10. *Vertical Displacements in Regions Markedly Deformed.*

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1. In the eastern part of Tokyo, precise levels have often been run on a number of lines during several years. From these levellings, as already reported[1], marked subsidence of several bench-marks were found. A bench-mark situated near the Nakagawa (river), for example, subsided about 155 mm during 1937~1938. If it could be regarded as having subsided at a constant rate, the rate of subsidence is roughly 0·4 mm per day.

However, to level a line that includes a number of bench-marks, requires several days of field work. In one case, part of a line was levelled after one year or more, when another section of the same line was levelled, and the heights of the bench-marks on the first line were deduced by assuming that both these levels were done at the same time. The vertical displacements of the bench-marks deduced by comparing the heights thus obtained with those obtained by levelling in a following epoch may therefore contain errors due to the movements of the earth's surface during the period of time that elapsed between the epoch in which a section of the line was levelled and that in which the second section was levelled. Such errors however are omitted in the vertical displacements deduced by comparing the results of successive surveys.

Errors of this kind increase and become larger when the time interval between the different epochs, in each of which epoch different sections of the line were levelled, was not small enough compared with the time intervals that elapsed between the successive surveys.

Such was actually the case in the relevellings that were done in the eastern part of Tokyo.

2. In order to correct the errors in the vertical displacements due to the cause just mentioned, the following method was used.

The height of the n-th bench-mark relative to the reference bench-mark is given by

1) N. MIYABE, *Bull. Earthq. Res. Inst.*, **10** (1932), 844~857; **13** (1935), 587~591; **16** (1938), 721~756.

$$H_n = \Sigma h_n, \tag{1}$$

where h_n is the height-difference between the successive bench-marks, that is, $h_n = H_n - H_{n-1}$.

The change in the height of this bench-mark during the time interval $t_2 \sim t_1$ is given by

$$H_{n,\,t_2} - H_{n,\,t_1} = \Sigma \left\{ h_{n,\,t_2} - h_{n,\,t_1} \right\}.$$

In the case cited above, t_2, t_1 are not the same for different sections of the line of levels, that is, they are functions of n.

Putting $t_2 = t_1 + \Delta t$,

$$h_{n,\,t_2} = h_{n,\,t_1} + \left(\frac{\partial h_n}{\partial t}\right)_{t_1} \Delta t + \frac{1}{2}\left(\frac{\partial^2 h_n}{\partial t^2}\right)(\Delta t)^2 + \ldots\ldots,$$

whence we have

$$h_{n,\,t_2} - h_{n,\,t_1} = \left(\frac{\partial h_n}{\partial t}\right)_{t_1} \Delta t + \frac{1}{2}\left(\frac{d^2 h_n}{\partial t^2}\right)(\Delta t)^2 + \ldots\ldots$$

The change in the height of the n-th bench-mark is therefore given by

$$H_{n,\,t_2} - H_{n,\,t_1} = \sum_n \left\{ \left(\frac{\partial h_n}{\partial t}\right) \Delta t + \frac{1}{2}\left(\frac{\partial^2 h_n}{\partial t^2}\right)(\Delta t)^2 + \ldots\ldots \right\}. \tag{2}$$

The numerical values of $\left(\frac{\partial h_n}{\partial t}\right)$, $\left(\frac{\partial^2 h_n}{\partial t^2}\right)$ in the above expression are easily calculated from the actual data of h for each section between the successive bench-marks of the line.

3. As an example, we dealt with the levelling data along the line of levels between Koiwa and Asakusa-basi, traversing the Kōtō region of Tokyo, including 9 bench-marks.

The distribution of these bench-marks is shown in Fig. 1.

Of these bench-marks, 3379, 3378, 3377 and 3376 were found from recent relevelling to have markedly subsided.

Table I gives the height-differences between the successive bench-marks, the dates of their measurements, and the changes in the height-differences during the successive surveys.

