22. *Aseismic Properties of a Daikoku-basira (Principal Column) or Similar Column in a Japanese Style House.*

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1. *Introduction.*

Almost at the centre, in houses of old Japanese style, there is a column (sometimes two) called *daikoku-basira* (principal column). The peculiarity of such a column[1] is that its diameter is much larger than that of any other column in the same building. Whereas in an oridinary dwelling house of past centuries, either of military family or of farmer class, there is only one principal column, in a palace or a temple there are usually two principal columns (not called daikoku-basira in this case). Although the special column now under consideration originally used to support the heavy weight of the roof or to batten horizontal members in every floor, the same column in the course of time tended to serve more of a decorative purpose than utility, and eventually lost its structural meaning. But, it came to be known that a house with this special column, even though its object was no more than for decoration, actually resisted wind and seismic forces. Although the resistance to such horizontal forces may depend on the structural condition of the column, since no satisfactory theory with respect to the problem has yet been offered, it would scarcely be possible to conclude that this thick column, to all apparent purposes for decoration only, is meaningless from the structural point of view. Mathematical investigations show that, even though the connection between this column and the horizontal members be fairly weak, the former still contributes greatly to the resistance of a building to seismic forces.

The general requirement for a high-class house of Japanese style is that it should have as far few walls as possible, the reason originally being probably ventilation during the hot weather. If, therefore, a strong extra thick column or any substitute for it were used in houses, such as just mentioned, it should be possible to render the house aseismic

1) From T. KAN's paper (*Jour. Arch.*, **47** (1933), 29) it appears that its quality is also better than that of any other column.

to a certain extent, even though it may be wall-less.

It is a common sense reasoning in engineering that all members in a structure should be distributed as uniformly as possible, it appearing therefore that the idea of daikoku-basira contradicts with the common sense under consideration.

There is the fact that in a strong earthquake, some houses rotated about their extra thick columns with but little structural failure. Although from this consideration it may be advisable to solve the case of a three-dimensional framed structure, namely, the case of space lattice, yet owing to the extreme complexity of the problem, it will be discussed two-dimensionally, certain special points only being treated three-dimensionally.

2. *A single-storied house, the problem being two-dimensional.*

To ascertain the aseismic properties of the daikoku-basira (extra thick column), it is advisable to consider the case of a finite number of columns. We have, thus, taken the case of seven columns, the middle one of which is the extra-thick column. Let the stiffness of the thick column, that of each of all the other ordinary columns, and that of the floor be E_0I_0, EI, $E'I'$ respectively. Let also the inertia mass at the panel point corresponding to the top of the thick column and that at every remaining panel point be m_0, m respectively. The column height and the length of every span are shown in the sketch.

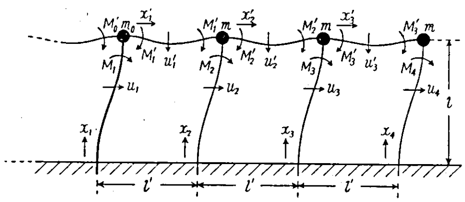

Fig. 1.

The expressions of vibratory motion for the columns and the floors are of the types

$$\left.\begin{aligned} u_s &= (A_s + B_s x_s + C_s x_s^2 + D_s x_s^3) e^{ipt}, \\ u'_s &= (A'_s + B'_s x'_s + C'_s x'^2_s + D'_s x'^3_s) e^{ipt}. \end{aligned}\right\} \quad (1)$$

The boundary conditions are such that

$$x_s = 0; \qquad u_s = b e^{ipt}, \quad \frac{du_s}{dx_s} = 0, \qquad (2), (3)$$

$$x_1 = l,\ x'_1 = 0; \qquad M_1 - 2M'_1 = 0, \quad u'_1 = 0, \quad \frac{du_1}{dx_1} = \frac{du'_1}{dx'_1}, \qquad (4), (5), (6)$$

$$x_2 = l,\ x'_1 = l',\ x'_2 = 0; \qquad M_2 + M'_1 - M'_2 = 0, \qquad (7)$$

$$u'_1 = u'_2 = 0, \quad \frac{du_2}{dx_2} = \frac{du'_1}{dx'_1} = \frac{du'_2}{dx'_2}, \qquad (8), (9)$$

$$x_3=l,\ x_2'=l',\ x_3'=0;\qquad M_3+M_2'-M_3'=0, \tag{10}$$

$$u_2'=u_3'=0,\quad \frac{du_3}{dx_3}=\frac{du_2'}{dx_2'}=\frac{du_3'}{dx_3'}, \tag{11), (12}$$

$$x_4=l,\ x_3'=l';\quad M_4+M_3'=0,\quad u_3'=0,\quad \frac{du_4}{dx_4}=\frac{du_3'}{dx_3'}, \tag{13), (14), (15}$$

$$x_s=l;\qquad u_1=u_2=u_3=u_4, \tag{16}$$

$$-\left\{E_0I_0\frac{d^3u_1}{dx_1^3}+2EI\left(\frac{d^3u_2}{dx_2^3}+\frac{d^3u_3}{dx_3^3}+\frac{d^3u_4}{dx_4^3}\right)\right\}=(m_0+6m)p^2u_1. \tag{17}$$

