7. *Determining Young's Modulus and the Solid Viscosity Coefficients of Rocks by the Vibration Method.*

By Kumizi IIDA,

Earthquake Research Institute.

(Read June 5, 1938.—Received Dec. 20, 1938.)

1. Introduction.

In our preceding paper,[1] we stressed the necessity of studying the physical properties of rocks, in order to understand fully such geophysical problems as the chronic deformation of the earth's crust and the propagation of seismic waves, etc., and also discussed the elastic properties as well as the viscous behaviour of a certain kind of rock (pumice) by subjecting it to vibration; and to forces, statically. We compared the elastic constants determined dynamically with those determined statically, and found that the former exceeded the latter. To obtain the elastic properties of materials, alone, accurately, it was found that the dynamical method of experiment was the one to use. The marked effect of moisture on the elastic and viscous behaviour was also investigated.

In arriving at the foregoing elastic properties, we used the same apparatus that was used in studying the elastic properties of soils. As the elastic constants, generally speaking, of soils and those of the pumice that were studied in our preceding paper are smaller than those of hard rocks, there was no objection to the use of the same apparatus. In order to study the elastic properties of rocks, dynamically, however, it is necessary to resort to other apparatus capable of dealing with a wider range of vibration frequencies than was used previously, seeing that the elastic constants of rocks in general are comparatively great.

The object of our present investigation was to devise some satisfactory dynamic method of determining the elastic constants of rocks and to find their elastic properties and solid viscosity coefficients as well. Although this investigation is quite preliminary, it is our desire to study the elastic properties as well as the viscous behaviour of rocks in the same way as those obtained for soils and pumice, and to compare the

1) K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939), 59.

elastic constants obtained by laboratory methods from samples of rocks with those obtained by direct field measurements, such as seismic prospecting now in vogue. It is our intention to study the properties of rocks under a variety of conditions, for if these constants could be obtained, it should be possible to infer from them the constituents of the earth's crust at depths.

The purpose of our present dynamical methods of experiment is the same as that described previously, namely, to impart vibrations of various frequencies to the base, or foot, of the rock specimen, and thus arrive at the fundamental resonance frequency by computing its elastic constants. Methods[2] similar to our present experiments have already been adopted elsewhere, but the apparatuses that were constructed for the present experiments differ from them in detail.

2. Description of the Specimen.

In order to find, tentatively, Young's modulus of the specimen by means of the apparatuses just mentioned, we took at first cylinders of various kinds of metal that are readily accessible, and whose Young's moduli are already known. The dimensions, the densities, and Young's moduli of these metal cylinders are given in Table I. We then took cylin-

Table I. Dimensions of the Specimens.

Kind of metal	Length (cm)	Diameter (cm)	Density	Young's modulus (C. G. S.)
Brass	60·1, 50·1, 40·1, 30·1	2·56	8·515	$8{\cdot}69\times10^{11}$
Aluminium	70·0, 50·0, 40·2, 30·2	2·50	2·714	$7{\cdot}20\times10^{11}$
Steel	70·2, 50·0, 40·0, 29·8	2·50	7·757	$1{\cdot}70\times10^{12}$

ders of the same material, but of various heights, in order to ascertain whether or not any variation in Young's modulus with height of the specimen is observable, and to investigate the nature of the solid viscosity in it. The diameters of these specimens were all about 2·5 cm.

Next we repeated the experiments with various kinds of rocks in the form of cylinders, about 30~20 cm in height and 5 cm in diameter.

2) J. M. IDE, *Proc. Nat. Acad. Sci.*, **22** (1936), 482, 502; *Rev. Sci. Instru.*, **6** (1935), 296; *Journ. Geol.*, **45** (1938), 689.
S. MORITA, *Matuda Kenkyû Zihô*, **11** (1936), 37, (in Japanese).

We used also the same rock specimen (pumice) that was used in the preceding experiments. The dimensions and densities of these rock specimens are shown in Table II.

Table II. Dimensions of Rock Specimens.