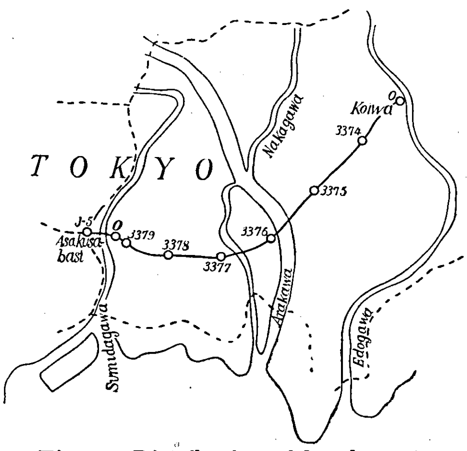

Fig. 1. Distribution of bench-marks.

Table I.

B. M. (Locality)	Dates of Level	h	Δh	Dates of Level	h	Δh
0 (Koiwa)		m	mm		m	mm
3374	1934·75	− 1·9330	+ 2·2	1936·92	− 1·9308	− 1·6
3375	〃	− 0·5440	−133·1	1936·67	− 0·6771	− 36·0
3376	1935·25	− 0·7693	− 23·5	1937·08	− 0·7928	+ 6·7
3377	1935·50	+ 0·8306	− 14·6	1936·33	+ 0·8160	− 27·5
3378	〃	+ 1·2344	+ 15·2	1936·25	+ 1·2496	+ 45·6
3379				1936·17	− 1·2275	+ 14·8
0				1935·92	+ 0·7135	+ 46·2
J-5 (Asakusabasi)				1935·83	+ 1·7642	+ 9·6

B. M. (Locality)	Dates of Level	h	Δh	Dates of Level	h
0 (Koiwa)		m	mm		m
3374	1937·17	− 1·9324	− 2·1	1938·25	− 1·9345
3375	〃	− 0·7131	− 79·8	〃	− 0·7927
3376	〃	− 0·7861	− 8·9	〃	− 0·7950
3377	〃	+ 0·7885	− 39·2	〃	+ 0·7493
3378	〃	+ 1·2952	+ 55·2	〃	+ 1·3504
3379	〃	− 1·2127	+ 24·3	〃	− 1·1884
0	〃	+ 0·7597	+ 34·7	〃	+ 0·7944
J-5 (Asakusabasi)	〃	+ 1·7738	+ 19·4	〃	+ 1·7932

As shown in this table, the dates of measurements of h in the lst and 2nd columns vary over wide ranges, compared with the time intervals between the dates of these measurements and subsequent measurements.

From the data in Table I, the values of $\left(\frac{\partial h}{\partial t}\right)$ are calculated as shown in Table II.

Table II.

B. M.	epoch	$\frac{\partial h}{\partial t}$	epoch	$\frac{\partial h}{\partial t}$	epoch	$\frac{\partial h}{\partial t}$
0		mm/year		mm/year		mm/year
3374	1935·84	1·01	1937·05	− 6·40	1937·17	− 1·94
3375	1935·71	− 69·02	1936·92	− 72·00	〃	− 73·89
3376	1936·22	− 9·13	1937·13	− 8·75	〃	− 8·24
3377	1935·92	− 17·59	1936·75	− 31·61	〃	− 36·30
3378	1935·88	+ 20·27	1936·71	+ 49·57	〃	+ 51·11
3379			1936·67	+ 14·80	〃	+ 22·50
0			1936·55	+ 36·96	〃	+ 32·13
J-5			1936·50	+ 7·16	〃	+ 17·96

In calculating the values of $\left(\frac{\partial h}{\partial t}\right)$ for the section of the line between bench-marks 3375 and 3376, the data of h were smoothed, because the large positive value of $\left(\frac{\partial^2 h}{\partial t^2}\right)$ for the section of the line mentioned above, which is obtainable without smoothing the data of h, is not due to the large amount of changes in the h's measured by levellings before and after the epoch under consideration, but to the shortness of the time interval between seccessive surveys. As another reason for having used smoothed values of h's, it will be seen that the values of $\frac{\partial h}{\partial t}$ for the section of the line under consideration in both the preceding and subsequent epochs are not only smaller in their absolute values, but also are of different signs when compared with the values for the epoch in question, obtainable from data that has not been smoothed.

By using the data given in Table II, Table III was constructed, which shows the values of $\frac{\partial^2 h}{\partial t^2}$.

Table III.

