6. *Elastic and Viscous Properties of a Certain Kind of Rock.*

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1. Introduction.

In order to clearly understand such geophysical problems as the chronic deformation of the earth's crust and the propagation of seismic waves through it, it is necessary to study first of all the physical properties of the materials that compose the earth's crust, such as rocks, etc. Unless the properties of such substances in the upper layer as well as in the interior of the earth are known, it is difficult to ascertain the exact mechanism by means of which the seismic waves are propagated in the earth's crust.

The writer has begun a study of the physical properties of the materials just mentioned, his experiments on the physical properties of certain soils and sands described in his previous papers[1] being the first step in this direction.

A number of the physical properties of rocks have been studied by H. Nagaoka[2], S. Kusakabe[3], F. D. Adams and E. G. Coker[4], P. W. Bridgman[5], W. A. Zisman[6], F. Birch and D. Bancroft[7], and others,[8] from the results of which it seems that, owing to the nature of rocks differing so much from that of the common metals, no perfectly elastic rock has yet been found, so that Hooke's law does not seem to hold even for very small stresses, the statically determined values of their modu-

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[1] M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **14** (1936), 632; **15** (1937), 671.
K. IIDA, *Bull. Earthq. Res. Inst.*, **15** (1937), 828; **16** (1938), 131; 391.
K. IIDA, *Zisin*, **9** (1937), 195; 243, (in Japanese).

[2] H. NAGAOKA, *Publ. Earthq. Inv. Comm.*, **4** (1900), 47.

[3] S. KUSAKABE, *Publ. Earthq. Inv. Comm.*, **14** (1902), 1; **17** (1904), 1; **22** (1906), 27.

[4] F. D. ADAMS and E. G. COKER, *Carnegie Inst. Washington Publ.*, **46** (1906), 69.

[5] P. W. BRIDGMAN, *Am. Journ. Sci.*, **7** (1925), 81. *Proc. Am. Acad. Arts. Sci.*, **63** (1929), 401; **64** (1929), 39. *Journ. Geol.*, **44** (1936), 653.

[6] W. A. ZISMAN, *Proc. Nat. Acad. Sci.*, **19** (1933), 653; 666; 680.

[7] F. BIRCH and D. BANCROFT, *Journ. Geol.*, **46** (1938), 59; 113.

[8] L. H. ADAMS and E. G. WILLIAMSON, *Journ. Frank. Inst.*, **195** (1923), 475.
A. L. DAY, R. S. SOSMAN, and J. C. HOSTETTER, *Amer. Journ. Sci.*, **37** (1914), 1.

li varying markedly with the applied forces. Recently M. Ide⁹⁾ studied the elastic properties of rocks by means of the dynamic method. S. Kusakabe and M. Ide show that there is a tendency for the statically determined values to be smaller than the dynamically determined values, and that the difference between them for the less compact sedimentary rocks is as large as thirty per cent in some cases. S. Kusakabe¹⁰⁾ further investigated the remarkable effect of moisture on the elastic behaviour of rocks. In his previous papers¹¹⁾, the writer already reported that the elastic properties of certain kinds of soils differ with their water contents.

The object of our present experiments was to ascertain the nature of rocks with reference to their elastic and viscous properties, for which purpose we studied not only their elastic behaviour by means of static and dynamic methods of experiment, but also their visco-elastic properties. We investigated, besides, the effect of water on their elastic and viscous behaviour. In order to compare our findings with those for soils, such as variation in elastic properties with water content and the relation between the normal-tangential viscosity ratio and Poisson's elasticity ratio, with those that could be detected in rocks, we selected a typical rock that possessed pores in abundance and would consequently absorb much water, namely, pumice.

2. Description of the Rock Specimen.

The pumice, with which the present experiments were made, was obtained from Niizima¹²⁾, one of the seven islands of Izu, Japan. The geological studies of the rocks of these islands have already been exhaustively treated by H. Tsuya¹³⁾, so that their geological characters are well known.

To facilitate the study of a rock of this kind, we had a stone-mason

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9) J. M. IDE, *Proc. Nat. Acad. Sci.*, **22** (1936), 81; 482.

10) S. KUSAKABE, *loc. cit.*, 3).

11) *loc. cit.*, 1)

12) These rock specimens were collected by members of the party led by Prof. T. Matuzawa, who visited Niizima after a strong earthquake occurred in its neighbourhood on December 27, 1936. We are indebted to Dr. H. Kawasumi, who presented us with the rock specimens. The damages of structure built of this kind of rock due to this earthquake were reported by some investigators: S. HAENO, H. KAWASUMI, F. KISHINOUYE, T. MATUZAWA, T. SUZUKI, R. YOSIYAMA, *Zisin* **9** (1937), 45. T. HAGIWARA and S. OMOTE, *Bull. Earthq. Res. Inst.*, **15** (1937), 559, (in Japanese).
The value of the strength of this rock was found to be $3{\cdot}37$ kgW/cm^2 by the extreme limit of its bending. This value is comparable to that of $3{\cdot}09\sim4{\cdot}35$ kgW/cm^2 obtained by Prof. F. Tanaka.

13) H. TSUYA, *Bull. Earthq. Res. Inst.*, **16** (1933), 171.

cut the rock into cylinders of about 48 cm in height and 5 cm in diameter. With these specimens both dynamic and static measurements were carried out. A photograph of the specimen is shown in Fig. 1 (a), its section being shown in Fig. 1 (b), from which it will be seen that the rock is very porous.

Fig. 1. (a) Photograph of rock specimen.

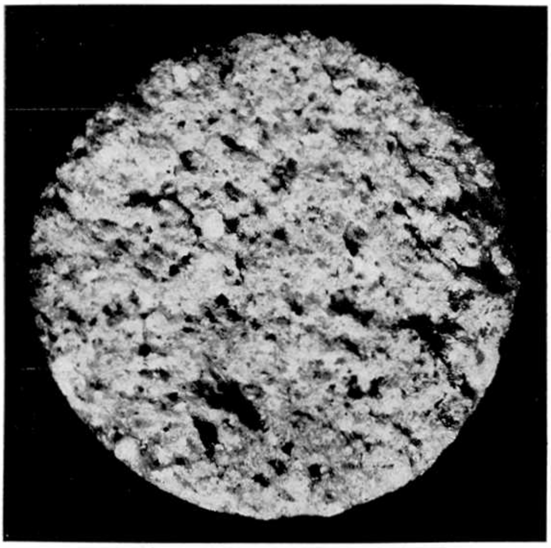

Fig. 1. (b) Section of rock cylinder.