Kind of rock	Length (cm)	Diameter (cm)	Density
Marble	30·4, 25·0, 20·9	5·06	2·725
Granite	30·0, 26·7, 21·5	5·04	2·610
Sandstone	30·4, 25·8, 20·3	5·06	1·921
Pumice	32·0, 17·6, 10·8	4·93	0·630

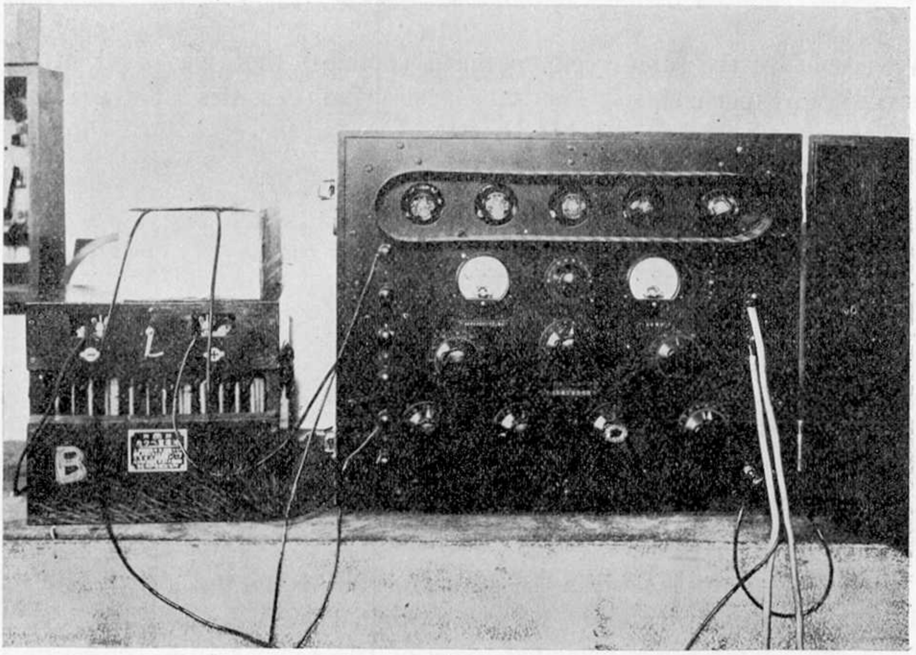
3. Experiments.

(a) Apparatus.

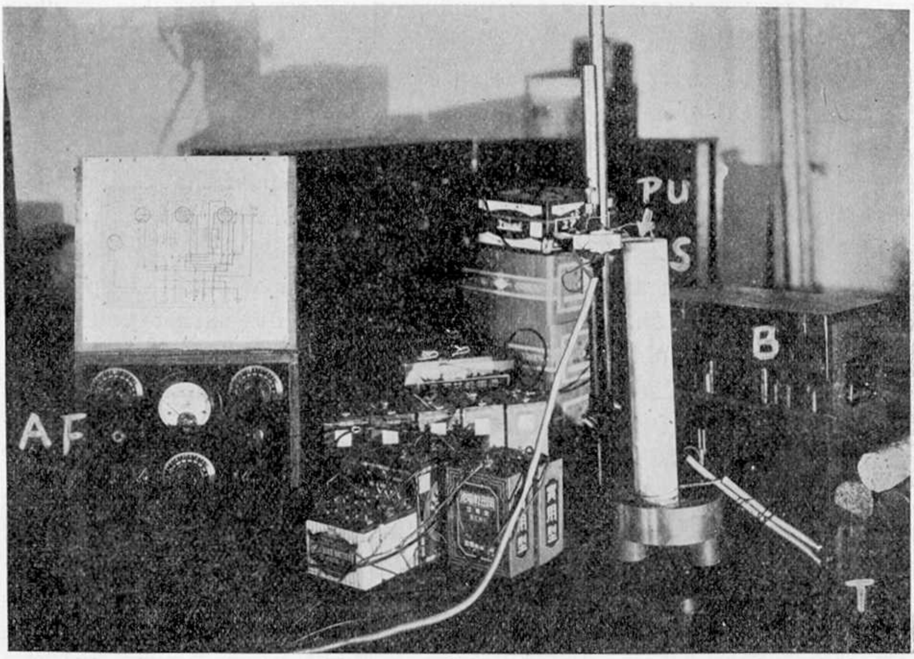
The apparatuses used in the present experiments are shown in Fig. 1 (a), (b), its arrangement being shown in Fig. 2. As will be seen from these figures, the principal parts of these apparatuses consist of a variable frequency oscillator for producing voltage of variable frequency to cause resonance vibration of the specimen, and a sensitive detector to detect the vibration of the specimen.