Substituting expressions of types (1) in (2)~(17), we get

$$C_1=-\frac{3\gamma b}{l^2\Phi}\left\{\eta_0(4+20\eta+30\eta^2+13\eta^3)+\eta(20+86\eta+114\eta^2+45\eta^3)\right\},$$

$$D_1=\frac{\gamma b}{l^3\Phi}\left\{\eta_0(4+20\eta+30\eta^2+13\eta^3)+2\eta(22+91\eta+117\eta^2+45\eta^3)\right\},$$

$$C_2=-\frac{3\gamma b}{2l^2\Phi}\left\{\eta_0(8+64\eta+120\eta^2+59\eta^3)+2\eta(8+56\eta+96\eta^2+45\eta^3)\right\},$$

$$D_2=\frac{\gamma b}{2l^3\Phi}\left\{\eta_0(8+112\eta+240\eta^2+125\eta^3)+4\eta(4+46\eta+90\eta^2+45\eta^3)\right\},$$

$$C_3=-\frac{\gamma b}{l^2\Phi}\left\{\eta_0(12+96\eta+171\eta^2+75\eta^3)+3\eta(8+62\eta+105\eta^2+45\eta^3)\right\},$$

$$D_3=\frac{\gamma b}{l^3\Phi}\left\{\eta_0(4+56\eta+111\eta^2+49\eta^3)+\eta(8+110\eta+207\eta^2+90\eta^3)\right\},$$

$$C_4=-\frac{3\gamma b}{2l^2\Phi}\left\{\eta_0(8+52\eta+96\eta^2+53\eta^3)+2\eta(8+50\eta+87\eta^2+45\eta^3)\right\},$$

$$D_4=\frac{\gamma b}{2l^3\Phi}\left\{\eta_0(8+76\eta+168\eta^2+107\eta^3)+2\eta(8+74\eta+153\eta^2+90\eta^3)\right\}, \tag{18}$$

where

$$\begin{aligned}\Phi=\gamma&\left\{2\eta_0(4+20\eta+30\eta^2+13\eta^3)+\eta(16+76\eta+108\eta^2+45\eta^3)\right\}\\&-6\left\{\eta_0^2(4+20\eta+30\eta^2+13\eta^3)+2\eta_0(12+172\eta+406\eta^2+282\eta^3+45\eta^4)\right.\\&\left.+12\eta(4+46\eta+90\eta^2+45\eta^3)\right\},\end{aligned} \tag{19}$$

$$\eta=\frac{E'I'l}{EIl'},\quad \eta_0=\frac{E_0I_0}{EI},\quad \gamma=\frac{(m_0+6m)\,p^2l^3}{EI}. \tag{20}$$

$\Phi=0$ is the frequency equation of the natural vibration.

The foregoing results may be obtained by the slope-deflection method, namely, if the deflections, slopes, shearing forces, and bending moments at the two ends of a member of span l and stiffness EI be u_A, u_B, θ_A, θ_B, F_A, F_B, M_A, M_B respectively, then the resultant of the moments at every panel point is zero and the resultant shearing forces for all the columns on that floor are equal to the sum of the (horizontal) inertia forces at all the panel points. The mathematical conditions are as follows:

$$M_A=\frac{2EI}{l}\left\{2\theta_A+\theta_B+\frac{3(u_A-u_B)}{l}\right\},\quad M_B=\frac{2EI}{l}\left\{\theta_A+2\theta_B+\frac{3(u_A-u_B)}{l}\right\},$$

$$-F_A=F_B=-\frac{6EI}{l^2}\left\{\theta_A+\theta_B+\frac{2(u_A-u_B)}{l}\right\},$$

$$\sum M \text{ at every panel point } =0\ ,$$

$$\sum F \text{ for all columns } =P. \tag{21}$$

The bending moments, shearing force, and inertia at every panel point are expressed by

