48. *The Effect of Viscosity on the Gravitational Stability of the Earth at its Liquid Cooling Stage.*

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1. *Introduction.*

In the previous paper[1] we discussed the gravitational stability of the earth at its liquid cooling stage without taking into account the liquid viscosity. Since, as a matter of fact, stability is a statical problem, not dynamical, we thought that the viscosity of the liquid would have no effect on stability. On the other hand, Lord Rayleigh[2] and Jeffreys[3] found that the stability of a layer of fluid that is heated below it is affected by its viscosity, which has raised the question whether or not viscosity also affects our problem. Since, however, the problems treated by Lord Rayleigh and Jeffreys are restricted to that case in which a liquid is bounded between two parallel rigid planes, and therefore not immediately available for our use, we have treated our case independently of that due to the authors just mentioned, although it is quite similar to that used in our previous paper.[4] The condition in our present problem which differs from that in our previous one is that, for simplicity, we deal with a two-dimensional problem of the earth's crust without regard to the surface curvature of the earth.

We shall first discuss the problem of the stability of a plane boundary between two liquids extending upward and downward to infinity and next treat the problem of the case in which there is a superficial layer of liquid on another liquid extending downwards to infinity.

The criterion of stability is such that, after discussing the vibratory motion of a boundary between two different liquids, the neutral state

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1) K. SEZAWA and K. KANAI, "Gravitational Stability of the Earth at its Liquid Cooling Stage", *Bull. Earthq. Res. Inst.*, **16** (1938), 234~243.

2) LORD RAYLEIGH, "On Convection Currents in a Horizontal Layer of Fluid, when Higher Temperature is on the Under Side", *Phil. Mag.*, **32** (1916), 529~546.

3) H. JEFFREYS, "The Stability of a Layer of Fluid heated below", *Phil. Mag.*, [7] **2** (1926), 833~844.

4) *loc. cit.* 1).

of the vibrational stability is specially examined. Since the physical conception of instability is that the vibratory deformation at every boundary between different liquids never assumes its original state, the criterion of the problem just mentioned is quite reasonable.

Our investigations show that viscosity affects gravitational stability, though not very much. Instability without viscosity would be changed to stability were the liquid viscous. Furthermore, for the stable condition in question to be realized, it is not necessary that the whole of the liquid shall be viscous. It is possible for the liquid to be in a stable condition provided a certain part of the same liquid near the boundary under consideration is viscous.

2. *Mathematical solutions for the case in which the two liquids extend to infinity.*

Let x, y be drawn in coincidence with and upward normal to the boundary between two liquids, and let u', v', u, v be the x- and y-components of the velocities of the upper and lower layers respectively. The equations of motion and continuity for the upper and the lower layer are

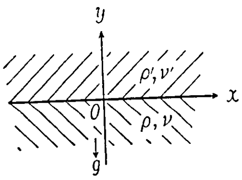

Fig. 1.

$$\left.\begin{aligned} \frac{\partial u'}{\partial t} &= -\frac{1}{\rho'}\frac{\partial p'}{\partial x} + \nu'\nabla^2 u', \quad \frac{\partial v'}{\partial t} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y} + \nu'\nabla^2 v', \\ &\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0, \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u, \quad \frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v, \\ &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \end{aligned}\right\} \quad (2)$$

where ρ', ρ; $\nu' = \mu'/\rho'$, $\nu = \mu/\rho$ are densities and the kinematic coefficients of the viscosities in the respective media. These equations are satisfied by

$$u' = -\frac{\partial \phi'}{\partial x} - \frac{\partial \psi'}{\partial y}, \quad v' = -\frac{\partial \phi'}{\partial y} + \frac{\partial \psi'}{\partial x}, \quad \frac{p'}{\rho'} = \frac{\partial \phi'}{\partial t} - g\eta', \quad (3)$$

$$u = -\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}, \quad \frac{p}{\rho} = \frac{\partial \phi}{\partial t} - g\eta, \quad (4)$$

under the condition that

$$\nabla^2\phi'=0\,,\quad \frac{\partial\psi'}{\partial t}=\nu'\nabla^2\psi',\tag{5}$$

$$\nabla^2\phi=0\,,\quad \frac{\partial\psi}{\partial t}=\nu\nabla^2\psi\,.\tag{6}$$

Solving (5), (6), we have

$$\phi'=Be^{-ky+ikx+nt},\quad \psi'=De^{-m'y+ikx+nt},\tag{7}$$

$$\phi=Ae^{ky+ikx+nt},\quad \psi=Ce^{my+ikx+nt},\tag{8}$$

where $m'^2=k^2+n/\nu'$, $m^2=k^2+n/\nu$.