The method of setting the specimen in vibration. To one end of the specimen S is securely affixed a metal foil, which forms one plate of an electrical condenser. In the case of a specimen that is a conductor of electricity, this foil is not necessary. A steel block C, which forms the other plate of the condenser, is separated from the specimen by a dielectric, such as a thin sheet of mica, on which the specimen stands upright, as shown in Figs. 1, 2. An alternating voltage of variable frequency, obtained from a variable frequency oscillator Os, is connected to the condenser thus formed. Since the metal foil that is fixed to one end of the rock specimen acts as a movable condenser plate, the end of the specimen partakes of the condenser vibration of the same frequency as the oscillator, and vibrates with maximum amplitude, when its characteristic period coincides with that of the driving voltage.

Variable frequency oscillator. An alternating voltage of variable frequency, which is useful in causing vibration of the specimen, is provided with a vacuum tube oscillator Os (Figs. 1(a), 2) of calibrated variable frequency. This frequency varies continuously in a range of from 50 to 50,000 cycles per second. These are read off the indexes of a set of five dials, as shown in Fig. 1(a). The error of these values



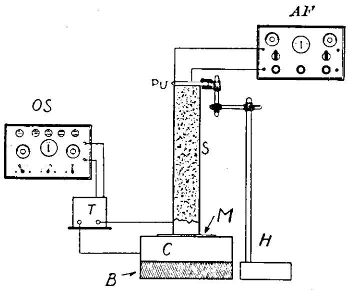
(a)



(b)

Fig. 1. Photographs of the Apparatus for the experiments.
(a) Variable frequency oscillator.
(b) Arrangement of apparatus for setting the specimen in vibration.
(The letters have the same meaning as in Fig. 2.)

is within one per cent. This instrument was made by the Yokokawa Electric Co.



S: Rock specimen.
Pu: Crystal of Rochelle salt.
M: Mica plate.
T: Transformer.
C: Steel block.
Os: Variable frequency oscillator.
AF: Amplifier.
B: Rubber ring.
H: Holder.

Fig. 2. Schematic diagram of arrangement of the apparatus for setting the specimen in longitudinal vibration.

The wiring diagram of the oscillator is shown in Fig. 3. One vacuum tube is used as the generator of oscillation, the three remaining

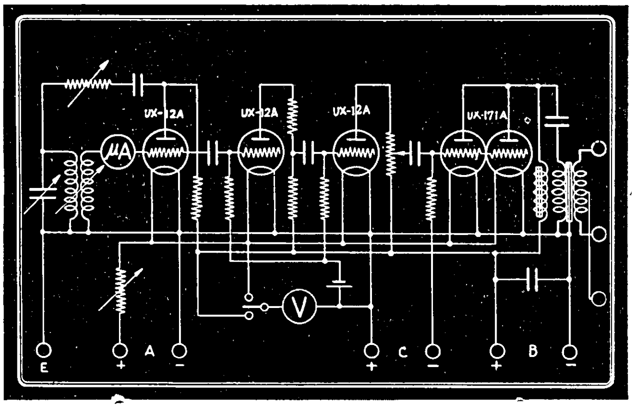


Fig. 3. Wiring diagram of variable frequency oscillator.

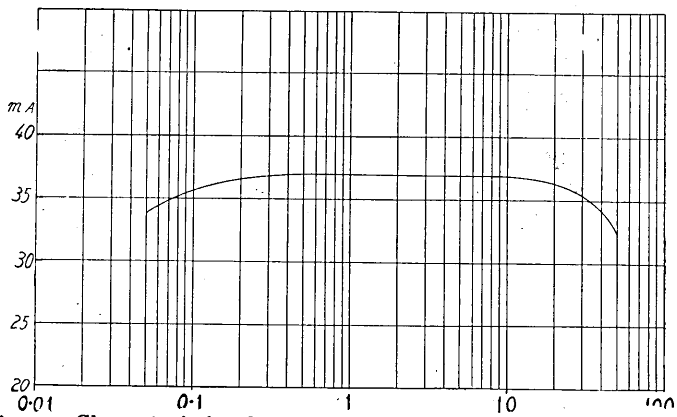


Fig. 4. Characteristic of the driving current from the oscillator. Abscissa; frequency (K. Cycle per sec).

vacuum tubes being used as amplifiers. The driving electric current is 36 mA at maximum under 600 ohms resistance. The variations in the driving current with the oscillator frequency are shown in Fig. 4, from which it may be concluded that the driving current is almost constant between the observed frequencies, especially frequencies of 300 and 30,000 cycles per second.