$$\left.\begin{aligned}\text{I:}\quad & M_1=\frac{2E_0I_0}{l}\left(2\theta_1-\frac{3u_1}{l}+\frac{3b}{l}\right),\quad M_1'=M_1''=\frac{2E'I'}{l'}(2\theta_1+\theta_2),\\ & -F_1=-\frac{6E_0I_0}{l^2}\left(\theta_1-\frac{2u_1}{l}+\frac{2b}{l}\right),\quad P=-m_0p^2u_1,\end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned}\text{II:}\quad & M_2=\frac{2EI}{l}\left(2\theta_2-\frac{2u_2}{l}+\frac{3b}{l}\right),\quad M_2'=\frac{2E'I'}{l'}(\theta_1+2\theta_2),\\ & M_2''=\frac{2E'I'}{l'}(2\theta_2+\theta_3),\\ & -F_2=-\frac{6EI}{l^2}\left(\theta_2-\frac{2u_2}{l}+\frac{2b}{l}\right),\quad P=-mp^2u_2,\end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned}\text{III:}\quad & M_3=\frac{2EI}{l}\left(2\theta_3-\frac{3u_3}{l}+\frac{3b}{l}\right),\quad M_3'=\frac{2E'I'}{l'}(\theta_2+2\theta_3),\\ & M_3''=\frac{2E'I'}{l'}(2\theta_3+\theta_4),\end{aligned}\right\} \tag{24}$$

$$\left.\begin{aligned}
&-F_3=-\frac{6EI}{l^2}\left(\theta_3-\frac{2u_3}{l}+\frac{2b}{l}\right), \quad P=-mp^2u_3, \\
\text{VI:}\quad &M_4=\frac{2EI}{l}\left(2\theta_4-\frac{2u_4}{l}+\frac{3b}{l}\right), \quad M_4'=\frac{2E'I'}{l'}(\theta_3+2\theta_4), \\
&-F_4=-\frac{6EI}{l^2}\left(\theta_4-\frac{2u_4}{l}+\frac{2b}{l}\right), \quad P=-mp^2u_4.
\end{aligned}\right\} \tag{25}$$

From the conditions (22)$\sim$(25) and the relations

$$\left.\begin{aligned}
&\textstyle\sum M \text{ at every panel point } =0, \quad \sum F \text{ for all columns } =\sum P, \\
&u_1=u_2=u_3=u_4,
\end{aligned}\right\} \tag{26}$$

we get the same solutions as in (18)—(19).

As examples, we shall take four cases of $E'I'/EI$, namely, $E'I'/EI=0$, 1, 10, and ∞. The vibrational frequencies of every case for different ratios of E_0I_0/EI are shown in Fig. 2. From this figure it will be seen that for every case of floor stiffness, a small increase in stiffness in the extra-thick column fairly increases the natural frequency of the building. The same figure also shows that in the case of extremely large stiffness of the thick column, considerable increase in floor stiffness is not yet sufficient for the building to acquire a sensible increase in its natural vibrations. This shows also how efficiently the thick column serves in rendering a house rigid.

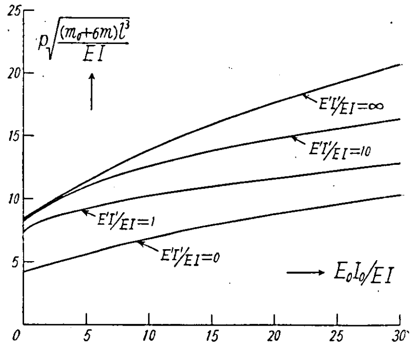

Fig. 2.

It is well-known that, other conditions being the same, increase in natural frequency affords good resistance to seismic vibrations in every condition (except the resonance condition) and also to wind forces. The damping forces (except dissipation) usually increase with increase in the natural frequency of a structure.

We shall next ascertain the amplitudes of the floor (roof) for different vibrational frequencies, including the resonance condition. Since in the present case no damping nor dissipative property is taken into account, it is impossible to obtain the general features of the problem resulting from the structure thus stiffened; it still shows that the more the house is stiffened the greater the increase of such a range of frequencies that the amplitudes of the floor (roof) are not specially large (see Figs. 3, 4, 5, 6).

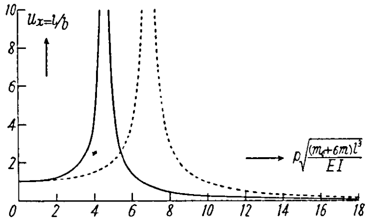

Fig. 3. $E'I'/EI=0$. Full and broken lines represent $E_0I_0/EI=1$, 10 respectively.

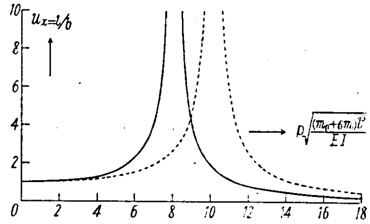

Fig. 4. $E'I'/EI=1$. Full and broken lines represent $E_0I_0/EI=1$, 10 respectively.