The normal displacements at the surface and the corresponding normal pressures are such that

$$\left.\begin{aligned}\eta'&=\frac{k}{n}(B+iD)e^{ikx+nt},\\ \frac{p'}{\rho'}&=\frac{\partial\phi'}{\partial t}-g\eta'=\left\{\frac{n^2-gk}{n}B-\frac{igk}{n}D\right\}e^{ikx+nt},\end{aligned}\right\}\tag{9}$$

$$\left.\begin{aligned}\eta&=-\frac{k}{n}(A-iC)e^{ikx+nt},\\ \frac{p}{\rho}&=\frac{\partial\phi}{\partial t}-g\eta=\left\{\frac{n^2+gk}{n}A-\frac{igk}{n}C\right\}e^{ikx+nt}.\end{aligned}\right\}\tag{10}$$

The velocity components in both media are

$$\left.\begin{aligned}u'&=-\frac{\partial\phi'}{\partial x}-\frac{\partial\psi'}{\partial y}=-\left\{ikBe^{-ky}-m'De^{-m'y}\right\}e^{ikx+nt},\\ v'&=-\frac{\partial\phi'}{\partial y}+\frac{\partial\psi'}{\partial x}=-\left\{-kBe^{-ky}-ikDe^{-m'y}\right\}e^{ikx+nt},\end{aligned}\right\}\tag{11}$$

$$\left.\begin{aligned}u&=-\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial y}=-\left\{ikAe^{ky}+mCe^{my}\right\}e^{ikx+nt},\\ v&=-\frac{\partial\phi}{\partial y}+\frac{\partial\psi}{\partial x}=-\left\{kAe^{ky}-ikCe^{my}\right\}e^{ikx+nt},\end{aligned}\right\}\tag{12}$$

The boundary conditions at $y=0$ are

$$\eta'=\eta\,,\quad -p'+2\mu'\frac{\partial v'}{\partial y}=-p+2\mu\frac{\partial v}{\partial y},\quad \mu'\left(\frac{\partial v'}{\partial x}+\frac{\partial u'}{\partial y}\right)=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)=0\,.\tag{13}$$

These relations imply the conditions that although all stresses and normal velocities are continuous respectively, there is a relative tangential motion at the boundary, that is to say, both media are slidable at the same boundary. If one of the media is inviscid, this condition is obviously satisfied.

Substituting (9), (10), (11), (12) in the boundary conditions (13), we get an equation in n as follows

$$n^2(1+\alpha)+4nk^2(\nu+\alpha\nu')+\left\{4k^4(\nu^2+\alpha\nu'^2)+gk(1-\alpha)\right\}$$
$$-4k^3(\nu^2 m+\alpha\nu'^2 m')=0, \quad (14)$$

where $\alpha=\rho'/\rho$. Since $m'^2=k^2+n/\nu'$, $m^2=k^2+n/\nu$, the accurate treatment of this equation is, generally, fairly difficult.

In the special case, $\mu'\to 0$, $\mu\to 0$, we have

$$n^2(\rho+\rho')+gk(\rho-\rho')=0, \quad (15)$$

which is the same as that shown by Lamb.[5]

In the special case, $\alpha=0$, we have

$$(n+2\nu k^2)^2+gk=4\nu^2k^3m\,, \quad (16)$$

which is of the same form as that shown by Lamb[6] for the case of a semi-infinite fluid.

3. *Approximate solution of the above case.*

Although it is possible to solve (14) in elimination of m, m' by using the relations $m^2=k^2+n/\nu$, $m'^2=k^2+n/\nu'$, since the equation thus obtained is of the eighth degree in n, we solve (14) by omitting m, m'.

Even in such a solution the correction due to m, m' is of the second order in $\nu k^2/n$.