B. M.	epoch	$\frac{\partial^2 h}{\partial t^2}$	epoch	$\frac{\partial^2 h}{\partial t^2}$
0		mm/(year)^2		mm/(year)^2
3374	1936·50	− 6·12	1937·39	+ 6·76
3375	1936·32	− 2·46	1937·32	− 2·39
3376	1936·67	+ 0·38	1937·42	+ 0·57
3377	1936·36	− 17·95	1937·23	− 4·90
3378	1936·29	+ 35·30	1937·21	+ 1·54
3379			1937·19	+ 7·40
0			1937·13	− 4·16
J-5			1937·11	+ 8·93

It will be noticed that, in this table, the values of $\left(\frac{\partial^2 h_n}{\partial t^2}\right)$ of the section of the lines of levels, 3374∼3375, 3375∼3376,[2] have not changed greatly during the different epochs, which shows that the first two terms in Taylor's expansion (expression 2) are sufficient for calculating the values of $(H_{n,t_2}-H_{n,t_1})$. In contrast to this, the values of $\frac{\partial^2 h}{\partial t^2}$ of sections 3376∼3377, 3377∼3378 vary, which shows that the higher terms in Taylor's expansion (2) are available for calculating the values

2) This may be due to the fact that smoothed data was used in calculating the values of $\frac{\partial h}{\partial t}$.

of $(H_{n,t_2}-H_{n,t_1})$ with sufficient accuracy.

The numerical results shown in Tables II and III are then combined, and the values of Δh for the time intervals 1934·75～1936·92, and 1936·92～1937·17 were calculated, with results as shown in Table IV.

Table IV.

B. M.	1934·75～1936·92		1936·92～1937·17	
	Δh (mm)	ΔH (mm)	Δh (mm)	ΔH (mm)
0		0·0		0·0
3374	− 2·7	− 2·7	− 1·7	− 1·7
3375	− 153·9	− 156·6	− 20·4	− 22·1
3376	− 20·9	− 177·5	− 0·7	− 22·8
3377	− 36·0	− 213·5	− 10·3	− 33·1
3378	+ 45·6	− 157·9	+ 16·6	− 16·5
3379			+ 5·1	− 11·4
0			+ 13·9	+ 2·5
J-5			+ 2·8	+ 5·3

In this table, the corrected vertical displacements of the bench-marks for the corresponding time intervals mentioned above are also given.[3]

The vertical displacements of these bench-marks during the time interval 1937·17～1938·25 need not be corrected, seeing that the heights of the bench-marks were measured during these recent levellings at approximately the same time.

In Fig. 2, the vertical displacements thus corrected are shown graphically compared with the uncorrected vertical displacements. The curves in full lines show the corrected vertical displacements and those in dotted lines the uncorrected ones.

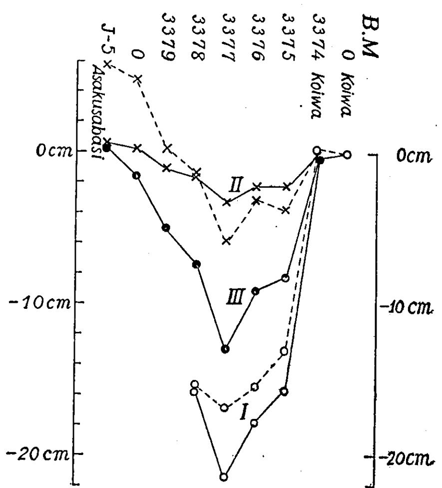

Fig. 2. Corrected (full lines) and uncorrected (dotted lines) vertical displacements of bench-marks along the line between Koiwa and Asakusabasi.

It will be noticed in Fig. 2 that curves I, II, of full lines have forms similar to curve III. and that the vertical displacements of bench-marks for the time interval 1934·75～

3) In calculating the vertical displacements, the displacement of B. M. 0 at Koiwa was taken as zero, as the vertical displacement of this bench-mark during 1930～1938 was very small, referred to the standard datum point.

1936·92 are approximately double those for the time interval 1937·17～1938·25, and 8 times those for the time interval 1936·92～1927·17, all of which correspond to the ratios of the time intervals.

4. A graphic method[4] can also be used for calculating Δh in order to correct the errors in the actual Δh, due to differences in the dates of surveys for different sections of the lines of levels.

