38. *The Formation of Boundary Waves at the Surface of a Discontinuity within the Earth's Crust. I.*

By Katsutada SEZAWA and Kiyoshi KANAI,

Earthquake Research Institute.

(Read June 21, 1938.—Received June 20, 1938.)

I. *Introduction.*

1. Stoneley,[1] in a recent paper, showed that, although it is possible for Rayleigh-type waves to be transmitted along the surface of separation of two solids (for some conditions of two solids), it is impossible for Love-type waves to be so transmitted. It appears that Stoneley's result just mentioned has an important bearing on the fact that the horizontal amplitudes of distortional waves are much larger than vertical amplitudes of the same distortional waves as well as amplitudes of dilatational waves. In the case of dilatational waves, or distortional waves with amplitudes orientated in a vertical plane, the energy of the waves is partly converted into that of boundary waves in passing through every discontinuous surface within the earth's crust, whereas in the case of distortional waves with amplitudes orientated horizontally, no boundary wave is formed at the said discontinuous surface so that the conditions of the problem for the two cases are quite different.

In the present paper we shall show, mathematically, how bodily waves generated from a point source in the solid excite boundary waves at any discontinuous surface in the same solid, the problem being two-dimensional. The method of calculation is somewhat similar to that which Sommerfeld[2] used in his paper on the transmission of electromagnetic waves, by means of which it was possible for us to get the solutions even in the case of some difficult conditions of the problem. We shall first deal with the behaviour of waves in the case of dilatational primary waves. The case of distortional primary waves, which

1) R. STONELEY, "Elastic Waves at the Surface of Separation of Two Solids", *Proc. Roy. Soc., London,* **106** (1924), 416~428.

2) A. SOMMERFELD, "Über die Ausbreitung der Wellen in der drahtlosen Telegraphie", *Ann. Phys.*, [4], **28** (1909), 665~738.

will next be discussed, will be divided into two subcases, namely, the one in which the amplitudes of the primary waves are orientated in a vertical plane and the other in which the amplitudes in question are orientated horizontally.

II. *The Case of Dilatational Primary Waves.*

2. Let the axes of x and y be drawn in coincidence with and perpendicular to the surface of discontinuity, the densities and elastic constants in the media on the positive and negative sides of y being ρ, λ, μ; ρ', λ', μ' respectively. In the case of plane primary waves

$$\phi_0 = \mathfrak{A} e^{ry+ifx-ipt}, \tag{1}$$

where $\tan^{-1}(f/ir)$ is the angle of incidence, the reflected and refracted dilatational and distortional waves assume the forms

$$\left.\begin{aligned} \phi &= Ae^{-ry+ifx-ipt}, & \psi &= Be^{-sy+ifx-ipt}, \\ \phi' &= Ce^{r'y+ifx-ipt}, & \psi' &= De^{s'y+ifx-ipt}, \end{aligned}\right\} \tag{2}$$

where

$$\left.\begin{aligned} r^2 &= f^2-h^2, & s^2 &= f^2-k^2, & r'^2 &= f^2-h'^2, & s'^2 &= f^2-k'^2, \\ h^2 &= \frac{\rho p^2}{\lambda+2\mu}, & k^2 &= \frac{\rho p^2}{\mu}, & h'^2 &= \frac{\rho' p^2}{\lambda'+2\mu'}, & k'^2 &= \frac{\rho' p^2}{\mu'}. \end{aligned}\right\} \tag{3}$$

The displacements in both media are then expressed by

$$\left.\begin{aligned} u &= \frac{\partial}{\partial x}(\phi_0+\phi)+\frac{\partial\psi}{\partial y}, & v &= \frac{\partial}{\partial y}(\phi_0+\phi)-\frac{\partial\psi}{\partial x}, \\ u' &= \frac{\partial\phi'}{\partial x}+\frac{\partial\psi'}{\partial y}, & v' &= \frac{\partial\phi'}{\partial y}-\frac{\partial\psi'}{\partial x}. \end{aligned}\right\} \tag{4}$$

Since the two media are continuous, the conditions at the boundary $y=0$, are such that

$$\left.\begin{gathered} u=u', \qquad v=v', \\ \lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial v}{\partial y} = \lambda'\left(\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right)+2\mu'\frac{\partial v'}{\partial y}, \\ \mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) = \mu'\left(\frac{\partial v'}{\partial x}+\frac{\partial u'}{\partial y}\right). \end{gathered}\right\} \tag{5}$$

Substituting (1), (2) in (5), we get

$$\left.\begin{aligned}
\frac{A\Phi}{\mathfrak{A}} &= \mu'^2 (f^2+\sqrt{f^2-h^2}\sqrt{f^2-k^2})\left\{4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-(2f^2-k'^2)^2\right\} \\
&\quad + \mu\mu' k^2 k'^2 (\sqrt{f^2-k^2}\sqrt{f^2-h'^2}-\sqrt{f^2-h^2}\sqrt{f^2-k'^2}) \\
&\quad + 2\mu\mu' f^2 (2f^2-k^2+2\sqrt{f^2-h^2}\sqrt{f^2-k^2}) \\
&\quad \cdot (2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\
&\quad + \mu^2 (\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-f^2)\left\{4f^2\sqrt{f^2-h^2}\sqrt{f^2-k^2}-(2f^2-k^2)^2\right\}, \\
\frac{B\Phi}{\mathfrak{A}} &= 2if\sqrt{f^2-h^2}\Big[\mu'^2\left\{4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-(2f^2-k'^2)^2\right\} \\
&\quad + \mu\mu' (4f^2-k^2)(2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\
&\qquad\qquad + 2\mu^2 (2f^2-k^2)(\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-f^2)\Big], \\
\frac{C\Phi}{\mathfrak{A}} &= 2\mu k^2\sqrt{f^2-h^2}\Big[\mu'\left\{(2f^2-k'^2)\sqrt{f^2-k^2}-2f^2\sqrt{f^2-k'^2}\right\} \\
&\qquad\qquad + \mu\left\{(2f^2-k^2)\sqrt{f^2-k'^2}-2f^2\sqrt{f^2-k^2}\right\}\Big], \\
\frac{D\Phi}{\mathfrak{A}} &= 2i\mu fk^2\sqrt{f^2-h^2}\Big\{\mu'(2f^2-k'^2-2\sqrt{f^2-k^2}\sqrt{f^2-h'^2}) \\
&\qquad\qquad + \mu(2\sqrt{f^2-k^2}\sqrt{f^2-h'^2}+k^2-2f^2)\Big],
\end{aligned}\right\} \quad (6)$$

where

$$\begin{aligned}
\Phi &= \mu'^2 (f^2-\sqrt{f^2-h^2}\sqrt{f^2-k^2})\left\{(2f^2-k'^2)^2-4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}\right\} \\
&\quad - \mu\mu' k^2 k'^2 (\sqrt{f^2-h^2}\sqrt{f^2-k'^2}+\sqrt{f^2-k^2}\sqrt{f^2-h'^2}) \\
&\quad + 2\mu\mu' f^2 (2\sqrt{f^2-h^2}\sqrt{f^2-k^2}+k^2-2f^2)(2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\
&\quad + \mu^2 (f^2-\sqrt{f^2-h'^2}\sqrt{f^2-k'^2})\left\{(2f^2-k^2)^2-4f^2\sqrt{f^2-h^2}\sqrt{f^2-k^2}\right\}. \quad (7)
\end{aligned}$$

Although we have discussed the problem of reflection and refraction of waves at a plane boundary, yet owing to the type of the expressions in (1), (2), the disturbances may be assumed to be a kind of boundary waves near $y=0$. The condition that $\Phi=0$, exactly corresponds to the equation for determining the velocity of the waves that are possibly transmitted along the boundary under consideration. The velocity equation obtained by Stoneley[3] is

$$\begin{aligned}
& c^4\left\{(\rho_1-\rho_2)^2-(\rho_1 A_2+\rho_2 A_1)(\rho_1 B_2+\rho_2 B_1)\right\} \\
&\quad + 2Kc^2\left\{\rho_1 A_2 B_2-\rho_2 A_1 B_1-\rho_1+\rho_2\right\}+K^2(A_1 B_1-1)(A_2 B_2-1)=0. \quad (8)
\end{aligned}$$

3) *loc. cit.* 1).

If the symbols used by Stoneley are replaced by the corresponding ones in the present case, such that

$$\left.\begin{aligned}
&c^2=\frac{p^2}{f^2},\quad \alpha_1^2=\frac{\lambda_1+2\mu_1}{\rho_1}=\frac{1}{p^2h^2},\quad \beta_1^2=\frac{\mu_1}{\rho_1}=\frac{1}{p^2k^2},\quad \rho_1=\rho,\quad \lambda_1=\lambda,\quad \mu_1=\mu,\\
&\qquad \alpha_2^2=\frac{\lambda_2+2\mu_2}{\rho_2}=\frac{1}{p^2h'^2},\quad \beta_2^2=\frac{\mu_2}{\rho_2}=\frac{1}{p^2k'^2},\quad \rho_2=\rho',\quad \lambda_2=\lambda',\quad \mu_2=\mu',\\
&A_1=\left(1-\frac{c^2}{\alpha_1^2}\right)^{\frac{1}{2}},\quad A_2=\left(1-\frac{c^2}{\alpha_2^2}\right)^{\frac{1}{2}},\quad B_1=\left(1-\frac{c^2}{\beta_1^2}\right)^{\frac{1}{2}},\quad B_2=\left(1-\frac{c^2}{\beta_2^2}\right)^{\frac{1}{2}},\\
&\qquad\qquad K=2\,(\rho_1\beta_1^2-\rho_2\beta_2^2),
\end{aligned}\right\}\quad (9)$$

the form of the expression in (8) reduces to that in (7).