Now, the approximate solution of (14) is

$$n=-\xi\pm i\sigma \quad (17)$$

where

$$\xi=\frac{2k^2(\nu+\alpha\nu')}{1+\alpha}, \quad \sigma=\frac{1}{1+\alpha}\sqrt{4k^4\alpha(\nu-\nu')^2+gk(1-\alpha^2)}. \quad (18)$$

If

$$\left\{4k^4(\nu^2+\alpha\nu'^2)+gk(1-\alpha)\right\}>0\,, \quad (19)$$

the oscillation of the boundary is stable, whereas if

$$\left\{4k^4(\nu^2+\alpha\nu'^2)+gk(1-\alpha)\right\}<0\,, \quad (19')$$

5) H. Lamb, *Hydrodynamics*, § 267.
6) H. Lamb, *ibid.*, § 349.

the same oscillation is unstable.

If we put, specially, $\nu=\nu'$, the criterion of stability becomes

$$\alpha=\frac{\rho'}{\rho}<\frac{gk+4k^4\nu^2}{gk-4k^4\nu'^2}\approx 1+\frac{8\nu^2k^3}{g}. \tag{20}$$

On the other hand, Lord Rayleigh[7] has given the criterion of stability for a fluid contained between two parallel plates, the result being that in the disturbed condition of density of fluid due to its temperature change, the condition of stability is defined by

$$\alpha=\frac{\rho'}{\rho}<1+\frac{27\pi^4\kappa\nu}{4g\zeta^3}, \tag{21}$$

where κ = diffusibility for temperature $=5\nu/2$ for air from Maxwell's theory and ζ is the distance between parallel plates. Since, furthermore, the case of maximum instability in a two-dimensional problem is given by the condition, $2\pi/k=2\zeta$, where $2\pi/k$ is the wave length corresponding to the disturbed state of the fluid taken along the lengths of the plates, the relation (21) reduces to

$$\alpha<1+\frac{135\pi}{8g}\nu^2k^3 \tag{21'}$$

for air,which is nearly four times as great as that which we obtained for the fluid extending in both vertical directions to infinity.

It will be seen that, if either one or both of the liquids be viscous, the critical condition of stability is raised somewhat beyond that for the case of non-viscous fluids. The greater the kinematic viscosities or the smaller the wave length of the disturbance, the greater the increase in the critical condition of the stability.

Finally, the components of the disturbed velocities can be derived from

$$\left.\begin{aligned}\phi&=Ae^{-\xi t+ky+i(kx\pm\sigma t)}, & \psi&=Ce^{-\xi t+my+i(kx\pm\sigma t)},\\ \phi'&=Be^{-\xi t-ky+i(kx\pm\sigma t)}, & \psi'&=De^{-\xi t-m'y+i(kx\pm\sigma t)},\end{aligned}\right\} \tag{22}$$

where

$$\frac{C}{A}\approx\frac{-2i\nu k^2}{n+2\nu k^2},\quad \frac{B}{A}\approx\frac{-(n+2\nu' k^2)}{n+2\nu k^2},\quad \frac{D}{A}\approx\frac{-2i\nu' k^2}{n+2\nu k^2}, \tag{23}$$

so that the velocity components corresponding to the viscous state of

7) LORD RAYLEIGH, *loc. cit.* 2).

the fluids are very small compared with those corresponding to the inviscid state of the same fluids.

4. *Mathematical solutions for the case in which an inviscid liquid layer is on a viscous liquid extending to infinity.*

In this case the equations and the solutions for the fluid in the subjacent liquid are the same as those shown in Section 2, whereas the solutions for the superficial layer are

$$\phi' = \left\{A'e^{ky} + B'e^{-ky}\right\} e^{ikx+nt}, \qquad \psi' = 0\,, \tag{24}$$

$$u' = -\frac{\partial \phi'}{\partial x}, \quad v' = -\frac{\partial \phi'}{\partial y}, \quad \frac{p'}{\rho'} = \frac{\partial \phi'}{\partial t} - g\eta', \quad p'_{yy} = -p'. \tag{25}$$

