25. *A "Free-Air" Reduction of Local Magnetic Anomalies.*

By Takesi NAGATA,

Earthquake Research Institute.

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1. *Introduction*

In my previous paper,[1] I gave a method for determining the subterranean structure directly from the distribution of magnetic anomalies on the earth's surface, due to the subterranean anomalous mass distribution. So long as we assume that the earth's surface is nearly a horizontal plane, the method proved valuable in analyzing actual local magnetic anomalies. If, however, the undulation of the earth's surface cannot be taken as a horizontal plane, it may not be possible to apply the method directly to the results of observations on the surface, in which case, the observed values must be reduced with the aid of a suitable correction formula to those on a horizontal plane.

A method of height correction has been proposed by A. Tanakadate[2] for calculating, with the aid of the conditions of "Divergenzfrei" and "Rotationsfrei" in the magnetic field, the normal values of the earth's magnetism on sea level. Tanakadate's method, however, may not be sufficient for reducing local magnetic anomalies in which the vertical gradient of the magnetic force is not independent of the height of the station, because the source of the magnetic anomalies lies near the earth's surface.

In this paper, the writer describes a method for reducing the observed values on an undulating surface to those on a horizontal plane with the aid of Fourier's analysis of the undulation in two-dimensional cases.

2. *Theory*

In my previous paper, the vertical and horizontal components of the magnetic force due to the subterranean magnetized body in a two-dimensional case were given by

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1) T. NAGATA, *Bull. Earthq. Res. Inst.*, **16** (1938), 550.
T. NAGATA, *Proc. Imp. Acad. Japan*, **14** (1938), 176.

2) A. TANAKADATE, *Journ. Coll. Sci. Tokyo Imp. Univ.*, **14** (1904), 1.

$$Z(x)=2\pi\sum_{n=0}^{\infty}A_n e^{-n(d-z)}\cos nx-2\pi\sum_{n=1}^{\infty}B_n e^{-n(d-z)}\sin nx$$

$$H(x)=2\pi\sum_{n=0}^{\infty}B_n e^{-n(d-z)}\cos nx+2\pi\sum_{n=1}^{\infty}A_n e^{-n(d-z)}\sin nx\,, \tag{1}$$

where A_n and B_n in the cases of a magnetized plane and a magnetized infinite dyke respectively are

$$\begin{aligned}
A_n&=n(a_n\sin\theta+b_n\cos\theta),\\
B_n&=n(a_n\cos\theta-b_n\sin\theta),\\
A_n&=\cos\varphi\{a_n\sin(\theta+\varphi)+b_n\cos(\theta+\varphi)\},\\
B_n&=\cos\varphi\{a_n\cos(\theta+\varphi)-b_n\sin(\theta+\varphi)\},
\end{aligned} \tag{2}$$

where a_n, b_n, θ, and φ are the Fourier coefficients of the distribution of the magnetic intensity on the subterranean elementary plane, the inclination of the magnetic force, and the dip angle of the dyke. Equation (1) holds also in the case of a horizontal magnetized layer or of a magnetized dyke of finite depth, etc., though the mathematical expressions of A_n and B_n are more complex in these cases than in the foregoing two.

Since the formulae of $Z(x)$ and $H(x)$ are similar, the Fourier coefficients A_n, B_n being merely exchanged, we shall, in what follows, mainly study the case of the vertical magnetic force $Z(x)$.

We take the heights of undulation of the earth's surface, measured from a suitable horizontal plane, as a function of x, that is,

$$z=f(x)\,.$$

If then putting

$$F(x)=e^{f(x)}\,, \tag{3}$$

we take $z=0$ to be the horizontal plane on which we desire to know the distribution of $Z(x)$ (on which plane all the relations described in my previous paper are based), we get

$$Z(x)=2\pi\sum_{n=0}^{\infty}A_n e^{-nd}F^n(x)\cos nx-2\pi\sum_{n=1}^{\infty}B_n e^{-nd}F^n(x)\sin nx^{3)}\,. \tag{4}$$