For studying the visco-elastic properties of this kind of porous rock, we had made for us another rock cylinder, 10 cm in length and 1·7 cm in diameter.

3. Experiments on Variation in Rock Density and Porosity with Water Content.

In order to investigate the physical properties of the rocks in question under a variety of conditions, we experimented with them with various water contents, to obtain which, the specimens were left in water for about two months, after which they were removed from the water and gradually dried in ordinary room temperature. In this way, we obtained specimens in various degrees of water content.

To observe the variation in rock density and porosity with water content, the specimens were first weighed in their most moist state, after which they were weighed in various water contents. We also computed their volume from their dimensions. Then we calculated their densities ρ from the ratio of their weight to their volume. Further, by means of a special kind of specific gravity bottle, we measured the true density of the specimens by crushing them into small grains. The value of the true density was 2·314. Next we computed the porosity P from the true density and the bulk density, that is

$$P = 1 - \frac{G}{G_s} \qquad (1)$$

where G_s is the true density, G a bulk density corresponding to a water content of w per cent. These results are shown in Table I and in Fig. 2,

Table I. Bulk Density, Porosity and Water Content of Pumice-specimen.

No.	Water cont. w (%)	Density ρ	Porosity P (%)
1	0	0·630	72·8
2	1·2	0·636	72·5
3	3·0	0·646	72·1
4	4·3	0·650	71·9
5	6·0	0·665	71·2
6	8·2	0·682	70·5
7	9·1	0·690	70·2
8	11·4	0·711	69·2
9	13·4	0·726	68·6
10	14·6	0·737	68·1
11	15·9	0·749	67·6
12	18·0	0·769	66·7
13	21·4	0·801	65·4
14	22·9	0·817	64·7
15	24·9	0·840	63·7

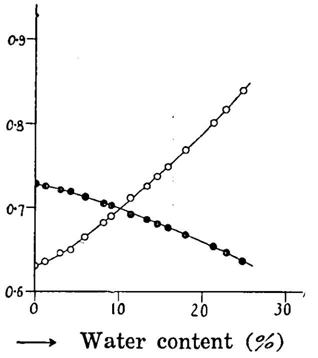

Fig. 2. Variation in rock density and porosity with water content.
Ordinate: Density and porosity. Abscissa: Water content (%). ● Density. ○ Porosity.

in which the densities and porosities were plotted as ordinates and the water contents as abscissae. As will be seen from this figure, the densities seem gradually to increase with increase in water content up to 10 per cent, after which the increase is linear with increase in water content, while the porosity gradually decreases with increase in water content, owing to a certain number of pores being filled with water. However the density did not exceed 1, while the value at which the porosity becomes zero could not be observed, so far as the present experiments were concerned, whence it follows that the larger number of pores were not filled entirely with water. It may, therefore, be concluded that water does not penetrate every pore in the inner part of the rock specimen.

4. Determinations of the Elastic Constants and the Normal-Tangential Solid Viscosity Coefficients by means of Dynamic Methods of Experiment.

To investigate the elastic properties of rocks by means of vibration methods, we employed the same longitudinal- and torsional vibration equipment that was used previously in our experiments with soils[14], having first of all satisfied ourselves by trials that it would suit the present dynamic measurements.

The methods of experiment were the same as those used previously, namely, by first setting the rock specimen on the vibrating plate of the longitudinal-vibration apparatus, we obtained a photographic record while applying forced longitudinal vibrations with continuous increased frequencies to the specimen up to the limit of vibration frequency; then by setting this same specimen on the vibrating disc of the torsional-vibration apparatus, we again obtained a photographic record while applying forced torsional vibrations up to the same limit. The procedure was repeated on identical material, but placed at various heights that were reduced by about equal amounts. We thus obtained the relations between the fundamental resonance period and the height of the specimens in the two cases of longitudinal and torsional vibrations, for rocks under the same conditions. From these relations obtained by experiments, we computed the two constants of elasticity and of Poisson's ratio, as well as the two kinds of solid viscosity coefficients of the specimen.

The diagram showing the relations between the fundamental longitudinal and torsional resonance periods and the height of the rock speci-

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[14] *loc. cit.*, 1).

men are shown in Fig. 3, in which the period has been taken as ordinate and the height as abscissa.

In the present case, we determined both elastic constants E, μ and the normal and tangential solid viscosity coefficients, γ_l, γ_t by means of least squares with the aid of equations

$$\left.\begin{aligned} T_l &= \frac{4h}{\sqrt{\dfrac{E}{\rho}-\dfrac{\pi^2\gamma_l^2}{16\rho^2h^2}}}\,, \\ T_t &= \frac{4h}{\sqrt{\dfrac{\mu}{\rho}-\dfrac{\pi^2\gamma_t^2}{16\rho^2h^2}}}\,, \end{aligned}\right\} \tag{2}$$

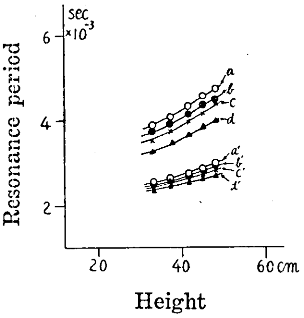


Fig. 3. Relation between the fundamental resonance period of vibration and the height of rock specimen. (a)~(d): torsional vibration. (a')~(d'): longitudinal vibration. (a), (a') water content $w=24{\cdot}9\%$. (b), (b') $w=21{\cdot}4\%$. (c), (c') $w=11{\cdot}4\%$. (d), (d') $w=0{\cdot}0\%$.

into which we put the experimental values, such as T_l, T_t corresponding to each height h. We then computed the longitudinal and the transverse wave-velocities V_l, V_t in the rock specimen by means of the relations,

$$V_l=\sqrt{\frac{E}{\rho}}\,, \quad V_t=\sqrt{\frac{\mu}{\rho}}\,. \tag{3}$$

All the values thus obtained are shown in Tables II, III, from which

Table II. Longitudinal and Torsional Wave-velocities, Normal and Tangential Viscosity Coefficients, and Water Content.

