frac{\partial U'}{\partial y}\right),\\&\lambda\left(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}\right)+2\mu\frac{\partial V}{\partial y}=\lambda'\left(\frac{\partial U'}{\partial x}+\frac{\partial V'}{\partial y}\right)+2\mu'\frac{\partial V'}{\partial y},\end{aligned}\right\} \qquad (14)$$

and at $y=-H$,

$$\left.\begin{aligned}&\lambda'\left(\frac{\partial U'}{\partial x}+\frac{\partial V'}{\partial y}\right)+2\mu'\frac{\partial V'}{\partial y}=0,\\&\mu'\left(\frac{\partial V'}{\partial x}+\frac{\partial U'}{\partial y}\right)=0,\end{aligned}\right\} \qquad (15)$$

so that, from (10), (11), (12), (13), (14), (15), we get

$$\left.\begin{aligned} A&=\frac{\Delta_A}{\Delta}\mathfrak{A}, \quad B=\frac{\Delta_B}{\Delta}\mathfrak{A}, \quad C=\frac{\Delta_C}{\Delta}\mathfrak{A}, \\ D&=\frac{\Delta_D}{\Delta}\mathfrak{A}, \quad E=\frac{\Delta_E}{\Delta}\mathfrak{A}, \quad F=\frac{\Delta_F}{\Delta}\mathfrak{A}, \end{aligned}\right\} \tag{16}$$

where

$$\begin{aligned}
\Delta=16\frac{r's'}{f^2}\Big(1-\frac{s'^2}{f^2}\Big)\Big[&-2\mu'^2\Big(1+\frac{rs}{f^2}\Big)\Big(1-\frac{s'^2}{f^2}\Big) \\
&+\mu\mu'\Big(1-\frac{s^2}{f^2}+2\frac{rs}{f^2}\Big)\Big(3-\frac{s'^2}{f^2}\Big)-\mu^2\Big\{\Big(1-\frac{s^2}{f^2}\Big)^2+4\frac{rs}{f^2}\Big\}\Big] \\
+\exp\Big\{i(r'+s')H\Big\}\cdot&\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2+4\frac{r's'}{f^2}\Big\}\Big[\mu'^2\Big(1+\frac{r's'}{f^2}\Big)\cdot \\
&\cdot\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2+4\frac{r's'}{f^2}\Big\}-\mu\mu'\Big\{\Big(1+\frac{s^2}{f^2}\Big)\Big(1+\frac{s'^2}{f^2}\Big)\cdot \\
&\quad\cdot\Big(\frac{rs'}{f^2}+\frac{r's}{f^2}\Big)+2\Big(1-\frac{s^2}{f^2}+2\frac{rs}{f^2}\Big)\Big(1-\frac{s'^2}{f^2}+2\frac{r's'}{f^2}\Big)\Big\} \\
&\quad\quad+\mu^2\Big(1+\frac{r's'}{f^2}\Big)\Big\{\Big(1-\frac{s^2}{f^2}\Big)^2+4\frac{rs}{f^2}\Big\}\Big] \\
+\exp\Big\{i(r'-s')H\Big\}\cdot&\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2-4\frac{r's'}{f^2}\Big\}\Big[-\mu'^2\Big(1+\frac{rs}{f^2}\Big)\cdot \\
&\cdot\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2-4\frac{r's'}{f^2}\Big\}+\mu\mu'\Big\{\Big(1+\frac{s^2}{f^2}\Big)\Big(1+\frac{s'^2}{f^2}\Big)\cdot \\
&\quad\cdot\Big(-\frac{rs'}{f^2}+\frac{r's}{f^2}\Big)+2\Big(1-\frac{s^2}{f^2}+2\frac{rs}{f^2}\Big)\Big(1-\frac{s'^2}{f^2}-2\frac{r's'}{f^2}\Big)\Big\} \\
&\quad\quad-\mu^2\Big(1-\frac{r's'}{f^2}\Big)\Big\{\Big(1-\frac{s^2}{f^2}\Big)^2+4\frac{rs}{f^2}\Big\}\Big] \\
+\exp\Big\{i(-r'+s')H\Big\}\cdot&\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2-4\frac{r's'}{f^2}\Big\}\Big[-\mu'^2\Big(1+\frac{rs}{f^2}\Big)\cdot \\
&\cdot\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2-4\frac{r's'}{f^2}\Big\}+\mu\mu'\Big\{\Big(1+\frac{s^2}{f^2}\Big)\Big(1+\frac{s'^2}{f^2}\Big)\cdot
\end{aligned}$$

$$\cdot\left(\frac{rs'}{f^2}-\frac{r's}{f^2}\right)+2\left(1-\frac{s^2}{f^2}+2\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}-2\frac{r's'}{f^2}\right)\Bigg\}$$

$$-\mu^2\left(1-\frac{r's'}{f^2}\right)\left\{\left(1-\frac{s^2}{f^2}\right)^2+4\frac{rs}{f^2}\right\}\Bigg]$$

$$+\exp\left\{i(-r'-s')H\right\}\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\Bigg[\mu'^2\left(1+\frac{rs}{f^2}\right)\cdot$$

$$\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}+\mu\mu'\left\{\left(1+\frac{s^2}{f^2}\right)\left(1+\frac{s'^2}{f^2}\right)\cdot\right.$$

$$\left.\cdot\left(\frac{rs'}{f^2}+\frac{r's}{f^2}\right)-2\left(1-\frac{s^2}{f^2}+2\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}+2\frac{r's'}{f^2}\right)\right\}$$

$$+\mu^2\left(1+\frac{r's'}{f^2}\right)\left\{\left(1-\frac{s^2}{f^2}\right)^2+4\frac{rs}{f^2}\right\}\Bigg], \qquad (17)$$

$$\Delta_A=\mu'^2\left\{-64\frac{r'ss'}{f^3}\left(1-\frac{s'^2}{f^2}\right)^2\right\}+\mu\mu'\left\{16\frac{r'ss'}{f^3}\left(3-\frac{s'^2}{f^2}\right)\cdot\right.$$

$$\left.\cdot\left(1-\frac{s'^2}{f^2}\right)\left(3-\frac{s^2}{f^2}\right)\right\}+\mu^2\left\{-64\frac{r'ss'}{f^3}\left(1-\frac{s^2}{f^2}\right)\left(1-\frac{s'^2}{f^2}\right)\right\}$$

$$+\exp\left\{i(-r'+s')H\right\}\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\Bigg[-\mu'^2 2\frac{s}{f}\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}+\mu\mu'\left\{2\frac{s}{f}\left(3-\frac{s^2}{f^2}\right)\cdot\right.$$

$$\left.\cdot\left(1-\frac{s'^2}{f^2}-2\frac{r's'}{f^2}\right)\right\}-\mu^2 4\frac{s}{f}\left(1-\frac{s^2}{f^2}\right)\left(1-\frac{r's'}{f^2}\right)\Bigg]$$

$$+\exp\left\{-i(r'+s')H\right\}\cdot 2\frac{s}{f}\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\Bigg[\mu'^2\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}-\mu\mu'\left(3-\frac{s^2}{f^2}\right)\cdot$$

$$\cdot\left(1-\frac{s'^2}{f^2}+2\frac{r's'}{f^2}\right)+\mu^2 2\left(1-\frac{s^2}{f^2}\right)\left(1+\frac{r's'}{f^2}\right)\Bigg]$$

$$+\exp\left\{i(r'+s')H\right\}\cdot 2\frac{s}{f}\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu'^2\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}-\mu\mu'\left(3-\frac{s^2}{f^2}\right)\cdot\right.$$

$$\left.\cdot\left(1-\frac{s'^2}{f^2}+2\frac{r's'}{f^2}\right)+\mu^2 2\left(1-\frac{s^2}{f^2}\right)\left(1+\frac{r's'}{f^2}\right)\right]$$

$$+\exp\left\{i(r'-s')H\right\}\cdot 2\frac{s}{f}\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu'^2\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}+\mu\mu'\left(3-\frac{s^2}{f^2}\right)\cdot\right.$$

$$\left.\cdot\left(1-\frac{s'^2}{f^2}-2\frac{r's'}{f^2}\right)-2\mu^2\left(1-\frac{s^2}{f^2}\right)\left(1-\frac{r's'}{f^2}\right)\right], \tag{18}$$

$$\Delta_n=32\mu'^2\frac{r's'}{f^2}\left(1-\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}\right)^2-16\mu\mu'\frac{r's'}{f^2}\left(1-\frac{s^2}{f^2}-2\frac{rs}{f^2}\right)\cdot$$

$$\cdot\left(1-\frac{s'^2}{f^2}\right)\left(3-\frac{s'^2}{f^2}\right)+16\mu^2\frac{r's'}{f^2}\left(1-\frac{s'^2}{f^2}\right)\left\{\left(1-\frac{s^2}{f^2}\right)^2-4\frac{rs}{f^2}\right\}$$

$$+\exp\left\{i(-r'+s')H\right\}\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\left[\mu'^2\left(1-\frac{rs}{f^2}\right)\cdot\right.$$

$$\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}-\mu\mu'\left\{\left(1+\frac{s^2}{f^2}\right)\left(1+\frac{s'^2}{f^2}\right)\left(\frac{rs'}{f^2}+\frac{r's}{f^2}\right)\right.$$

$$\left.+2\left(1-\frac{s^2}{f^2}-2\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}-2\frac{r's'}{f^2}\right)\right\}$$

$$\left.+\mu^2\left(1-\frac{r's'}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right)\right]$$

$$+\exp\left\{-i(r'+s')H\right\}\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\left[-\mu'^2\left(1-\frac{rs}{f^2}\right)\cdot\right.$$

$$\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}+\mu\mu'\left\{-\left(1+\frac{s^2}{f^2}\right)\left(1+\frac{s'^2}{f^2}\right)\left(\frac{rs'}{f^2}-\frac{r's}{f^2}\right)\right.$$

$$+2\left(1-\frac{s^2}{f^2}-2\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}+2\frac{r's'}{f^2}\right)\Big\}$$

$$-\mu^2\left(1+\frac{r's'}{f^2}\right)\left\{\left(1-\frac{s^2}{f^2}\right)^2-4\frac{rs}{f^2}\right\}\Bigg]$$

$$+\exp\left\{i(r'+s')H\right\}\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\Bigg[-\mu'^2\left(1-\frac{rs}{f^2}\right)\cdot$$

$$\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}+\mu\mu'\Big\{\left(1+\frac{s^2}{f^2}\right)\left(1+\frac{s'^2}{f^2}\right)\left(\frac{rs'}{f^2}-\frac{r's}{f^2}\right)$$

$$+2\left(1-\frac{s^2}{f^2}-2\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}+2\frac{r's'}{f^2}\right)\Big\}$$

$$-\mu^2\left(1+\frac{r's'}{f^2}\right)\left\{\left(1-\frac{s^2}{f^2}\right)^2-4\frac{rs}{f^2}\right\}\Bigg]$$

$$+\exp\left\{i(r'-s')H\right\}\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\Bigg[\mu'^2\left(1-\frac{rs}{f^2}\right)\cdot$$

$$\cdot\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}+\mu\mu'\Big\{\left(1+\frac{s^2}{f^2}\right)\left(1+\frac{s'^2}{f^2}\right)\left(\frac{rs'}{f^2}+\frac{r's}{f^2}\right)$$

$$-2\left(1-\frac{s^2}{f^2}-2\frac{rs}{f^2}\right)\left(1-\frac{s'^2}{f^2}-2\frac{r's'}{f^2}\right)\Big\}$$

$$+\mu^2\left(1-\frac{r's'}{f^2}\right)\left\{\left(1-\frac{s^2}{f^2}\right)^2-4\frac{rs}{f^2}\right\}\Bigg], \tag{19}$$

$$\Delta_C=8\frac{ss'}{f^2}\left(1+\frac{s^2}{f^2}\right)\left(1-\frac{s'^2}{f^2}\right)\cdot$$

$$\cdot\left[\mu\mu'\left\{\frac{r}{f}\left(1-\frac{s'^2}{f^2}\right)+2\frac{r'}{f}\right\}-\mu^2\left\{\frac{r'}{f}\left(1-\frac{s^2}{f^2}\right)+2\frac{r}{f}\right\}\right]$$

$$+\exp\left\{i(-r'+s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu\mu'\left(1-\frac{s'^2}{f^2}-2\frac{rs'}{f^2}\right)-\mu^2\left(1-\frac{s^2}{f^2}-2\frac{rs'}{f^2}\right)\right]$$