Some examples of curves showing the wave-form of alternating

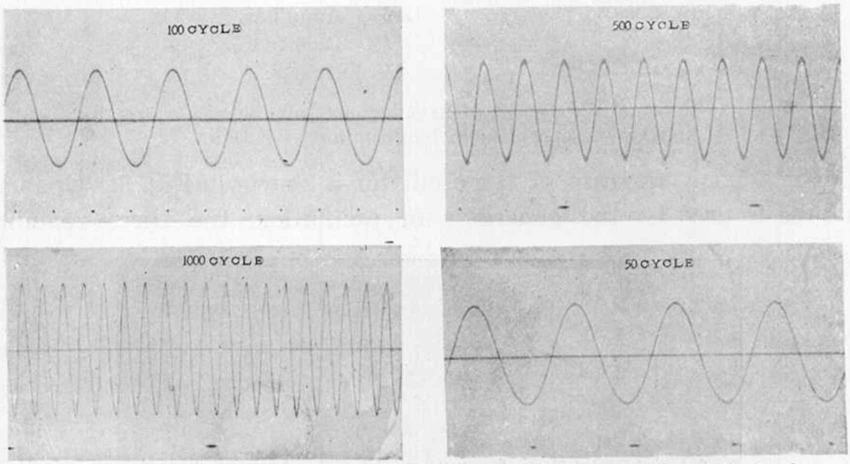


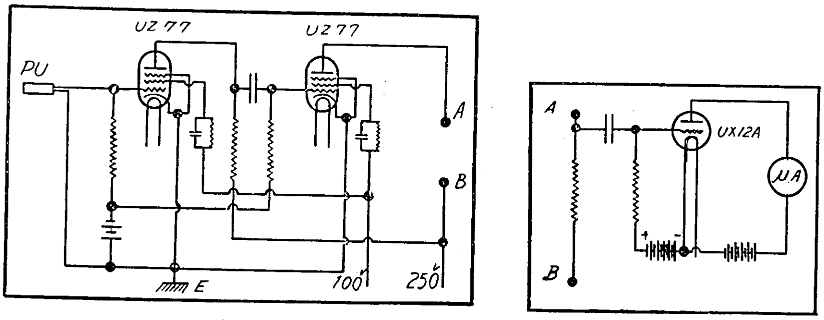
Fig. 5. Wave-form of alternating current driven from the output of the oscillator.

current driven from the output of the oscillator are shown in Fig. 5, from which it will be seen that it is almost of sine form.

Detection of vibration. If a driving voltage with variable frequency is supplied to the specimen from the variable frequency oscillator, it vibrates with maximum amplitude at its own natural longitudinal vibration period. In order to detect this vibration in the specimen, a sensitive detector *Pu*, consisting of a small crystal of Rochelle salt cemented with Canada balsam to the upper end of the specimen, was used. The vibration of the specimen generated piezo-electricity in the Rochelle salt. The accuracy of this method is limited by the loading effect of the attached crystal. As the ratio of the crystal to the rock mass in our experiments was about 1: 2000, the weight of this detector was negligible compared with that of the specimen, so that the loading effect of the crystal may be ignored. The sensitive detector was, needless to say, shielded carefully from electrostatic effects.

Amplification of the piezoelectric voltage. To amplify the piezoelectric voltage driven across this Rochelle salt, an amplifier was used, the schematic wiring diagrams of which are shown in Figs. 6 (a), (b).

By combining these arrangements as in Figs. 6 (a), (b), it is possible to generate an electric current by the vibration of the specimens. This electric current is read on an ammeter (μA), as shown in Fig. 6 (b),



(a) (b)

Fig. 6. Wiring diagram of amplifier.

and the vibration of the specimen thus observed. Its vibration could also be detected by any type of sound receiver that would detect directly the sound produced by its vibration, provided its vibration is sufficiently amplified and it lies within the range of audible frequency.

(b) Testing the apparatus.

(1) In order to eliminate as far as possible the natural frequency of the steel block C from that of the specimen, after a number of trials with short steel cylinders of various sizes, one 5 cm in height and 15 cm in diameter was selected for purpose of test, when it was proved that its natural frequency differed greatly from that of the specimen.

(2) To observe the effect of the thickness of a sheet of mica on the natural frequency of the specimen, sheets of various thicknesses were tested, with the result that when the thickness of the mica is less than 0·1 mm, the natural frequency of the specimen is distorted, whereas if it is very much thicker, it is difficult to get a suitable vibration. For this experiment, a thickness of up to 2 mm is the most suitable. The base of the specimen must be in close contact with the plate on which the specimen is mounted, while the sheet of mica also must be in close contact with them so as to permit no gap between them.