No.	Water cont. w (%)	Long. vel. V_l (m/sec)	Tors. vel. V_t (m/sec)	Norm. visco. coef. γ_l (C.G.S.)	Tang. visco. coef. γ_t (C.G.S.)	$\frac{V_l}{V_t}$
				$\times 10^6$	$\times 10^6$	
1	0	1186·1±120·1	772·8±80·0	4·962±0·315	1·041±0·096	1·536
2	1·2	1174·5±115·6	759·8±72·1	3·869±0·263	0·883±0·073	1·546
3	3·0	1177·4± 87·7	755·4±50·3	3·406±0·178	0·839±0·084	1·558
4	4·3	1160·0±100·3	746·7±81·2	3·133±0·330	0·778±0·091	1·556
5	6·0	1154·2± 98·7	740·9±65·3	2·775±0·296	0·711±0·079	1·557
6	8·2	1142·6±138·6	730·8±75·2	2·523±0·300	0·673±0·068	1 564
7	9·1	1139·7±150·0	729·3±71·5	2·481±0·198	0·652±0·052	1·562
8	11·4	1136·8±147·1	723·5±70·3	2·166±0·203	0·624±0·049	1·571
9	13·4	1139·7±120·8	723·5±71·6	2·124±0·175	0·631±0·050	1·575
10	14·6	1128·1±112·1	713·4±73·9	1·808±0·150	0·553±0·039	1·583
11	15·9	1125·2±110·0	709·0±46·7	1·724±0·181	0·523±0·041	1·588
12	18·0	1128·1±108·7	711·9±82·3	1·661±0·121	0·500±0·028	1·582
13	21·4	1123·7±105·9	703·2±66·4	1·661±0·098	0·503±0·053	1·599
14	22·9	1110·7±121·7	694·6±53·8	1·556±0·089	0·507±0·061	1·599
15	24·9	1075·9±100·2	664·1±60·9	1·367±0·100	0·473±0·037	1·620

Table III. Elastic Constants, γ_l/γ_t, and E/μ.

No.	Young's modulus E (C. G. S.)	Mod. of rigidity μ (C. G. S.)	Poisson's ratio σ	λ (C. G. S.)	$\frac{\gamma_l}{\gamma_t}$	$\frac{E}{\mu}$
	$\times 10^9$	$\times 10^9$		$\times 10^9$		
1	8·893±0·240	0·763±0·160	0·18	2·124	4·768	2·36
2	8·767±0·231	3·679±0·144	0·19	2·334	4·379	2·38
3	8·957±0·175	3·679±0·101	0·21	2·754	4·073	2·42
4	8·746±0·200	3·616±0·162	0·21	2·628	4·023	2·42
5	8·851±0·197	3·637±0·131	0·21	2·712	3·886	2·42
6	8·894±0·277	3·637±0·150	0·22	2·901	3·754	2·44
7	8·957±0·300	3·658±0·142	0·22	2·880	3·678	2·44
8	9·188±0·294	3·721±0·141	0·23	2·280	3·472	2·46
9	9·440±0·241	3·784±0·163	0·24	3·511	3·362	2·48
10	9·377±0·224	3·742±0·107	0·25	3·869	3·282	2·50
11	9·482±0·220	3·763±0·093	0·26	4·141	3·257	2·52
12	9·777±0·217	3·890±0·165	0·25	3·974	3·300	2·50
13	10·092±0·211	3·953±0·133	0·26	4·478	3·150	2·52
14	10·071±0 243	3·932±0·107	0·28	4·857	3·064	2·56
15	9·735±0·200	3·700±0·121	0·31	6·076	2·866	2·62

it will be seen that the probable errors in these values range from about 2 per cent to 20 per cent. The resonance periods, however, were determined, as usual, with an accuracy of from 1 to 2 per cent.

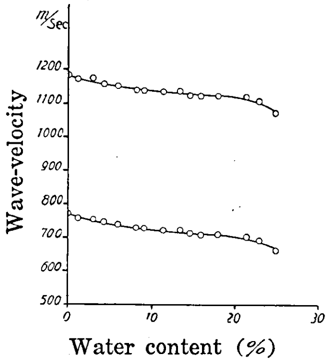


Fig. 4. Variation in velocity with water content.
Upper curve: longitudinal wave-velocity.
Lower curve: transverse wave-velocity.

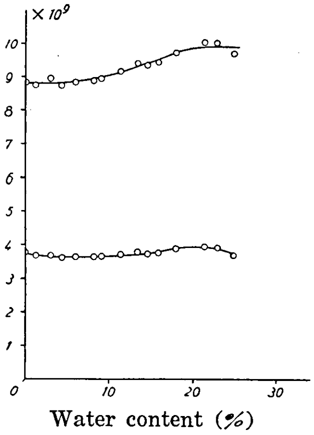
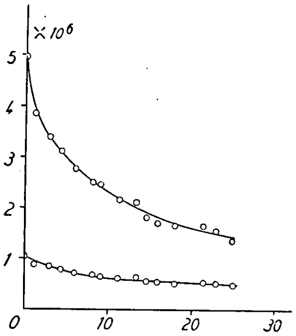


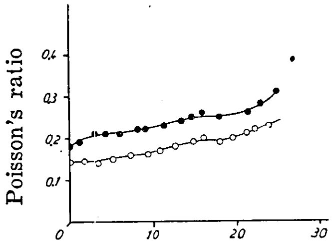
Fig. 5. Relation between either Young's modulus E or modulus of rigidity μ and water content.
Ordinate: E and μ.
Upper curve: Young's modulus.
Lower curve: modulus of rigidity.