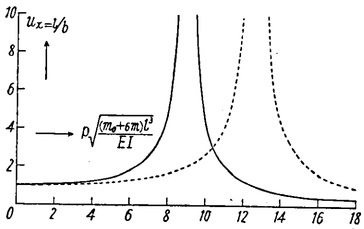

Fig. 5. $E'I'/EI=10$. Full and broken lines represent $E_0I_0/EI=1$, 10 respectively.

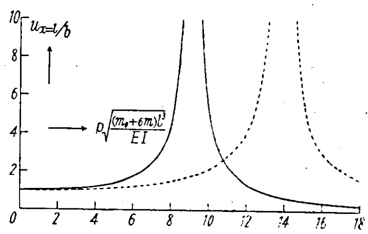

Fig. 6. $E'I'/EI=\infty$. Full and broken lines represent $E_0I_0/EI=1$, 10 respectively.

With a view to ascertaining the extent to which the stresses in columns are reduced by the addition of the extra-thick column, we shall let the abscissa of Fig. 7 represent the ratio of the diameter (z_0) of the thick column to that (z) of the ordinary columns. With increase in diameter of the thick-column, the stresses in the ordinary columns enormously diminish, and even the stress in the thick column itself (for the range $E_0I_0/EI>1$) also decreases. In these conditions, owing to the larger diameter of the thick column, the total area (a_0+6a) of cross sections of all the columns gradually increases, as shown by the chain-line in the figure. If we increase the area of every column without any special increase in the area of the thick column, the stresses in every column then decrease in the manner shown by the dotted line in the figure. The stresses in this condition are much greater than the stresses in the ordinary columns, and still somewhat higher than that in the thick column in the case just mentioned.

In an actual case, if the thick column were of some material other than wood say steel, in built-up form or in reinforced concrete, the stresses in the thick column would not necessarily be small; the aseismic properties of the thick column would then be particularly marked at the same time. It is highly commended that hotels and similar house of Japanese style have special columns (steel-framed, if possible) in some inner part of the building.

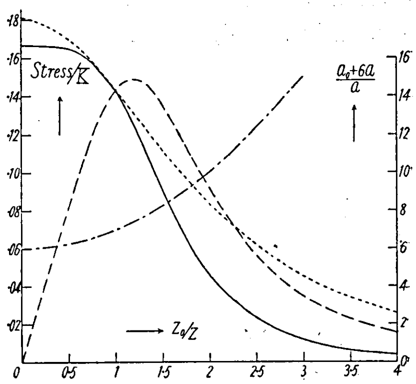

Fig. 7. Full and broken lines represent stresses in ordinary and thick columns respectively, total cross section area being shown by chain line. Dotted line represent stress in every column in the case that the sectional area of every column is the same, the total area being kept in the condition above given.

3. *Special cases of a two-storied buildings, the problem being two-dimensional.*

We shall next consider some special cases of a two-storied house, namely the case of extremely rigid floors (or roof) and that of extremely flexible floors or roof.

Let the stiffness of the thick column and that of the oridinary columns below the first floor and those below the roof be E_0I_0, EI; $E_0'I_0'$, $E'I'$ respectively. Let also the masses concentrated on the panel points corresponding to the tops of the respective columns just mentioned be m_0, m; m_0', m' respectively. If the deflections of the thick column and the oridinary columns, below the first floor and below the roof are u_1; u_2, ..., u_7; u_1'; u_2', ..., u_7', then the expressions for the vibrations of columns have the same forms as those in (1), and the boundary conditions are such that, s being 1, 2, ..., 7,

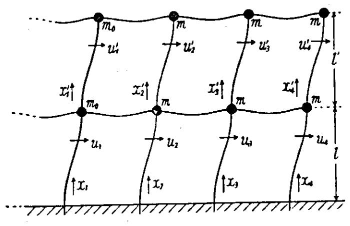

Fig. 8.

$$x_s=0; \qquad u_s=b\cos pt, \quad \frac{\partial u_s}{\partial x_s}=0, \tag{27), (28}$$

$$x_s=l,\ x_s'=0; \qquad u_s=u_s', \quad u_1=u_2=\cdots\cdots=u_s=\cdots\cdots=u_7, \tag{29), (30}$$

$$-\left\{E_0I_0\frac{\partial^3 u_1}{\partial x_1^3}+2EI\left(\frac{\partial^3 u_2}{\partial x_2^3}+\frac{\partial^3 u_3}{\partial x_3^3}+\frac{\partial^3 u_4}{\partial x_4^3}\right)\right\}$$