(3) To prevent any disturbance to this vibrating system from the outside, the steel block was separated from the table on which the system rested by means of three small thick rubber rings.

(4) The driving frequency of the variable frequency oscillator was tested by resonance methods, that is, the frequency in question was made to resonate another known frequency. This alternating frequency in the driving voltage from a variable frequency oscillator is almost

invariable, even if the various other lead-wires of the apparatus are connected to the circuit of the output of the oscillator.

(5) The amplifier was calibrated by changing the vacuum tube, the condenser, and the various resistances.

(6) The receiver being sensitive, it was necessary to shield the whole of the foregoing apparatus from possible electrostatic effects.

(c) Methods.

First, to one end of the prepared rock specimen was cemented a thin metal foil with the aid of a dielectric liquid, and the specimen set upright on a sheet of mica.

An alternating voltage of continuously increasing variable frequency was sent from the output of the oscillator to the condenser that was formed by one end of the specimen and the steel block. It was by such sinulsoidal alternating driving force that the specimen was set in longitudinal vibration. During this process, the variation in the vibration amplitude of the specimen was observed by means of deflection of the index of the ammeter attached to the amplifier, μA. Since, when the period of the specimen coincides with that of the driving voltage, the specimen vibrates with maximum amplitude, the deflection of the index of the ammeter becomes also maximum, from which frequencies we could find the characteristic property of the specimen in question. In this way we obtained the resonance curve of the specimen.

We further determined the density of the specimen by measuring its mass and volume from its dimensions, as usual.

(d) Results.

Some examples of the relation between the readings of the ammeter that indicated the vibration amplitude of a number of rock specimens and the driving frequencies thus obtained are shown in Fig. 7.

These curves we shall call the "resonance curve". As will be seen from these curves, the characteristics of the resonance are almost alike, but the frequency corresponding to the resonance position differs with the kind of rock. In the present experiment, the resonance frequency assumed the greatest value in the case of marble and the smallest value in that of pumice—facts showing that the elastic constant of marble is greater than that of pumice, and also that the elastic constant of sandstone is smaller than those of granite and marble. We succeeded in obtaining not only the fundamental resonance frequency, but also those of higher orders of vibration of the specimen. The resonance curves of higher orders of vibration have a more flattened shape than those of the fundamental.

As will be seen from these curves, the resonance frequencies of higher orders of vibration are about an even number of times that of the fundamental resonance frequencies. It seems that the mode of vibration in the present case corresponds to that of a bar free at both ends. We can, therefore, determine, the longitudinal velocity V in the specimen from the simple relation

$$V=2hf,$$

in which h is the height of the specimen, and f the fundamental resonance frequency.

Since

$$V=\sqrt{\frac{E}{\rho}},$$

where ρ is its density, Young's modulus E of the specimen can also be determined.

The results are shown in Table III.

It seems one of the most important part of the investigation to ascertain whether or not these elastic constants, thus obtained, assume reasonable values. For this purpose we took pumice, the elastic constants of which were determined in our preceding paper.[3] In the case of pumice, the two results are in good agreement, notwithstanding the difference in the methods of experiments. There are, besides, the various readily accesible metal cylinders, the general values of Young's modulus of which are already known by other means. In this case, we compare the results obtained

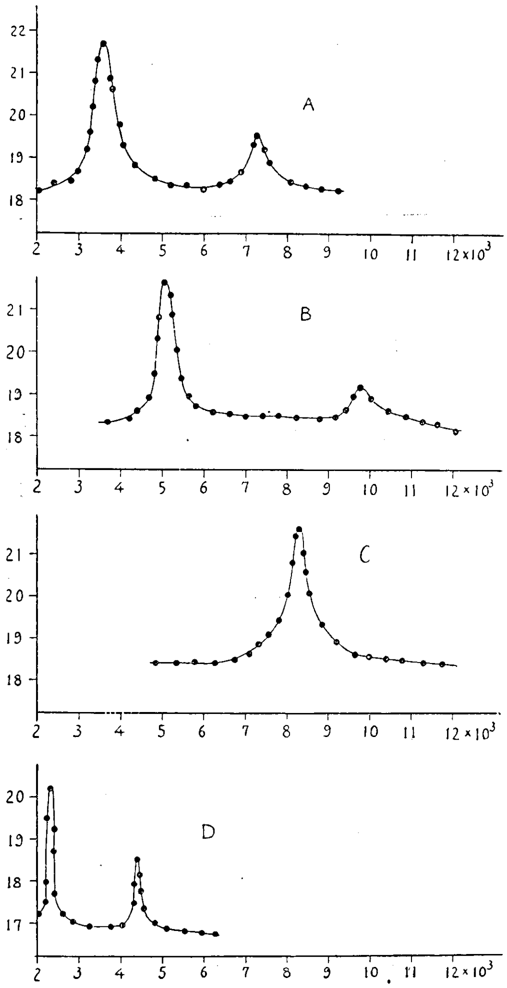


Fig. 7. Resonance curves of rocks.
Ordinate: amplitude in arbitrary unit.
Abscissa: frequency per sec.
A: Sandstone. B: Granite.
C: Marble. D: Pumice.