Let us next consider the condition in which the primary dilatational waves are radiated from a point source at $x=0$, $y=\xi$. Then

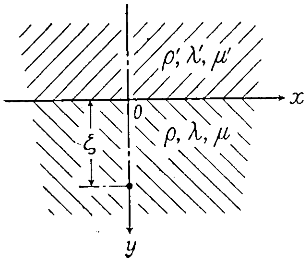

Fig. 1.

$$\left.\begin{aligned}
\phi_0&=\Re\{e^{-ipt}H_0^{(1)}(hR)\}\\
&=\frac{\Re\{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{r(y-\xi)+ifx}}{r}\,df,\quad [y<\xi]\\
&=\frac{\Re\{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{-r(y-\xi)+ifx}}{r}\,df.\quad [y>\xi]
\end{aligned}\right\}\quad (10)$$

From (1), (2) it follows that the reflected and refracted waves assume the forms

$$\left.\begin{aligned}
\phi&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Ae^{-r(y+\xi)+ifx}}{r}\,df,\quad \psi=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Be^{-sy-r\xi+ifx}}{r}\,df,\\
\phi'&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Ce^{r'y-r\xi+ifx}}{r}\,df,\quad \psi'=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{De^{s'y-r\xi+ifx}}{r}\,df.
\end{aligned}\right\}\quad (11)$$

The displacements at any point are obtained by using (10), (11), (4).

3. To evaluate the integral expressions of displacements, we consider such integrals in which the f-value in (10), (11) is replaced by a complex quantity $Z=X+iY$. The paths of integration are: (i) the real axis in the Z plane from $-\infty$ to ∞, (ii) a circular arc of infinite radius in the first and second quadrants in the same plane, (iii) an arc around the pole $Z=\kappa$, which satisfies the condition $\Phi(\kappa)=0$, (iv) four branch lines connecting the respective branch points with $Z=i\infty$. The sum of the four kinds of contour integrals vanishes, from which it is possible to obtain the integrals performed along the real axis from $-\infty$ to ∞. The integral performed along the semi-circular arc vanishes owing to the factor $e^{-Rx\sin\theta}$, whereas the integral of

$$\int\frac{\chi(Z)}{\Phi(Z)}dZ \tag{12}$$

around the pole $Z=\kappa$, is merely written

$$-2\pi i\frac{\chi(\kappa)}{\Phi'(\kappa)}. \tag{13}$$

There remains the integral along every branch line.

The branch lines assumed here are similar to those used by Sommerfeld. Let $Z=h,\ k,\ h',\ k'$ be four branch points. Four branch lines are drawn in such a way that the values of $\sqrt{Z^2-h^2}$, $\sqrt{Z^2-k^2}$, $\sqrt{Z^2-h'^2}$, $\sqrt{Z^2-k'^2}$ along the respective branch lines are purely imaginary for any Z on the same branch lines. $Z=i\infty$ satisfies this condition, at least at infinity. Every branch line near its branch point makes angle $\theta(=\pi-\varphi)$ with the X-axis, where φ is the angle of inclination of the line passing through the branch point and the origin $X=0,\ Y=0$. To prove this, we shall take, for example, the line through $Z=h$. Then $Z-h=\zeta_h=|\zeta_h|e^{i\theta}$, $h=|h|e^{i\varphi}$, so that $\sqrt{Z^2-h^2}=\sqrt{2|h|\zeta_h|}e^{\frac{i(\varphi+\theta)}{2}}$. In order that the value of $\sqrt{Z^2-h^2}$ shall be purely imaginary, $\varphi+\theta$ should be π, the problem being thus proved. Although the condition of $\sqrt{Z^2-h^2}$ being purely imaginary for intermediate points is not verified, the existence of such special points, arranged in a certain line, is quite possible. It should also be borne in mind that the two values of $\sqrt{Z^2-h^2}$ on both sides of the branch line have opposite signs. The conditions here given are valid for every branch line in the Z-plane.

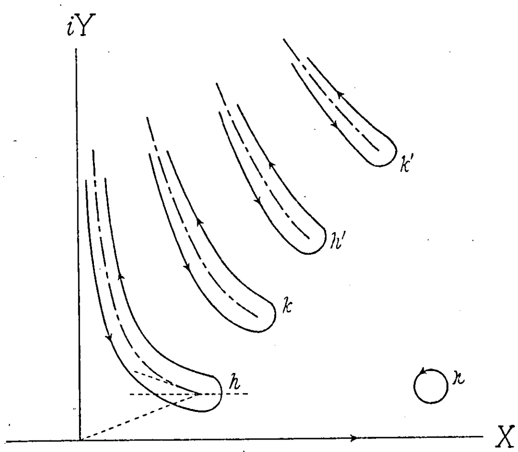

Fig. 2.

4. In the present paper there are six cases of representative integrations along branch lines, the respective ones of which will be shown successively.

In every case the sense of the integration along the contour is opposite to that along the X-axis.

(i) $$\int_{i\infty}^{\eta} \frac{U_1 e^{\sqrt{Z^2-\eta^2}y+iZx}}{-\sqrt{Z^2-\eta^2}}\, dZ + \int_{\eta}^{i\infty} \frac{U_2 e^{-\sqrt{Z^2-\eta^2}y+iZx}}{\sqrt{Z^2-\eta^2}}\, dZ \qquad (14)$$

along the branch line through $Z=\eta$. We change the variable, such that

$$\pm\sqrt{Z^2-\eta^2}=\pm i\tau, \quad Z=i\sqrt{\tau^2-\eta^2}, \quad dZ=\frac{i\tau d\tau}{\sqrt{\tau^2-\eta^2}}, \qquad (15)$$

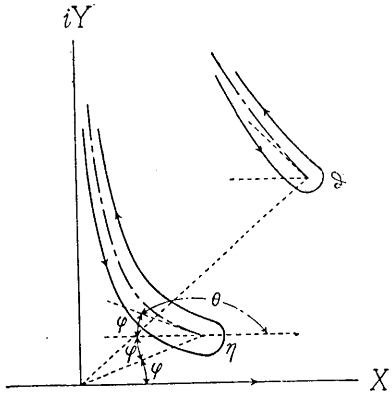

Fig. 3.

with the condition that $y \gtrless 0$, $x \gtrless 0$.
The limits of integration $i\infty$, η, in Z-plane become ∞ and 0 in the τ-plane. The integrals (14) then assume the forms

$$\int_{\infty}^{0} \frac{U_1' e^{i\tau y-\sqrt{\tau^2-\eta^2}x}}{-i\tau} \frac{i\tau d\tau}{\sqrt{\tau^2-\eta^2}} + \int_{0}^{\infty} \frac{U_2' e^{-i\tau y-\sqrt{\tau^2-\eta^2}x}}{i\tau} \frac{i\tau d\tau}{\sqrt{\tau^2-\eta^2}}. \qquad (14')$$

If we write $\tau=-\tau'$ in the first integral, the integrals transform to

$$\int_{-\infty}^{0} \frac{U_1'' e^{-i\tau' y-\sqrt{\tau'^2-\eta^2}x}\, d\tau'}{\sqrt{\tau'^2-\eta^2}} + \int_{0}^{\infty} \frac{U_2'' e^{-i\tau y-\sqrt{\tau^2-\eta^2}x}\, d\tau}{\sqrt{\tau^2-\eta^2}}, \qquad (14'')$$

U_1'' being equal to U_2'' for $\tau=0$. If U_1'' is important merely in the vicinity of the branch point $\tau=0$, the integral (14″) is evidently

$$\pi U_1'' H_0^{(1)}(\eta R), \qquad (16)$$

where $R^2=x^2+y^2$, $y \gtrless 0$, $x \gtrless 0$.

(ii) $$\int_{i\infty}^{\kappa} \frac{U_1 e^{\sqrt{Z^2-\eta^2}y+iZx}}{-\sqrt{Z^2-\eta^2}}\, dz + \int_{\kappa}^{i\infty} \frac{U_2 e^{-\sqrt{Z^2-\eta^2}y+iZx}}{\sqrt{Z^2-\eta^2}}\, dZ \qquad (17)$$

along the branch line through $Z=\kappa$. In this case, we write

$$\pm\sqrt{Z^2-\kappa^2}=\pm i\tau, \quad Z=i\sqrt{\tau^2-\kappa^2}, \quad dZ=\frac{i\tau d\tau}{\sqrt{\tau^2-\kappa^2}}, \qquad (18)$$

so that the limits of integration $i\infty$, κ in the Z-plane become ∞ and 0 in the τ-plane. The integrals (17) transform to

$$\int_{\infty}^{0} \frac{U_1' e^{-i\sqrt{\tau^2-\kappa^2+\eta^2}y-\sqrt{\tau^2-\kappa^2}x}}{\sqrt{\tau^2-\kappa^2+\eta^2}} \frac{\tau d\tau}{\sqrt{\tau^2-\kappa^2}} + \int_{0}^{\infty} \frac{U_2' e^{-i\sqrt{\tau^2-\kappa^2+\eta^2}y-\sqrt{\tau^2-\kappa^2}x}}{\sqrt{\tau^2-\kappa^2+\eta^2}} \frac{\tau d\tau}{\sqrt{\tau^2-\kappa^2}}. \tag{17'}$$

In the present problem the values of U_1', U_2' are important near $\tau=0$, in which case $U_1'=U_2'$, and both the integrals in (17′) then cancel each other.