$$+\left\{E_0'I_0'\frac{\partial^3 u_1'}{\partial x_1'^3}+2E'I'\left(\frac{\partial^3 u_2'}{\partial x_2'^3}+\frac{\partial^3 u_3'}{\partial x_3'^3}+\frac{\partial^3 u_4'}{\partial x_4'^3}\right)\right\}-(m_0+6m)p^2u_1=0, \tag{31}$$

$$\left.\begin{aligned}&\frac{\partial u_s}{\partial x_s}=\frac{\partial u'_s}{\partial x'_s}=0 \text{ for the case of extremely rigid floors,}\\ &\frac{\partial u_s}{\partial x_s}=\frac{\partial u'_s}{\partial x'_s},\quad EI\frac{\partial^2 u_s}{\partial x_s^2}=E'I'\frac{\partial^2 u'_s}{\partial x'^2_s}\\ &\qquad\qquad \text{for the case of extremely flexible floors,}\end{aligned}\right\} \quad (32),\ (33)$$

$$x'_1=l'; \qquad u'_1=u'_2=\cdots\cdots=u'_s=\cdots\cdots=u'_7, \tag{34}$$

$$-\left\{E'_0I'_0\frac{\partial^3 u'_1}{\partial x'^3_1}+2E'I'\left(\frac{\partial^3 u'_2}{\partial x'^3_2}+\frac{\partial^3 u'_3}{\partial x'^3_3}+\frac{\partial^3 u'_4}{\partial x'^3_4}\right)\right\}-(m'_0+6m')p^2u'_1=0, \tag{35}$$

$$\left.\begin{aligned}&\frac{\partial u'_s}{\partial x'_s}=0 \text{ for the case of extremely rigid floors,}\\ &E'I'\frac{\partial^2 u'_s}{\partial x'^2_s}=0 \text{ for the case of extremely flexible floors.}\end{aligned}\right\} \tag{36}$$

Substituting expressions of types (1) in (27)$\sim$(36), we get
(i) for the case of extremely rigid floors,

$$u=be^{ipt}\left\{1-\frac{1}{\Phi}\left[\xi^3\gamma_1\left\{\gamma_2-12(\eta_4+6)\right\}\right.\right.$$
$$\left.\left.-12\gamma_2(\eta_2+6\eta_3)\right]\left\{3\left(\frac{x}{l}\right)^2-2\left(\frac{x}{l}\right)^3\right\}\right\},$$

$$u'=be^{ipt}\left\{1-\frac{1}{\Phi}\left[\xi^3\gamma_1\left\{\gamma_2-12(\eta_4+6)\right\}-12\gamma_2(\eta_2+6\eta_3)\right]\right.$$
$$\left.+\frac{12}{\Phi}\xi^3\gamma_2(\eta_1+6)\left\{3\left(\frac{x'}{l'}\right)^3-2\left(\frac{x'}{l'}\right)^3\right\}\right\}, \tag{37}$$

where

$$\Phi=\xi^3\left\{\gamma_1-12(\eta_1+6)\right\}\left\{\gamma_2-12(\eta_4+6)\right\}-12\gamma_2(\eta_2+6\eta_3), \tag{38}$$

$$\eta_1=\frac{E_0I_0}{EI},\quad \eta_2=\frac{E'_0I'_0}{EI},\quad \eta_3=\frac{E'I'}{EI},\quad \eta_4=\frac{E'_0I'_0}{E'I'},$$

$$\gamma_1=\frac{(m_0+6m)p^2l^3}{EI},\quad \gamma_2=\frac{(m'_0+6m')p^2l'^3}{E'I'},\quad \xi=\frac{l'}{l}, \tag{39}$$

$\Phi=0$ being the frequency equation of natural vibrations;
(ii) for the case of extremely flexible floors,

$$u=be^{ipt}\left\{1+3\xi\left[\xi^2\gamma_1\gamma_2(3\eta_3+2\xi)-6\gamma_1(\eta_4+6)\right.\right.$$

$$\left.-6\eta_3\gamma_2\left\{\xi(\eta_1+6)+(\eta_4+6)\right\}\right]\frac{1}{\Phi}\left(\frac{x}{l}\right)^2$$

$$\left.+2\xi\left[3\eta_3\gamma_2(\eta_4+6)+3\gamma_1(\eta_4+6)-\xi^2\gamma_1\gamma_2(3\eta_3+\xi)\right]\frac{1}{\Phi}\left(\frac{x}{l}\right)^3\right\},$$

$$u'=be^{ipt}\left\{1+\frac{\xi}{\Phi}\left[\xi^2\gamma_1\gamma_2(3\eta_3+4\xi)-12\gamma_1(\eta_4+6)\right.\right.$$