We further assume that (4) is expressed in Fourier's series of x, that is

$$Z(x)=\sum_{m=0}^{\infty}\alpha_m\cos mx+\sum_{m=1}^{\infty}\beta_m\sin mx\,, \tag{5}$$

while, as is well known from the theory of Fourier's series,

3) If $F(x)$ is larger than one, $F^n(x)$ diverges when n becomes larger. We can, however, make the right-hand side of eq. (4) converge, provided we change the zero position of z coordinate so as to make $f(x)$ negative throughout the whole region of x.

$$\alpha_0=\frac{1}{2\pi}\int_0^{2\pi} Z(\lambda)\,d\lambda$$

$$\alpha_m=\frac{1}{\pi}\int_0^{2\pi} Z(x)\cos m\lambda d\lambda\,, \qquad B_m=\frac{1}{\pi}\int_0^{2\pi} Z(\lambda)\sin m\lambda d\lambda\,. \tag{6}$$

If $F^n(x)$ is also expressed in Fourier's series as

$$F^n(x)=\sum_{k=0}^{\infty} S_k^{(n)}\cos kx+\sum_{k=1}^{\infty} t_k^{(n)}\sin kx\,, \tag{7}$$

then

$$\begin{aligned}\alpha_0=&\sum_{n=0}^{\infty} e^{-nd}A_k\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\cos\lambda d\lambda\\
&+\sum_{n=0}^{\infty} e^{-nd}A_n\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\cos n\lambda d\lambda\\
&-\sum_{n=1}^{\infty} e^{-nd}B_n\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\sin n\lambda d\lambda\\
&-\sum_{n=1}^{\infty} e^{-nd}B_n\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\cos n\lambda d\lambda\,,\end{aligned}$$

$$\begin{aligned}\alpha_m=&2\sum_{n=0}^{\infty} e^{-nd}A_n\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\cos n\lambda\cos m\lambda d\lambda\\
&+2\sum_{n=0}^{\infty} e^{-dn}A_n\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\cos n\lambda\cos m\lambda d\lambda\\
&-2\sum_{n=1}^{\infty} e^{-nd}B_n\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\sin n\lambda\cos m\lambda d\lambda\\
&-2\sum_{n=1}^{\infty} e^{-nd}B_n\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\sin n\lambda\cos m\lambda d\lambda\,,\end{aligned}$$

$$\begin{aligned}\beta_m=&2\sum_{n=0}^{\infty} e^{-nd}A_n\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\cos n\lambda\sin m\lambda\,d\lambda\\
&+2\sum_{n=0}^{\infty} e^{-nd}A_n\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\cos n\lambda\sin m\lambda\,d\lambda\end{aligned}$$

$$-2\sum_{n=1}^{\infty} e^{-\nu d}B_n\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\sin n\lambda\sin m\lambda d\lambda$$

$$-2\sum_{n=1}^{\infty} e^{-nd}B_n\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\sin n\lambda\sin m\lambda d\lambda. \tag{8}$$

With the aid of the relations

$$\int_0^{2\pi}\cos l\lambda d\lambda=\begin{cases}2\pi & (l=0)\\ 0 & (l\neq 0)\end{cases}$$

$$\int_0^{2\pi}\sin l\lambda d\lambda=0,$$

under the conditions $n>0$, $m>0$ and $k>0$, we get

$$\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\cos n\lambda\cos m\lambda d\lambda=\frac{\pi}{2}\left\{S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\},$$

$$\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\sin n\lambda\cos m\lambda d\lambda=\frac{\pi}{2}\left\{t_{m+n}^{(n)}-t_{[m-n]}^{(n)}+t_{[n-m]}^{(n)}\right\},$$

$$\sum_{k=1}^{\infty} t_k^{(n)}\int_0^{2\pi}\sin k\lambda\cos n\lambda\sin m\lambda d\lambda=\frac{\pi}{2}\left\{t_{m+n}^{(n)}+t_{[m-n]}^{(n)}-t_{[n-m]}^{(n)}\right\},$$