The boundary conditions are

$$y = H; \qquad p'_{yy} = 0\,, \tag{26}$$

$$y = 0; \qquad p'_{yy} = p_{yy}, \quad p_{xy} = 0\,, \quad v = v'. \tag{27}$$

Substituting (9), (10), (12), (25) in (26), (27), we get the frequency equation

$$\begin{aligned} n^4(1+\alpha\,\mathrm{th}\,kH) + n^3(4\nu k^2) + n^2\left\{gk(1+\mathrm{th}\,kH) + 4\nu^2k^4\right\} \\ + n\left\{4\nu k^2 gk\,\mathrm{th}\,kH\right\} + gk\left\{gk(1-\alpha) + 4\nu^2k^4\right\}\mathrm{th}\,kH \\ = 4\nu^2k^3m\left\{n^2 + gk\,\mathrm{th}\,kH\right\}, \end{aligned} \tag{28}$$

where H is the thickness of the surface layer, $\alpha = \rho'/\rho$, and ν the kinematic coefficient of viscosity of the subjacent medium. Since, in the present case, too, the effect of m is negligible, we shall solve equation (28), omitting the term on the right-hand side of the same equation. Even in such a simplified condition of the problem, since the solution of a linear equation of fourth degree is somewhat difficult, we shall use the following approximation. Let

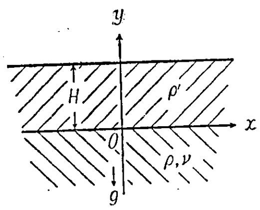

Fig. 2.

$$n_1 = -\xi_1 \pm i\sigma_1, \quad n_2 = -\xi_2 \pm i\sigma_2, \tag{29}$$

then, there are such relations as

$$2(\xi_1 + \xi_2) = \frac{-4\nu k^2}{1+\alpha\,\mathrm{th}\,kH}, \tag{30}$$

$$\xi_1^2+\sigma_1^2+4\xi_1\xi_2+\xi_2^2+\sigma_2^2=\frac{gk(1+\text{th}\,kH)+4\nu^2k^4}{1+\alpha\,\text{th}\,kH}, \tag{31}$$

$$2\xi_1(\xi_2^2+\sigma_2^2)+2\xi_2(\xi_1^2+\sigma_1^2)=\frac{-4\nu k^2gk\,\text{th}\,kH}{1+\alpha\,\text{th}\,kH}, \tag{32}$$

$$(\xi_1^2+\sigma_1^2)(\xi_2^2+\sigma_2^2)=\frac{gk\{gk(1-\sigma)+4\nu^2k^4\}\,\text{th}\,kH}{1+\alpha\,\text{th}\,kH}. \tag{33}$$

From these conditions we shall first solve

$$n^4+n^2\frac{gk(1+\text{th}\,kH)+4\nu^2k^4}{1+\alpha\,\text{th}\,kH}+\frac{gk\{gk(1-\alpha)+4\nu^2k^4\}\,\text{th}\,kH}{1+\alpha\,\text{th}\,kH}=0, \tag{34}$$

under the assumption that νk^2 is small, that is to say, $\xi_1=0$, $\xi_2=0$. Solving (34), we get

$$\sigma_1^2,\ \sigma_2^2=\frac{gk(1+\text{th}\,kH)+4\nu^2k^4}{2(1+\alpha\,\text{th}\,kH)} \mp\sqrt{\left\{\frac{gk(1+\text{th}\,kH)+4\nu^2k^4}{2(1+\alpha\,\text{th}\,kH)}\right\}^2-\frac{\{gk(1-\alpha)+4\nu^2k^4\}\,gk\,\text{th}\,kH}{1+\alpha\,\text{th}\,kH}} \tag{35}$$

approximately. Substituting these values of σ_1, σ_2 in (32), and solving (30) and (32), we get

$$\left.\begin{aligned}\xi_1&=\frac{\nu k^2}{1+\alpha\,\text{th}\,kH}\left[1-\frac{1}{\sqrt{M^2-N}}\left\{M-2(1+\alpha\,\text{th}\,kH)\,\text{th}\,kH\right\}\right],\\ \xi_2&=\frac{\nu k^2}{1+\alpha\,\text{th}\,kH}\left[1+\frac{1}{\sqrt{M^2-N}}\left\{M-2(1+\alpha\,\text{th}\,kH)\,\text{th}\,kH\right\}\right],\end{aligned}\right\} \tag{36}$$

where

$$M=1+\text{th}\,kH,\quad N=4(1-\alpha)(1+\alpha\,\text{th}\,kH)\,\text{th}\,kH. \tag{37}$$