It is possible to obtain from Tables II, III several diagrams representing the relations between the elastic constants and the solid viscosity coefficients and the water contents. They are shown in Figs. 4~8, in which the abscissa always represents the water content and the ordinate one of the following values, namely, the wave-velocities, Young's modulus E and the modulus of rigidity μ, Poisson's ratio σ, the normal and tangential viscosity coefficients γ_l and γ_t, the ratio of γ_l to γ_t, and the ratio of E to μ.



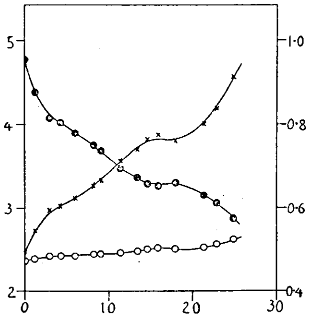
Water content (%)

Fig. 6. Showing that the normal and tangential solid viscosity coefficients diminish with increase in water content.
Ordinate: γ_l and γ_t.
Upper curve: γ_l.
Lower curve: γ_t.



Water content (%)

Fig. 7. Showing that Poisson's ratio increases with increase in water content.
• Dynamically determined values.
○ Statically determined values.



Water content (%)

Fig. 8. Relation between the water content and E/μ, γ_l/γ_t, and m.
○ E/μ. • γ_l/γ_t. × m.
The ordinate of curves in the case of E/μ and γ_l/γ_t are shown on the left-hand side scale; that in the case of m on the right-hand side scale.
Ordinate: E/μ, γ_l/γ_t, and m.

The relations between any one V_l, V_t, σ, γ_l, γ_t and the water content are similar to those previously obtained in soils, namely, the terms V_l, V_t, γ_l, all somewhat rapidly diminish with increase in water content, while γ_t diminishes somewhat gradually with increase in water content (Figs. 4, 6), whereas the variation in elastic constants E and μ with water content differs from those obtained in soils. As will be seen from Fig. 6, the modulus of rigidity is almost the same for the various water contents, whereas in contrast to it, Young's modulus increases somewhat with increase in water content. It was found that, especially, when the water content exceeded 10 per cent, Young's modulus increases somewhat rapidly. These results seem to show the characteristics of sponge-like substances, such as pumice, the skeleton of which alone reacts in the case of torsional vibration, while the water has scarecely any effect. In the case of longitudinal vibration, the effects of water are very marked, and the properties of the specimen plus those of water could be clearly observed, σ and E/μ increasing with increase in water content, while γ_l/γ_t diminished in value with increase in water content. If, experimentally, there were always such a linear relation between γ_l/γ_t and E/μ for a certain water content, it could be assumed that the relation is of the form as that already reported in our previous paper,[15] namely,

$$\frac{E}{\mu} = m\frac{\gamma_l}{\gamma_t}, \tag{4}$$

where m is a constant depending on the water content. The values of m may be determined from the experimental results shown in Tables II, III, those thus determined (Table IV) being plotted agaist the water content (Fig. 8), showing that the more water in the rock specimen the larger the value of m.

Table IV. The Value of m and Water Content.

No.	Water content w (%)	m	No.	Water content w (%)	m
1	0	0·495	9	13·4	0·739
2	1·2	0·544	10	14·6	0·762
3	3·0	0·594	11	15·9	0·774
4	4·3	0·603	12	18·0	0·758
5	6·0	0·623	13	21·4	0·800
6	8·2	0·650	14	22·9	0·837
7	9·1	0·664	15	24·9	0·913
8	11·4	0·710			

It is possible to determine the viscous coefficients λ' and μ', analogous to Lamé constants λ and μ, in the case of elasticity. As already reported in our previous paper[15], the relations between Poisson's elasticity ratio and the normal-tangential viscosity ratio is

$$\frac{\gamma_l}{\gamma_t} = \frac{\lambda\mu\left(\frac{\lambda'}{\lambda} - \frac{\mu'}{\mu}\right) + \frac{\mu'}{\mu}(\lambda+\mu)(3\lambda+2\mu)}{\frac{\mu'}{\mu}(\lambda+\mu)^2}, \tag{5}$$

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[15] K. Iida, *Bull. Earthq. Res. Inst.*, **16** (1938), 391.

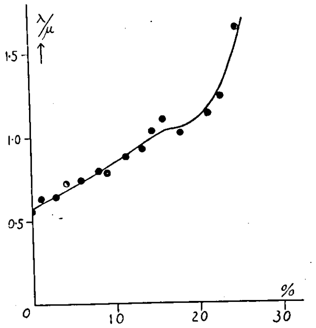
where

$$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}, \qquad \mu = \frac{E}{2(1+\sigma)}. \tag{6}$$

Substituting (6) in (5), we get

$$\frac{\gamma_l}{\gamma_t} = (1-2\sigma)^2 \frac{\lambda'}{\mu'} + 2(2\sigma^2+1). \tag{7}$$

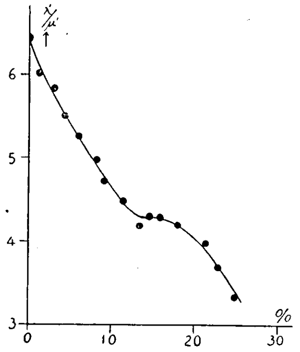
As the values of both γ_l/γ_t and σ are found experimentally, we can determine, by means of equation (7), the viscous coefficients λ' and the values of λ'/μ' for various water contents, the results being shown in Table IV. The relations between λ'/μ' and the water content thus found are shown in Fig. 9, λ'/μ' and the water content being taken as ordinate and abscissa respectively. As will be seen from this figure, λ'/μ' becomes smaller with increase in water content, although, so far as the present experiment is concerned, the value of λ'/μ', unlike that found in the case of soils, did not become negative with increase in water content.



Water content (%)

Fig. 9. Relation between λ/μ and the water content.

Further, the relations between λ/μ and the water content are shown in Fig. 10, in which λ/μ is taken as ordinate and the water content as abscissa. From Fig. 10 we see that the more water in the rock the larger the value of λ/μ. This is chiefly due to the value of λ.