(iii)
$$\int_{i\infty}^{\kappa} U_1 e^{\sqrt{Z^2-\kappa^2}y+iZx} dZ + \int_{\kappa}^{i\infty} U_2 e^{-\sqrt{Z^2-\kappa^2}y+iZx} dZ. \tag{19}$$

Writing $\pm\sqrt{Z^2-\kappa^2}=\pm i\tau$, this reduces to

$$\int_{\infty}^{0} U_1' e^{i\tau y-\sqrt{\tau^2-\kappa^2}x} \frac{i\tau d\tau}{\sqrt{\tau^2-\kappa^2}} + \int_{0}^{\infty} U_2' e^{-i\tau y-\sqrt{\tau^2-\kappa^2}x} \frac{i\tau d\tau}{\sqrt{\tau^2-\kappa^2}}. \tag{19'}$$

If we put $\tau=-\tau'$ in the first integral of (19′) and put $\tau=0$ in U_1', U_2', then (19′) becomes

$$\int_{-\infty}^{\infty} U_1' e^{-i\tau y-\sqrt{\tau^2-\kappa^2}x} \frac{i\tau d\tau}{\sqrt{\tau^2-\kappa^2}}. \tag{19''}$$

Since U_1' is a constant and

$$-\int_{-\infty}^{\infty} \frac{e^{-i\tau y-\sqrt{\tau^2-\kappa^2}x}}{\sqrt{\tau^2-\kappa^2}} i\tau d\tau = \pi \frac{\partial H_0^{(1)}(\kappa R)}{\partial y}, \tag{20}$$

(19″) reduces to

$$\pi U_1' \kappa H_1^{(1)}(\kappa R) \frac{y}{R}, \tag{21}$$

where $R^2=x^2+y^2, x\neq 0,\ y\neq 0$.

(iv)
$$\int_{i\infty}^{\eta} \frac{U_1 e^{\sqrt{Z^2-\eta^2}y+iZx}}{-\sqrt{Z^2-\eta^2}} Z dZ + \int_{\eta}^{i\infty} \frac{U_2 e^{-\sqrt{Z^2-\eta^2}y+iZx}}{\sqrt{Z^2-\eta^2}} Z dZ. \tag{22}$$

In the same way as in the preceding case, the integral transforms to

$$i\int_{-\infty}^{\infty} U_1' e^{-i\tau y-\sqrt{\tau^2-\eta^2}x} d\tau. \tag{22'}$$

By means of the condition

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} e^{i\tau y-\sqrt{\tau^2-\eta^2}x} d\tau = \frac{\partial H_0^{(1)}(\eta R)}{\partial x}, \tag{23}$$

the integral finally assumes the form

$$i\pi U_1'\gamma H_1^{(1)}(\gamma R)\frac{x}{R}. \qquad [x \neq 0, \quad y \neq 0] \tag{24}$$

(v)

$$\int_{i\infty}^{\varkappa} \frac{U_1 e^{\sqrt{Z^2-\gamma^2}y+iZx}}{-\sqrt{Z^2-\gamma^2}} ZdZ + \int_{\varkappa}^{i\infty} \frac{U_2 e^{-\sqrt{Z^2-\gamma^2}y+iZx}}{\sqrt{Z^2-\gamma^2}} ZdZ. \tag{25}$$

In this case the integrals transform to

$$-\int_{\infty}^{0} \frac{U_1' e^{-i\sqrt{\tau^2-\varkappa^2+\gamma^2}y-\sqrt{\tau^2-\varkappa^2}x}}{i\sqrt{\tau^2-\kappa^2+\gamma^2}}\tau d\tau - \int_0^{\infty} \frac{U_2' e^{-i\sqrt{\tau^2-\varkappa^2+\gamma^2}y-\sqrt{\tau^2-\varkappa^2}x}}{\sqrt{\tau^2-\kappa^2+\gamma^2}}\tau d\tau. \tag{25}$$

Thus, when $U_1'=U_2'$ for $\tau=0$, both the integrals cancel each other.

(vi)

$$\int_{i\infty}^{\varkappa} U_1 e^{\sqrt{Z^2-\varkappa^2}y+iZx} ZdZ + \int_{\varkappa}^{i\infty} U_2 e^{-\sqrt{Z^2-\varkappa^2}y+iZx} ZdZ. \tag{26}$$

Proceeding in the same way as before, the integrals reduce to

$$-\int_{-\infty}^{0} U_1' e^{-i\tau' y-\sqrt{\tau'^2-\varkappa^2}x}\tau' d\tau' - \int_0^{\infty} U_2' e^{-i\tau y-\sqrt{\tau^2-\varkappa^2}x}\tau d\tau. \tag{26'}$$

When $U_1'=U_2'$ for $\tau=0$, the integrals transform to

$$-\int_{-\infty}^{\infty} U_1' e^{-i\tau y-\sqrt{\tau^2-\varkappa^2}x}\tau d\tau. \tag{26''}$$

Using the relation

$$\frac{i}{\pi}\int_{-\infty}^{\infty} e^{-i\tau y-\sqrt{\tau^2-\varkappa^2}x}\tau d\tau = \frac{\partial^2}{\partial x\partial y}H_0^{(1)}(\kappa R) = -\kappa^2\frac{xy}{R^2}\left\{H_0^{(1)}(\kappa R) - \frac{1}{\kappa R}H_1^{(1)}(\kappa R)\right\}, \tag{27}$$

the integral (26″) is approximately

$$-i\pi U_1'\kappa^2\frac{xy}{R^2}H_0^{(1)}(\kappa R). \qquad [x \neq 0, \quad y \neq 0] \tag{28}$$

5. Although in the condition of boundary waves any one among ϕ_0, ϕ, ψ, ϕ_1' ψ_1' by itself, would indicate no wave, in the condition of bodily waves, on the other hand, the respective ones of such quantities represent separate waves. Besides physically, the displacements corresponding to ϕ_0 and ϕ are inseparable, in consequence of which it would be advisable to get such expression of $\phi_0+\phi$ as has the form

$$\begin{aligned}\phi_0+\phi &= e^{ifx-ipt}\left\{\mathfrak{A}e^{ry}+Ae^{-ry}\right\} \\ &= \frac{\mathfrak{A}}{\Phi}e^{ifx-ipt}\left\{P\cosh ry+Q\sinh ry\right\},\end{aligned} \tag{29}$$

where

$$\left.\begin{aligned}\frac{\Phi P}{\mathfrak{A}} &= \mu'^2\sqrt{f^2-k^2}\left\{4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-(2f^2-k'^2)^2\right\} \\ &\quad -\mu\mu' k^2k'^2\sqrt{f^2-k'^2}+4\mu\mu' f^2\sqrt{f^2-k^2}\,(2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\ &\qquad +4\mu^2 f^2\sqrt{f^2-k^2}\,(\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-f^2)\,, \\ \frac{\Phi Q}{\mathfrak{A}} &= \mu'^2 f^2\left\{(2f^2-k'^2)^2-4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}\right\} \\ &\quad -\mu\mu' k^2k'^2\sqrt{f^2-k^2}\sqrt{f^2-h'^2} \\ &\quad +2\mu\mu' f^2(k^2-2f^2)\,(2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\ &\qquad +\mu^2(2f^2-k^2)^2(f^2-\sqrt{f^2-h'^2}\sqrt{f^2-k'^2})\,.\end{aligned}\right\} \tag{30}$$

The expressions of $\phi_0+\phi$ in the case of a point source thus assume the forms

$$\left.\begin{aligned}\phi_0+\phi &= \frac{2e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{-ry+ifx}}{r}\left\{P\sqrt{f^2-h^2}\cosh r\xi+Q\sinh r\xi\right\}df, \quad [y>\xi] \\ \phi_0+\phi &= \frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\left\{\frac{\mathfrak{A}e^{ry-r\xi+ifx}}{r}+\frac{Ae^{-ry-r\xi+ifx}}{r}\right\}df. \quad [y<\xi]\end{aligned}\right\} \tag{31}$$

6. From the results shown in Sections 3, 4, 5, it is now possible to get the general expressions of waves at relatively large values of x, y as follows:

$$\left.\begin{aligned}\phi_0 &= \mathfrak{A}e^{-ipt}H_0^{(1)}(hR), \qquad [R^2=x^2+(y-\xi)^2] \\ \phi_0+\phi &= e^{-ipt}\left[\frac{2i\beta\alpha'\beta'}{\kappa}A_1e^{-\alpha y-\alpha\xi+i\kappa x}+\mathfrak{A}H_0^{(1)}(hR_1)+A_2H_0^{(1)}(hR_2)\right], \\ &\qquad [y<\xi,\quad x\neq 0,\quad y\neq 0,\quad R_1^2=x^2+(y-\xi)^2,\quad R_2^2=x^2+(y+\xi)^2] \\ \phi_0+\phi &= 2e^{-ipt}\left[\frac{2i\beta\alpha'\beta'}{\kappa}e^{-\alpha y+i\kappa x}(P_1\alpha\cosh\alpha\xi+Q_1\sinh\alpha\xi)\right. \\ &\qquad \left.+hP_2\frac{y}{R}H_1^{(1)}(hR)\right], \quad [y>\xi,\quad R^2=x^2+y^2]\end{aligned}\right\} \tag{32}$$

$$
\left.\begin{aligned}
\psi &= 2e^{-ipt}\left[-2\alpha\beta\alpha'\beta' B_1 e^{-\alpha\xi-\beta y+i\kappa x} + k^2 e^{-\sqrt{\kappa^2-h^2}\,\xi}\, B_2 \frac{xy}{R^2} H_0^{(1)}(kR)\right], \\
&\qquad\qquad [y>0,\quad x\neq 0,\quad y\neq 0,\quad R^2=x^2+y^2] \\
\phi' &= 2\mu k^2 e^{-ipt}\left[\frac{2i\alpha\beta\alpha'\beta'}{\kappa} C_1 e^{-\alpha\xi+\alpha' y+i\kappa x} + h' e^{-\sqrt{h'^2-h^2}\,\xi}\, C_2 \frac{y}{R} H_1^{(1)}(h'R)\right], \\
\psi' &= 2\mu k^2 e^{-ipt}\left[-2\alpha\beta\alpha'\beta' D_1 e^{-\alpha\xi+\beta' y+i\kappa x} + k'^2 e^{-\sqrt{k'^2-h^2}\,\xi}\, D_2 \frac{xy}{R^2} H_0^{(1)}(k'R)\right], \\
&\qquad\qquad [y<0,\quad x\neq 0,\quad y\neq 0,\quad R^2=x^2+y^2]
\end{aligned}\right\} \quad (33)
$$