Again, substituting the values of ξ_1, ξ_2 in (36) in those of (31), (33), we get more accurate values of σ_1, σ_2 as follows

$$\left.\begin{aligned}\sigma_1&=\sqrt{\frac{1}{2}\left\{P_1-\sqrt{P_1^2-4Q_1}\right\}},\\ \sigma_2&=\sqrt{\frac{1}{2}\left\{P_2+\sqrt{P_2^2-4Q_2}\right\}},\end{aligned}\right\} \tag{38}$$

where

$$P_1=\frac{gkM+4\nu^2k^4}{1+\alpha\,\mathrm{th}\,kH}-\frac{\nu^2k^4}{(1+\alpha\,\mathrm{th}\,kH)^2}\left[3-\frac{1}{\sqrt{M^2-N}}\Big\{M-2(1+\alpha\,\mathrm{th}kH)\,\mathrm{th}\,kH\Big\}\right]\left[1-\frac{1}{\sqrt{M^2-N}}\Big\{M-2(1+\alpha\,\mathrm{th}\,kH)\,\mathrm{th}\,kH\Big\}\right],$$

$$Q_1=\frac{\{gk(1-\alpha)+4\nu^2k^4\}\,gk\,\mathrm{th}\,kH}{1+\alpha\,\mathrm{th}\,kH}-\frac{gk\nu^2k^4M}{(1+\alpha\,\mathrm{th}\,kH)^2}\left[1-\frac{1}{\sqrt{M^2-N}}\Big\{M-2(1+\alpha\,\mathrm{th}\,kH)\,\mathrm{th}\,kH\Big\}\right]^2,$$

$$P_2=\frac{gkM+4\nu^2k^4}{1+\alpha\,\mathrm{th}\,kH}-\frac{\nu^2k^4}{(1+\alpha\,\mathrm{th}\,kH)^2}\left[3+\frac{1}{\sqrt{M^2-N}}\Big\{M-2(1+\alpha\,\mathrm{th}kH)\,\mathrm{th}\,kH\Big\}\right]\left[1+\frac{1}{\sqrt{M^2-N}}\Big\{M-2(1+\alpha\,\mathrm{th}\,kH)\,\mathrm{th}\,kH\Big\}\right],$$

$$Q_2=\frac{\{gk(1-\alpha)+4\nu^2k^4\}\,gk\,\mathrm{th}\,kH}{1+\alpha\,\mathrm{th}\,kH}-\frac{gk\nu^2k^4M}{(1+\alpha\,\mathrm{th}\,kH)^2}\left[1+\frac{1}{\sqrt{M^2-N}}\Big\{M-2(1+\alpha\,\mathrm{th}\,kH)\,\mathrm{th}\,kH\Big\}\right]^2. \qquad (39)$$

From the expressions of σ_1, σ_2 in (38), it is obvious that the vibration corresponding to $n_2=-\xi_2\pm i\sigma_2$ does not participate in the problem of stability. Thus, $n_1=-\xi_1\pm i\sigma_1$ only concerns the stability problem, its condition being

$$\left.\begin{aligned}&P_1-\sqrt{P_1^2-4Q_1}+2\xi_1^2>0\ \text{ for stable condition,}\\&P_1-\sqrt{P_1^2-4Q_1}+2\xi_1^2<0\ \text{ for unstable condition.}\end{aligned}\right\}\qquad (40)$$

The deformations or velocities of the fluids are determined from

$$\left.\begin{aligned}&\phi=Ae^{-\xi t+ky+i(kx\pm\sigma t)},\qquad \psi=Ce^{-\xi t+my+i(kx\pm\sigma t)},\\&\phi'=A'e^{-\xi t+ky+i(kx\pm\sigma t)}+B'e^{-\xi t-ky+i(kx\pm\sigma t)},\end{aligned}\right\}\qquad (41)$$

where

$$\left.\begin{aligned}&\frac{C}{A}=\frac{-2i\nu k^2}{n+2\nu k^2},\\&\frac{A'}{A}=\frac{n(n^2-gk)e^{-kH}}{2(n+2\nu k^2)(n^2\mathrm{ch}\,kH+gk\,\mathrm{sh}\,kH)},\\&\frac{B'}{A}=\frac{-n(n^2+gk)e^{kH}}{2(n+2\nu k^2)(n^2\mathrm{ch}\,kH+gk\,\mathrm{sh}\,kH)}.\end{aligned}\right\}\qquad (42)$$

It will be seen that in this case too, the component of velocity corresponding to the viscous state of the fluid is very small compared with that due to the inviscid state of the same fluid.