$$\sum_{k=0}^{\infty} S_k^{(n)}\int_0^{2\pi}\cos k\lambda\sin n\lambda\sin m\lambda d\lambda=\frac{\pi}{2}\left\{-S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\},$$

and

$$\int_0^{2\pi}\sin k\lambda\cos n\lambda\cos m\lambda d\lambda=\int_0^{2\pi}\cos k\lambda\sin n\lambda\cos m\lambda d\lambda$$

$$=\int_0^{2\pi}\cos k\lambda\cos n\lambda\sin m\lambda d\lambda=\int_0^{2\pi}\sin k\lambda\sin m\lambda\sin n\lambda d\lambda=0,$$

where

$$S_{[m-n]}^{(n)}=\begin{cases}S_{m-n}^{(n)} & (m-n\geqq 0)\\ 0 & (m-n<0),\end{cases}$$

$$S_{[n-m]}^{(n)}=\begin{cases}S_{n-m}^{(n)} & (n-m\geqq 0)\\ 0 & (n-m<0),\end{cases}$$

$$t_{[m-n]}^{(n)}=\begin{cases}t_{m-n}^{(n)} & (m-n\geqq 0)\\ 0 & (m-n<0),\end{cases}$$

$$t_{[n-m]}^{(n)}=\begin{cases}t_{n-m}^{(n)} & (n-m\geqq 0)\\ 0 & (n-m<0).\end{cases} \tag{10}$$

Putting the relations given by (9) into (8), we get

$$\alpha_m=\pi\sum_{n=1}^{\infty}A_n e^{-nd}\left\{S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\}-\pi\sum_{n=1}^{\infty}B_n e^{-nd}\left\{t_{m+n}^{(n)}-t_{[m-n]}^{(n)}+t_{[n-m]}^{(n)}\right\}$$

$$\beta_m=\pi\sum_{n=1}^{\infty}A_n e^{-nd}\left\{t_{m+n}^{(n)}+t_{[m-n]}^{(n)}-t_{[n-m]}^{(n)}\right\}-\pi\sum_{n=1}^{\infty}B_n e^{-nd}\left\{-S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\}, \tag{11}$$

while by calculating (6), we obtain

$$\alpha_0=2\pi A_0+\pi\sum_{n=1}^{\infty}A_n e^{-nd}S_n^{(n)}-\pi\sum_{n=1}^{\infty}B_n e^{-nd}t_n^{(n)}. \tag{12}$$

Since the constant term of the Fourier coefficient given by (12) is not suitable for our method of analysis, we shall not discuss (12) any further.

If we let

$$\alpha_m^0=2\pi A_m e^{-md}, \qquad \beta_m^0=2\pi B_m e^{-md}, \tag{13}$$

the distribution of vertical component $Z(x)$ on the $z=0$ plane is given by

$$Z_0(x)=\sum_{m=1}^{\infty}\alpha_m^0\cos mx+\sum_{m=1}^{\infty}\beta_m^0\sin mx+\text{const}, \tag{14}$$

while from (11) and (13) we get

$$\alpha_m=\frac{1}{2}\sum_{n=1}^{\infty}\alpha_n^0\left\{S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\}+\frac{1}{2}\sum_{n=1}^{\infty}\beta_n^0\left\{t_{m+n}^{(n)}-t_{[m-n]}^{(n)}+t_{[n-m]}^{(n)}\right\},$$

$$\beta_m=\frac{1}{2}\sum_{n=1}^{\infty}\alpha_n^0\left\{t_{m+n}^{(n)}+t_{[m-n]}^{(n)}-t_{[n-m]}^{(n)}\right\}+\frac{1}{2}\sum_{n=1}^{\infty}\beta_n^0\left\{-S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\}.$$

$$(m\geqq 1) \tag{15}$$

In a special case of $f(x)=0$, that is, where there are no undulations of the earth's surface,