$$+\exp\left\{-i(r'+s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[-\mu\mu'\left(1-\frac{s'^2}{f^2}+2\frac{rs'}{f^2}\right)+\mu^2\left(1-\frac{s^2}{f^2}+2\frac{rs'}{f^2}\right)\right], \qquad (20)$$

$$\Delta_D=8\frac{ss'}{f^2}\left(1+\frac{s^2}{f^2}\right)\left(1-\frac{s'^2}{f^2}\right)\cdot$$

$$\cdot\left[\mu\mu'\left\{2\frac{r'}{f}-\frac{r}{f}\left(1-\frac{s'^2}{f^2}\right)\right\}+\mu^2\left\{2\frac{r}{f}-\frac{r'}{f}\left(1-\frac{s^2}{f^2}\right)\right\}\right]$$

$$+\exp\left\{i(r'+s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu\mu'\left(2\frac{rs'}{f^2}-1+\frac{s'^2}{f^2}\right)+\mu^2\left(-2\frac{rs'}{f^2}+1-\frac{s^2}{f^2}\right)\right]$$

$$+\exp\left\{i(r'-s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu\mu'\left(2\frac{rs'}{f^2}+1-\frac{s'^2}{f^2}\right)-\mu^2\left(2\frac{rs'}{f^2}+1-\frac{s^2}{f^2}\right)\right], \qquad (21)$$

$$\Delta_E=8\frac{sr'}{f^2}\left(1+\frac{s^2}{f^2}\right)\left(1-\frac{s'^2}{f^2}\right)\cdot$$

$$\cdot\left[\mu\mu'\left\{2\frac{rs'}{f^2}-1+\frac{s'^2}{f^2}\right\}+\mu^2\left(-2\frac{rs'}{f^2}+1-\frac{s^2}{f^2}\right)\right]$$

$$+\exp\left\{i(r'-s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu\mu'\left\{\frac{r}{f}\left(1-\frac{s'^2}{f^2}\right)+2\frac{r'}{f}\right\}-\mu^2\left\{\frac{r'}{f}\left(1-\frac{s^2}{f^2}\right)+2\frac{r}{f}\right\}\right]$$

$$+\exp\left\{-i(r'+s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\cdot$$

$$\cdot\left[\mu\mu'\left\{-\frac{r}{f}\left(1-\frac{s'^2}{f^2}\right)+2\frac{r'}{f}\right\}+\mu^2\left\{-\frac{r'}{f}\left(1-\frac{s^2}{f^2}\right)+2\frac{r}{f}\right\}\right], \qquad (22)$$

$$
\begin{aligned}
\Delta_{\mu'}=&8\frac{sr'}{f^2}\Big(1+\frac{s^2}{f^2}\Big)\Big(1-\frac{s'^2}{f^2}\Big)\cdot\\
&\cdot\Big[\mu\mu'\Big(2\frac{rs'}{f^2}+1-\frac{s'^2}{f^2}\Big)+\mu^2\Big(-2\frac{rs'}{f^2}-1+\frac{s^2}{f^2}\Big)\Big]\\
&+\exp\Big\{i(r'+s')H\Big\}\cdot 2\frac{s}{f}\Big(1+\frac{s^2}{f^2}\Big)\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2+4\frac{r's'}{f^2}\Big\}\cdot\\
&\cdot\Big[\mu\mu'\Big\{\frac{r}{f}\Big(1-\frac{s'^2}{f^2}\Big)+2\frac{r'}{f}\Big\}-\mu^2\Big\{\frac{r'}{f}\Big(1-\frac{s^2}{f^2}\Big)+2\frac{r}{f}\Big\}\Big]\\
&+\exp\Big\{i(-r'+s')H\Big\}\cdot 2\frac{s}{f}\Big(1+\frac{s^2}{f^2}\Big)\Big\{\Big(1-\frac{s'^2}{f^2}\Big)^2-4\frac{r's'}{f^2}\Big\}\cdot\\
&\cdot\Big[\mu\mu'\Big\{\frac{r}{f}\Big(1-\frac{s'^2}{f^2}\Big)-2\frac{r'}{f}\Big\}+\mu^2\Big\{\frac{r'}{f}\Big(1-\frac{s^2}{f^2}\Big)-2\frac{r}{f}\Big\}\Big]. \qquad (23)
\end{aligned}
$$

As in that case[5] in which a longitudinal wave of harmonic type is primarily incident on the bottom surface of the surface-layer, all the constants A, B, C, D, E, F, thus determined, are transcendental functions, the variables of which consist of the incidence angle of the primary distortional wave, the wave-length of that wave, and also the densities, the elastic constants of both media, and, naturally, the thickness of the surface-layer.

Obviously, each orbital motion of a particle in both media, due to the respective seven waves, expressed by (1), (3)~(8), is simple harmonic, and each orbit becomes a straight line. We can see, however, from expressions (16)~(23), that all the constants, A, B, C, D, E, F become complex values, so that each wave has an amplitude and phase-difference that are transcendental functions, having the same variables as the six constants, while the direction of the straight line orbit of the seven waves generally do not coincide with one another. For these reasons, every particle in both media generally pursues an elliptic orbital motion, which will be studied in the next section, as the result of the interference effect of all the waves. The major and minor axes of this elliptic orbit and the inclination angle which the major axis of the orbit makes with the horizontal surface of the solids are also transcendental functions, the variables of which consist of the same elements as the six constants.

5) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

3. In the next section, we shall study the orbital motions of a particle in the surface-layer for the case when both media, the surface-layer and the subjacent medium, are assumed to satisfy Cauchy's elasticity condition, and the ratio of the rigidity of the surface-layer to that of the subjacent medium is 1/10, and the densities of the materials are exactly the same for both media, and also when the incidence angle of the primary distortional wave is 45°. For convenience, we shall show all the quantities Δ, Δ_A, Δ_B, Δ_C, Δ_D, Δ_E, Δ_F, for determining the values of A, B, C, D, E, F for the case when $\lambda=\mu$, $\lambda'=\mu'$, $\rho=\rho'$ $\mu'/\mu=1/10$, and $\theta=45°$, as in the following forms:

$$\frac{\Delta}{f^{10}}=\{-1075{\cdot}81+7358{\cdot}67\sin a'-5687{\cdot}44\sin b'+2671{\cdot}88\cos a'-1596{\cdot}07\cos b'\}+i\{11801{\cdot}27-6960{\cdot}61\sin a'-5379{\cdot}78\sin b'+20283{\cdot}3\cos a'+6256{\cdot}16\cos b'\}, \tag{24}$$

$$\frac{\Delta_A}{f^{10}}=\{7411{\cdot}14+4538{\cdot}91\cos a'-11950{\cdot}05\cos b'\}, \tag{25}$$

$$\frac{\Delta_B}{f^{10}}=\{1075{\cdot}81-7358{\cdot}67\sin a'+5687{\cdot}44\sin b'-2671{\cdot}88\cos a'+1596{\cdot}07\cos b'\}+i\{11801{\cdot}27-6960{\cdot}61\sin a'-5379{\cdot}78\sin b'+20283{\cdot}3\cos a'+6256{\cdot}16\cos b'\}, \tag{26}$$

$$\frac{\Delta_C}{f^{10}}=\{-597{\cdot}672+6622{\cdot}81\sin a'-5118{\cdot}70\sin b'+2631{\cdot}64\cos a'-2033{\cdot}96\cos b'\}+i\{2754{\cdot}18-2631{\cdot}64\sin a'-2033{\cdot}96\sin b'+6622{\cdot}81\cos a'+5118{\cdot}70\cos b'\}, \tag{27}$$

$$\frac{\Delta_D}{f^{10}}=\{-597{\cdot}672+6622{\cdot}81\sin a'-5118{\cdot}70\sin b'+2631{\cdot}64\cos a'-2033{\cdot}96\cos b'\}+i\{-2754{\cdot}18+2631{\cdot}64\sin a'+2033{\cdot}96\sin b'-6622{\cdot}805\cos a'-5118{\cdot}70\cos b'\}, \tag{28}$$

$$\frac{\Delta_E}{f^{10}}=\{-1234{\cdot}04+3207{\cdot}57\sin a'-2479{\cdot}10\sin b'+696{\cdot}061\cos a'+537{\cdot}978\cos b'\}+i\{3105{\cdot}60-696{\cdot}061\sin a'-537{\cdot}978\sin b'+3207{\cdot}57\cos a'-2479{\cdot}10\cos b'\}, \tag{29}$$

$$\frac{\Delta_F}{f^{10}}=\{1234{\cdot}04-3207{\cdot}57\sin a'+2479{\cdot}098\sin b'$$
$$-696{\cdot}061\cos a'-537{\cdot}978\cos b'\}$$
$$+i\{3105{\cdot}60-696{\cdot}061\sin a'-537{\cdot}978\sin b'$$
$$+3207{\cdot}57\cos a'-2479{\cdot}10\cos b'\}, \qquad (30)$$

where $a'=29{\cdot}9423H/L$, $b'=8{\cdot}78990H/L$. Using these formulae, we numerically calculated the values A, B, C, D, E, and F for various values of H/L, as shown in Table I.

Table I. ($\lambda=\mu$, $\lambda'=\mu'$, $\mu'/\mu=1/10$, $\rho=\rho'$, $\theta=45°$.)

$\frac{H}{L}$	A	B	C	D	E	F
0·01	$-0{\cdot}000197$ $+i0{\cdot}0044$	$0{\cdot}9960$ $+i0{\cdot}0890$	$0{\cdot}3801$ $-i0{\cdot}0230$	$-0{\cdot}3765$ $-i0{\cdot}05676$	$0{\cdot}09958$ $-i0{\cdot}01548$	$0{\cdot}09781$ $+i0{\cdot}02428$
0·02	$-0{\cdot}00183$ $+i0{\cdot}0201$	$0{\cdot}9836$ $+i0{\cdot}1802$	$0{\cdot}3869$ $-i0{\cdot}0456$	$-0{\cdot}3724$ $-i0{\cdot}1145$	$0{\cdot}09775$ $-i0{\cdot}03281$	$0{\cdot}09023$ $+i0{\cdot}04991$
0·03	$-0{\cdot}00757$ $+i0{\cdot}0539$	$0{\cdot}9612$ $+i0{\cdot}2754$	$0{\cdot}4003$ $-i0{\cdot}0675$	$-0{\cdot}3661$ $-i0{\cdot}1752$	$0{\cdot}09335$ $-i0{\cdot}05383$	$0{\cdot}07489$ $+i0{\cdot}07747$
0·04	$-0{\cdot}0244$ $+i0{\cdot}1239$	$0{\cdot}9254$ $+i0{\cdot}3788$	$0{\cdot}4249$ $-i0{\cdot}0870$	$-0{\cdot}3602$ $-i0{\cdot}2415$	$0{\cdot}08296$ $-i0{\cdot}08278$	$0{\cdot}04544$ $+i0{\cdot}1080$
0·05	$-0{\cdot}0786$ $+i0{\cdot}2944$	$0{\cdot}8669$ $+i0{\cdot}4986$	$0{\cdot}4765$ $-i0{\cdot}0972$	$-0{\cdot}3648$ $-i0{\cdot}3219$	$0{\cdot}05398$ $-i0{\cdot}1344$	$-0{\cdot}0202$ $+i0{\cdot}1434$
0·055	$-0{\cdot}1583$ $+i0{\cdot}5025$	$0{\cdot}8195$ $+i0{\cdot}5730$	$0{\cdot}5330$ $-i0{\cdot}08778$	$-0{\cdot}3865$ $-i0{\cdot}3774$	$0{\cdot}01620$ $-i0{\cdot}1892$	$-0{\cdot}09516$ $+i0{\cdot}1644$
0·06	$-0{\cdot}4240$ $+i1{\cdot}067$	$0{\cdot}7276$ $+i0{\cdot}6858$	$0{\cdot}6737$ $-i0{\cdot}0280$	$-0{\cdot}4710$ $-i0{\cdot}4824$	$-0{\cdot}08754$ $-i0{\cdot}3355$	$-0{\cdot}2939$ $+i0{\cdot}1842$
0·065	$-8{\cdot}119$ $+i5{\cdot}275$	$-0{\cdot}4067$ $+i0{\cdot}9132$	$1{\cdot}553$ $+i1{\cdot}894$	$-1{\cdot}099$ $-i2{\cdot}188$	$-0{\cdot}1962$ $-i2{\cdot}807$	$-2{\cdot}484$ $-i1{\cdot}323$
0·066	$-6{\cdot}114$ $-i10{\cdot}51$	$0{\cdot}4940$ $-i0{\cdot}8695$				
0·067	$0{\cdot}1049$ $-i5{\cdot}233$	$0{\cdot}9992$ $+i0{\cdot}0401$				
0·0675	$0{\cdot}4150$ $-i3{\cdot}978$	$0{\cdot}9784$ $+i0{\cdot}2064$				
0·0677	$0{\cdot}4661$ $-i3{\cdot}619$	$0{\cdot}9673$ $+i0{\cdot}2533$				
0·0678	$0{\cdot}4786$ $-i3{\cdot}462$	$0{\cdot}9624$ $+i0{\cdot}2713$				
0·06784	$0{\cdot}48921$ $-i3{\cdot}40577$	$0{\cdot}95957$ $+i0{\cdot}28148$	$-0{\cdot}3709$ $-i0{\cdot}3573$	$0{\cdot}4565$ $-i0{\cdot}2384$	$0{\cdot}9411$ $+i0{\cdot}4322$	$1{\cdot}025$ $-i0\ 1497$
0·0679	$0{\cdot}4968$ $-i3{\cdot}322$	$0{\cdot}9562$ $+i0{\cdot}2925$				
0·0680	$0{\cdot}5050$ $-i3{\cdot}198$	$0{\cdot}9514$ $+i0{\cdot}3082$				
0·0685	$0{\cdot}5219$ $-i2{\cdot}673$	$0{\cdot}9265$ $+i0{\cdot}3761$				

(to be continued.)