5. *Interpretation of the two kinds of vibrations in the above case.*

It has already been ascertained that the vibration corresponding to expression $n_1=-\xi_1\pm i\sigma_1$ only concerns the problem of stability. We shall now examine the kinds of disturbances that the vibrations $n_1=-\xi_1\pm i\sigma_1$, $n_2=-\xi_2\pm i\sigma_2$ represent.

Let us take the case $kH\to 0$, then

$$\left.\begin{array}{lll}\xi_1=0\,, & \sigma_1^2=gk+4\nu^2k^4, & 0\,, \\ \xi_2=2\nu k^2, & \sigma_2^2=gk\,, & -4\nu^2k^4.\end{array}\right\} \tag{43}$$

so that $n_2=-\xi_2\pm i\sigma_2=-2\nu k^2\pm i\sqrt{gk}$ represents the gravitational waves on a semi-infinite viscous fluid as already shown by Lamb.[8] In other words, the vibration $n_2=-\xi_2\pm i\sigma_2$ corresponds to the vibration of the free surface of the case of a very thin layer, the vibration being stable even in the present special case.

Let us next take the case $kH\to\infty$, then

$$\xi_1=\frac{2\nu k^2}{1+\alpha}\,,\quad \xi_2=0\,,\quad \sigma_2^2=gk\,, \tag{44}$$

the expression of σ_1 being rather complex. Since vibration $n_2=-\xi_2\pm i\sigma_2$ in the present case represents the gravitational waves that are transmitted over the surface of an inviscid fluid, the vibration in question corresponds to the one on the free surface and has no place in the problem of stability.

It is now established that vibration $n_1=-\xi_1\pm i\sigma_1$ concerns the condition at the boundary between the two fluids and contributes much to gravitational stability, whereas vibration $n_2=-\xi_2\pm i\sigma_2$ represents the waves on the free surface and is outside the problem of stability.

6. *The calculation for the case in which there is an inviscid surface layer.*

Using the equations shown in Section 4, we have calculated the ratio of ρ'/ρ, namely α, corresponding to the critical condition of stability for different ratios of L/H, $(L=2\pi/k)$, and gk(and νk^2). The values of gk taken here are $gk=0{\cdot}06$ and $0{\cdot}02$, both of which corre-

8) H. LAMB, *loc. cit.*, 6).

spond to the waves of lengths, 1 km and 1/3 km respectively. The values of ν, on the other hand, roughly corresponds to the viscosity of rocks or of soils. It should be borne in mind that νk^2 should change in agreement with the change in gk.

The calculation for the two conditions (I) $gk=0{\cdot}06$, $\nu k^2=0{\cdot}01$ and (II) $gk=0{\cdot}02$, $\nu k^2=0{\cdot}00111$ are shown in Tables I, II and Fig. 3. It will be seen that the value of ρ'/ρ corresponding to the critical stability decreases as the ratio of L/H increases, that is to say, as the thickness of the surface layer decreases, tending to approach the asymptotic value:

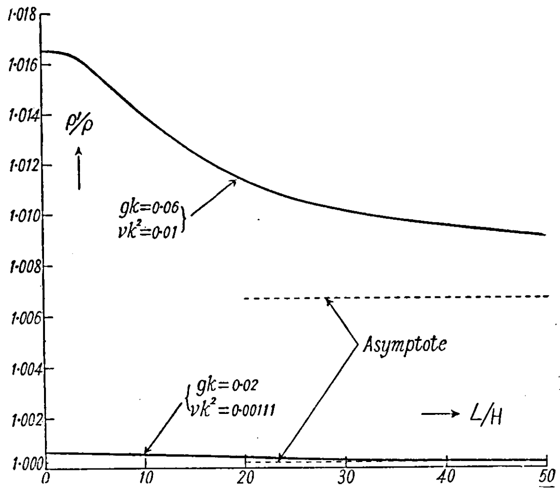

Fig. 3. Critical conditions of stability for two cases.

$$a=1+4\nu^2k^3/g \tag{45}$$

for $L/H\to\infty$. This asymptotic value incidentally coincides with the value of a for $\nu'=0$ that has been obtained in using equation (19) in Section 3. The value of a for $L/H=0$ does not agree with that obtained in the manner in Section 2. This results from the assumption in neglecting higher orders of νk^2 in both the cases.