$$S_0^{(n)}=1,\quad S_p^{(n)}=0\quad (p\geqq 1),$$
$$t_q^{(n)}=0\quad (q\geqq 0),$$

whence

$$\alpha_m=\frac{1}{2}\alpha_m^0\left\{S_0^{(n)}+S_0^{(n)}\right\}=\alpha_m^0$$

$$\beta_m=\frac{1}{2}\beta_m^0\left\{S_0^{(n)}+S_0^{(n)}\right\}=\beta_m^0.$$

Thus it is proved that α_m and β_m given by (15) agree in the special case with Fourier coefficients on the horizontal plane, α^0_m and β^0_m, given by (14).

In (15), α_m, β_m, $S^{(n)}_p$ and $t^{(n)}_q$ are known values, seeing that α_m and β_m are given by Fourier analyses of the observed values on the earth's surface, while $S^{(n)}_p$ and $t^{(n)}_q$ are also given by Fourier analyses of the function of the undulation of the surface, i. e. $e^{nf(x)}$.

Theoretically speaking, therefore, we can obtain the values of the unknown quantities α^0_n, β^0_n by solving the infinite simultaneous equations of the first order involving infinite unknown quantities that are given by (15). Although it seems to be difficult to obtain rigorous solutions of (15), we can estimate the values of these unknown quantities with the aid of successive approximations, because in many of the actual problems, the absolute magnitudes of $S^{(n)}_0$ are much larger compared with the other terms of $S^{(n)}_p$ and $t^{(n)}_q$.

Hence, the values of α^0_n and β^0_n in the first approximation are given by

$$\begin{aligned}
\frac{1}{2}\left\{S^{(m)}_{2m}+2S^{(m)}_0\right\}\alpha^0_m &= \alpha_m-\frac{1}{2}\sum_{n\neq m}\alpha_n\left\{S^{(n)}_{m+n}+S^{(n)}_{[m-n]}+S^{(n)}_{[n-m]}\right\} \\
&\quad -\frac{1}{2}\sum_{n}\beta_n\left\{t^{(n)}_{m+n}+t^{(n)}_{[m-n]}+t^{(n)}_{[n-m]}\right\}, \\
\frac{1}{2}\left\{-S^{(m)}_{2m}+2S^{(m)}_0\right\}\beta^0_m &= \beta_m-\frac{1}{2}\sum_{n}\alpha_n\left\{t^{(n)}_{m+n}+t^{(n)}_{[m-n]}-t^{(n)}_{[n-m]}\right\} \\
&\quad -\frac{1}{2}\sum_{n\neq m}\beta_m\left\{-S^{(n)}_{m+n}+S^{(n)}_{[m-n]}+S^{(n)}_{[n-m]}\right\}.
\end{aligned} \tag{16}$$

Taking the values of α^0_n and β^0_n instead of α_n and β_n on the right-hand side of (16), we can obtain sufficiently accurate values of these quantities. In many cases of actual calculations, however, the foregoing second approximation is not always necessary; details of the practical treatment will be discussed in connexion with the calculation of an actual example in the following paragraph.

3. *An actual example*

In order to check the feasibility of the present method, it will be applied to a case in which the subterranean structure is already known. Fig. 1 shows the subterranean structure and the undulation of the earth's surface. We assume that the dyke shown in the figure is magnetized in the direction dipping 45°, while the distribution of $Z(x)$ on the undulating earth's surface is given as a functions of x alone, as shown in Fig. 2, the heights of the undulation of the earth's surface being measured from the $z=0$ plane.

Analysing $Z(x)$ with the aid of practical Fourier analysis of the twelveth degree, we get the values of the Fourier coefficients α_m, β_m, as shown in Table I.

Table I.

n	1	2	3	4	5	6	7	8	9	10	11	12
α_n	−117·1	+22·6	+27·3	−35·6	+45·7	−20·7	+ 6·4	− 2·9	+12·1	− 8·4	+ 0·8	− 0.5
β_n	+191·6	−107·2	+78·3	−50·8	+21·3	− 0·8	−15·2	+15·5	− 5·3	−13·5	+ 5·8	—

On the other hand, we express $e^{nf(x)}$ also in Fourier series, where $f(x)$ is the undulation of the earth's surface, the coefficients $S_k^{(n)}$, $t_k^{(n)}$ being given in Table II.