3) K. IIDA, *loc. cit.*, 1).

Table III. Young's Modulus of Rock Specimens.

Kind of rock	h (cm)	Resonance frequency per sec)	Longitudinal wave-velocity (km/sec)	Young's modulus (C. G. S.)
Marble	30·4	8160	4·96	$6{\cdot}70\times10^{11}$
Granite	30·0	5080	3·04	$2{\cdot}32\times10^{11}$
Sandstone	30·4	3600	2·19	$9{\cdot}20\times10^{10}$
Pumice	32·0	1720	1·10	$7{\cdot}62\times10^{9}$

by the present experiment with the old ones.

The resonance curves of these metal cylinders are shown in Figs. 8, 9, 10. By means of the foregoing treatments, Young's moduli of these metal cylinders were determined, with results as shown in Table IV, the old values being shown in Table I. By comparing the values, we concluded that the results obtained by the present experiments are almost comparable with the old ones.

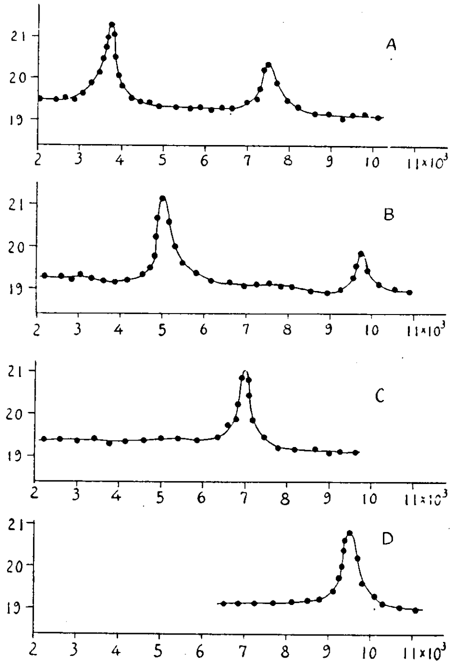


Fig. 8. Resonance curves of aluminium cylinder.
Ordinate: amplitude in arbitrary unit.
Abscissa: frequency per sec.
A: h=70·0 cm. B: h=50·0 cm.
C: h=40·2 cm. D: h=30·2 cm.

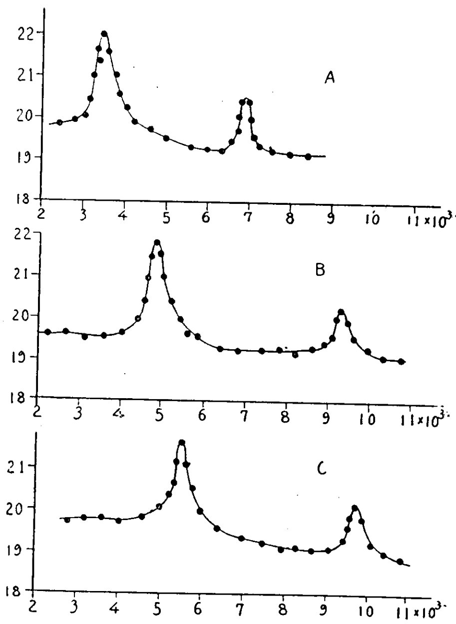


Fig. 9. Resonance curves of steel cylinder.
Ordinate: amplitude in arbitrary unit.
Abscissa: frequency per sec.
A: h=70·2 cm. B: h=50·0 cm.
C: h=40·0 cm.

From the foregoing two cases of agreement it may be concluded that the values obtained by the present methods of experiment are reasonable.