Water content (%)

Fig. 10. Relation between λ'/μ' and the water content.

It was found that the curves showing decreases in the values of λ'/μ' or λ/μ with increase in water content became irregular where the water content was between 14 and 16 per cent.

We, moreover, specially investigated experimentally the values of n when it has the relation

$$\frac{\lambda'}{\lambda} = n\frac{\mu'}{\mu}. \tag{8}$$

If $\frac{\lambda'}{\lambda} = n\frac{\mu'}{\mu}$, then (7) becomes

$$\frac{\gamma_l}{\gamma_t}=2\left\{n\sigma(1-2\sigma)+(1+2\sigma^2)\right\}. \qquad (9)$$

Table V. The Values of λ/μ, λ'/μ', n, and Water Content.

No.	Water content w (%)	$\frac{\lambda}{\mu}$	$\frac{\lambda'}{\mu'}$	n
1	0	0·564	6·43	11·40
2	1·2	0·636	6·01	9·45
3	3·0	0·646	5·83	9·05
4	4·3	0·726	5·50	7·58
5	6·0	0·743	5·26	7·07
6	8·2	0·798	4·97	6·23
7	9·1	0·785	4·72	6·02
8	11·4	0·880	4·48	5·10
9	13·4	0·928	4·19	4·52
10	14·6	1·033	4·30	4·06
11	15·9	1·100	4·29	3·90
12	18·0	1·020	4·20	4·11
13	21·4	1·133	3·98	3·52
14	22·9	1·236	3·69	2·99
15	24·9	1·641	3·35	2·04

The results are shown in Table V. The values of n were plotted against the water content, as shown in Fig. 11, from which it was found that the value of n assumes a fairly larger positive value with increase in water content than those in the case of soils, the values of n being about 11 if the specimen is dry. From the foregoing experiments, it was found that the possible relations between Poisson's elasticity ratio and the normal-tangential viscosity ratio are complicated, whence relations, such as

$$\frac{\lambda'}{\mu'}=\frac{\lambda}{\mu}, \quad \lambda'+\frac{2}{3}\mu'=0, \quad \frac{\gamma_l}{\gamma_t}=\frac{2}{3}\frac{E}{\mu}, \qquad (10)$$

do not hold for this kind of rock within the scope of the present experiments. These relations were explained in his previous paper.[16]

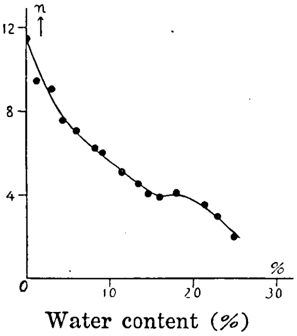


Fig. 11. Relation between n and the water content.

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[16] K. Iida, *loc. cit.*, 15).

5. Determination of the Elastic Constants by means of Static Methods of Experiment.

According to some of the foregoing investigators17), the nature of rocks differs so much from that of the common metals that no perfectly elastic rock has yet been found, so that Hooke's law does not seem to hold even for very small forces, hence the difficulty of obtaining the purely elastic constants of rocks by means of static methods of experiment. To compare the modulus of elasticity obtained dynamically with that obtained statically, we, however, applied very small forces to the rock specimen by the ordinary static methods of experiment, and thus obtained the stress-strain diagram, from which the elastic constant can be derived.

The schematic arrangement of the apparatus for the static measurements is shown in Fig. 12, in which the purpose of the upper part is to impart bending to the specimen and the lower part torsion.

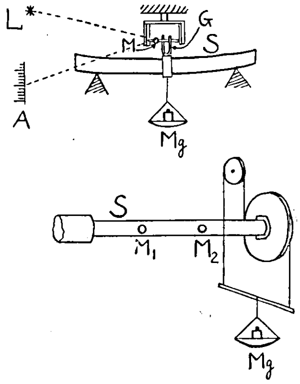

Fig. 12. Schematic diagram of apparatus for the static measurements.
The upper part for bending test.
The lower part for torsion test.

In order to impart bending to the rock specimen S, the two parts near its two ends are supported on the knife-edge of a clamped body, and the forces applied to its middle part as shown in Fig. 12. To observe the bending, we contrived a device for magnifying the deformation due to bending by adopting a pivot and a small magnet G, the arm of which latter was clamped to the middle part of the specimen. The pivot, 1·99 mm in diameter, is in contact with the upper part of the magnet. On the pivot we affixed a small lens mirror M of a focal length of about 50 cm, the rotation of which pivot deflected the light beam that was reflected by this lens mirror. A beam of light radiating from a slit in the light source L is reflected to the lens mirror and then focussed on a scale A. In this way we succeeded in reading the amount of deformation δ_b due to bending of the middle of the specimen.

To impart torsion to the specimen, one end of it was clamped and torsional couple applied to its other end. The torsional couple was produced with the aid of the weight Mg, as shown in Fig. 12. The

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17) *loc. cit.*, 2), 3), 4), 5), 6).

specimen was twisted as the result of this torsional couple. The amount of torsional angle δ_t caused in the specimen S was measured by observing the deflection of the slit image of a light source, the beam radiating from which is reflected by two small lens mirrors M_1, M_2.

Since in this way, the strains δ_b, δ_t corresponding to each applied force are experimentally obtained, a stress-strain diagram could be drawn by taking the strain that has developed as ordinate and the stress as abscissa. The same procedure was repeated on rock specimens with various water contents. Some examples of the stress-strain diagram are shown in Fig. 13. It being found that when the applied forces become greater, there being a tendency in the relations between the stress and the strain to deviate from a straight line passing through the coordinate origin, the elastic constant was determined from certain points related to the coordinate origin in Fig. 13. Young's modulus and the modulus of rigidity can then be determined by means of the following equations.