where κ is the root of $\Phi(\kappa)=0$ shown in (7) and

$$
\begin{aligned}
R^2=x^2+y^2,\quad \alpha=\sqrt{\kappa^2-h^2},\quad \beta=\sqrt{\kappa^2-k^2},\\
\alpha'=\sqrt{\kappa^2-h'^2},\quad \beta'=\sqrt{\kappa^2-k'^2},
\end{aligned} \qquad (34)
$$

$$
\begin{aligned}
\frac{A_1\Phi_1}{\mathfrak{A}} &= \mu'^2(\kappa^2+\alpha\beta)\left\{4\kappa^2\alpha'\beta'-(2\kappa^2-k'^2)^2\right\}+\mu\mu' k^2k'^2(\beta\alpha'-\alpha\beta') \\
&\quad +2\mu\mu'\kappa^2(2\kappa^2-k^2+2\alpha\beta)(2\kappa^2-k'^2-2\alpha'\beta') \\
&\quad +\mu^2(\alpha'\beta'-\kappa^2)\left\{4\kappa^2\alpha\beta+(2\kappa^2-k^2)^2\right\}, \\
\frac{P_1\Phi_1}{\mathfrak{A}} &= \mu'^2\beta\left\{4\kappa^2\alpha'\beta'-(2\kappa^2-k'^2)^2\right\}-\mu\mu' k^2k'^2\beta' \\
&\quad +4\mu\mu'\kappa^2\beta(2\kappa^2-k'^2-2\alpha'\beta')+4\mu^2\kappa^2\beta(\alpha'\beta'-\kappa^2), \\
\frac{Q_1\Phi_1}{\mathfrak{A}} &= \mu'^2\kappa^2\left\{(2\kappa^2-k'^2)^2-4\kappa^2\alpha'\beta'\right\}-\mu\mu' k^2k'^2\beta\alpha' \\
&\quad +2\mu\mu'\kappa^2(k^2-2\kappa^2)(2\kappa^2-k'^2-2\alpha'\beta')+\mu^2(2\kappa^2-k^2)^2(\kappa^2-\alpha'\beta'), \\
\frac{B_1\Phi_1}{\mathfrak{A}} &= \mu'^2\left\{4\kappa^2\alpha'\beta'-(2\kappa^2-k'^2)^2\right\}+\mu\mu'(4\kappa^2-k^2)(2\kappa^2-k'^2-2\alpha'\beta') \\
&\quad +2\mu^2(2\kappa^2-k^2)(\alpha'\beta'-\kappa^2), \\
\frac{C_1\Phi_1}{\mathfrak{A}} &= \mu'\left\{(2\kappa^2-k'^2)\beta-2\kappa^2\beta'\right\}+\mu\left\{(2\kappa^2-k^2)\beta'-2\kappa^2\beta\right\}, \\
\frac{D_1\Phi_1}{\mathfrak{A}} &= \mu'(2\kappa^2-k'^2-2\beta\alpha')+\mu(2\beta\alpha'+k^2-2\kappa^2),
\end{aligned} \qquad (35)
$$

$$
\begin{aligned}
\Phi_1 &= 4\mu'^2\alpha\beta(\kappa^2-\alpha\beta)\left\{2(2\kappa^2-k'^2)\alpha'\beta'-2\alpha'^2\beta'^2-\kappa^2\beta'^2-\kappa^2\alpha'^2\right\} \\
&\quad +\mu'^2\alpha'\beta'(\alpha-\beta)^2\left\{4\kappa^2\alpha'\beta'-(2\kappa^2-k'^2)^2\right\} \\
&\quad -\mu\mu' k^2k'^2(\beta\alpha'\beta'+\alpha\beta\alpha'+\alpha\alpha'\beta'+\alpha\beta\beta') \\
&\quad +4\mu\mu'\alpha\beta\alpha'\beta'(2\alpha\beta+k^2-2\kappa^2)(2\kappa^2-k'^2-2\alpha'\beta') \\
&\quad +2\mu\mu'\kappa\alpha'\beta'(2\kappa\beta^2+2\kappa\alpha^2+k^2\alpha\beta-4\kappa\alpha\beta)(2\kappa^2-k'^2-2\alpha'\beta')
\end{aligned}
$$

$$+2\mu\mu'\kappa\alpha\beta(2\alpha\beta+k^2-2\kappa^2)(4\kappa\alpha'\beta'-k'^2\alpha'\beta'-2\kappa\beta'^2-2\kappa\alpha'^2)$$
$$+\mu^2\alpha\beta(\alpha'-\beta')^2\left\{4\kappa^2\alpha\beta-(2\kappa^2-k^2)^2\right\}$$
$$+4\mu^2\alpha'\beta'(\kappa^2-\alpha'\beta')\left\{2(2\kappa^2-k^2)\alpha\beta-2\alpha^2\beta^2-\kappa^2\beta^2-\kappa^2\alpha^2\right\}, \quad (36)$$

$$\left.\begin{aligned}
\frac{A_2\Phi_{A_2}}{\mathfrak{A}} &= \mu'^2h^2\left\{4h^2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}-(2h^2-k'^2)^2\right\}\\
&\quad+\mu\mu'k^2k'^2\sqrt{h^2-k^2}\sqrt{h^2-h'^2}\\
&\quad+2\mu\mu'h^2(2h^2-k^2)(2h^2-k'^2-2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2})\\
&\qquad+\mu^2(\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}-h^2)(2h^2-k^2)^2,\\
\frac{P_2\Phi_{P_2}}{\mathfrak{A}} &= \mu'^2\sqrt{h^2-k^2}\left\{4h^2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}-(2h^2-k'^2)^2\right\}\\
&\quad-\mu\mu'k^2k'^2\sqrt{h^2-k'^2}+4\mu^2h^2\sqrt{h^2-k^2}(\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}-h^2)\\
&\qquad+4\mu\mu'h^2\sqrt{h^2-k^2}(2h^2-k'^2-2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}),
\end{aligned}\right\}(37)$$

$$\begin{aligned}
\Phi_{A_2}=\Phi_{P_2} &= \mu'^2h^2\left\{(2h^2-k'^2)^2-4h^2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}\right\}\\
&\quad-\mu\mu'k^2k'^2\sqrt{h^2-k^2}\sqrt{h^2-h'^2}\\
&\quad+2\mu\mu'h^2(k^2-2h^2)(2h^2-k'^2-2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2})\\
&\qquad+\mu^2(h^2-\sqrt{h^2-h'^2}\sqrt{h^2-k'^2})(2h^2-k^2)^2.
\end{aligned}\quad(38)$$

$$\left.\begin{aligned}
\frac{B_2\Phi_{B_2}}{\mathfrak{A}} &= \mu'^2\left\{4k^2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}-(2k^2-k'^2)^2\right\}\\
&\quad+3\mu\mu'k^2(2k^2-k'^2-2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2})\\
&\qquad+2\mu^2k^2(\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}-k^2),\\
\Phi_{B_2} &= \mu'^2k^2\left\{(2k^2-k'^2)^2-4k^2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}\right\}\\
&\quad-\mu\mu'k^2k'^2\sqrt{k^2-h^2}\sqrt{k^2-k'^2}+\mu^2k^4(k^2-\sqrt{k^2-h'^2}\sqrt{k^2-k'^2})\\
&\qquad+2\mu\mu'k^4(2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}+k'^2-2f^2),
\end{aligned}\right\}(39)$$

$$\left.\begin{aligned}
\frac{C_2\Phi_{C_2}}{\mathfrak{A}} &= \mu'\left\{(2h'^2-k'^2)\sqrt{h'^2-k^2}-2h'^2\sqrt{h'^2-k'^2}\right\}\\
&\qquad+\mu\left\{(2h'^2-k^2)\sqrt{h'^2-k'^2}-2h'^2\sqrt{h'^2-k^2}\right\},\\
\Phi_{C_2} &= \mu'^2(h'^2-\sqrt{h'^2-h^2}\sqrt{h'^2-k^2})(2h'^2-k'^2)^2\\
&\quad-\mu\mu'k^2k'^2\sqrt{h'^2-h^2}\sqrt{h'^2-k'^2}\\
&\quad+2\mu\mu'h'^2(2\sqrt{h'^2-h^2}\sqrt{h'^2-k^2}+k^2-2h'^2)(2h'^2-k'^2)\\
&\qquad+\mu^2h'^2\left\{(2h'^2-k^2)^2-4h'^2\sqrt{h'^2-h^2}\sqrt{h'^2-k^2}\right\},
\end{aligned}\right\}(40)$$

$$\left.\begin{aligned}
\frac{D_2\Phi_{p_2}}{\mathfrak{A}} &= \mu'\ (k'^2-2\sqrt{k'^2-k^2}\sqrt{k'^2-h'^2}) \\
&\qquad\qquad +\mu(2\sqrt{k'^2-k^2}\sqrt{k'^2-h'^2}+k^2-2k'^2), \\
\Phi_{p_2} &= \mu'^2(k'^2-\sqrt{k'^2-h^2}\sqrt{k'^2-k^2})k'^4 \\
&\quad -\mu\mu' k^2k'^2\sqrt{k'^2-k^2}\sqrt{k'^2-h'^2} \\
&\quad +2\mu\mu' k'^4(2\sqrt{k'^2-h^2}\sqrt{k'^2-k^2}+k^2-2k'^2) \\
&\qquad +\mu^2k'^2\left\{(2k'^2-k^2)^2-4k'^2\sqrt{k'^2-h^2}\sqrt{k'^2-k^2}\right\}.
\end{aligned}\right\} \tag{41}$$