In any case, if the liquid viscosity is not zero, the value of ρ'/ρ corresponding to the critical stability is greater than 1.

Table I. Case I. $gk=0{\cdot}06$, $\nu k^2=0{\cdot}01$.

kH	0	0·1	0·2	0·5	1·0	2·1	π	∞
L/H	∞	62·832	31·416	12·566	6·283	2·994	2·000	0
ρ'/ρ	1·0067	1·0087	1·0100	1·0132	1·0152	1·0164	1·0165	1·0165

Table II. Case II. $gk=0\cdot02$, $\nu k^2=0\cdot00111$.

kH	0	0·1	0·2	1	2	∞
L/H	∞	62·832	31·416	6·283	3·142	0
ρ'/ρ	1·00025	1·00026	1·00027	1·00057	1·00059	1·00065

7. *Concluding remarks.*

From the mathematical calculation it is shown that in stratified liquids, even should the upper liquid be denser than the lower liquid, the dynamical condition may be stablilized to a certain extent by means of the viscosity of one or both liquids. It was also found that although the disturbance at the boundary between the liquids contributes only to the problem of stability under consideration, disturbance at the free surface, on the other hand, does not participate in the same problem.

In the liquid stage of the earth, as soon as the part near the free surface, as the result of cooling, becomes slightly denser than the subjacent medium, the same part becomes gravitationally unstable and sinks. But, since at such a stage, the liquids were perhaps fairly viscous, stability is likely to have been maintained to a certain extent even if the density relation in the liquid layers was in unstable condition.

The general feature of equilibrium seen in the present state of the earth's crust, is probably a continuation of the stage just mentioned. Some local changes in the gravitational forces may at times result from the unstable displacements of layers in the earth's crust. If that is the case, it ought not to be hopeless to attempt to examine the origin of earthquakes by means of careful observations of crustal deformation, or of changes in the gravitational forces, or even of changes in terrestrial magnetism.

48. 粘性が地球の液狀冷却時代に於ける重力的安定に及ぼす影響

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地球が液狀をなして冷却した時代は相當長く續いたものと見てよろしく，その場合に表面から冷却する結果として種々の重力的安定の問題があり得るのである．大體に於ては上層の物質が内部層のそれよりも重くなると，重力的不安定が起る譯であるが，粘性が働く場合には上部の層が少し位重くても安定の狀態が保たれ得るのである．この問題は Rayleigh や Jeffreys も取扱つた事があるけれども，二つの層が接し且つ兩方の層の一方又は兩方が無限に續いてをる場合や自由面のある場合，即ち地球の問題に直接關係のありそうな場合は未だ手がつけられてをらなかつたので，玆にこの問題を研究して見たのである．

表面層のある場合を理論的に研究して見ると，表面層の自由面の振動と兩方の層の境界に起る振動とになる．表面の振動は如何なる場合にも安定であるけれども境界の振動は兩方の層の比重と粘性とによつて不安定になるのである．

無限に厚い層が接する場合には，勿論兩方の境界面の振動の安定不安定が問題となるのである．

粘性はいつでも安定狀態を助けることになる．而も液體の全部が粘性をもつ必要がなく，その一方だけに粘性があつても不安定の狀態が安定の狀態になり得るのである．之は表面の層が薄い場合にも無限に厚い場合にもあり得るものである．

現在の地殼は液體の狀態からは殆ど脱してをるとはいへ，極めて遲い運動については可なり液狀の運動があるものと見てもよい譯である．又，冷却して行く狀態も既になくなつてをるとはいへ，地殼の水平壓力のため輕い層が重い層の下に入り込むこともあり得る譯である．大陸の邊緣等にはこのやうな狀態があるであらう．その場合の重力的不安定の問題はたしかに地震の原因と結びつけられるものと思はれる．又，下層の物質に於ける熱力學的又は放射能的變化の結果として密度の變化が起ることも考へられる．玆にも重力的の不安定の問題があり得るのである．地殼の變形乃至は地球上の重力，地磁氣，地電流等の時間的變化を測定する事は以上の點に於て大いに意味があるものと考へられる．但し粘性の影響も大いにある事をも記憶して問題を取扱ふ必要があるのである．