As will be seen from this table, $S_0^{(n)}$ is usually much larger than the other quantities of $S_k^{(n)}$ and $t_k^{(n)}$, at any rate in the lower degree of n. Hence, putting these values of $S_k^{(1)}$ and $t_k^{(1)}$ into (16), it is possible to get sufficiently accurate values of α_1^0 and β_1^0. In the case of a higher degree of n than 0, the following relation may give a sufficiently reliable approximation,

$$\frac{1}{2}\left\{S_{2m}^{(m)}+2S_0\right\}$$
$$=\alpha_m-\frac{1}{2}\sum_{n=1}^{m-1}\alpha_n^0\left\{S_{m+n}^{(n)}+S_{m-n}^{(n)}\right\}-\frac{1}{2}\sum_{n=m+1}^{N}\alpha_n\left\{S_{m+n}^{(n)}+S_{n-m}^{(n)}\right\}$$
$$-\frac{1}{2}\sum_{n=1}^{m-1}\beta_n^0\left\{t_{m+n}^{(n)}-t_{m-n}^{(n)}\right\}-\frac{1}{2}\sum_{n=m+1}^{N}\beta_n\left\{t_{m+n}^{(n)}+t_{n-m}^{(n)}\right\},$$

$$\frac{1}{2}\left\{-S_{2m}^{(m)}+2S_0\right\}$$
$$=\beta_m-\frac{1}{2}\sum_{n=1}^{m-1}\alpha_n^0\left\{t_{m+n}^{(n)}+t_{m-n}^{(n)}\right\}-\frac{1}{2}\sum_{n=m+1}^{N}\alpha_n\left\{t_{m+n}^{(n)}-t_{n-m}^{(n)}\right\}$$
$$-\frac{1}{2}\sum_{n=1}^{m-1}\beta_n^0\left\{-S_{m+n}^{(n)}+S_{m-n}^{(n)}\right\}-\frac{1}{2}\sum_{n=m+1}^{N}\beta_n\left\{-S_{m+n}^{(n)}+S_{n-m}^{(n)}\right\}. \tag{17}$$

The values of α_m^0 and β_m^0 thus obtained and the distribution of $Z_0(x)$ on the horizontal plane calculated from these values are shown in Table III and Fig. 2 respectively.

Table II (a). Values of $S_k^{(n)}$.

$k \backslash n$	1	2	3	4	5	6	7	8	9	10	11	12
0	+0·888	+0·799	+0·731	+0·675	+0·632	+0·597	+0·570	+0·546	+0·530	+0·518	+0·506	+0·498
1	−0·007	−0·016	−0·028	−0·043	−0·058	−0·077	−0·095	−0·109	−0·129	−0·149	−0·168	−0·195
2	+0·128	+0·224	+0·297	+0·356	+0·403	+0 440	+0·472	+0·498	+0·522	+0·555	+0·571	+0·591
3	−0·036	−0·064	−0·092	−0·106	−0·123	−0·138	−0·153	−0·167	−0·181	−0·198	−0·214	−0·223
4	+0·005	+0·019	+0·037	+0·060	+0·082	+0·108	+0·128	+0·153	+0·175	−0·194	+0·220	+0·243
5	+0·010	+0·013	+0·011	+0·007	−0·003	−0·007	−0·017	−0·027	−0·038	−0·051	−0·063	−0·071
6	+0·003	+0·004	+0·008	+0·012	+0·018	+0·024	+0·033	+0·038	+0·047	+0·058	+0·068	+0·078
7	+0·002	+0·006	+0·009	+0·012	+0·017	+0·015	+0·016	+0·017	+0·017	+0·018	+0·017	−0 011
8	0·000	+0·001	0·000	0·000	0·000	−0·003	−0·001	−0·001	0·000	+0·003	+0·002	+0·001
9	+0·003	+0·006	+0 015	+0·014	+0·015	+0·017	+0·022	+0·028	+0·033	+0·033	+0·041	+0·041
10	+0·003	+0·004	+0 005	+0·006	+0·006	+0·003	+0·003	+0·003	0·000	−0·012	−0·008	−0·013
11	+0·005	+0·008	+0·012	+0·014	+0·018	+0·025	+0·030	+0·029	+0·034	+0·039	+0·043	+0·055
12	−0·001	−0·002	−0·003	−0·003	−0·004	−0·002	−0·005	−0·006	−0·008	−0·010	−0·010	−0·012