The expressions of displacements are as follows:

$$\left.\begin{aligned}
u_1 &= \frac{\partial}{\partial x}(\phi_0+\phi) = e^{-ipt}\Big[-2\beta\alpha'\beta' A_1 e^{\alpha-y-\alpha\xi+i\kappa x} \\
&\qquad\qquad -h\mathfrak{A}\frac{x}{R}H_1^{(1)}(hR_1)\ -hA_2\frac{x}{R}H_1^{(1)}(hR_2)\Big], \\
v_1 &= \frac{\partial}{\partial y}(\phi_0+\phi) = e^{-ipt}\Big[-\frac{2i\alpha\beta\alpha'\beta'}{\kappa}A_1 e^{-\alpha y-\alpha\xi+i\kappa x} \\
&\qquad\qquad -h\mathfrak{A}\frac{y}{R}H_1^{(1)}(hR_1)-hA_2\frac{y}{R}H_1^{(1)}(hR_2)\Big], \\
&[R_1^2=x^2+(y-\xi)^2,\quad R_2^2=x^2+(y+\xi)^2,\quad x\neq 0,\quad y\neq 0,\quad y<\xi]
\end{aligned}\right\} \tag{42}$$

$$\left.\begin{aligned}
u_1 &= \frac{\partial}{\partial x}\ (\phi_0+\phi) = 2e^{-ipt}\Big[-2\beta\alpha'\beta' e^{-\alpha y+i\kappa x}\left\{P_1\alpha\cosh\alpha\xi+Q_1\sinh\alpha\xi\right\} \\
&\qquad\qquad -h^2P_2\frac{xy}{R^2}H_0^{(1)}(hR)\Big], \\
v_1 &= \frac{\partial}{\partial y}\ (\phi_0+\phi) = 2e^{-ipt}\Big[\frac{2i\alpha\beta\alpha'\beta'}{\kappa}e^{-\alpha y+i\kappa x}\left\{P_1\alpha\sinh\alpha\xi+Q_1\cosh\alpha\xi\right\} \\
&\qquad\qquad -h^2P_2\frac{y^2}{R^2}H_0^{(1)}(hR)\Big], \\
&[R^2=x^2+y^2,\quad x\neq 0,\quad y>\xi]
\end{aligned}\right\} \tag{42'}$$

$$\left.\begin{aligned}
u_2 &= \frac{\partial\psi}{\partial y} = 2e^{-ipt}\Big[2\alpha\beta^2\alpha'\beta' B_1 e^{-\alpha\xi-\beta y+i\kappa x} \\
&\qquad\qquad -k^3e^{-\sqrt{k^2-h^2}\xi}B_2\frac{xy^2}{R^3}H_1^{(1)}(kR)\Big], \\
v_2 &= -\frac{\partial\psi}{\partial x} = 2e^{-ipt}\Big[2i\kappa\alpha\beta\alpha'\beta' B_1 e^{-\alpha\xi-\beta y+i\kappa x} \\
&\qquad\qquad +k^3e^{-\sqrt{k^2-h^2}\xi}B_2\frac{x^2y}{R^3}H_1^{(1)}(kR)\Big], \\
&[R^2=x^2+y^2,\quad x\neq 0,\quad y\neq 0,\quad y>0]
\end{aligned}\right\} \tag{43}$$

$$
\left.
\begin{aligned}
u_1' &= \frac{\partial \phi'}{\partial x} = 2\mu k^2 e^{-ipt}\Big[-2\alpha\beta\alpha'\beta' C_1 e^{-\alpha\xi+\alpha' y+i\kappa x} \\
&\qquad - h'^2 e^{-\sqrt{h'^2-h^2}\,\xi} C_2 \frac{xy}{R^2} H_0^{(1)}(h'R)\Big], \\
v_1' &= \frac{\partial \phi'}{\partial y} = 2\mu k^2 e^{-ipt}\Big[\frac{2i\alpha\beta\alpha'^2\beta'}{\kappa} C_1 e^{-\alpha\xi+\alpha' y+i\kappa x} \\
&\qquad - h'^2 e^{-\sqrt{h'^2-h^2}\,\xi} C_2 \frac{y^2}{R^2} H_0^{(1)}(h'R)\Big], \\
u_2' &= \frac{\partial \psi'}{\partial y} = 2\mu k^2 e^{-ipt}\Big[-2\alpha\beta\alpha'\beta'^2 D_1 e^{-\alpha\xi+\beta' y+i\kappa x} \\
&\qquad - k'^3 e^{-\sqrt{k'^2-h^2}\,\xi} D_2 \frac{xy^2}{R^3} H_1^{(1)}(k'R)\Big], \\
v_2' &= -\frac{\partial \psi'}{\partial x} = 2\mu k^2 e^{-ipt}\Big[2i\kappa\alpha\beta\alpha'\beta' D_1 e^{-\alpha\xi+\beta' y+i\kappa x} \\
&\qquad + k'^3 e^{-\sqrt{k'^2-h^2}\,\xi} D_2 \frac{x^2 y}{R^3} H_1^{(1)}(k'R)\Big]. \\
&\qquad [R^2=x^2+y^2,\quad x\neq 0,\quad y\neq 0,\quad y<0]
\end{aligned}
\right\} \qquad (44)
$$

The first term within every pair of brackets represents the boundary surface waves (for some conditions of two media) and the second one the bodily waves. It will be seen that, whereas the amplitudes of boundary surface waves do not vary with changes in the epicentral distance, those of bodily waves decrease rapidly with increase in horizontal distance and, furthermore, the horizontal components of the same bodily waves also decrease fairly rapidly with increase in vertical distance. The law of decrease in the amplitudes of bodily waves for a given $y(\equiv Y)$ and for a given $x(\equiv X)$ is shown below.

Table I. Amplitudes of bodily waves/$H^{(1)}$.

	u_1	v_1	u_2	v_2	u_1'	v_1'	u_2'	v_2'
At Y	$\frac{xY}{R^2}$	$\frac{Y^2}{R^2}$	$\frac{xY^2}{R^3}$	$\frac{x^2Y}{R^3}$	$\frac{xY}{R^2}$	$\frac{Y^2}{R^2}$	$\frac{xY^2}{R^3}$	$\frac{x^2Y}{R^3}$
At X	$\frac{Xy}{R^2}$	$\frac{y^2}{R^2}$	$\frac{Xy^2}{R^3}$	$\frac{X^2y}{R^3}$	$\frac{Xy}{R^2}$	$\frac{y^2}{R^2}$	$\frac{Xy^2}{R^3}$	$\frac{X^2y}{R^3}$

At a relatively large x for a given Y, the values of u_1, v_2, u_1', v_2' are proportional to $R^{-3/2}$, and the values of v_1, u_2, v_1', u_2' are proportional to $R^{-5/2}$. At a relatively large y for a given X, the values of u_1, u_2, u_1', u_2' are proportional to $R^{-3/2}$ and the values of v_2, v^2 are proportional to $R^{-5/2}$, the exception being that at these values of y, X; v_1, v_1' are proportional to $R^{-1/2}$.

It should be borne in mind that the decrease in amplitudes with increase in radial distance R is always proportional to $R^{-1/2}$. More detailed feature will be discussed in Chapter V.

III. *The Case of Distortional Primary Waves with Amplitudes Orientated in a Vertical Plane.*

7. Using the same axes of x, y and ρ, λ, μ; ρ', λ', μ', as in the preceding section, it is possible to solve the present problem. Even in the case of plane primary waves

$$\psi_0 = Be^{sy+ifx-ipt}, \tag{45}$$

where $\tan^{-1}(f/is)$ is the angle of incidence, the reflected and refracted dilatational and distortional waves are of the same expressions as those in (2). Proceeding in the same way as in Chapter II, and remembering that the expressions for the displacements are

$$\left.\begin{aligned} u &= \frac{\partial\phi}{\partial x} + \frac{\partial(\psi_0+\psi)}{\partial y}, & v &= \frac{\partial\phi}{\partial y} - \frac{\partial(\psi_0+\psi)}{\partial x}, \\ u' &= \frac{\partial\phi'}{\partial x} + \frac{\partial\psi'}{\partial y}, & v' &= \frac{\partial\phi'}{\partial y} - \frac{\partial\psi}{\partial x}, \end{aligned}\right\} \tag{46}$$

it is possible to determine the constants A, B, C, D. The result is given below.

$$\begin{aligned} A\Phi = 2if\sqrt{f^2-k^2}\Big[& 2\mu^2(2f^2-k^2)(f^2-\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\ &+\mu\mu'(2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2})(k^2-4f^2) \\ &\qquad +\mu'^2\left\{(2f^2-k'^2)^2-4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}\right\}\Big], \end{aligned}$$

$$\begin{aligned} B\Phi = \mu'^2(f^2+\sqrt{f^2-h^2}\sqrt{f^2-k^2})&\left\{4f^2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-(2f^2-k'^2)^2\right\} \\ &+\mu\mu'k^2k'^2(\sqrt{f^2-h^2}\sqrt{f^2-k'^2}-\sqrt{f^2-k^2}\sqrt{f^2-h'^2}) \\ &+2\mu\mu'f^2(2f^2-k^2+2\sqrt{f^2-h^2}\sqrt{f^2-k^2})(2f^2-k'^2-2\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}) \\ &+\mu^2(\sqrt{f^2-h'^2}\sqrt{f^2-k'^2}-f^2)\left\{(2f^2-k^2)^2+4f^2\sqrt{f^2-h^2}\sqrt{f^2-k^2}\right\}\Big], \end{aligned}$$

$$\begin{aligned} C\Phi = 2i\mu fk^2\sqrt{f^2-k^2}\Big\{&\mu(2f^2-k^2-2\sqrt{f^2-h^2}\sqrt{f^2-k'^2}) \\ &+\mu'(2\sqrt{f^2-h^2}\sqrt{f^2-k'^2}+k'^2-2f^2)\Big\}, \end{aligned}$$

$$D\Phi=2\mu k^2\sqrt{f^2-k^2}\Big[\mu\big\{(2f^2-k^2)\sqrt{f^2-h'^2}-2f^2\sqrt{f^2-h^2}\big\}$$
$$+\mu'\big\{(2f^2-k'^2)\sqrt{f^2-h^2}-2f^2\sqrt{f^2-h'^2}\big\}\Big], \quad (47)$$

where Φ is of the same form as (7), i. e. the velocity equation of the boundary waves.