Taking a certain epoch at which the line of levels under consideration was run as the time origin, changes in the height-differences are plotted against the time intervals of the corresponding epoch from the epoch taken as the time reference. In applying this method to the present example, for convenience, the data of 1937·17 was taken as the time origin.

We then have curves showing the changes in h (height-differences between successive bench-marks) as shown in Fig. 3. Each of the curves in Fig. 3 corresponds to each section of the lines of levels.

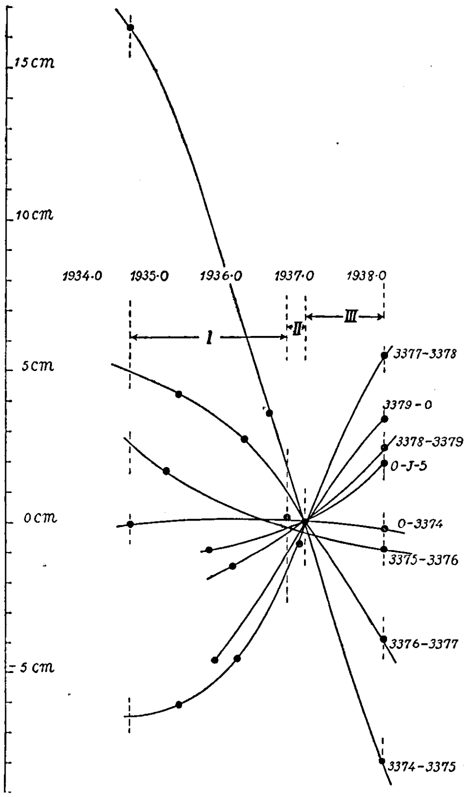

Fig. 3. Changes in height-differences.(Graph used for estimating vertical displacements)

From the graphs in Fig. 3, the values of Δh, i.e., changes in the

Table V.

B. M.	1934·75～1936·92		1936·92～1937·17	
	Δh	ΔH	Δh	ΔH
	mm	mm	mm	mm
0		0·0		0·0
	+ 1·8		− 1·2	
3374		+ 1·8		− 1·2
	− 152·4		− 16·7	
3375		− 150·6		− 17·9
	− 22·3		− 2·1	
3376		− 172·9		− 20·0
	− 37·5		− 10·2	
3377		− 210·4		− 30·2
	+ 52·7		+ 17·4	
3378		− 157·7		− 12·8
			+ 4·1	
3379				− 8·7
			+ 9·2	
0				+ 0·5
			+ 2·7	
J-5				+ 3·2

4) Practically the same method was used by Tsuboi in discussing the crustal deformation in the east coast of Idu Peninsula; C. Tsuboi, *Bull. Earthq. Res. Inst.*, **11** (1933), 488～499.

height-differences, are obtained, the results being shown in Table V.

The results thus obtained and the vertical displacements of the bench-marks deduced therefrom are nearly the same as those obtained by calculating from the data of $\left(\frac{\partial h}{\partial t}\right)$ and $\left(\frac{\partial^2 h_n}{\partial t^2}\right)$, as given in the preceding paragraph. This graphical method of calculating the values of Δh however is not independent of the method of numerical calculation, given in the preceding paragraph. Hence the concordance of the results, obtained by apparently different methods, should not be surprising.

The vertical displacements of the bench-marks in regions markedly deformed, especially those that occurred during shorter time intervals, should be corrected as shown in the manner given above.

In conclusion, the writer wishes to express his best thanks to Mr. H. Yogo for his assistance in preparing the numerical data.

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10. 變動著しき地域に於ける變動量の修正

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東京市内の一部，江東の地域では著しく地盤が沈下してゐる．斯様な地域に於いて，水準測量を施行し，地表面の變形を測定する場合，一回の測量に要する日子が，この測量結果を比較して變動量を算出すべき次の測量までの經過日數に比較して充分少なくないことがある．その様な場合には，測量に要した若干の日數内に生じた變動量をも考慮に入れなければ，眞の，若しくは眞に近い變動量を算定することは出來ない．

實例として，小岩の0號水準點から淺草橋の交5號水準點に至る8鎖部の區間につき，上述の如き修正量を加へた結果を示し，その修正しない變動量に較べて眞に近いらしいことを述べた．