In order to verify whether or not the resonance frequency varies with height, experiments were made with specimens of various heights, the method being similar to those described in previous papers.[4] An example of the relation between the fundamental resonance period and the height of the specimen is shown in Fig. 11. Seeing that the curves showing these relations do not pass through the origin, it was concluded that these results are due to the solid viscosity of the specimen, in which case the previous methods could be used in the present investigations for determining the solid viscosity coefficient γ and Young's modulus E. In the present case, the fundamental resonance period T is expressed by

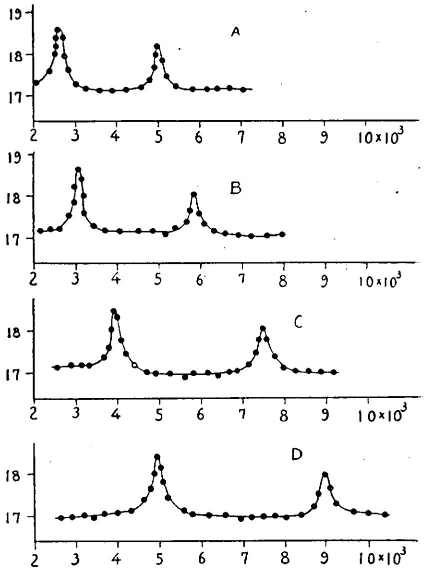


Fig. 10. Resonance curves of brass cylinder.
Ordinate: amplitude in arbitrary unit.
Abscissa: frequency per sec.
A: h=60·1 cm. B: h=50·1 cm.
C: h=40·1 cm. D: h=30·1 cm.

Table IV. Young's Modulus of Metal Cylinders.

Kind of metal	h (cm)	Resonance frequency (per sec)	Longitudinal wave-velocity (km/sec)	Young's modulus (C. G. S.)
Brass	60·1	2680	3·22	$8{\cdot}81\times10^{11}$
	50·1	3180	3·19	8 66 〃
	40·2	3880	3·11	8·24 〃
	30·1	4980	3·00	7·66 〃
Aluminium	70·0	3900	5·48	$8{\cdot}15\times10^{11}$
	50·0	5200	5·20	7·06 〃
	40·2	6250	5·03	6·85 〃
	30·2	8100	4·90	6·51 〃
Steel	70·2	3500	4·91	$1{\cdot}87\times10^{12}$
	50·0	4800	4·80	1·79 〃
	40·0	5500	4·40	1·50 〃
	29·8	7800	4·65	1·68 〃

4) M. Ishimoto and K. Iida, *Bull. Earthq. Res. Inst.*, **14** (1936), 632; **15** (1937), 67. K. Iida, *Bull. Earthq. Res. Inst.*, **16** (1938), 391.

$$T=\frac{2h}{\sqrt{\frac{E}{\rho}-\frac{\pi^2\gamma^2}{4\rho^2h^2}}},$$

where the symbols have the same meaning as before, this relation being derived from equation

$$\rho\frac{\partial^2\zeta}{\partial t^2}=E\frac{\partial^2\zeta}{\partial z^2}+\gamma\frac{\partial^3\zeta}{\partial z^2\partial t}.$$

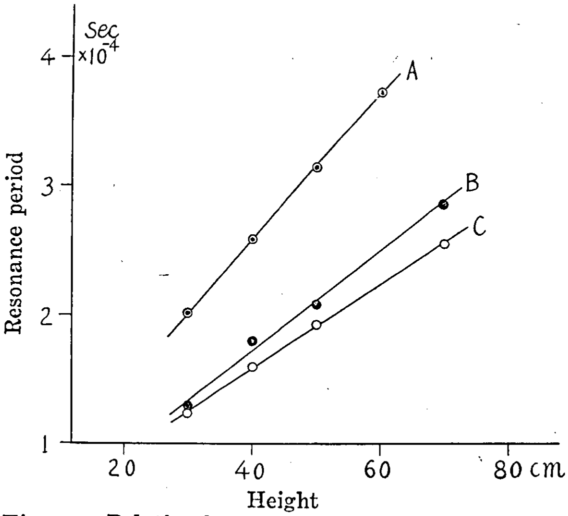


Fig. 11. Relation between the fundamental period of resonance vibration and the height of the specimen.
A: Brass and granite cylinder.
B: Steel and marble cylinder.
C: Aluminium cylinder.

The solid viscosity coefficients and Young's modulus thus obtained are shown in Table V, from which it will be seen that the solid viscosity coefficients are of the order of 10^6 in C. G. S. units, and Young's moduli of the order of 10^{11}, in the same units, for specimens of marble and granite. These values for the

Table V. Young's Modulus, and Solid Viscosity Coefficient of the specimens.