Table II (b). Values of $t_k^{(n)}$.

$k \backslash n$	1	2	3	4	5	6	7	8	9	10	11	12
1	−0·083	−0·133	−0·166	−0·187	−0·198	−0·199	−0·201	−0·202	−0·200	−0·198	−0·196	−0·188
2	−0·008	−0·018	−0·027	−0·034	−0·040	−0·039	−0·040	−0·045	−0·043	−0·040	−0·036	−0·026
3	+0·004	−0 006	−0 021	−0·040	−0 061	−0·082	−0·102	−0·123	−0·142	−0·163	−0·178	−0·191
4	−0·001	+0·003	+0·007	+0·013	+0·020	+0·028	+0·034	+0·039	+0·048	+0·053	+0·062	+0·068
5	−0·012	−0·020	−0·031	−0·038	−0·045	−0·057	−0·068	−0·079	−0·092	−0·103	−0·118	−0·138
6	+0·006	+0·008	+0·011	+0·013	+0·016	+0·018	+0 023	+0·029	+0·036	+0·047	+0·055	+0·061
7	−0·005	−0·010	−0·016	−0·023	−0·028	−0·037	−0·048	−0·053	−0·062	−0·072	−0·083	−0·098
8	−0·001	0·000	+0·002	+0·003	+0·007	+0·011	−0·013	+0·018	+0·023	+0·028	+0·035	+0·042
9	−0·001	−0·003	−0·004	−0·007	−0·012	−0·013	−0·017	−0·021	−0·027	−0·036	−0·043	−0·046
10	0·000	−0·002	0·000	−0·001	0·000	+0·004	+0·005	+0·005	+0·007	+0·010	+0·014	+0·019
11	−0·002	−0·003	−0·006	−0·008	−0·008	−0·009	−0·011	−0·012	−0·013	−0·017	−0·021	−0·018

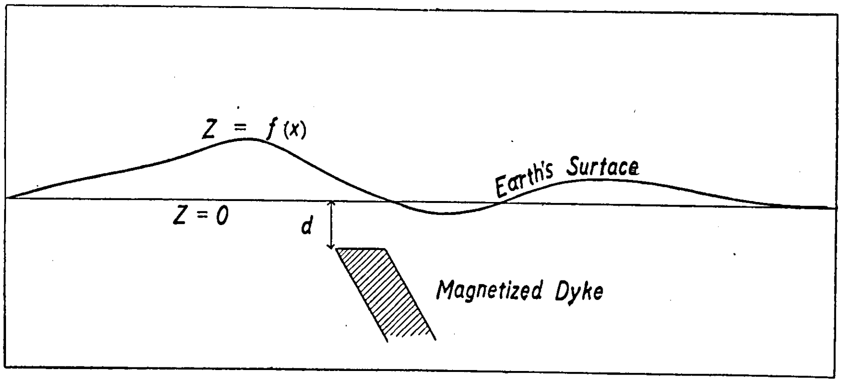

Fig. 1. Undulation of earth's surface and magnetized subterranean body.