Table I. (*continued.*)

$\frac{H}{L}$	A	B	C	D	E	F
0·069	0·5169 $-i$2·3013	0·9039 $+i$0·4276				
0·07	0·4803 $-i$1·807	0·8682 $+i$0·4960	0·0200 $-i$0·3311	0·1468 $-i$0·2973	0·5419 $+i$0·2280	0·5836 $+i$0·0707
0·075	0·3432 $-i$0·9171	0·7541 $+i$0·6562	0·2482 $-i$0·2965	0·00752 $-i$0·3866	0·3439 $+i$0·07355	0·3076 $+i$0·1703
0·077	0·3162 $-i$0·7790	0·7172 $+i$0·6969				
0 0775	0·3107 $-i$0·7516	0·7082 $+i$0·7060				
0·078	0·3045 $-i$0·7255	0·7001 $+i$0·7138				
0·0781	0·3045 $-i$0·7215	0·6973 $+i$0·7166				
0·07817	0·30391 $-i$0·71800	0·69612 $+i$0·71792	0·3073 $-i$0·2931	$-$0·0034 $-i$0·4247	0·3062 $+i$0·0327	0·2366 $+i$0·1971
0·0782	0·3034 $-i$0·7162	0·6955 $+i$0·7182				
0·0783	0·3025 $-i$0·7119	0·6941 $+i$0·7199				
0·0785	0·3011 $-i$0·7029	0·6899 $+i$0·7239				
0·079	0·2965 $-i$0·6811	0·6813 $+i$0·7319				
0·08	0·2889 $-i$0·6418	0·6625 $+i$0·7485	0·3328 $-i$0·2944	$-$0·0001 $-i$0·4443	0·2935 $+i$0·0155	0·2061 $+i$0·2094
0·085	0·2649 $-i$0·5032	0·5659 $+i$0·8244	0·3886 $-i$0·3012	0·02839 $-i$0·4909	0·2763 $-i$0·01760	0·1418 $+i$0·2377
0·09	0·2535 $-i$0·4142	0·4537 $+i$0·8914	0·4381 $-i$0·3118	0·0793 $-i$0·5319	0·2732 $-i$0·04095	0·08732 $+i$0·2621
0·10	0·2460 $-i$0·2888	0·1594 $+i$0·9869	0·5727 $-i$0·3036	0·2084 $-i$0·6137	0·2907 $-i$0·07207	$-$0·0248 $+i$0·2985
0·11	0·2315 $-i$0·1793	$-$0·2504 $+i$0·9679	0·6744 $-i$0·3149	0·4736 $-i$0·5742	0·3326 $-i$0 08388	$-$0·1645 $+i$0·3010
0·12	0·1778 $-i$0·07150	$-$0·7211 $+i$0·6926	0·8198 $-i$0·2253	0·7473 $-i$0·4052	0·3893 $-i$0·06274	$-$0·3243 $+i$0·2244
0·128	0·08961 $-i$0·01017	$-$0·9745 $+i$0·2242				
0·129	0·07616 $-i$0·00591	$-$0·9880 $+i$0·1545				
0·13	0·06234 $-i$0·00258	$-$0·9967 $+i$0·08268	0·9020 $-i$0·03612	0·9019 $-i$0·03856	0·4297 $+i$0·002522	$-$0·4280 $+i$0·03803
0·1305	0·05526 $-i$0·00128	$-$0·9990 $+i$0·04631	0·9023 $-i$0·0249	0·9023 $-i$0·01691	0·4303 $+i$0·00663	$-$0·4296 $+i$0·02658
0·131	0·04836 $-i$0·0003	$-$1·000 $+i$0·01072				

(*to be continued.*)

$$
\begin{aligned}
\Delta_{F}=&8\frac{sr'}{f^2}\left(1+\frac{s^2}{f^2}\right)\left(1-\frac{s'^2}{f^2}\right)\cdot\\
&\cdot\left[\mu\mu'\left(2\frac{rs'}{f^2}+1-\frac{s'^2}{f^2}\right)+\mu^2\left(-2\frac{rs'}{f^2}-1+\frac{s^2}{f^2}\right)\right]\\
&+\exp\left\{i(r'+s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2+4\frac{r's'}{f^2}\right\}\cdot\\
&\cdot\left[\mu\mu'\left\{\frac{r}{f}\left(1-\frac{s'^2}{f^2}\right)+2\frac{r'}{f}\right\}-\mu^2\left\{\frac{r'}{f}\left(1-\frac{s^2}{f^2}\right)+2\frac{r}{f}\right\}\right]\\
&+\exp\left\{i(-r'+s')H\right\}\cdot 2\frac{s}{f}\left(1+\frac{s^2}{f^2}\right)\left\{\left(1-\frac{s'^2}{f^2}\right)^2-4\frac{r's'}{f^2}\right\}\cdot\\
&\cdot\left[\mu\mu'\left\{\frac{r}{f}\left(1-\frac{s'^2}{f^2}\right)-2\frac{r'}{f}\right\}+\mu^2\left\{\frac{r'}{f}\left(1-\frac{s^2}{f^2}\right)-2\frac{r}{f}\right\}\right].
\end{aligned}
\tag{23}
$$

As in that case[5] in which a longitudinal wave of harmonic type is primarily incident on the bottom surface of the surface-layer, all the constants A, B, C, D, E, F, thus determined, are transcendental functions, the variables of which consist of the incidence angle of the primary distortional wave, the wave-length of that wave, and also the densities, the elastic constants of both media, and, naturally, the thickness of the surface-layer.

Obviously, each orbital motion of a particle in both media, due to the respective seven waves, expressed by (1), (3)$\sim$(8), is simple harmonic, and each orbit becomes a straight line. We can see, however, from expressions (16)$\sim$(23), that all the constants, A, B, C, D, E, F become complex values, so that each wave has an amplitude and phase-difference that are transcendental functions, having the same variables as the six constants, while the direction of the straight line orbit of the seven waves generally do not coincide with one another. For these reasons, every particle in both media generally pursues an elliptic orbital motion, which will be studied in the next section, as the result of the interference effect of all the waves. The major and minor axes of this elliptic orbit and the inclination angle which the major axis of the orbit makes with the horizontal surface of the solids are also transcendental functions, the variables of which consist of the same elements as the six constants.

5) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

Table I. (*continued.*)

$\frac{H}{L}$	A	B	C	D	E	F
0·069	0·5169 −*i*2·3013	0·9039 +*i*0·4276				
0·07	0·4803 −*i*1·807	0·8682 +*i*0·4960	0·0200 −*i*0·3311	0·1468 −*i*0·2973	0·5419 +*i*0·2280	0·5836 +*i*0·0707
0·075	0·3432 −*i*0·9171	0·7541 +*i*0·6562	0·2482 −*i*0·2965	0·00752 −*i*0·3866	0·3439 +*i*0·07355	0·3076 +*i*0·1703
0·077	0·3162 −*i*0·7790	0·7172 +*i*0·6969				
0·0775	0·3107 −*i*0·7516	0·7082 +*i*0·7060				
0·078	0·3045 −*i*0·7255	0·7001 +*i*0·7138				
0·0781	0·3045 −*i*0·7215	0·6973 +*i*0·7166				
0·07817	0·30391 −*i*0·71800	0·69612 +*i*0·71792	0·3073 −*i*0·2931	−0·0034 −*i*0·4247	0·3062 +*i*0·0327	0·2366 +*i*0·1971
0·0782	0·3034 −*i*0·7162	0·6955 +*i*0·7182				
0·0783	0·3025 −*i*0·7119	0·6941 +*i*0·7199				
0·0785	0·3011 −*i*0·7029	0·6899 +*i*0·7239				
0·079	0·2965 −*i*0·6811	0·6813 +*i*0·7319				
0·08	0·2889 −*i*0·6418	0·6625 +*i*0·7485	0·3328 −*i*0·2944	−0·0001 −*i*0·4443	0·2935 +*i*0·0155	0·2061 +*i*0·2094
0·085	0·2649 −*i*0·5032	0·5659 +*i*0·8244	0·3886 −*i*0·3012	0·02839 −*i*0·4909	0·2763 −*i*0·01760	0·1418 +*i*0·2377
0·09	0·2535 −*i*0·4142	0·4537 +*i*0·8914	0·4381 −*i*0·3118	0·0793 −*i*0·5319	0·2732 −*i*0·04095	0·08732 +*i*0·2621
0·10	0·2460 −*i*0·2888	0·1594 +*i*0·9869	0·5727 −*i*0·3036	0·2084 −*i*0·6137	0·2907 −*i*0·07207	−0·0248 +*i*0·2985
0·11	0·2315 −*i*0·1793	−0·2504 +*i*0·9679	0·6744 −*i*0·3149	0·4736 −*i*0·5742	0·3326 −*i*0·08388	−0·1645 +*i*0·3010
0·12	0·1778 −*i*0·07150	−0·7211 +*i*0·6926	0·8198 −*i*0·2253	0·7473 −*i*0·4052	0·3893 −*i*0·06274	−0·3243 +*i*0·2244
0·128	0·08961 −*i*0·01017	−0·9745 +*i*0·2242				
0·129	0·07616 −*i*0·00591	−0·9880 +*i*0·1545				
0·13	0·06234 −*i*0·00258	−0·9967 +*i*0·08268	0·9020 −*i*0·03612	0·9019 −*i*0·03856	0·4297 +*i*0·002522	−0·4280 +*i*0·03803
0·1305	0·05526 −*i*0·00128	−0·9990 +*i*0·04631	0·9023 −*i*0·0249	0·9023 −*i*0·01691	0·4303 +*i*0·00663	−0·4296 +*i*0·02658
0·131	0·04836 −*i*0·0003	−1·000 +*i*0·01072				

(*to be continued.*)

$$\begin{aligned}\frac{\Delta_F}{f^{10}}=&\{1234{\cdot}04-3207{\cdot}57\sin a'+2479{\cdot}098\sin b'\\&-696{\cdot}061\cos a'-537{\cdot}978\cos b'\}\\&+i\{3105{\cdot}60-696{\cdot}061\sin a'-537{\cdot}978\sin b'\\&+3207{\cdot}57\cos a'-2479{\cdot}10\cos b'\},\end{aligned}\tag{30}$$

where $a'=29{\cdot}9423H/L$, $b'=8{\cdot}78990H/L$. Using these formulae, we numerically calculated the values A, B, C, D, E, and F for various values of H/L, as shown in Table I.

Table I. ($\lambda=\mu$, $\lambda'=\mu'$, $\mu'/\mu=1/10$, $\rho=\rho'$, $\theta=45°$.)