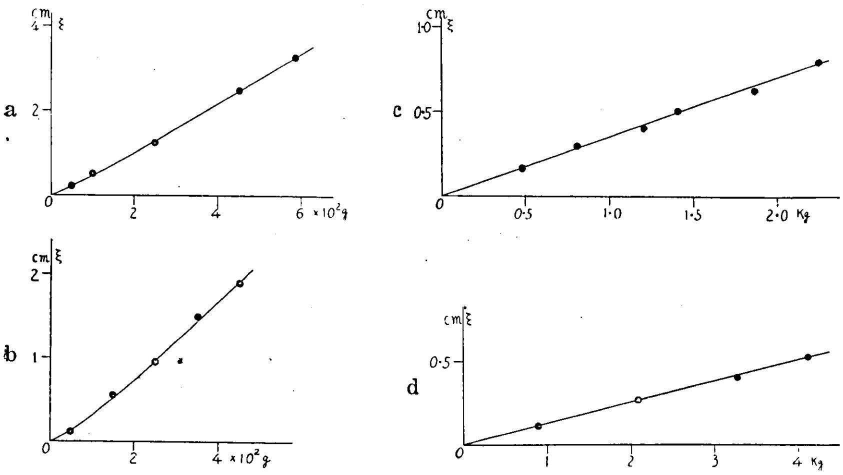

Fig. 13. Stress-strain diagrams for bending and torsion tests.
a), b) Bending test. c), d) Torsion test.
a) Water content $w=22{\cdot}1\%$. b) $w=19{\cdot}5\%$. c) $w=19{\cdot}5\%$, d) $w=3{\cdot}4\%$,
Ordinate: Amount of deformation ξ. Actual deformations are $\delta_b=\xi\times4{\cdot}93\times10^{-4}$ cm in the case of bending test, and $\delta_t=\xi\times5\times10^{-3}$ radian in the case of torsion test.
Abscissa: Load.

a) *Young's modulus.* At horizontal distance x from a supported end of the specimen as shown in Fig. 12, a relation holds, such that

$$\frac{d^2\delta_b}{dx^2}=\frac{Mgx}{2EI}, \tag{10}$$

where I is the moment of inertia of the cross-section, E the Young's modulus, and Mg the weight. The solution of (10), that is, the displacement δ_b at $x=\frac{l}{2}$ is given by

$$\delta_b=\frac{Mgl^3}{48EI}, \tag{11}$$

where

$$I=\frac{\pi d^4}{64}, \tag{12}$$

and we obtain

$$E=\frac{4Mgl^3}{3\pi\delta_b d^4}, \tag{13}$$

where l is the distance between the two supported points of the specimen, and d the diameter of the rock cylinder.

b) *Modulus of rigidity.* We can determine the modulus of rigidity by measuring the torsional couple C and the torsional angle δ_t. The torsional couple is represented by

$$C=\int_0^{d/2}\frac{2\pi\mu\delta_t}{l}r^3dr=\frac{\pi\mu\delta_t d^4}{32l}, \tag{14}$$

whence we obtain

$$\mu=\frac{32l}{\pi d^4\delta_t}C, \tag{15}$$

where μ is the modulus of rigidity, C the torsional couple, l the distance between the two small lens mirrors, δ_t the torsional angle of the specimen, and d its diameter.

Table VI. Statically Determined Elastic Constants and Water Content.

No.	Water content w (%)	Young's modulus E (C. G. S.)	Mod. of rigidity μ (C. G. S.)	Poisson's ratio σ
		$\times 10^9$	$\times 10^9$	
1	0	5·499	2·392	0·15
2	1·8	5·060	2·202	0·15
3	3·4	4·620	2·029	0·14
4	5·2	4·460	1·941	0·15
5	7·3	4·175	1·80	0·16
6	9·4	3·807	1·64	0·16
7	10·8	3·766	1·62	0·17
8	12·7	3·682	1·56	0·18
9	14·8	3·664	1·54	0·19
10	16·0	3·600	1·50	0·20
11	18·0	3·380	1·42	0·19
12	19·5	3·360	1·40	0·20
13	21·2	3·242	1·34	0·21
14	22·1	3·123	1·28	0·22
15	23·9	3 050	1·24	0·23

The results thus obtained are shown in Table VI. We take Young's modulus, the modulus of rigidity and Poisson's ratio of the specimens as ordinate and the water content as abscissa, as shown in Figs. 7, 14. From these figures it will be seen that the values of both E and μ diminish somewhat rapidly with increase in water content, which phenomenon was already observed in the case of soils. The more water there is in the rock the smaller the values of E and μ. We thus ascertained that, in the case of static measurements water reduces considerably the value of the elastic constants, which reason, it seems, contributed to the yielding of the specimen on account of the effect of water. The value of σ increases with increase in water content.

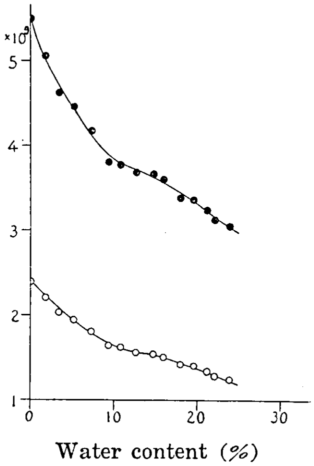

Fig. 14. Relation between statically determined Young's modulus as well as modulus of rigidity and the water content.
• Young's modulus. ∘ Modulus of rigidity.
Ordinate: E and μ.

6. Visco-elastic Properties of the Rock.

In our previous papers[18], we discussed the visco-elastic properties of pitch-like materials. In this case we applied the same methods of experiment. As found by statical methods of experiment, the ratio of stress to strain is not constant, but varies with the stress, so that this kind of rock differs from perfectly elastic material, in that it seems to possess two properties, elasticity and viscosity. The writer therefore desired to know their viscous properties when they are subjected to forces for a prolonged period, and to distinguish clearly the elastic and viscous properties of this rock under certain conditions.

The apparatus and the methods used in this experiment were the same as those used previously. In order to ascertain the angular deformation of the specimen, we applied to the specimen torque that was proportional to the rotating angle of steel wire for a prolonged period, that is, they were allowed to remain in that state for about a week. The record for a week, thus obtained, is shown in Fig. 15, in which the angular deformation is taken as ordinate and the time as abscissa.