8. If primary distortional waves are radiated from a point source at $x=0$, $y=\xi$, then

$$\left.\begin{aligned}\psi_0&=\frac{\mathfrak{B}e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{s(y-\xi)+ifx}}{s}\,df, &&[y<\xi]\\ \psi_0&=\frac{\mathfrak{B}e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{-s(y-\xi)+ifx}}{s}\,df, &&[y>\xi]\end{aligned}\right\}\quad(48)$$

so that the reflected and refracted waves assume the forms

$$\left.\begin{aligned}\psi&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Be^{-s(y+\xi)+ifx}}{s}\,df, &\phi&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Ae^{-ry-s\xi+ifx}}{s}\,df, &&[y>0]\\ \phi'&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Ce^{r'y-s\xi+ifx}}{s}\,df, &\psi'&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{De^{s'y-s\xi+ifx}}{s}\,df, &&[y<0]\end{aligned}\right\}(50)$$

and the superposed values of $\psi_0+\psi$ are

$$\left.\begin{aligned}\psi_0+\psi&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{-s\xi+ifx}}{s}(\mathfrak{B}e^{sy}+Be^{-sy})\,df, &&[y<\xi]\\ \psi_0+\psi&=\frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{-sy+ifx}}{s}(\mathfrak{B}e^{s\xi}+Be^{-s\xi})\,df. &&[y>\xi]\end{aligned}\right\}\quad(50)$$

After calculating as in the preceding chapter, we get

$$\left.\begin{aligned}&\psi_0=\mathfrak{B}e^{-ipt}H_0^{(1)}(kR), \qquad [R^2=x^2+(y-\xi)^2]\\ &\psi_0+\psi=e^{-ipt}\Big[\frac{2i\alpha\alpha'\beta'}{\kappa}B_1e^{-\beta y-\beta\xi+i\kappa x}+\mathfrak{B}H_0^{(1)}(kR_1)+B_2H_0^{(1)}(kR_2)\Big],\\ &\qquad [y<\xi,\quad x\neq0,\quad y\neq0,\quad R_1^2=x^2+(y-\xi)^2,\quad R_2^2=x^2+(y+\xi)^2]\\ &\psi_0+\psi=2e^{-ipt}\Big[\frac{2i\alpha\alpha'\beta'}{\kappa}e^{-\beta y+i\kappa x}(P_1\beta\cosh\beta\xi+Q_1\sinh\beta\xi)+kP_2\frac{y}{R}H_1^{(1)}(kR)\Big],\\ &\qquad [y>\xi,\quad x\neq0,\quad R^2=x^2+y^2]\end{aligned}\right\}(51)$$

$$\left.\begin{aligned}
&\phi=2e^{-ipt}\left[-2\alpha\beta\alpha'\beta' A_1 e^{-\alpha y-\beta\xi+i\kappa x}+h^2 e^{-\sqrt{h^2-k^2}\xi} A_2 \frac{xy}{R^2} H_0^{(1)}(hR)\right],\\
&\qquad\qquad\qquad\qquad [y>0,\quad x\neq 0,\quad y\neq 0,\quad R^2=x^2+y^2]\\
&\phi'=2\mu k^2 e^{-ipt}\left[-2\alpha\beta\alpha'\beta' C_1 e^{\alpha' y-\beta\xi+i\kappa x}+h'^2 e^{-\sqrt{h'^2-k^2}\xi} C_2 \frac{xy}{R^2} H_0^{(1)}(h'R)\right],\\
&\qquad\qquad\qquad\qquad [y>0,\quad R^2=x^2+y^2]\\
&\psi'=2\mu k^2 e^{-ipt}\left[\frac{2i\alpha\beta\alpha'\beta'}{\kappa} D_1 e^{\beta' y-\beta\xi+i\kappa x}+k' e^{-\sqrt{k'^2-k^2}\xi} D_2 \frac{y}{R} H_1^{(1)}(k'R)\right],\\
&\qquad\qquad\qquad\qquad [y<0,\quad x\neq 0,\quad y\neq 0,\quad R^2=x^2+y^2]
\end{aligned}\right\}\quad(52)$$

where κ is the root of $\Phi(\kappa)=0$ shown in (7) and

$$\alpha=\sqrt{\kappa^2-h^2},\quad \beta=\sqrt{\kappa^2-k^2},\quad \alpha'=\sqrt{\kappa^2-h'^2},\quad \beta'=\sqrt{\kappa^2-k'^2},$$

$$\left.\begin{aligned}
\frac{B_1\Phi_1}{\mathfrak{B}}&=\mu'^2(\kappa^2+\alpha\beta)\left\{4\kappa^2\alpha'\beta'-(2\kappa^2-k'^2)^2\right\}+\mu\mu' k^2k'^2(\alpha\beta'-\beta\alpha')\\
&\quad+2\mu\mu'\kappa^2(2\kappa^2-k^2+2\alpha\beta)(2\kappa^2-k'^2-2\alpha'\beta')\\
&\quad+\mu^2(\alpha'\beta'-\kappa^2)\left\{(2\kappa^2-k^2)^2+4\kappa^2\alpha\beta\right\},\\
\frac{P_1\Phi_1}{\mathfrak{B}}&=\mu'^2\alpha\left\{4\kappa^2\alpha'\beta'-(2\kappa^2-k'^2)^2\right\}-\mu\mu' k^2k'^2\alpha'\\
&\quad+4\mu\mu'\kappa^2\alpha(2\kappa^2-k'^2-2\alpha'\beta')+4\mu^2\kappa^2\alpha(\alpha'\beta'-\kappa^2),\\
\frac{Q_1\Phi_1}{\mathfrak{B}}&=\mu'^2\kappa^2\left\{(2\kappa^2-k'^2)^2-4\kappa^2\alpha'\beta'\right\}-\mu\mu' k^2k'^2\alpha\beta'\\
&\quad+2\mu\mu'\kappa^2(2\kappa^2-k^2-2\alpha'\beta')(k^2-2\kappa^2)+\mu^2(\kappa^2-\alpha'\beta')(2\kappa^2-k^2)^2,\\
\frac{A_1\Phi_1}{\mathfrak{B}}&=2\mu^2(2\kappa^2-k^2)(\kappa^2-\alpha'\beta')+\mu\mu'(2\kappa^2-k'^2-2\alpha'\beta')(k^2-4\kappa^2)\\
&\quad+\mu'^2\left\{(2\kappa^2-k'^2)^2-4\kappa^2\alpha'\beta'\right\},\\
\frac{C_1\Phi_1}{\mathfrak{B}}&=\mu(2\kappa^2-k^2-2\alpha\beta')+\mu'(2\alpha\beta'+k'^2-2\kappa^2),\\
\frac{D_1\Phi_1}{\mathfrak{B}}&=\mu\left\{(2\kappa^2-k^2)\alpha'-2\kappa^2\alpha\right\}+\mu'\left\{(2\kappa^2-k'^2)\alpha-2\kappa^2\alpha'\right\},
\end{aligned}\right\}\quad(53)$$

$$\begin{aligned}
\Phi_1&=\mu'^2(\kappa^2-\alpha\beta)\left\{(2\kappa^2-k'^2)^2-4\kappa^2\alpha'\beta'\right\}-\mu\mu' k^2k'^2(\alpha\beta'+\beta\alpha')\\
&\quad+2\mu\mu'\kappa^2(2\alpha\beta+k^2-2\kappa^2)(2\kappa^2-k'^2-2\alpha'\beta')\\
&\quad+\mu^2(\kappa^2-\alpha'\beta')\left\{(2\kappa^2-k^2)^2-4\kappa^2\alpha\beta\right\},
\end{aligned}\quad(54)$$

$$\left.\begin{aligned}
\frac{B_2\Phi_{B_2}}{\mathfrak{B}} &= \mu'^2 k^2\left\{4k^2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}-(2k^2-k'^2)^2\right\} \\
&\quad +\mu\mu' k^2k'^2\sqrt{k^2-h^2}\sqrt{k^2-k'^2}+\mu^2(\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}-k^2)k^4 \\
&\quad +2\mu\mu' k^4(2k^2-k'^2-2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}), \\
\frac{P_2\Phi_{P_2}}{\mathfrak{B}} &= \mu'^2\left\{4k^2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}-(2k^2-k'^2)^2\sqrt{k^2-h^2}\right. \\
&\quad +4\mu\mu' k^2\sqrt{k^2-h^2}(2k^2-k'^2-2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}) \\
&\quad -\mu\mu' k^2k'^2\sqrt{k^2-h'^2}+4\mu^2k^2\sqrt{k^2-h^2}(\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}-k^2),
\end{aligned}\right\}\quad(55)$$

$$\begin{aligned}
\Phi_{B_2}=\Phi_{P_2} &= \mu'^2k^2\left\{(2k^2-k'^2)^2-4k^2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}\right\} \\
&\quad -2\mu\mu' k^4(2k^2-k'^2-2\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}) \\
&\quad -\mu\mu' k^2k'^2\sqrt{k^2-h^2}\sqrt{k^2-k'^2}+\mu^2k^4(k^2-\sqrt{k^2-h'^2}\sqrt{k^2-k'^2}),
\end{aligned}\quad(56)$$