$\frac{H}{L}$	A	B	C	D	E	F
0·01	$-0{\cdot}000197$ $+i0{\cdot}0044$	$0{\cdot}9960$ $+i0{\cdot}0890$	$0{\cdot}3801$ $-i0{\cdot}0230$	$-0{\cdot}3765$ $-i0{\cdot}05676$	$0{\cdot}09958$ $-i0{\cdot}01548$	$0{\cdot}09781$ $+i0{\cdot}02428$
0·02	$-0{\cdot}00183$ $+i0{\cdot}0201$	$0{\cdot}9836$ $+i0{\cdot}1802$	$0{\cdot}3869$ $-i0{\cdot}0456$	$-0{\cdot}3724$ $-i0{\cdot}1145$	$0{\cdot}09775$ $-i0{\cdot}03281$	$0{\cdot}09023$ $+i0{\cdot}04991$
0·03	$-0{\cdot}00757$ $+i0{\cdot}0539$	$0{\cdot}9612$ $+i0{\cdot}2754$	$0{\cdot}4003$ $-i0{\cdot}0675$	$-0{\cdot}3661$ $-i0{\cdot}1752$	$0{\cdot}09335$ $-i0{\cdot}05383$	$0{\cdot}07489$ $+i0{\cdot}07747$
0·04	$-0{\cdot}0244$ $+i0{\cdot}1239$	$0{\cdot}9254$ $+i0{\cdot}3788$	$0{\cdot}4249$ $-i0{\cdot}0870$	$-0{\cdot}3602$ $-i0{\cdot}2415$	$0{\cdot}08296$ $-i0{\cdot}08278$	$0{\cdot}04544$ $+i0{\cdot}1080$
0·05	$-0{\cdot}0786$ $+i0{\cdot}2944$	$0{\cdot}8669$ $+i0{\cdot}4986$	$0{\cdot}4765$ $-i0{\cdot}0972$	$-0{\cdot}3648$ $-i0{\cdot}3219$	$0{\cdot}05398$ $-i0{\cdot}1344$	$-0{\cdot}0202$ $+i0{\cdot}1434$
0·055	$-0{\cdot}1583$ $+i0{\cdot}5025$	$0{\cdot}8195$ $+i0{\cdot}5730$	$0{\cdot}5330$ $-i0{\cdot}08778$	$-0{\cdot}3865$ $-i0{\cdot}3774$	$0{\cdot}01620$ $-i0{\cdot}1892$	$-0{\cdot}09516$ $+i0{\cdot}1644$
0·06	$-0{\cdot}4240$ $+i1{\cdot}067$	$0{\cdot}7276$ $+i0{\cdot}6858$	$0{\cdot}6737$ $-i0{\cdot}0280$	$-0{\cdot}4710$ $-i0{\cdot}4824$	$-0{\cdot}08754$ $-i0{\cdot}3355$	$-0{\cdot}2939$ $+i0{\cdot}1842$
0·065	$-8{\cdot}119$ $+i5{\cdot}275$	$-0{\cdot}4067$ $+i0{\cdot}9132$	$1{\cdot}553$ $+i1{\cdot}894$	$-1{\cdot}099$ $-i2{\cdot}188$	$-0{\cdot}1962$ $-i2{\cdot}807$	$-2{\cdot}484$ $-i1{\cdot}323$
0·066	$-6{\cdot}114$ $-i10{\cdot}51$	$0{\cdot}4940$ $-i0{\cdot}8695$				
0·067	$0{\cdot}1049$ $-i5{\cdot}233$	$0{\cdot}9992$ $+i0{\cdot}0401$				
0·0675	$0{\cdot}4150$ $-i3{\cdot}978$	$0{\cdot}9784$ $+i0{\cdot}2064$				
0·0677	$0{\cdot}4661$ $-i3{\cdot}619$	$0{\cdot}9673$ $+i0{\cdot}2533$				
0 0678	$0{\cdot}4786$ $-i3{\cdot}462$	$0{\cdot}9624$ $+i0{\cdot}2713$				
0·06784	$0{\cdot}48921$ $-i3{\cdot}40577$	$0{\cdot}95957$ $+i0{\cdot}28148$	$-0{\cdot}3709$ $-i0{\cdot}3573$	$0{\cdot}4565$ $-i0{\cdot}2384$	$0{\cdot}9411$ $+i0{\cdot}4322$	$1{\cdot}025$ $-i0\ 1497$
0·0679	$0{\cdot}4968$ $-i3{\cdot}322$	$0{\cdot}9562$ $+i0{\cdot}2925$				
0·0680	$0{\cdot}5050$ $-i3{\cdot}198$	$0{\cdot}9514$ $+i0{\cdot}3082$				
0·0685	$0{\cdot}5219$ $-i2{\cdot}673$	$0{\cdot}9265$ $+i0{\cdot}3761$				

(*to be continued.*)

3. In the next section, we shall study the orbital motions of a particle in the surface-layer for the case when both media, the surface-layer and the subjacent medium, are assumed to satisfy Cauchy's elasticity condition, and the ratio of the rigidity of the surface-layer to that of the subjacent medium is 1/10, and the densities of the materials are exactly the same for both media, and also when the incidence angle of the primary distortional wave is 45°. For convenience, we shall show all the quantities Δ, Δ_A, Δ_B, Δ_C, Δ_D, Δ_E, Δ_F, for determining the values of A, B, C, D, E, F for the case when $\lambda=\mu$, $\lambda'=\mu'$, $\rho=\rho'$ $\mu'/\mu=1/10$, and $\theta=45°$, as in the following forms:

$$\frac{\Delta}{f^{10}}=\{-1075{\cdot}81+7358{\cdot}67\sin a'-5687{\cdot}44\sin b'+2671{\cdot}88\cos a'-1596{\cdot}07\cos b'\}+i\{11801{\cdot}27-6960{\cdot}61\sin a'-5379{\cdot}78\sin b'+20283{\cdot}3\cos a'+6256{\cdot}16\cos b'\}, \tag{24}$$

$$\frac{\Delta_A}{f^{10}}=\{7411{\cdot}14+4538{\cdot}91\cos a'-11950{\cdot}05\cos b'\}, \tag{25}$$

$$\frac{\Delta_B}{f^{10}}=\{1075{\cdot}81-7358{\cdot}67\sin a'+5687{\cdot}44\sin b'-2671{\cdot}88\cos a'+1596{\cdot}07\cos b'\}+i\{11801{\cdot}27-6960{\cdot}61\sin a'-5379{\cdot}78\sin b'+20283{\cdot}3\cos a'+6256{\cdot}16\cos b'\}, \tag{26}$$

$$\frac{\Delta_C}{f^{10}}=\{-597{\cdot}672+6622{\cdot}81\sin a'-5118{\cdot}70\sin b'+2631{\cdot}64\cos a'-2033{\cdot}96\cos b'\}+i\{2754{\cdot}18-2631{\cdot}64\sin a'-2033{\cdot}96\sin b'+6622{\cdot}81\cos a'+5118{\cdot}70\cos b'\}, \tag{27}$$

$$\frac{\Delta_D}{f^{10}}=\{-597{\cdot}672+6622{\cdot}81\sin a'-5118{\cdot}70\sin b'+2631{\cdot}64\cos a'-2033{\cdot}96\cos b'\}+i\{-2754{\cdot}18+2631{\cdot}64\sin a'+2033{\cdot}96\sin b'-6622{\cdot}805\cos a'-5118{\cdot}70\cos b'\}, \tag{28}$$

$$\frac{\Delta_E}{f^{10}}=\{-1234{\cdot}04+3207{\cdot}57\sin a'-2479{\cdot}10\sin b'+696{\cdot}061\cos a'+537{\cdot}978\cos b'\}+i\{3105{\cdot}60-696{\cdot}061\sin a'-537{\cdot}978\sin b'+3207{\cdot}57\cos a'-2479{\cdot}10\cos b'\}, \tag{29}$$

Table I. (*continued.*)

H/L	A	B	C	D	E	F
0·132	0·03422 +i0·00107	−0·9981 −i0·06220				
0·14	−0·07187 −i0·02334	−0·8094 −i0·5873	0·8413 +i0·1687	0·7800 +i0·3577	0·4173 +i0·08233	−0·3861 −i0·1786
0·15	−0·1545 −i0·1100	−0·3272 −i0·9451	0·6916 +i0·2768	0·4878 +i0·5629	0·3675 +i0·1300	−0·2431 −i0·3048
0·17	−0·1741 −i0·2855	0·4581 −i0·8890	0·4450 +i0·2578	0·02535 +i0·5137	0·2833 +i0·1336	0·01102 −i0·3129
0·18	−0·1567 −i0·3552	0·6740 −i0·7387	0·3735 +i0·2112	−0·09574 +i0·4182	0·2632 +i0·1200	0·08873 −i0·2753
0·19	−0·1352 −i0·4229	0·8144 −i0·5802	0·3223 +i0·1614	−0·1688 +i0·3185	0·2551 +i0·1060	0·1463 −i0·2343
0·20	−0·1104 −i0·4974	0·9061 −i0·4232	0·2825 +i0·1110	−0·2090 +i0·2201	0·2564 +i0·09377	0·1927 −i0·1935
0·21	−0·08027 −i0·5928	0·9640 −i0·2660	0·2469 +i0·05832	−0·2225 +i0·1219	0·2684 +i0·08502	0·2361 −i0·1533
0·22	−0·03593 −i0·7307	0·9952 −i0·09809	0·2092 −i0·002539	−0·2084 +i0·01799	0·2944 +i0·08263	0·2849 −i0·1111
0·225	−0·001695 −i0·8291	1·000 −i0·00409	0·1869 −i0·03999	−0·1871 −i0·03923	0·3156 +i0·08584	0·3153 −i0·08712
0·23	0·04967 −i0·9608	0·9947 +i0·1031	0·1606 −i0·08640	−0·1508 −i0·1025	0 3454 +i0·09477	0·3534 −i0·05866
0·235	0·1258 −i1·1077	0·9745 +i0·2243	0·1234 −i0·1442	−0·08793 −i0·1682	0·3759 +i0·1089	0·3908 −i0·02185
0·24	0·2937 −i1·4165	0·9175 +i0·3975	0·08779 −i0·2421	0·01572 −i0·2571	0·4516 +i0·1558	0·4763 +i0·03654
0·245	0·6469 −i1·8365	0·7792 +i0·6266	0·04485 −i0·4043	0·2185 −i0·3431	0·5476 +i0·2525	0·5849 +i0·1465
0·248	1·1011 −i2·181	0·5936 +i0·8047				
0·2485	1·2074 −i2·239	0·5492 +i0·8354				
0·249	1·3285 −i2·2988	0·4992 +i0·8664				
0·2495	1·4612 −i2·3549	0·4440 +i0·8963				
0·2497	1·5264 −i2·3797	0·4169 +i0·9089				
0·2498	1·5575 −i2·3910	0·4042 +i0·9147				
0·2499	1·5878 −i2·4008	0·3915 +i0·9202				
0·24992	1·5948 −i2·4040	0·38878 +i0·92133	0·0635 −i0·7282	0·6462 −i0·3416	0·6627 +i0·5077	0·7252 +i0·4131
0·25	1·6218 −i2·4124	0·3774 +i0·9260	0·06574 −i0·7361	0·6571 −i0·3387	0·6643 +i0·5148	0·7273 +i0·4210

(*to be continued.*)

Table I. (*continued.*)

$\frac{H}{L}$	A	B	C	D	E	F
0·2502	1·6879 $-i$2·433	0·3500 $+i$0·9367				
0·2505	1·7936 $-i$2·4592	0·3055 $+i$0·4761				
0·251	1·9845 $-i$2·498	0·2260 $+i$0·9742				
0·255	4·217 $-i$1·7066	$-$0·7188 $+i$0·6956	0·6441 $-i$1·2380	1·324 $+i$0·4419	0·3970 $+i$1·155	0·5180 $+i$1·106
0·26	3·5854 $+i$2·1384	$-$0·4741 $-i$0·8800	1·4347 $-i$0·4898	0·2506 $+i$1·4952	$-$0·5658 $+i$0·8384	$-$0·4685 $+i$0·8960
0·265	1·357 $+i$2·243	0·4643 $-i$0·8855	1·103 $+i$0·03050	$-$0·4849 $+i$0 9905	$-$0·5330 $+i$0·2695	$-$0·4861 $+i$0·3469
0·27	0·5828 $+i$1·7097	0·7918 $-i$0·6107	0·8555 $+i$0·1103	$-$0·6100 $+i$0·6099	$-$0·3591 $+i$0·07395	$-$0·3295 $+i$0·1608
0·28	0·1397 $+i$1·1170	0·9692 $-i$0·2462	0·6505 $+i$0·08787	$-$0·6089 $+i$0·2453	$-$0·1925 $-i$0·03854	$-$0·1770 $+i$0·08474
0·29	0·003961 $+i$0·8708	1·0000 $-i$0·009099	0·5790 $+i$0·04491	$-$0·5786 $+i$0·05017	$-$0·1210 $-i$0·08730	$-$0·1202 $+i$0·08840
0·30	$-$0·07472 $+i$0·7758	0·9817 $+i$0·1909	0·5580 $+i$0·00619	$-$0·5487 $-i$0·1004	$-$0·09074 $-i$0·1316	$-$0·1142 $+i$0·1118
0·31	$-$0·1587 $+i$0·7742	0·9195 $+i$0·3936	0·5695 $-i$0·02734	$-$0·5129 $-i$0·2492	$-$0·07123 $-i$0·1872	$-$0·1391 $+i$0·1441