$$\left.-6\eta_3\gamma_2\left\{3\xi(\eta_1+6)+2(\eta_4+6)\right\}\right]+6\xi^2\left[\xi^3\gamma_1\gamma_2-3\gamma_1(\eta_4+6)\right.$$

$$\left.-3\eta_3\gamma_2\left\{2\xi(\eta_1+6)+(\eta_4+6)\right\}\right]\frac{1}{\Phi}\left(\frac{x'}{l'}\right)$$

$$\left.-9\xi^4\gamma_2\left\{\xi\gamma_1+2(\eta_1+6)\right\}\frac{1}{\Phi}\left(\frac{x'}{l'}\right)^2+3\xi^4\gamma_2\left\{\xi\gamma_1+2(\eta_1+6)\right\}\frac{1}{\Phi}\left(\frac{x'}{l'}\right)^3\right\}, \tag{40}$$

where

$$\Phi=\left\{\xi\gamma_1+2(\eta_4+6)\right\}\left[18(\eta_4+6)-3\xi\gamma_2\left\{\eta_3(1+3\xi)+2\xi^2\right\}\right]$$

$$-\left[6(\eta_4+6)-\xi\gamma_2\left\{3\eta_3(1+2\xi)+2\xi^2\right\}\right]\left[\xi\left\{6(\eta_1+6)+\gamma_1\right\}+6(\eta_4+6)\right], \tag{41}$$

$$\eta_1=\frac{E_0I_0}{EI},\quad \eta_2=\frac{E_0'I_0'}{EI},\quad \eta_3=\frac{E'I'}{EI},\quad \eta_4=\frac{E_0'I_0'}{E'I'},$$

$$\gamma_1=\frac{(m_0+6m)p^2l^3}{EI},\quad \gamma_2=\frac{(m_0'+6m')\,p^2l'^3}{E'I'},\quad \xi=\frac{l'}{l}. \tag{42}$$

In the special condition wherein $\xi=1$, $\eta_1=\eta_2=\eta_4\equiv\eta$, $\eta_3=1$, $\gamma_1=\gamma_2\equiv\gamma$, we get the frequency equations (i), (ii) below.

(i) $$\gamma^2-36\gamma(\eta+6)+144(\eta+6)^2=0,$$

so that $$\gamma=(\eta+6)[4{\cdot}584,\ 31{\cdot}416], \tag{43}$$

(ii) $$7\gamma^2-108(\eta+6)\gamma+36(\eta+6)^2=0,$$

so that $$\gamma=(\eta+6)[0{\cdot}341,\ 15{\cdot}088]. \tag{44}$$

The results of calculations for various ratios of E_0I_0/EI are shown in Fig. 9. It will be seen that, even in the case of a two-storied house, the presence of a thick-column increases the vibrational frequency of that house. Although the absolute frequencies of the fundamental vibrations are very low compared with those in the case of a one-storied house, the rate of increase in frequency with increase in the stiffness of the thick column is the same as that in the one-storied house under consideration.

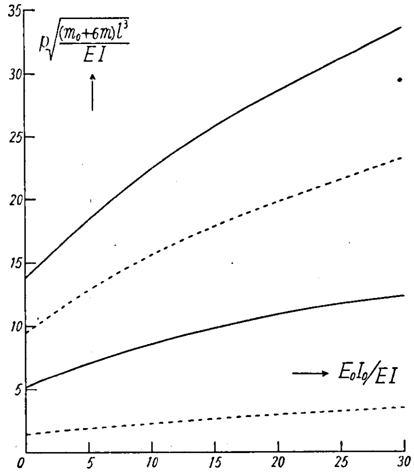

Fig. 9. Full and broken lines represent the cases of rigid floors and flexible floors respectively. There are two vibrational frequencies (fundamental and second) in every case.

4. *The problem of a space-latticed framed structure (solid Rahmen) in connection with an extra thick column.*

It is well known that the problem of space-latticed framed structure is extremely difficult. Although Reissner,[2] Prager,[3] Ban,[4] Takimoto,[5] Waltking,[6] Hohenemser,[7] Reinitzhuber,[8] and others have studied this problem, there seems little chance of their results being are used in the case of vibration problems, whereas in some cases it is even impossible to apply them to practical cases. Notwithstanding the complexity of the problem, since the principle of its dynamical conditions is rather simple, we shall discuss the problem from the elementary stage of its treatment.

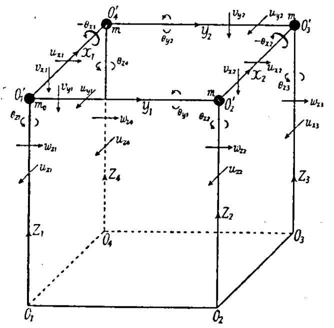

Fig. 10.