$$\left.\begin{aligned}
\frac{A_2\Phi_{A_2}}{\mathfrak{B}} &= 2\mu^2(2h^2-k^2)(h^2-\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}) \\
&\quad +\mu\mu'(2h^2-k'^2-2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2})(k^2-4h^2) \\
&\quad +\mu'^2\left\{(2h^2-k'^2)^2-4h^2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}\right\}, \\
\Phi_{A_2} &= \mu'^2h^2\left\{(2h^2-k'^2)^2-4h^2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}\right\} \\
&\quad -\mu\mu' k^2k'^2\sqrt{h^2-k^2}\sqrt{h^2-h'^2} \\
&\quad +2\mu\mu' h^2(k^2-2h^2)(2h^2-k'^2-2\sqrt{h^2-h'^2}\sqrt{h^2-k'^2}) \\
&\quad +\mu^2(h^2-\sqrt{h^2-h'^2}\sqrt{h^2-k'^2})(2h^2-k^2)^2,
\end{aligned}\right\}\quad(57)$$

$$\left.\begin{aligned}
\frac{C_2\Phi_{C_2}}{\mathfrak{B}} &= \mu(2h'^2-k^2-2\sqrt{h'^2-h^2}\sqrt{h'^2-k'^2}) \\
&\quad +\mu'(2\sqrt{h'^2-h^2}\sqrt{h'^2-k'^2}-k'^2), \\
\Phi_{C_2} &= \mu'^2(h'^2-\sqrt{h'^2-h^2}\sqrt{h'^2-k^2})(2h'^2-k'^2)^2 \\
&\quad -\mu\mu' k^2k'^2\sqrt{h'^2-h^2}\sqrt{h'^2-k'^2} \\
&\quad +2\mu\mu' h'^2(2\sqrt{h'^2-h^2}\sqrt{h'^2-k^2}+k^2-2h'^2)(2h'^2-k'^2) \\
&\quad +\mu^2h'^2\left\{(2h'^2-k^2)^2-4h'^2\sqrt{h'^2-h^2}\sqrt{h'^2-k^2}\right\},
\end{aligned}\right\}\quad(58)$$

$$\left.\begin{aligned}
\frac{D_2\Phi_{D_2}}{\mathfrak{B}} &= \mu\left\{(2k'^2-k^2)\sqrt{k'^2-h'^2}-2k'^2\sqrt{k'^2-h^2}\right\} \\
&\quad +\mu' k'^2\left\{\sqrt{k'^2-h^2}-2\sqrt{k'^2-h'^2}\right\}, \\
\Phi_{D_2} &= \mu'^2k'^4(k'^2-\sqrt{k'^2-h^2}\sqrt{k'^2-k^2})-\mu\mu' k^2k'^2\sqrt{k'^2-k^2}\sqrt{k'^2-h'^2} \\
&\quad +2\mu\mu' k'^4(2\sqrt{k'^2-h^2}\sqrt{k'^2-k^2}+k^2-2k'^2) \\
&\quad +\mu^2k'^2\left\{(2k'^2-k^2)^2-4k'^2\sqrt{k'^2-h^2}\sqrt{k'^2-k^2}\right\}.
\end{aligned}\right\}\quad(59)$$

The expressions for the displacements are shown below.

$$\left.\begin{aligned} u_1 &= \frac{\partial\phi}{\partial x} = 2e^{-ipt}\left[-2i\kappa\alpha\beta\alpha'\beta' A_1 e^{-\alpha y-\beta\xi+i\kappa x} - h^3 e^{-\sqrt{h^2-k^2}\xi} A_2 \frac{x^2 y}{R^3} H_1^{(1)}(hR)\right], \\ v_1 &= \frac{\partial\phi}{\partial y} = 2e^{-ipt}\left[2\alpha^2\beta\alpha'\beta' A_1 e^{-\alpha y-\beta\xi+i\kappa x} - h^3 e^{-\sqrt{h^2-k^2}\xi} A_2 \frac{xy^2}{R^3} H_1^{(1)}(hR)\right], \\ &\qquad [R^2 = x^2+y^2, \quad x \neq 0, \quad y \neq 0, \quad y>0] \end{aligned}\right\} \quad (60)$$

$$\left.\begin{aligned} u_2 &= \frac{\partial(\psi_0+\psi)}{\partial y} = e^{-ipt}\Big[-\frac{2i\alpha\beta\alpha'\beta'}{\kappa} B_1 e^{-\beta y-\beta\xi+i\kappa x} \\ &\qquad - k\mathfrak{B}\frac{y}{R} H_1^{(1)}(kR_1) - kB_2\frac{y}{R} H_1^{(1)}(kR_2)\Big], \\ v_2 &= \frac{-\partial(\psi_0+\psi)}{\partial x}\, e^{-ipt}\Big[2\alpha\alpha'\beta' B_1 e^{-\beta y-\beta\xi+i\kappa x} \\ &\qquad + k\mathfrak{B}\frac{x}{R} H_1^{(1)}(kR_1) + kB_2\frac{x}{R} H_1^{(1)}(kR_2)\Big], \\ &[y<\xi, \quad x\neq 0, \quad y\neq 0, \quad R_1^2 = x^2+(y-\xi)^2, \quad R_2^2 = x^2+(y+\xi)^2] \end{aligned}\right\} \quad (61)$$

$$\left.\begin{aligned} u_2 &= \frac{\partial(\psi_0+\psi)}{\partial y} = 2e^{-ipt}\Big[-\frac{2i\alpha\beta\alpha'\beta'}{\kappa} e^{-\beta y+i\kappa x}\,(P_1\beta\cosh\beta\xi + Q_1\sinh\beta\xi) \\ &\qquad - k^2 P_2 \frac{y^2}{R^2} H_0^{(1)}(kR)\Big], \\ v_2 &= \frac{-\partial(\psi_0+\psi)}{\partial x} = 2e^{-ipt}\Big[2\alpha\alpha'\beta' e^{-\beta y+i\kappa x}(P_1\beta\cosh\beta\xi + Q_1\sinh\beta\xi) \\ &\qquad + k^2 P_2\frac{xy}{R^2} H_0^{(1)}(kR)\Big], \\ &\qquad [y>\xi, \quad x\neq 0, \quad R^2 = x^2+y^2] \end{aligned}\right\} \quad (61')$$

$$\left.\begin{aligned} u_1' &= \frac{\partial\phi'}{\partial x} = 2\mu k^2 e^{-ipt}\Big[-2i\kappa\alpha\beta\alpha'\beta' C_1 e^{\alpha' y-\beta\xi+i\kappa x} \\ &\qquad - h'^3 e^{-\sqrt{h'^2-k^2}\xi} C_2 \frac{x^2 y}{R^3} H_1^{(1)}(h'R)\Big], \\ v_1' &= \frac{\partial\phi'}{\partial y} = 2\mu k^2 e^{-ipt}\Big[-2\alpha\beta\alpha'^2\beta' C_1 e^{\alpha' y-\beta\xi+i\kappa x} \\ &\qquad - h'^3 e^{-\sqrt{h'^2-k^2}\xi} C_2 \frac{xy^2}{R^3} H_1^{(1)}(h'R)\Big], \\ u_2' &= \frac{\partial\psi'}{\partial y} = 2\mu k^2 e^{-ipt}\Big[\frac{2i\alpha\beta\alpha'\beta'^2}{\kappa} D_1 e^{\beta' y-\beta\xi+i\kappa x} \\ &\qquad - k'^2 e^{-\sqrt{k'^2-k^2}\xi} D_2 \frac{y^2}{R^2} H_0^{(1)}(k'R)\Big], \end{aligned}\right\} \quad (62)$$

$$v_2' = -\frac{\partial \psi'}{\partial x} = 2\mu k^2 e^{-ipt}\Big[2\alpha\beta\alpha'\beta' D_1 e^{\beta' y + -\beta\xi + i\kappa x} + k'^2 e^{-\sqrt{k'^2-k^2}\xi} D_2 \frac{xy}{R^2} H_0^{(1)}(k'R)\Big].$$

$$[y<0, \quad x \neq 0, \quad y \neq 0, \quad R^2 = x^2 + y^2]$$

In the present case, too, the first term within every pair of brackets indicates the boundary surface waves (for some conditions of two media) and the second one the bodily waves.

Although the amplitudes of the boundary waves do not decrease with epicentral distance, those of bodily waves decay in some manner; the law of decrease of amplitudes of the same waves for a given $y(\equiv Y)$ and for a given $x(\equiv X)$ is shown below.

Table II. Amplitudes of bodily waves/$H^{(1)}$

	u_1	v_1	u_2	v_2	u_1'	v_1'	u_2'	v_2'
At Y	$\frac{x^2Y}{R^3}$	$\frac{xY^2}{R^3}$	$\frac{Y^2}{R^2}$	$\frac{xY}{R^2}$	$\frac{x^2Y}{R^3}$	$\frac{xY^2}{R^3}$	$\frac{Y^2}{R^2}$	$\frac{xY}{R^2}$
At X	$\frac{X^2y}{R^3}$	$\frac{Xy^2}{R^3}$	$\frac{y^2}{R^2}$	$\frac{Xy}{R^3}$	$\frac{X^2y}{R^3}$	$\frac{Xy^2}{R^3}$	$\frac{y^2}{R^2}$	$\frac{Xy}{R^2}$

At a relatively large x for a given Y, the values of u_1, v_2, u_1', v_2' are proportional to $R^{-3/2}$, while the values of v_1, u_2, v_1', u_2' are proportional to $R^{-5/2}$. At a relatively large y for a given X, the values of v_1, v_2, v_1', v_2' are proportional to $R^{-3/2}$, while the values of u_1, u_1' are proportional to $R^{-5/2}$, the exception being that, at these values of y, X; u_2, u_2' are proportional to $R^{-1/2}$. More detailed feature will be discussed in chapter V.

The amplitudes distribution at a relatively large x for a given Y in the present case is quite similar to that in the case in which the primary waves are dilatational, whereas the amplitude distribution at a relatively large y for a given X in the present case would become similar to that of the last one, provided the letters u's are replaced by v's, the letters v_2, v_2' by u_1, u_1', and the letters v_1, v_1' by u_2, u_2'.