4. For an example, we put $\lambda=\mu$, $\lambda'=\mu'$, $\rho=\rho'$, and $(\mu'/\mu)=(1/10)$, and also $\theta=45°$ in expressions (10), (11) in the previous section. Then, taking the real parts only, we get the following horizontal and vertical oscillations U' and V' of the particles in the surface-layer when $x=0$:

$$\left.\frac{LU'}{2\pi\mathfrak{A}}\right]_{x=0}=\cos\theta\cos z\Big[(D'-C')\sin a-(C''+D'')\cos a+\frac{s'}{f}\Big\{(E'+F')\sin b+(E''-F'')\cos b\Big\}\Big]$$
$$+\cos\theta\sin z\Big[(-C''+D'')\sin a+(C'+D')\cos a+\frac{s'}{f}\Big\{(E''+F'')\sin b+(-E'+F')\cos b\Big\}\Big], \quad (31)$$

$$\left.\frac{LV'}{2\pi\mathfrak{A}}\right]_{x=0}=\cos\theta\cos z\Big[(E'-F')\sin b+(E''+F'')\cos b$$

$$+\frac{r'}{f}\left\{(C'+D')\sin a+(C''-D'')\cos a\right\}\Bigg]$$

$$+\cos\theta\sin z\Bigg[(E''-F'')\sin b-(E'+F')\cos b$$

$$+\frac{r'}{f}\left\{(C''+D'')\sin a-(C'-D')\cos a\right\}\Bigg], \qquad (32)$$

where $z=\frac{2\pi}{T}t$, and C', D', C'', D'', E', E'', F', F'' are real quantities connected with C, D, E, F as in the following forms:

$$\left.\begin{array}{ll} C=C'+iC'', & D=D'+iD'', \\ E=E'+iE'', & F=F'+iF'', \end{array}\right\} \qquad (33)$$

and

$$\frac{r'}{f}=2{\cdot}3804765, \quad \frac{s'}{f}=4{\cdot}3588994,$$

$$a=29{\cdot}9423\frac{y}{L} \quad b=8{\cdot}78990\frac{y}{L},$$

$$\cos\theta=0{\cdot}7071067.$$

Expressions (31), (32) are rewritten

$$\left.\frac{LU'}{2\pi\mathfrak{A}}\right]_{x=0}=\cos\theta\sqrt{\alpha^2+\beta^2}\sin\left\{z+\tan^{-1}\frac{\beta}{\alpha}\right\}, \qquad (34)$$

$$\left.\frac{LV'}{2\pi\mathfrak{A}}\right]_{x=0}=\cos\theta\sqrt{\alpha'^2+\beta'^2}\sin\left\{z+\tan^{-1}\frac{\beta'}{\alpha'}\right\}, \qquad (35)$$

where

$$\alpha=\left\{(C''-D'')\sin a+(C'+D')\cos a\right\}$$

$$+\frac{s'}{f}\left\{-(E''+F'')\sin b+(-E'+F')\cos b\right\}, \qquad (36)$$

$$\beta=\left\{(C'-D')\sin a-(C''+D'')\cos a\right\}$$

$$+\frac{s'}{f}\left\{-(E'+F')\sin b+(E''-F'')\cos b\right\}, \qquad (37)$$

$$\alpha'=\frac{r'}{f}\left\{-(C''+D'')\sin a+(-C'+D')\cos a\right\}$$

$$+\left\{-(E''-F'')\sin b-(E'+F')\cos b\right\}, \tag{38}$$

$$\beta'=\frac{r'}{f}\left\{-(C'+D')\sin a+(C''-D'')\cos a\right\}$$

$$+\left\{-(E'-F')\sin b+(E''+F'')\cos b\right\}, \tag{39}$$

and $\tan^{-1}(\beta/\alpha)$, $\tan^{-1}(\beta'/\alpha')$ should satisfy the conditions that

$$\sqrt{\alpha^2+\beta^2}\cos\gamma=\alpha,\quad \sqrt{\alpha^2+\beta^2}\sin\gamma=\beta,\quad \gamma=\tan^{-1}(\beta/\alpha), \tag{40}$$

$$\sqrt{\alpha'^2+\beta'^2}\cos\gamma'=\alpha',\quad \sqrt{\alpha'^2+\beta'^2}\sin\gamma'=\beta',\quad \gamma'=\tan^{-1}(\beta'/\alpha'). \tag{41}$$

From expressions (34), (35), it will be seen that forced stationary harmonic vibrations (horizontal and vertical) are caused in the surface-layer by the incident distortional wave, as in the case[6] when the dilatational wave of harmonic type is obliquely incident on the bottom surface of that layer. The orbital motions of the particles in the surface-layer may be studied by

$$\frac{U'^2}{(\alpha^2+\beta^2)}+\frac{V'^2}{(\alpha'^2+\beta'^2)}-\frac{2\cos\varepsilon}{\sqrt{\alpha^2+\beta^2}\sqrt{\alpha'^2+\beta'^2}}U'V'=\frac{4\pi^2\mathfrak{A}^2}{L^2}\sin^2\varepsilon, \tag{42}$$

where

$$\varepsilon=\tan^{-1}(\beta'/\alpha')-\tan^{-1}(\beta/\alpha), \tag{43}$$

which are obtained by eliminating the quantity z from the two expressions (34), (35).

Expression (42) shows that the orbit of the particles in the surface-layer generally pursues an ellipse, whose major and minor axes ξ, η are respectively given by

$$\xi=\sin\varepsilon/\sqrt{\gamma},\quad \eta=\sin\varepsilon/\sqrt{\delta}, \tag{44}$$

where γ, δ are the roots of the equation

$$\zeta^2-\frac{(\alpha^2+\beta^2+\alpha'^2+\beta'^2)}{(\alpha^2+\beta^2)(\alpha'^2+\beta'^2)}\zeta+\frac{\sin^2\varepsilon}{(\alpha^2+\beta^2)(\alpha'^2+\beta'^2)}=0, \tag{45}$$

the inclination angle τ which the major axis makes with the horizontal plane surface being obtained from the expression

6) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

$$\tan 2\tau = -\frac{2\cos\varepsilon\sqrt{(\alpha^2+\beta^2)(\alpha'^2+\beta'^2)}}{(\alpha'^2+\beta'^2)-(\alpha^2+\beta^2)}. \tag{46}$$

All the quantities such that the major and minor axes of the elliptic orbit of the particles in the surface-layer, and the inclination angle of the same elliptic orbit, are transcendental functions, only H/L being variable for the present example. The next section, in which the resonance properties of the surface-layer are discussed, deals with expressions (42), (46) for several cases of H/L that are related to the resonance phenomenon of the surface-layer.

5. Since in the previous section, we studied, generally, the stationary forced vibrations of the surface-layer when the distortional wave is obliquely incident on the bottom surface of it, it should now be easy to obtain the stationary vibrations of the particles on both the top and the bottom surfaces of the surface-layer from expressions (34), (35). Instead of expressions (34), (35), we may use the following expressions which show the stationary vibrations of the bottom surface of that layer, and which are easily reduced from expressions (12), (13):

$$\left.\frac{UL}{2\pi\mathfrak{A}}\right]_{\substack{x=0\\y=0}} = \cos\theta\sqrt{\left(1+A'+\frac{s}{f}B'\right)^2+\left(A''+\frac{s}{f}B''\right)^2}\sin(z+\alpha), \tag{47}$$

$$\left.\frac{VL}{2\pi\mathfrak{A}}\right]_{\substack{x=0\\y=0}} = \cos\theta\sqrt{\left(-\frac{r}{f}+\frac{r}{f}A'-B'\right)^2+\left(B''-\frac{r}{f}A''\right)^2}\sin(z+\alpha'), \tag{48}$$

where

$$\alpha = \tan^{-1}\left\{-\left(A''+\frac{s}{f}B''\right)\Big/\left(1+A'+\frac{s}{f}B'\right)\right\}, \tag{49}$$

$$\alpha' = \tan^{-1}\left\{\left(B''-\frac{r}{f}A''\right)\Big/\left(-\frac{r}{f}+\frac{r}{f}A'-B'\right)\right\}, \tag{50}$$

and $\cos\theta = 0{\cdot}7071067$, $r/f = i\,0{\cdot}5773502$, $s/f = 1$. Obviously α and α' should satisfy

$$\left.\begin{aligned}\sqrt{\left(1+A'+\frac{s}{f}B'\right)^2+\left(A''+\frac{s}{f}B''\right)^2}\sin\alpha &= -\left(A''+\frac{s}{f}B''\right),\\ \sqrt{\left(1+A'+\frac{s}{f}B'\right)^2+\left(A''+\frac{s}{f}B''\right)^2}\cos\alpha &= 1+A'+\frac{s}{f}B',\end{aligned}\right\} \tag{51}$$

$$\left.\begin{aligned}\sqrt{\left(-\frac{r}{f}+\frac{r}{f}A'-B'\right)^2+\left(B''-\frac{r}{f}A''\right)^2}\sin\alpha'&=\left(B''-\frac{r}{f}A''\right),\\ \sqrt{\left(-\frac{r}{f}+\frac{r}{f}A'-B'\right)^2+\left(B''-\frac{r}{f}A''\right)^2}\cos\alpha'&=\left(-\frac{r}{f}+\frac{r}{f}A'-B'\right).\end{aligned}\right\}\quad(52)$$

In these expressions (47)~(52), A', B', A'', B'' are real quantities connected with A and B as in the following forms:

$$A=A'+iA'',\quad B=B'+iB'',\tag{53}$$

the values A and B being given in Table I with their respective values of H/L.

Calculating the amplitudes (horizontal and vertical) both on the top and the bottom surfaces of the surface-layer with the respective values of the quantity H/L, we get Tables II, III, the results being shown in Figs. 2, 3.

Table II. (Amplitudes on the Top Surface.)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0·01	0·0035	1·425	0·17	1·945	1·891
0·02	0·0194	1·456	0·18	1·837	1·530
0·03	0·0601	1·517	0·19	1·790	1·264
0·04	0·1620	1·620	0·20	1·801	1·030
0·05	0·4567	1·812	0·21	1·885	0·806
0·055	0·903	2·010	0·22	2·075	0·557
0·06	2·050	2·465	0·225	2·230	0·404
0·065			0·23	2·450	0·212
0·06784	6·73	0·498	0·235	2·68	0·0428
0·07	4·010	0·489	0·24	3·267	0·410
0·075	2·32	1·309	0·245	4·11	1·021
0·07817	1·980	1·506	0·24992	5·61	2·290
0·08	1·860	1·587	0·25	5·670	2·290
0·085	1·685	1·781	0·255	7·81	4·775
0·09	1·634	1·964	0·26	6·46	5·39
0 10	1·670	2·393	0·265	3·595	4 020
0·11	1·938	2·740	0·27	2·018	3·228
0·12	2·236	3·129	0·28	0·694	2·438
0·13	2·466	3·310	0·29	0·0142	2·168
0·1305	2·470	3·322	0·30	0·425	2·082
0·14	2·452	3·130	0·31	0·912	2·117
0·15	2·330	2·717			

Table III. (Amplitudes on the Bottom-surface.)