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18) K. IIDA, *Bull. Earthq. Res. Inst.*, **13** (1935), 198; 443.

When the torque is first applied to the specimen, there is a rapid movement, which, however, gradually subsides and eventually reaches a steady state. This steady state of deformation appears as if it would continue indefinitely, but its final state approaches a definite asymptotic value, balancing the torque of the wire in accordance with the property of this rock.

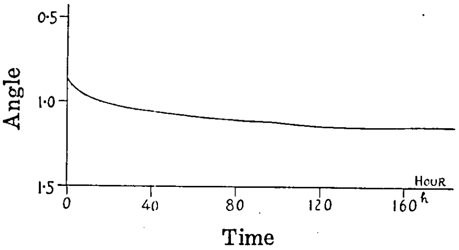

Fig. 15. Deformation curve of twisted rock cylinder.
0·1 on the scale of ordinate corresponds to 0·031°.
The initial twisted angle of wire $\varphi = 1{\cdot}53°$.

Using the same analysis of the curve as that made previously, we analyzed the curve in Fig. 15 with the intention of determining the elastic and the viscous constants separately, namely,

$$k=\frac{C(\varphi-\theta_{t=\infty})}{\theta_{t=\infty}}, \qquad \varepsilon=\frac{k+C}{a}, \qquad \mu=\frac{2l}{\pi r^4}k, \qquad \eta=A\varepsilon,$$

in which the symbols have the same meaning as previously: C is the torsional couple, φ the initial twisted angle, $\theta_{t=\infty}$ the final twisted angle, l the length, r the radius, A the constant, and a the constant depending on the initial state of the deformation curve. As $C=6{\cdot}12\times10^7$ (C. G. S.), $\theta_{t=\infty}=22{\cdot}4'$, $\varphi=1{\cdot}53°$, $a=0{\cdot}00512$ for the case of experiment No. 1, the results are shown in the following table,

No.	Temp. °C	Modulus of rigidity μ (C. G. S.)	Viscosity coef. η (C. G. S.)	Water cont. (%)
1	21·2	$2{\cdot}30\times10^9$	$1{\cdot}60\times10^{14}$	0
2	21·3	2·21 〃	1·43 〃	2·1
3	21·5	2·11 〃	1·23 〃	3·0

from which it was found that these elastic constants are smaller than those obtained in the dynamic measurement and that they were of the same order as those obtained in the static measurement, the viscosity coefficient thus obtained being of the order of 10^{14} in C. G. S. units. It was ascertained that the rock possesses the dual properties of viscosity and elasticity. Consequently the structure of this rock could be explained by imagining a sponge, the cavities of the elastic skeleton of which are filled with a viscous fluid. The rock would therefore behave like an elastic substance when subjected to sudden forces for only such a short period of time as in the foregoing dynamic measurements, and like a liquid of high viscosity when subjected to forces for a long period of time. The values of μ and η decrease with increase in water content.

7. The Statically and Dynamically Determined Elastic Constants Compared.

According to some investigators, the dynamically determined elastic constants are larger than those statically determined, the two methods giving the most divergent results for rocks that have cracks and pores which yield under static stress due to the gradual closing of the cracks and cavities. It should, therefore, be interesting to compare the dynamically determined elastic constants with the statically determined elastic constants of such a typical porous rock as this pumice. The elastic constants obtained by the two methods for the specimens in various degrees of water content can be determined by interpolation from the values, as shown in Figs. 5, 14. The results of the comparison are given in Table VII, the most obvious general conclusion to be derived

Table VII. Comparison of E_D, μ_D and E_S, μ_S.

Water content (%)	E_D (C. G. S.)	E_S (C. G. S.)	$\frac{E_D-E_S}{E_D}$	μ_D (C. G. S.)	μ_S (C. G. S.)	$\frac{\mu_D-\mu_S}{\mu_D}$
	$\times 10^9$	$\times 10^9$	%	$\times 10^9$	$\times 10^9$	%
0	8·89	5·50	38·2	3·76	2·39	36·4
1	8·84	5·22	40·7	3·70	2·28	38·4
2	8·84	4·97	43·7	3·67	2·20	40.0
3	8·85	4·72	46·6	3·66	2·11	42·3
4	8·85	4·52	49·0	3·64	2·01	44·8
5	8·85	4·40	50·3	3·64	1·96	46·1
6	8·85	4·28	51·6	3·64	1·90	47·8
7	8·87	4·16	53·1	3·65	1·82	50·1
8	8·88	4·04	54·5	3·65	1·76	51·7
9	8·96	3·92	56·3	3·66	1·70	53·5
10	9·06	3·82	57·9	3·67	1·65	53·7
11	9·15	3·74	59·2	3·68	1·60	56·5
12	9·24	3·70	60·0	3·71	1·58	57·4
13	9·31	3·66	60·7	3·74	1·54	58·7
14	9·39	3·64	61·2	3·76	1·52	59·5
15	9·47	3·60	62·0	3·77	1·50	60·1
16	9·54	3·56	62·7	3·79	1·48	60·9
17	9·66	3·49	63·8	3·80	1·45	61·8
18	9·78	3·42	65·0	3·86	1·42	63·1
19	9·88	3·37	65·9	3·86	1·40	63·7
20	9·94	3·32	66·6	3·89	1·38	64·5
21	9·96	3·26	67·3	3·92	1·34	65·8
22	10·00	3·20	68·0	3·92	1·30	66·8
23	10·02	3·16	68·6	3·92	1·28	67·4
24	10·00	3·10	69·0	3·92	1·25	68·1
25	9·96	3·05	69·4	3·91	1·24	68·2

from which is that the elastic constants obtained by dynamic methods are from 36 to 69 per cent higher than those obtained by the static methods. We shall now consider the causes of this, the chief of which, in the case of the present experiments, is believed to be the porous nature of this rock specimen. Owing to these numerous pores, the elastic constants statically determined are bound to be very low. Moreover, the effects of elastic-after working, hysteresis, viscous flow, etc, are introduced into the static measurements, whereas in dynamic measurements they should be entirely free from such effects.