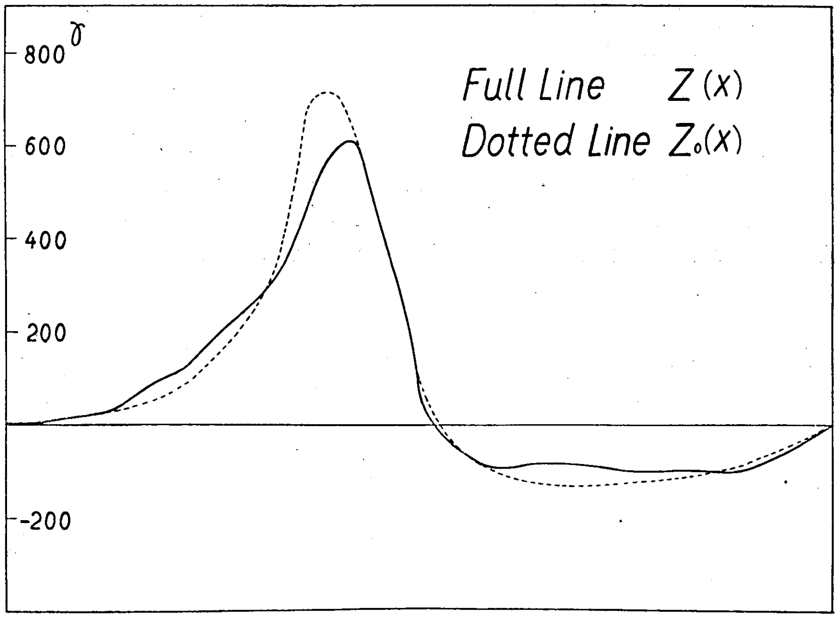

Fig. 2. $Z(x)$; Vertical Magnetic Force on $z=f(x)$
$Z_0(x)$; Vertical Magnetic Force on $z=0$, reduced from $Z(x)$.

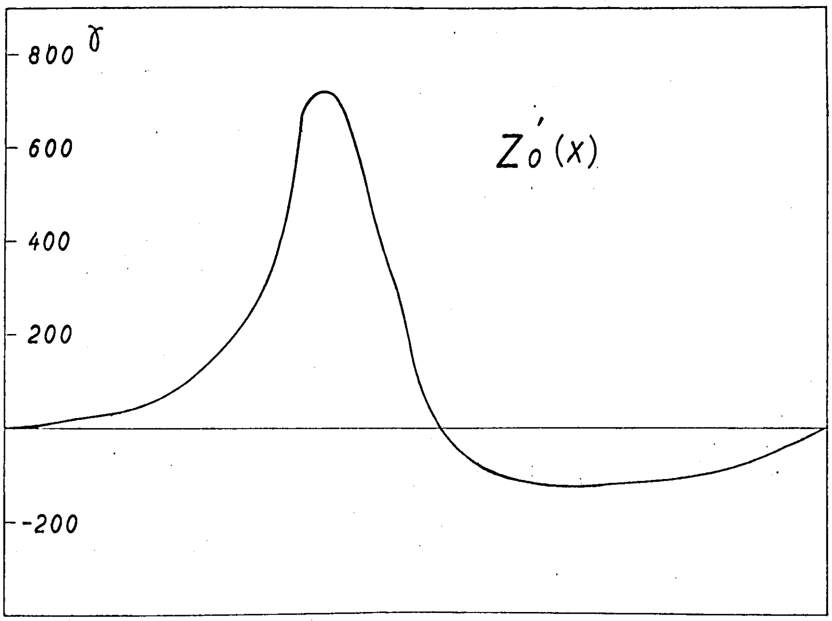

Fig. 3. $Z_0'(x)$, Vertical Magnetic Force on $z=0$, directly calculated from subterranean mass.

It has already been proved that it is possible to estimate the subterranean mass distribution from these reduced values of α_m^0 and β_m^0 with the aid of analysis – the method given in my previous paper.

On the other hand, the distribution of $Z_0'(x)$, calculated directly from the given mass distribution, is shown in Fig. 3. Comparing these two curves shown in Figs. 2 and 3, we notice that these two are in good agreement.