$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude	$\frac{H}{L}$	Horizontal amplitude	Vertical amplitude
0·01	0·0661	1·414	0·13	1·369	0·0325
0·02	0·1420	1·417	0·1305	1·373	0·0102
0·03	0·2348	1·422	0·132	1·390	0 0580
0·04	0·3629	1·440	0·14	1·399	0·4057
0·05	0·581	1·475	0·15	1·286	0·741
0·055	0·797	1·565	0·16	1 200	0·880
0·06	1·333	1·783	0·17	0·974	1·071
0·065	8·04	4·721	0·18	0·846	1·135
0·066	9·31	3·740	0·19	0·745	1·167
0·067	3·675	0·724	0·20	0·667	1·167
0·0675		0·227	0·21	0·613	1·157
0·0677		0·0878	0·22	0·593	1·114
0·06780		0·026	0·225	0·589	1·075
0·06784	2·230	0·00483	0·230	0·615	1·019
0·0679		0·0271	0·235	0·628	0·951
0·0680		0·0743	0·24	0·737	0·794
0·0685		0·2756	0·245	0·907	0·539
0·069	1·359	0·4165	0·248	1·090	0·264
0·070	0·960	0·595	0·2485		0·2056
0·075	0·197	0·925	0·2495		0·070
0·077	0·0627	0·967	0 2497		0·0363
0·0775	0·0349		0·2499		0·0048
0·078	0·0088	0·983	0·24992	1·258	0·00073
0·0781	0·0037		0·250	1·267	0·0146
0·07817	0·0000	0·984	0·2502		0·047
0·0782	0·0016		0·251	1·377	0·1947
0·0783	0·0062		0·255	1·902	1·325
0·0785	0·01615		0·260	1·739	2·430
0·079	0·0392		0·265	1·122	2·290
0·080	0·0828	1·005	0·270	0·821	2·075
0·085	0·2565	1·020	0·28	0·663	1·861
0·09	0·396	1·008	0·29	0·609	1·770
0·10	0·648	0·923	0 30	0 687	1·726
0·11	0·9101	0·749	0·31	0·843	1·708
0·12	1·175	0·500	0·2490		0·1402
0·128	1·342	0·1226	0·2498		0·0173
0·129		0·0785			0·0474

It will be seen from Figs. 2, 3 that the horizontal and the vertical amplitudes of forced vibrations on the top surface are usually larger than the respective amplitudes of the particles on the bottom surface of the surface-layer. When H/L becomes relatively small, the horizontal amplitude on the top surface, however, is a little smaller than that on

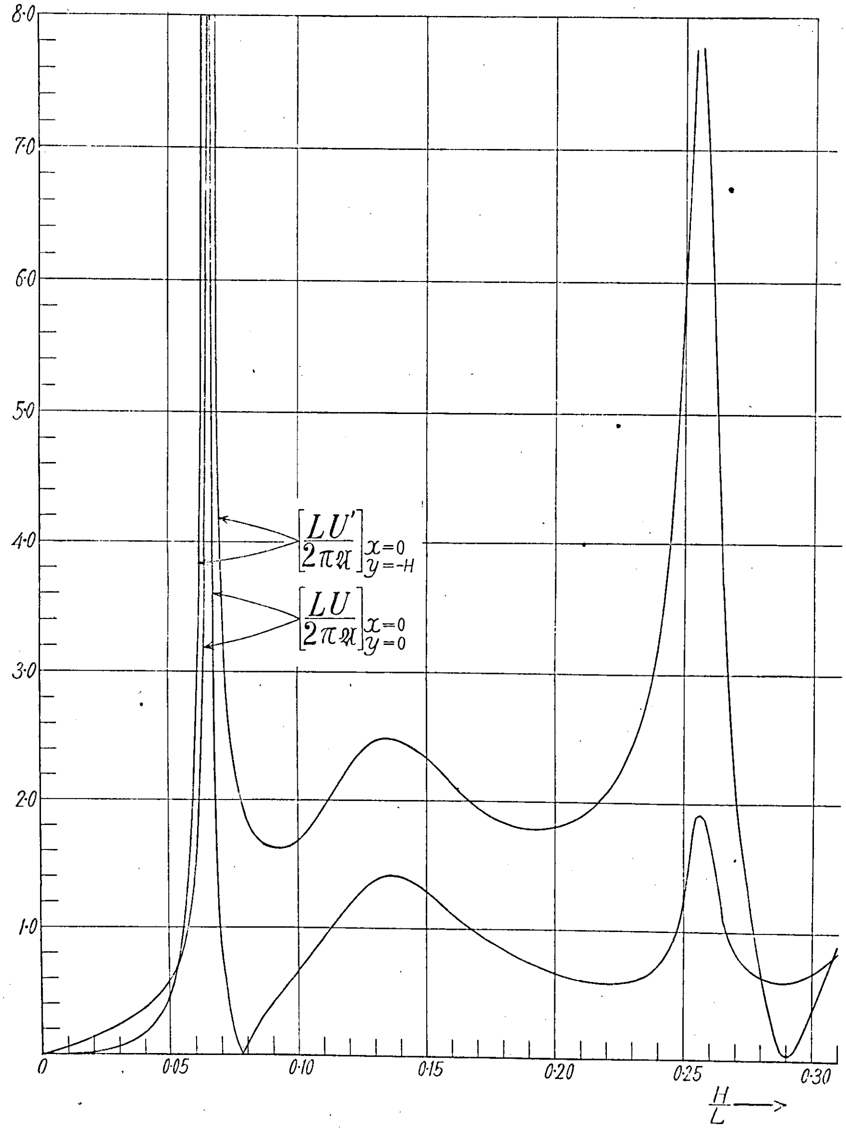


Fig. 2. Horizontal displacements of particles on the top and bottom surfaces.

the bottom surface. When H/L becomes zero, or when the period of the primary incident distortional wave is very long, the horizontal and the vertical amplitudes on the top surface become respectively equal to those on the bottom surface. It will be seen, therefore, that when the wave-length of the primary incident wave becomes very large compared with the thickness of the surface-layer, the layer has little effect on the

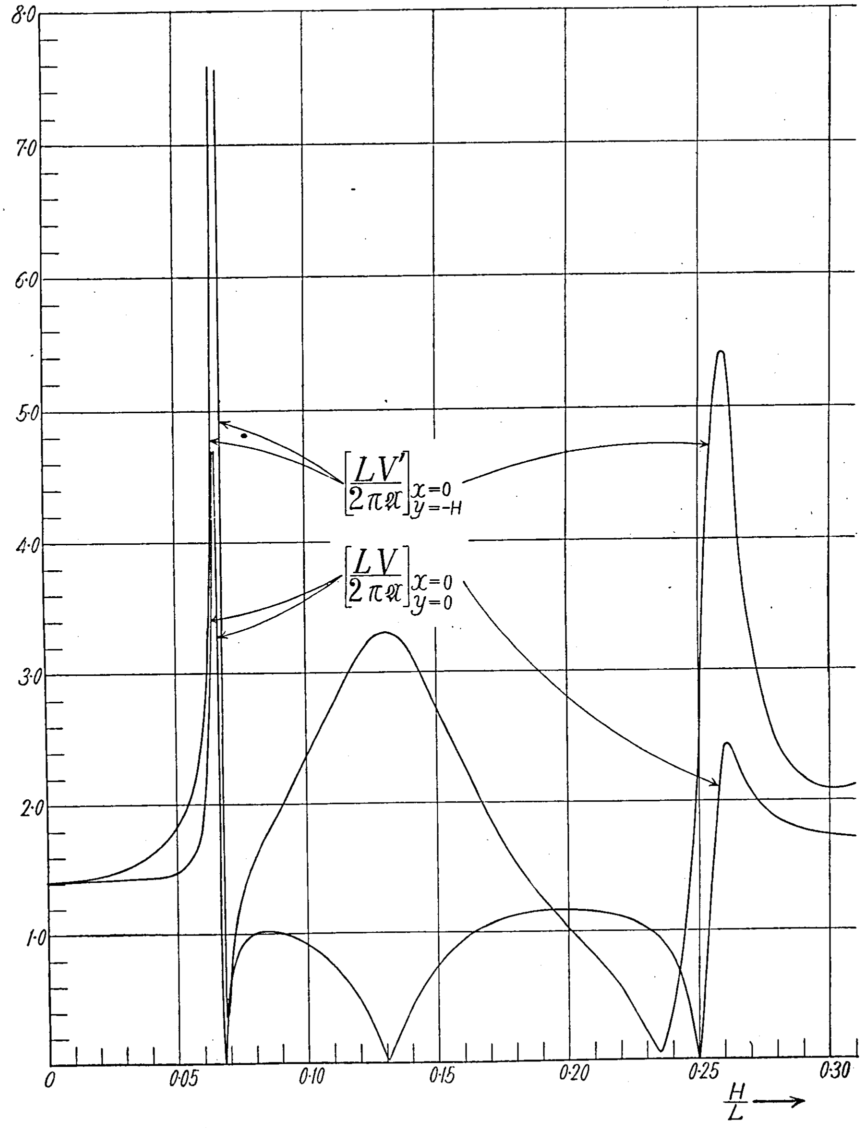


Fig. 3. Vertical displacements of particles on the top and bottom surfaces.

amplitude of vibration of the particles.[7] In this actual example, moreover, the horizontal amplitudes become zero when $H/L=0$, so that the

7) This fact has also been shown for the case when the primary incident wave is dilatational and of harmonic type.
G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

particles in the surface-layer pursue only a vertical movement, notwithstanding that the incidence angle of the primary wave is 45°.

We now have the fact[8] that, when the primary incident wave is dilatational, the respective positions of the maxima and minima of the horizontal or vertical vibration amplitudes with respect to the same values of H/L in their resonance curves are completely changed in both amplitudes on the top surface and those on the bottom surface of the surface-layer. Generally speaking, the amplitude on the bottom surface of the surface-layer becomes minimum, and very small for the values of H/L, for which the amplitude on the top surface becomes maximum and very large. This fact, however, may not necessarily be obtained in the present example when the primary wave is distortional as will be seen from Figs. 2, 3. From Fig. 2, it will be seen that the respective positions of the maxima and minima of the horizontal vibration amplitudes with respect to the same values of H/L are approximately the same in both amplitudes on the top surface and those on the bottom surface of the surface-layer, and we find nearly a parallelism in the variation between the two curves with variation in H/L. From Fig. 3, we shall also see, usually, the same properties as in the horizontal amplitude curves shown in Fig. 2, with, however, the exception that the vertical amplitude on the bottom surface of the surface-layer becomes minimum and very small, but not zero for $H/L=0{\cdot}1305$, for which the vertical amplitude on the top surface becomes maximum and comparatively large.

From the foregoing fact it will be seen that the resonance periods of the surface-layer cannot easily be obtained from the resonance curves on the top surface of the surface-layer alone; for example, although $H/L=0{\cdot}065$ gives the first maximum to both the horizontal and vertical amplitudes of vibration on the top surface of the surface-layer, it will not be correct to take $H/L=0{\cdot}065$ as the value giving the gravest resonance period of both the horizontal and vertical vibrations of the surface-layer. Theoretically the resonance periods of the horizontal and vertical vibrations of the gravest mode of this layer must respectively be obtained from the relations

$$H/L=0{\cdot}078 \text{ and } H/L=0{\cdot}131.$$

When $H/L=0{\cdot}078$, the horizontal vibration amplitude on the top surface is somewhat smaller than that for $H/L=0{\cdot}065$, as will be seen from Fig. 2, but it gives the resonance period of the horizontal vibration of the gravest mode, while the latter value $H/L=0{\cdot}131$ gives the secondary

8) G. NISHIMURA and T. TAKAYAMA, *loc. cit.*

maximum value to the vertical amplitude of vibration. Although this value is smaller than the first maximum value when $H/L=0{\cdot}065$, as will be seen from Fig. 3, it corresponds to the resonance period of the vertical oscillation of the gravest mode of the surface-layer. Although the value $H/L=0{\cdot}26$, gives the third maximum to the horizontal and vertical amplitudes on the top surface of the surface-layer, as will be seen from Figs. 2, 3, it also does not correspond to the respective resonance periods of the second modes of the horizontal and vertical vibrations of that layer. Obviously, the value $H/L=0{\cdot}135$ also does not give the proper period of the higher mode of the horizontal vibration of the surface-layer, although it gives the second maximum to the horizontal amplitude on the top surface. (See Fig. 2.)

To ascertain why we should take 0·078 and 0·131 as the values of the quantity H/L, which corresponds respectively to the resonance periods of the horizontal and vertical vibrations of the gravest mode of the surface-layer, we calculate the vibrational movements of particles in the surface-layer numerically from expressions (34), (35), and obtain Table VI and Figs. 4a, 4b, 4c, 5, 6. Phase-differences $\tan^{-1}(\beta/\alpha)$ and $\tan^{-}(\beta'/\alpha')$ are also shown in Table IV. Figs. 4a, 4b, 4c show the elliptic orbits of particles on the surfaces $y=0$, $-(1/8)H$, $-(2/8)H$, $-(3/8)H$,

Table IV.