Referring to Fig. 10, inertia masses, m_0, m, m, m, are concentrated at panel points, $0_1'$, $0_2'$, $0_3'$, $0_4'$. The bending stiffnesses and the torsional stiffnesses of the members $0_10_1'$; $0_20_2'$, $0_30_3'$, $0_40_4'$; $0_1'0_2'$, $0_2'0_3'$, $0_3'0_4'$, $0_4'0_1'$ are E_0I_0, EI, $E'I'$ respectively.

2) H. Reissner, *Zeits. Bauwesen*, **8** (1903), 135~162.
3) W. Prager, *Z. techn. Phys.*, **10** (1929), 275~280.
4) S. Ban, *Jour. Arch., Japan*, **44** (1930), 75~118.
5) G. Takimoto, *ditto*, **45** (1931), 743~771.
6) F. W. Waltking, *Ing. Arch.*, **2** (1931), 247~274.
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8) Reinitzhuber, *Ing. Arch.*, **8** (1937) 349~363.

Since the vibratory equations of every member in bending as well as in torsion are of the types

$$\left.\begin{array}{llll}\frac{\partial^4 u_{zs}}{\partial z_s^4}=0, & \frac{\partial^4 w_{zs}}{\partial z_s^4}=0, & \frac{\partial^2 \theta_{zs}}{\partial z_s^2}=0, & [s=1,\ 2,\ 3,\ 4] \\ \frac{\partial^4 u_{ys}}{\partial y_s^4}=0, & \frac{\partial^4 v_{ys}}{\partial y_s^4}=0, & \frac{\partial^2 \theta_{ys}}{\partial y_s^2}=0, & [s=1,\ 2] \\ \frac{\partial^4 u_{xs}}{\partial x_s^4}=0, & \frac{\partial^4 v_{xs}}{\partial x_s^4}=0, & \frac{\partial^2 \theta_{xs}}{\partial x_s^2}=0, & [s=1,\ 2]\end{array}\right\} \tag{45}$$

the solutions of the corresponding equations have the forms

$$\left.\begin{array}{l}u_{zs}=(A_{zs}+B_{zs}z_s+C_{zs}z_s^2+D_{zs}z_s^3)e^{ipt}, \\ w_{zs}=(E_{zs}+F_{zs}z_s+G_{zs}z_s^2+H_{zs}z_s^3)e^{ipt}, \\ \theta_{zs}=\alpha_{zs}+\beta_{zs}z_s,\end{array}\right\} [s=1,\ 2,\ 3,\ 4] \tag{46}$$

$$\left.\begin{array}{l}u_{ys}=(A_{ys}+B_{ys}y_s+C_{ys}y_s^2+D_{ys}y_s^3)e^{ipt}, \\ v_{ys}=(P_{ys}+Q_{ys}y_s+R_{ys}y_s^2+S_{ys}y_s^3)e^{ipt}, \\ \theta_{ys}=\alpha_{ys}+\beta_{ys}y_s,\end{array}\right\} [s=1,\ 2] \tag{47}$$

$$\left.\begin{array}{l}u_{xs}=(A_{xs}+B_{xs}x_s+C_{xs}x_s^2+D_{xs}x_s^3)e^{ipt}, \\ v_{xs}=(P_{xs}+Q_{xs}x_s+R_{xs}x_s^2+S_{xs}x_s^3)e^{ipt}, \\ \theta_{xs}=\alpha_{xs}+\beta_{xs}x_s.\end{array}\right\} [s=1,\ 2] \tag{48}$$

It is possible to assume that the lengthwise stiffness of every member is so large that the member cannot elongate or contract. The possible freedom of every member is therefore bendings in two perpendicular directions, and torsion.

If we choose the components of displacements in the senses as shown in Fig. 10, the boundary conditions assume the forms

$$z_s=0; \qquad u_{zs}=ae^{ipt}, \quad w_{zs}=be^{ipt}, \quad \frac{du_{zs}}{dz_s}=0, \quad \frac{dw_{zs}}{dz_s}=0, \quad \theta_{zs}=0,$$

$$[s=1,\ 2,\ 3,\ 4] \quad (49),\ (50),\ (51),\ (52)$$

$$z_1=l,\ y_1=0,\ x_1=0; \qquad v_{y_1}=0, \quad v_{x_1}=0, \quad u_{z_1}=u_{y_1}, \quad w_{z_1}=u_{x_1},$$