IV. *Primary Distortional Waves with Amplitudes Orientated Horizontally.*

9. In the present case λ, λ' do not participate in the problem. The expressions of incident, reflected, and refracted plane waves assume the forms

$$\left.\begin{aligned} v_0 &= \mathfrak{B}e^{sy+ifx-ipt}, \\ v &= Be^{-sy+ifx-ipt}, \qquad v' = B'e^{s'y+ifx-ipt}, \end{aligned}\right\} \tag{63}$$

where

$$B = \frac{\mu s - \mu' s'}{\mu s + \mu' s'}\mathfrak{B}, \quad B' = \frac{2\mu s}{\mu s + \mu' s'}\mathfrak{B}. \tag{64}$$

When the primary waves are radiated from a point source $x=0$, $y=\xi$, the displacements of the three kinds of waves assume the forms

$$\left.\begin{aligned} v_0 &= \mathfrak{B}e^{-ipt}H_0^{(1)}(kR) = \frac{\mathfrak{B}e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{s(y-\xi)+ifx}}{s}df, \qquad [y<\xi], \\ &= \frac{\mathfrak{B}e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{e^{-s(y-\xi)+ifx}}{s}df, \qquad [y>\xi], \\ v &= \frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{Be^{-s(y+\xi)+ifx}}{s}df, \qquad [y>0] \\ v' &= \frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\frac{B'e^{s'y-s\xi+ifx}}{s}df. \qquad [y<0] \end{aligned}\right\} \tag{65}$$

Since in every integral of the present case no principal value of the integrand exists, it is impossible for boundary waves to exist. The superposition of v_0+v gives

$$\left.\begin{aligned} v_1 = v_0 + v &= \frac{e^{-ipt}}{\pi}\int_{-\infty}^{\infty}\left[\frac{\mathfrak{B}e^{s(y-\xi)+ifx}}{s} + \frac{Be^{-s(y+\xi)+ifx}}{s}\right]df, \qquad [y<\xi] \\ &= \frac{2\mathfrak{B}e^{-ipt}}{\pi}\int_{-\infty}^{\infty}e^{-sy+ifx}\left[\frac{\mu}{\mu s+\mu' s'}\cosh s\xi + \frac{\mu' s'}{s(\mu s + \mu' s')}\sinh s\xi\right]df. \quad [y>\xi] \end{aligned}\right\} \tag{66}$$

Integrating (67) for $x \gtrless 0$, $y \gtrless 0$ in the same way as in the preceding cases, we finally obtain

$$\left.\begin{aligned} v_1 &= e^{-ipt}\left[\mathfrak{B}H_0^{(1)}(kR_1) - BH_0^{(1)}(kR_2)\right], \\ &\qquad [R_1^2 = x^2 + (y-\xi)^2, \quad R_2^2 = x^2 + (y+\xi)^2, \quad y \gtrless 0, \quad y<\xi] \\ v_1 &= 2\mathfrak{B}e^{-ipt}\frac{k\mu}{\mu'\sqrt{k^2-k'^2}}\frac{y}{R}H_1^{(1)}(kR), \qquad [R^2 = x^2+y^2, \quad y>\xi] \\ v' &= 2\mathfrak{B}e^{-ipt}\frac{k'}{\sqrt{k'^2-k^2}}\frac{y}{R}H_1^{(1)}(kR). \quad [R^2=x^2+y^2, \quad y\gtrless 0, \quad y>0] \end{aligned}\right\} \tag{67}$$

Since the calculation is rather approximate, the result in (67) is only qualitatively correct. It will be seen that the distribution of the amplitudes for v_1, v' are such that at a relatively large x for a given Y, they vary as Y/R, while at a relatively large y for a given X, they vary as y/R.

V. *Comparison of the Three Cases.*

10. In the cases of dilatational primary waves and distortional primary waves with amplitudes orientated in a vertical plane, the amplitudes in the epicentral region that decay the least with distance are proportional to $R^{-1/2}(y/R)^2$, whereas in the case of distortional primary waves with amplitudes orientated horizontally, the amplitudes that decay the least are proportional to $R^{-1/2}\,y/R$, from which it is possible to conclude that in the former condition of the two, the region (epicentral) in which large amplitudes of the waves appear is much narrower than that in the latter condition.

Similar conditions exist in the region of large epicentral distance, that is to say, at a relatively large x for a given Y. In the cases of dilatational primary waves and distortional primary waves with amplitudes orientated in a vertical plane, the amplitudes that decay the least with distance are proportional to $R^{-1/2}(x/R)(Y/R)$, whereas in the case of distortional primary waves with amplitudes orientated horizontally, the amplitudes that decay the least are proportional to $R^{-1/2}\,Y/R$. Thus, even from this feature, in the former condition of the two the region of a large horizontal focal distance in which the amplitudes decay the least, is much narrower than that in the latter condition.

In the actual condition, the phenomena just given occur repeatedly in passing through successive discontinuities, from which it is possible to expect that the regions (the epicentral region as well as the region of large horizontal focal distance) in which large amplitudes of the horizontal distortional waves appear, become increasingly greater than similar regions for dilatational waves and distortional waves with amplitudes orientated in a vertical plane.

Aside from the above condition, there is the feature that, in the case of dilatational primary waves and in the case of distortional primary waves with amplitudes orientated in a vertical plane, the energy of the bodily waves is converted, in a majority of cases, into that of boundary waves in passing through or reflected from a discontinuity, whereas in the case of distortional primary waves with amplitudes orientated horizontally, no boundary wave is formed even in passing through or reflected from any discontinuity. There is no doubt that there are a number of nearly horizontal discontinuous surfaces in the crust, so that the energy of bodily waves other than horizontal distortional waves are successively converted into that of boundary waves at such discontinuous surfaces. It is therefore obvious that the amplitudes of bodily waves, excepting those of horizontal distortional waves, become smaller and smaller in passing through or reflected from discontinuous surfaces.

Dynamically, it is possible to conclude that the condition that the region in which the large amplitudes of horizontal distortional waves appear, is broader than the similar regions for other kinds of waves, is equivalent to the condition whether energy of bodily waves is unchanged or changed into that of boundary waves.

It also appears that the reason for *ScS* waves being of relatively large amplitudes, is possibly explained to a certain extent by the present theory.

From the present investigation it has also been ascertained that Lamb's and Nakano's (and also our) mathematical condition that the amplitudes of bodily waves accompanying Rayleigh-waves on a semi-infinite body shall decay very rapidly, is nothing more than the condition on an azimuth difference in a spherical wave front.

In the present paper we have given only an approximate discussion of the two-dimensional case of the problem from the standpoint of the formation of Stoneley's boundary waves, the case of the three dimensional condition being now under investigation in a more rigorous manner.

Note on the paper No. 19 entitled "Anormolous Dispersion of Elastic Surface Waves" *Bull. Earthq. Res. Inst.*, **16** (1938), 225~233.

When the paper in printing had already been revised, some one asked us as to whether or not there is any part of dispersion curve within the range between $L/H=0$ and 4·635 in Fig. 3, p. 230. It was theoretically proved in the same paper that dispersion curve with ordinate higher than $p_1\sqrt{\rho/\mu}/f=1$ is impossible to exist at that range. Since, furthermore, the functions contained in the velocity equation (10) are all hyperbolic and not sinusoidal, the curve in thick line is the only dispersion curve lying below the ordinate $p_1\sqrt{\rho/\mu}/f=1$. The dotted line extending from thick curve is nothing more than the indication that the thick curve near its end (not the very point of the end) inclines in that sense.

Although in the case of very small ratio of L/H, Stoneley's boundary waves are possibly transmitted along the surface between two media, since in the present case the velocity of distortional waves in one of the media much differs from that in the other, Stoneley's waves are scarcely transmitted as has been proved by Stoneley. Thus, there is no evidence of transmission of waves of surface type at such range of L/H as less than 4·635. It should, however, be borne in mind that when $L/H=0$, Rayleigh-waves of such velocity as $0{\cdot}9194\sqrt{\mu'/\rho'}$ are physically possible to exist at the free surface, but since, in the present case, the conditions as to $(r/f)^2 \gtrless 1$, $(s/f)^2 \gtrless 0$ are restricted, the waves under consideration does not concern the problem.

Since, furthermore, we discussed the possible existence of waves of permanent type, the consideration of waves that shall decay with time in consequence of the boundary conditions is out of question. (This note was read by K. SEZAWA, June 21, 1938.)

---

38. 地殼內の不連續面に於ける境界波の生成（第1報）

地震研究所 妹澤克惟
金井清

2種の固體が平面で密着連續してをるとき，この境界層に沿うて Rayleigh 型の彈性波が傳播し得るけれども Love 型の彈性波は傳播し得ぬ事を Stoneley が述べてゐる．この論文では最初に傳はる固體波が夫々縱波，鉛直橫波，水平橫波である場合について前述の境界波が如何にして生成するか，又，反射，屈折の固體波は如何なるものであるかを數理的に取扱つて見たのである．

地殼の中には水平な不連續面が幾つもある事は極く明かであるが，その結果として，水平橫波が最初に傳はるときには結局境界波が現れ得ぬことになるのである．之に反して縱波や鉛直橫波が最初に傳はるときは，境界波が生成するから，多くの不連續面に於て通過し反射する每に固體波の波動勢力が境界波のそれに變化するのである．從て水平橫波の振幅が比較的に減衰しない事實と一致する物理的性質をあらはす．

直接數理的に示される振幅に就ては，水平橫波の場合の卓越せる振幅のある地殼區域が，他の波の場合のその區域に比して著しく廣いといふ事である．尙，Lamb 其他が論じたやうに表面波に伴ふ固體波が，距離と共に著しく減少するのは寧ろ，走波面の方向性の結果に過ぎぬ事もわかつた．