$\frac{H}{L}$	y	Horizontal amplitude	Vertical amplitude	$\tan^{-1}\frac{\beta}{\alpha}$	$\tan^{-1}\frac{\beta'}{\alpha'}$
0·065	$-H$	18·8	7·67	0·573	2·15
	$-7/8\,H$	18·9	7·40	〃	〃
	$-6/8\,H$	18·5	7·10	〃	〃
	$-5/8\,H$	17·7	6·71	〃	〃
	$-4/8\,H$	16·3	6·35	〃	〃
	$-3/8\,H$	14·8	5·92	〃	〃
	$-2/8\,H$	12·8	5·54	〃	〃
	$-1/8\,H$	10·4	5·12	〃	〃
	0	8·04	4·72	〃	〃
0·06784	$-H$	7·10	0·438	1·428	2·995
	$-7/8\,H$	7·02	0·349	〃	〃
	$-6/8\,H$	6·78	0·263	〃	〃
	$-5/8\,H$	6·36	0·176	〃	〃
	$-4/8\,H$	5·78	0·104	〃	〃
	$-2/8\,H$	4·22	0·0092	〃	〃
	0	2·23	0·0048	〃	〃

(*to be continued.*)

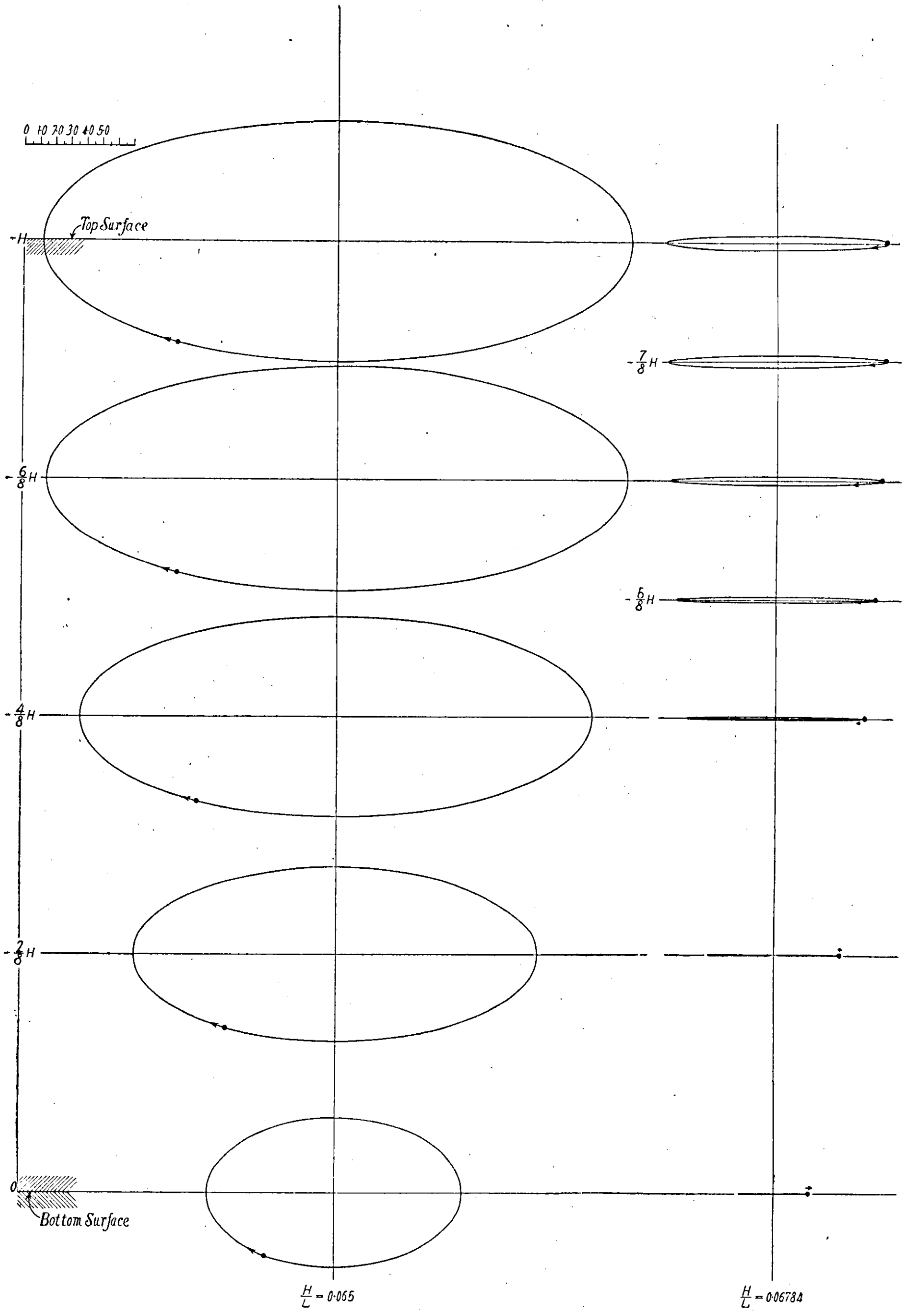
Table IV. (*continued.*)

$\frac{H}{L}$	y	Horizontal amplitude	Vertical amplitude	$\tan^{-1}\frac{\beta}{\alpha}$	$\tan^{-1}\frac{\beta'}{\alpha'}$
0·07817	$-H$	1·98	1·51	1·170	2·74
	$-3/4\,H$	1·72	1·52	″	″
	$-2/4\,H$	1·26	1·44	″	″
	$-1/4\,H$	0·649	1·26	″	″
	0	0	0·984	″	″
0·11	$-H$	1·94	2·74	0·658	2·230
	$-7/8\,H$	1·71	2·74	″	″
	$-6/8\,H$	1·33	2·67	″	″
	$-5/8\,H$	0·976	2·51	″	″
	$-4/8\,H$	0·532	2·29	″	″
	$-3/8\,H$	0·087	1·99	″	″
	$-2/8\,H$	0·321	1·62	″	″
	$-1/8\,H$	0·663	1·20	″	″
	0	0·911	0·747	″	″
0·1305	$-H$	2·47	3·32	0·0231	1·594
	$-7/8\,H$	2·13	3·31	″	″
	$-6/8\,H$	1·60	3·18	″	″
	$-5/8\,H$	0·960	2·90	″	″
	$-4/8\,H$	0·285	2·51	″	″
	$-3/8\,H$	0·353	2·00	″	″
	$-2/8\,H$	0·876	1·39	″	″
	$-1/8\,H$	1·23	0·719	″	″
	0	1·37	0·0102	″	″
0·20	$-H$	1·80	1·03	1·789	0·218
	$-7/8\,H$	1·50	1·05	″	″
	$-6/8\,H$	0·902	0·949	″	″
	$-5/8\,H$	0·149	0·726	″	″
	$-4/8\,H$	0·571	0·392	″	″
	$-3/8\,H$	1·09	0·0186	″	″
	$-2/8\,H$	1·30	0·456	″	″
	$-1/8\,H$	1·14	0·859	″	″
	0	0·658	1·17	″	″
0·235	$-H$	2·68	0·0428	1·458	3·03
	$-7/8\,H$	2·29	0·0623	″	″
	$-6/8\,H$	1·24	0·0976	″	″
	$-5/8\,H$	0·161	0·0137	″	″
	$-4/8\,H$	1·48	0·192	″	″

(*to be continued.*)

Table IV. (*continued.*)

$\frac{H}{L}$	y	Horizontal amplitude	Vertical amplitude	$\tan^{-1}\frac{\beta}{\alpha}$	$\tan^{-1}\frac{\beta'}{\alpha'}$
0·235	$-3/8\,H$	2·31	0·472	1·458	3·03
	$-2/8\,H$	2·42	0·742	〃	〃
	$-1/8\,H$	1·79	0·912	〃	〃
	0	0.628	0·904	〃	〃
0·24992	$-H$	5·61	2·29	0·985	2·555
	$-15/16\,H$	5·52		〃	〃
	$-7/8\,H$	4·96	1·92	〃	〃
	$-6/8\,H$	2·63	1·45	〃	〃
	$-5/8\,H$	0·662	1·07	〃	〃
	$-4/8\,H$	3·84	0·867	〃	〃
	$-3/8\,H$	5·76	0·800	〃	〃
0·24992	$-5/16\,H$	6·12		0·985	2·555
	$-2/8\,H$	5·94	0·739	〃	〃
	$-3/16\,H$		0·680	〃	〃
	$-1/8\,H$	4·23	0·5105	〃	〃
	$-1/16\,H$		0·295	〃	〃
	0	1·26	0·0007	〃	〃
0·255	$-H$	8·04	4·78	0·385	1·956
	$-15/16\,H$	8·00		〃	〃
	$-7/16\,H$	7·26	4·12	〃	〃
	$-6/8\,H$	3·85	3·14	〃	〃
	$-5/8\,H$	1·11	2·18	〃	〃
	$-4/8\,H$	5·93	1·40	〃	1·954
	$-3/8\,H$	9·11	0·841	〃	〃
	$-5/16\,H$	9·47		〃	〃
	$-2/8\,H$	9·22	0·328	〃	〃
	$-1/8\,H$	6·51	0·343	〃	〃
	0	1·90	1·33	〃	〃
0·260	$-H$	6·46	5·39	2·604	1·03
	$-7/8\,H$	5·96	4·75	〃	〃
	$-6/8\,H$	2·79	3·64	〃	〃
	$-5/8\,H$	1·03	2·41	〃	〃
	$-4/8\,H$	5·23	1·28	〃	〃
	$-3/8\,H$	7·87	0·314	〃	〃
	$-5/16\,H$	8·36			
	$-2/8\,H$	8·13	0·549	〃	〃
	$-1/8\,H$	5·92	1·44	〃	〃
	0	1·73	2·43	〃	〃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Top Surface
-H
$-\frac{7}{8}H$
$-\frac{6}{8}H$
$-\frac{5}{8}H$
$-\frac{4}{8}H$
$-\frac{2}{8}H$
0
Bottom Surface
$\frac{H}{L} = 0{\cdot}065$
$\frac{H}{L} = 0{\cdot}06784$

Fig. 4a.

$-(4/8)H$, $-(5/8)H$, $-(6/8)H$, $-(7/8)H$, $-H$ when $H/L=0{\cdot}065$, 0·06784, 0·07817, 0·110, 0·1305, 0·200, 0·235, 0·24992, 0·255, and 0·260 respectively;

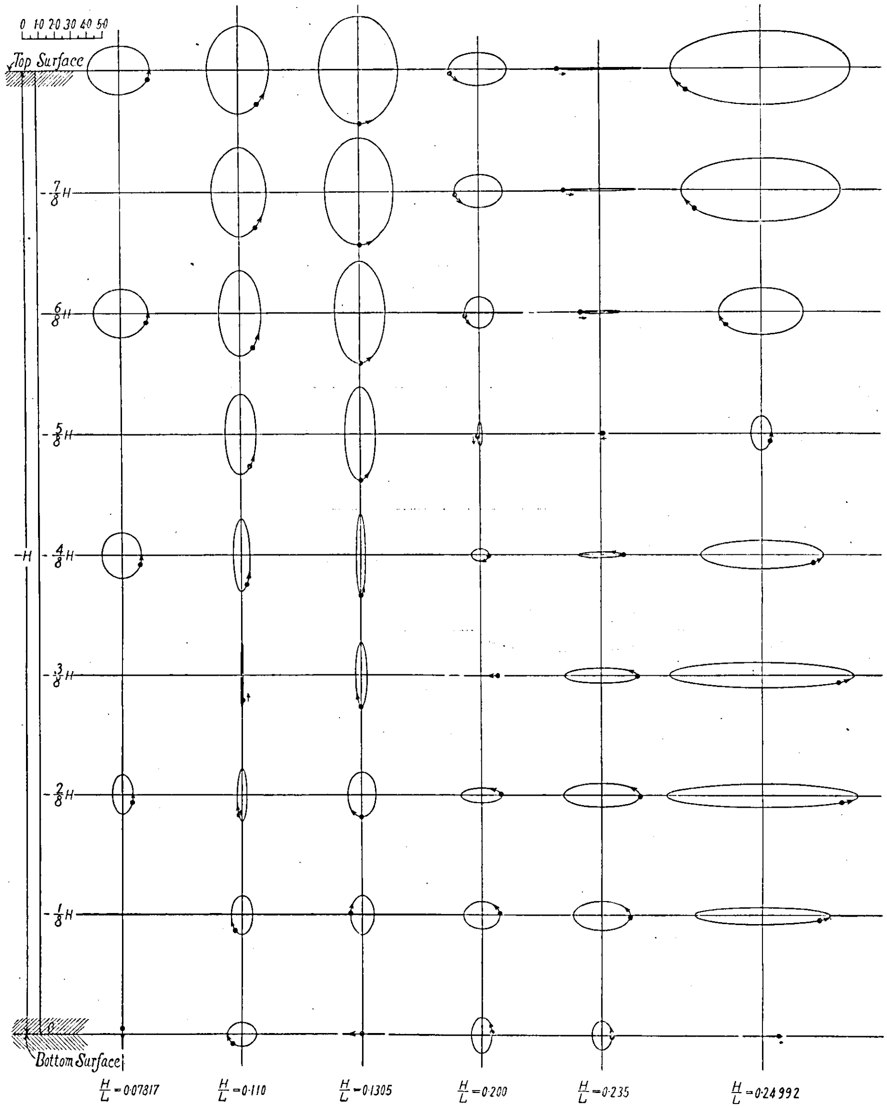

Fig. 4b.

and Figs. 5, 6 show the distributions of the horizontal and vertical amplitudes of vibration in the surface-layer when H/L has respectively the same values as in the case of Fig. 4.