Kind of specimen	Young's modulus (C.G.S.)	Solid viscosity coefficient	Density	Locality
Marble	$7{\cdot}21\times10^{11}$	$7{\cdot}18\times10^{6}$	2·725	Manzyo, Mitu district, Okayama prefecture.[5]
Granite	$2{\cdot}89\times10^{11}$	$3{\cdot}13\times10^{6}$	2·610	Akiyosi village, Mine district, Yamaguti prefecture.[6]
Sandstone	$9{\cdot}99\times10^{10}$	$2{\cdot}07\times10^{6}$	1·921	Kodaira, Tano district, Gumma prefecture.[7]
Pumice	$8{\cdot}13\times10^{9}$	$4{\cdot}12\times10^{6}$	0·630	Niizima, Izu Island.[8]
Brass	$9{\cdot}06\times10^{11}$	$2{\cdot}96\times10^{6}$	8·515	
Aluminium	$8{\cdot}35\times10^{11}$	$1{\cdot}78\times10^{6}$	2·714	
Steel	$1{\cdot}93\times10^{12}$	$7{\cdot}32\times10^{6}$	7·757	

specimen of pumice and sandstone are of the order of 10^6 and 10^{10}

5) 岡山縣御津郡萬成
6) 山口縣美禰郡秋吉村秋吉
7) 群馬縣多野郡小平
8) 伊豆七島新島

(C. G. S.) respectively, while these values for specimens of metal are of the order of 10^6 and 10^{12} (C. G. S.) respectively.

It was ascertained that the value of Young's modulus, obtained by the experiment with specimens of various heights, exceeds that obtained by experiments with only one constant height. This is chiefly due to the solid viscosity of the specimen.

3. Concluding Remarks.

A certain dynamic method and an apparatus for determining Young's modulus of rocks are described. To establish the reliability of this method, experiments with several metal cylinders and a certain kind of rock were carried out. From these experiments, it will be seen that the resulting values are reasonable.

We determined, dynamically, Young's modulus and the coefficients of solid viscosity of a number of rock specimens by means of the experiments with specimens of various heights. The Young's modulus and the solid viscosity coefficient of a certain kind of marble and granite are of the order of 10^{11} and 10^6 in C. G. S. units respectively, whereas in the case of a certain kind of sandstone and pumice, these values were of the order of 10^9 and 10^6, the same units.

This paper is a preliminary study on the elastic properties of rocks, it being our intention to continue the preceding studies with other rocks collected from various localities, with the solution of geophysical problems ever in mind.

In conclusion, the writer wishes to express his hearty thanks to Professor Mishio Ishimoto for his kind guidance and valuable advice in the course of this study. His sincere thanks are also due to Mr. S. Morita, engineer of the Tokyo Electric Company, for his valuable advice in the present work, and to Mr. M. Iwashita, for his assistance in the present experiments.

He also wishes to express his cordial thanks to the Foundation for the Promotion of Scientific and Industrial Research of Japan and the South Manchurian Railway Company, with the aid of whose grants the present study was made.

7. 振動方法による岩石の彈性學的研究

ヤング率及び固體粘性の測定

地震研究所　飯　田　汲　事

振動方法による物質の彈性測定法は既に此れを土，砂，特殊岩石等に應用し，それ等の彈性係數及び固體粘性等について論じた．而して此等の物質の如く比較的彈性係數の小なるものの研究には既に製作せし器械を應用して十分にその効果を擧げ得ると信ずるが，一般岩石の如く其の彈性係數が比較的大なるものに於いては，今迄使用せる器械を用ひて其の彈性係數を求める事は殆ど不可能である．從つて今回は一般岩石の彈性に關する研究の第1歩として，動力學的に其のヤング率の測定をなす事を試み，新たに別の器械を製作し使用した．

本論文に於いては岩石のヤング率，固體粘性係數等の測定の方法並びに數種の岩石について此等の値を測定したる結果について論じた．大理石や花崗岩のヤング率及び固體粘性係數は夫々 10^{11} 及 10^{6} (C. G. S. 單位) 等の値であり，砂岩の此等の値は夫々 10^{10} 及び 10^{6}(C. G. S.) である．此等の値の吟味は金屬や他の物質の實驗値から行はれたが，求められた値は何れも信用の置ける程度のものと考へて差支へない．

將來多くの岩石について研究し岩石の性質を闡明したい希望を有する．而して近時彈性波式地下探査も行はれてゐるから其の方面との關係をも保つて研究を進めたい．

本研究は日本學術振興會並びに南滿洲鐵道株式會社の援助により行はれたもので，こゝに厚く感謝の意を表する．