It must be added that water is a factor of great importance, increasing the differences between the elastic constants obtained by the two methods. The values of Young's modulus obtained dynamically increase with increase in water contents, while those obtained statically diminish with increase in water contents, so that the differences in question increase with increase in water content.

It was found that Poisson's ratio of this rock obtained by the dynamic methods exceeds that obtained by the static methods for the water content range of the present experiments. We, therefore, concluded that the relation of the dynamic and the static elastic constants E_D, E_S; μ_D, μ_S; σ_D, σ_S are respectively represented by

$$E_D > E_S, \quad \mu_D > \mu_S, \quad \sigma_D > \sigma_S.$$

8. Summary and Conclusion.

The experimental results in connection with certain physical properties, such as elasticity and viscosity, of pumice as exhibited by static and dynamic measurements, are described.

The relations between their properties and the water content are investigated. The wave-velocities propagated in the specimen and the solid viscosity coefficients diminish with increase in water content. Young's modulus and modulus of rigidity obtained by static measurement diminish somewhat rapidly with increase in water content, while Young's modulus, obtained dynamically, increased with increase in water content, the modulus of rigidity remaining almost unchanged despite the variation in water content. These results seem to show the characteristics of sponge-like materials, such as pumice, the skeleton of which alone reacts in the case of torsional vibration, the included water being scarcely effected, while in the case of the longitudinal vibration the effects of water are quite marked. Such facts must be noticed when studying the propagation of seismic waves in this kind of rock.

The coefficients of normal solid viscosity are greater than those of tangential solid viscosity.

The relations between the normal-tangential viscosity ratio and Poisson's elasticity ratio are investigated.

The value of m obtained under the assumption $E/\mu=m\ \gamma_l/\gamma_t$, varies with the water content, the greater the increase of water in the rock the larger the value of m.

The viscous coefficients λ' and μ', similar to both Lamé elastic constants λ, μ, are determined, and the variations in λ'/μ', λ/μ with water content are investigated. The value of λ'/μ' increases with increase in water content, and $\lambda'/\mu' > 0$, so far as the present experiments are concerned, while that of λ/μ diminishes with increase in water content. The quantity of n showing the relation between the ratio of λ/μ and λ'/μ' obtained in our experiments is investigated, with the result that $\frac{\lambda'}{\mu'}=n\frac{\lambda}{\mu}$, and n diminishes in value with increase in water content. The simple relations, such as $\lambda'/\lambda=\mu'/\mu$, $\lambda'+2\mu'/3=0$, $\gamma_l/\gamma_t=2,3\cdot E/\mu$, do not hold within the scope of the present experiment.

The visco-elastic properties of the rock are investigated. The elastic and viscous characteristics of the rock can be separately known. The elastic constant is of the order of 10^9 in C. G. S. units, and the viscosity coefficient of the order of 10^{14} poises.

The values of Young's modulus, modulus of rigidity, and Poisson's ratio obtained dynamically exceed those obtained statically. It was ascertained that the effect of water on the physical properties in question are quite marked. The relations between some of the physical properties of this rock and water content differ from those of soils obtained previously. The characteristics of this kind of rock, which has a special structure somewhat similar to that of a sponge, are investigated.

In conclusion, the writer wishes to express his hearty thanks to Professor Mishio Ishimoto and also to Professor Takeo Matuzawa for their valuable suggestions. His cordial thanks are also due to Dr. H. Kawasumi who presented him with the rock specimen. He also acknowledges his great indebtedness to the Foundation for the Promotion of Scientific and Industrial Research of Japan, with whose grant certain parts of the present study in connection with elastic constants of rocks by means of the vibration methods were made.

6. 岩石の彈性及び粘性に關する研究

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前論文に於いては，土の彈性及び固體粘性に關する研究を報告し，それ等の性質の水分に對する影響の著しい事を指摘した．今回は此等の性質が岩石に於いても見出されるか否かを研究するために先づ土と同じく多量の水分を吸收する特種の岩石卽ち浮石について研究を行つた．此の岩石は伊豆新島產のものにして，昭和 11 年 12 月 27 日の新島强震直後同島に地震の調査に赴かれた松澤博士一行によつて採取されたものである．此の地震の被害の多くは此の種の岩石によつて作られたる構造物であつた．從つて此の種の岩石の物理的性質の闡明は此の方面に於いても必要な事と思はれる．

此の岩石の彈性を靜力學的及び動力學的に研究をなし，尙その粘彈性性質をも研究した．既に取扱つた振動方法によつてその彈性係數を求めると靜力學的方法によつて求めた値の數十%の大なる値を示し，且つ振動方法による場合には水分の影響は次の如くである．卽ちその剛性率は水分によつて受ける影響は比較的少なく，水分と共にその値は餘り變化しないが，ヤング率は水分の增加と共に大となる傾向を有する事が判明した．之に反し，靜力學的方法によつて求めた此等の値は何れもその水分の增加と共に小さくなる事が知られた．以上の事實は彈性波傳播に對して相當考慮すべきものであると思はれる．斯樣に靜力學的に求めた値の小さいのは此の岩石は多孔質なるため靜力學的の力の作用のもとに yield し，岩石內の間隙の閉縮を生ずるため並びに此の岩石の粘性的變形等の影響に原因するものと考へられる．

又 2 種類の固體粘性係數を求めた．而して縱振動によつて求めた固體粘性係數は捩振動によつて求めた固體粘性係數より大なる事は土の場合と同樣である．なほ Lamé の彈性常數並びに λ'，μ' 等の粘性係數をも求めた．以上の諸量に對する水分の影響をも研究した．λ'/μ' は水分の增加と共に小さくなるが，約 30% の水分に對してもその値が負數とならない事は土の場合に求めた結果と違つて居る事も判明した．此等は物質の構造的差異に基くものと考へられる．

最後に此の岩石の粘性と彈性とを分離して測定した．而して此の岩石の如き海綿狀構造をなせる物質の示す特性を述べた．