The Fourier coefficients α_m^0, β_m^0 obtained from Fourier analysis of $Z_0'(x)$ given in Fig. 4 are shown in Table III.

Table III.

n	α_n	α_n^0	$\alpha_n^{0\prime}$	β_n	β_n^0	$\beta_n^{0\prime}$
1	−117	−135	−137	+192	+216	+216
2	+ 23	+ 35	+ 39	−107	−139	−154
3	+ 27	+ 37	+ 40	+ 78	+ 91	+ 98
4	− 36	− 54	− 48	− 51	− 47	− 38
5	+ 46	+ 53	+ 49	+ 21	+ 8	+ 1
6	− 21	− 12	− 15	− 1	+ 16	+ 14

Although the coefficients α_m^0, β_m^0 obtained from these two different methods differ somewhat, a part of the difference being due to the error in the calculations of $Z(x)$ and $Z_0'(x)$ directly from the given mass distribution with the aid of numerical integration, we may say that the effect of the difference on the general tendency of the distribution of magnetic anomaly consisting of these values is very small. Thus, we may conclude that the present reduction method is quite practicable in actual problems, provided we do not discuss the small quantities of the order of the error in the calculation above mentioned.

4. *Conclusion*

As the Fourier coefficients α_m', β_m' of the distribution of horizontal magnetic intensity are related to α_m and β_m by

$$\alpha_m' = \beta_m, \qquad \beta_m' = -\alpha_m,$$

exactly the same reduction method can be applied to the case of horizontal magnetic intensity.

The present method can be applied only to two-dimensional problems. The similar method for three-dimensional problems is so complex in mathematical expressions that the actual calculations are very laborious. I hope that a more simple method applicable to both two and three dimensional cases will be discovered in the near future.

In conclusion, the writer wishes to express his sincerest thanks to Dr. C. Tsuboi and Dr. R. Takahasi for much encouragement received.

25. 局部的地磁氣異常に對する高度補正

地震研究所　永　田　　武

二次元的取扱ひの出來る場合に，凹凸のある地表で測定した地磁氣の局部異常の値を水平面上の値に補正する一方法を提出する．水平面上での地磁氣異常の分布が解れば，それから地下構造が算出出來る事は前論文に述べた如くである．

地表を $z=f(x)$ で表はし，$F(x)=e^{f(x)}$ とする時，

$$F^n(x)=\sum_n S_k^{(n)}\cos kx+\sum_n t_k^{(n)}\sin kx$$

と展開する事が出來れば，地表 $Z=f(x)$ 上の垂直磁力を

$$Z(x)=\sum_n \alpha_n\cos nx+\sum_n \beta_n\sin nx$$

とし，求める水平面 $z=0$ 上の垂直磁力を

$$Z_0(x)=\sum_n \alpha_n^0\cos nx+\sum_n \beta_n^0\sin nx$$

とすれば

$$\alpha_m=\frac{1}{2}\sum_{n-1}^{\infty}\alpha_n^0\left\{S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\}+\frac{1}{2}\sum_{n=1}^{\infty}\beta_n^0\left\{t_{m+n}^{(n)}-t_{[m-n]}^{(n)}+t_{[n-m]}^{(n)}\right\}$$

$$\beta_m=\frac{1}{2}\sum_{n=1}^{\infty}\alpha_n^0\left\{t_{m+n}^{(n)}+t_{[m-n]}^{(n)}-t_{[n-m]}^{(n)}\right\}+\frac{1}{2}\sum_{n-1}^{\infty}\beta_n^0\left\{-S_{m+n}^{(n)}+S_{[m-n]}^{(n)}+S_{[n-m]}^{(n)}\right\}$$

なる關係が成立する．幸ひな事に一般に實際の場合には $S_0^{(n)}$ が他の $S_p^{(n)}$，$t_q^{(n)}$ の項に比して著しく大きいので上の式の近似解は容易に求められる．

實例によつて，上に述べた方法が充分の精度を與へる事を證明した．