Figs. 4a, 4b, 4c show that all the particles in the surface-layer pursue an elliptic orbit, of which the inclination angle that the major axis of the orbit makes with the horizontal plane surface becomes zero or 90°. Table IV shows that there are no phase-differences in the horizontal vibrations of the particles for their positions in the surface-

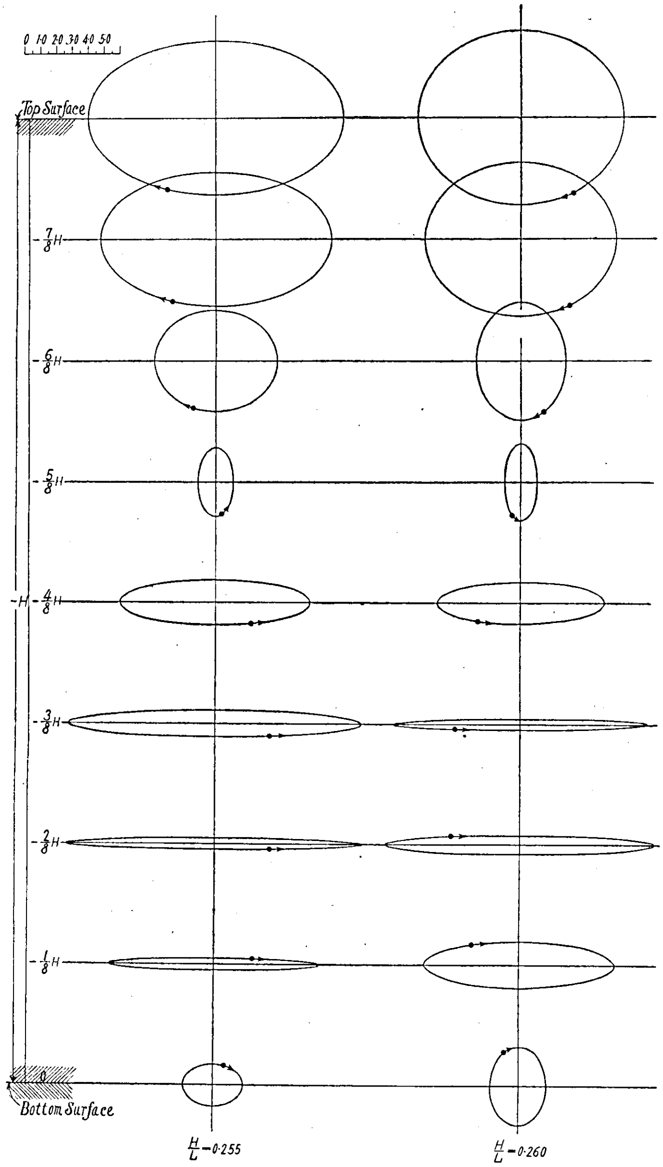

Fig. 4c.

layer,and also the same in the vertical vibration, so that Figs. 5 and 6 correspond respectively to the horizontal and the vertical vibration modes in the surface-layer for the respective cases of H/L.

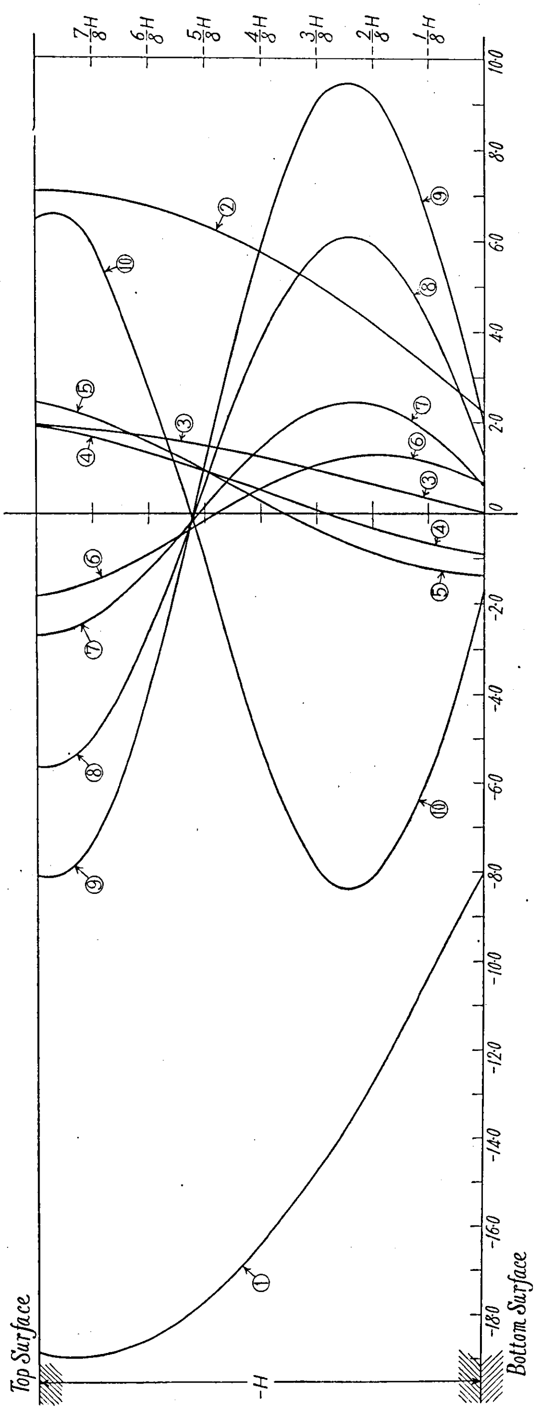

Fig. 5. Horizontal vibration modes in the surface-layer.

①, ②, ③, ④, ⑤, ⑥, ⑦, ⑧, ⑨, ⑩ show them for the respective cases when H/L=0·065, 0·06784, 0·07817, 0·110, 0·1305, 0·200, 0·235, 0·24992, 0·255, 0·260.

From Figs. 4a, 4b, 4c, 5, 6, it will be seen that it is only when H/L becomes 0·07817 that the horizontal mode of vibration in the surface-layer becomes most nearly equal to the gravest free vibration mode that may be excited in the surface-layer, when it may be assumed to be an

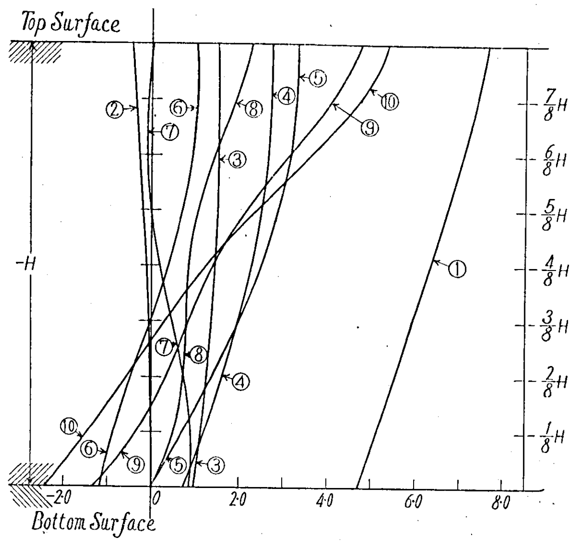

Fig. 6. Vertical vibration modes in the surface-layer.

①, ②, ③, ④, ⑤, ⑥, ⑦, ⑧, ⑨, ⑩ show them for the respective cases when $H/L=0{\cdot}065$, 0·06784, 0·07817, 0·110, 0·1305, 0·200, 0·235, 0·24992, 0·255, 0·260.

isolated elastic pendulum, while when H/L becomes smaller or greater than this value, i.e. when $H/L<0{\cdot}07817$, or $H/L>0{\cdot}07817$, the horizontal vibration mode differs greatly from this gravest free vibration mode. Next, when H/L becomes 0·1305, the vertical vibration mode becomes also nearly the gravest free vertical vibration modes that may be excited in the surface-layer, moreover, when $H/L<0{\cdot}1305$ or $H/L>0{\cdot}1305$, the vertical vibration mode differs considerably from this free vibration mode. It is a remarkable fact, moreover, that the periods calculated from these values, $H/L=0{\cdot}07817$ and 0·1305, become respectively equal to the resonance periods of the gravest modes[9] of the horizontal and

9) The writers studied the forced vibration in the surface-layer, due to a dilatational wave of harmonic type, obliquely incident on its bottom surface when $\lambda=\mu$, $\lambda'=\mu'$, $\rho=\rho'$, $\mu'/\mu=1/10$, $\theta=45°$, and obtained the values $H/L'=0{\cdot}048$, 0·080, which correspond to the resonance periods of the gravest mode of the horizontal and the vertical vibrations of the surface-layer, where L' is the wave-length of the primary incident dilatational wave. Let these resonance periods be T_1, T_2. Then $T_1=H/(0{\cdot}048\ v)=\sqrt{3}H/(0{\cdot}048\ V_2)$, and $T_2=\sqrt{3}H/(0{\cdot}080\ V_2)$, where v and V_2 respectively show the wave-velocities of the dilatational and distortional waves.

the vertical vibrations that are obtained when the dilatational wave of harmonic type is obliquely incident on the bottom surface of the surface-layer. For these reasons, therefore, the proper periods of the gravest mode of the horizontal and vertical vibrations of the surface-layer should respectively be obtained from $H/L=0{\cdot}07817$ and $H/L=0{\cdot}1305$. The proper periods of higher modes of vibration of the surface-layer may also be determined by the same method as that used to obtain those of the gravest mode in the present example. Discussions on the proper periods of higher modes are deferred to another occasion.

We may conclude from these discussions that there exist proper periods of vibration in the surface-layer that closely adheres to the subjacent semi-infinte medium, but when the primary incident wave is distortional, the amplitudes of the forced vibration (horizontal and vertical) on the top surface of the surface-layer do not necessarily become relatively large at such periods as are synchronous with the proper periods of that layer. These proper periods of vibration of the surface-layer usually differ in their horizontal and vertical vibrations; the proper period of the gravest mode of the horizontal vibration is usually longer than that of the vertical vibration.

When any dilatational or distortional wave, of which the periods are equal to the proper periods of horizontal or vertical vibrations of the surface-layer, is obliquely incident on the bottom surface of it, the respective vibration modes become nearly equal to the free vibration modes that may be excited in the surface-layer, when it may be regarded as an elastic solid that is isolated from the subjacent medium. It will be seen, however, that even if the period of the primary incident wave becomes synchronous with the proper periods of the vertical and horizontal vibrations of the surface-layer, the amplitudes of vibration on the top surface do not become infinite, as in the case of an isolated elastic body.

22. 斜めの入射波（調和波型横波）による表面層の振動（第1報）

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高山威雄

地殼表面層に入射する地震波が調和波型横波である場合にこの表面層に起される地震動を研究する事は大切な事である．本論文では入射波の入射角を 45° として取扱つたのであるが，入射波が調和波型縦波である場合に比較して地震動の性質に可なりの相違が存在する事を知つた．又この研究の結果と，入射波が調和波型縦波である場合の研究結果とから地殼表面層には入射波の種類には無關係な固有振動週期の存在する事が確められた．