$$(53),\ (54),\ (55),\ (56)$$

$$\frac{dw_{z_1}}{dz_1}=\frac{dv_{y_1}}{dv_1}=\theta_{x_1}, \quad \frac{du_{y_1}}{dy_1}=\frac{du_{x_1}}{dx_1}=-\theta_{z_1}, \quad -\frac{dv_{x_1}}{dx_1}=\frac{du_{z_1}}{dz_1}=\theta_{y_1},$$

$$(57),\ (58),\ (59)$$

$$-E_0I_0\frac{d^2w_{z_1}}{dz_1^2}+E'I'\frac{d^2v_{y_1}}{dy_1^2}+G'J'\frac{d\theta_{x_1}}{dx_1}=0, \tag{60}$$

$$E'I'\frac{d^2u_{y_1}}{dy_1^2}+E'I'\frac{d^2u_{x_1}}{dx_1^2}+G_0J_0\frac{d\theta_{z_1}}{dz_1}=0, \tag{61}$$

$$-E'I'\frac{d^2v_{x_1}}{dx_1^2}-E_0I_0\frac{d^2u_{z_1}}{dz_1^2}+G'J'\frac{d\theta_{y_1}}{dy_1}=0, \tag{62}$$

$z_2=l,\ y_1=l',\ x_2=0;$ $\quad v_{y_1}=0,\quad v_{x_2}=0,\quad u_{z_2}=u_{y_1},\quad w_{z_2}=u_{x_2},$

(63), (64), (65), (66)

$$\frac{dw_{z_2}}{dz_2}=\frac{dv_{y_1}}{dy_1}=\theta_{x_2},\quad \frac{du_{y_1}}{dy_1}=\frac{du_{x_2}}{dx_2}=-\theta_{z_2},\quad -\frac{dv_{x_2}}{dx_2}=\frac{du_{z_2}}{dz_2}=\theta_{y_1},$$

(67), (68), (69)

$$-EI\frac{d^2w_{z_2}}{dz_2^2}-E'I'\frac{d^2v_{y_1}}{dy_1^2}+G'J'\frac{d\theta_{x_2}}{dx_2}=0, \tag{70}$$

$$-E'I'\frac{d^2u_{y_1}}{dy_1^2}+E'I'\frac{d^2u_{x_2}}{dx_2^2}+GJ\frac{d\theta_{z_2}}{dz_2}=0, \tag{71}$$

$$E'I'\frac{d^2v_{x_2}}{dx_2^2}+EI\frac{d^2u_{z_2}}{dz_2^2}+G'J'\frac{d\theta_{y_1}}{dy_1}=0, \tag{72}$$

$z_3=l,\ y_2=l',\ x_2=l';$ $\quad v_{y_2}=0,\quad v_{x_2}=0,\quad u_{z_3}=u_{y_2},\quad w_{z_3}=u_{x_2},$

(73), (74), (75), (76)

$$\frac{dw_{z_3}}{dz_3}=\frac{dv_{y_2}}{dy_2}=\theta_{x_2},\quad \frac{du_{y_2}}{dy_2}=\frac{du_{x_2}}{dx_2}=-\theta_{z_3},\quad -\frac{dv_{x_2}}{dx_2}=\frac{du_{z_3}}{dz_3}=\theta_{y_2},$$

(77), (78), (79)

$$EI\frac{d^2w_{z_3}}{dz_3^2}+E'I'\frac{d^2v_{y_2}}{dy_2^2}+G'J'\frac{d\theta_{x_2}}{dx_2}=0, \tag{80}$$

$$-E'I'\frac{d^2u_{y_2}}{dy_2^2}-E'I'\frac{d^2u_{x_2}}{dx_2^2}+GJ\frac{d\theta_{z_3}}{dz_3}=0, \tag{81}$$

$$-E'I'\frac{d^2v_{x_2}}{dx_2^2}+EI\frac{d^2u_{z_3}}{dz_3^2}+G'J'\frac{d\theta_{y_2}}{dy_2}=0, \tag{82}$$

$z_4=l,\ y_2=0,\ x_1=l';$ $\quad v_{y_2}=0,\quad v_{x_1}=0,\quad u_{z_4}=u_{y_2},\quad w_{z_4}=u_{x_1},$

(83), (84), (85), (86)

$$\frac{dw_{z_4}}{dz_4}=\frac{dv_{y_2}}{dy_2}=\theta_{x_1},\quad \frac{du_{y_2}}{dy_2}=\frac{du_{x_1}}{dx_1}=-\theta_{z_4},\quad -\frac{dv_{x_1}}{dx_1}=\frac{du_{z_4}}{dz_4}=\theta_{y_2}, \tag{87), (88), (89}$$

$$EI\frac{d^2w_{z_4}}{dz_4^2}-E'I'\frac{d^2v_{y_2}}{dy_2^2}+G'J'\frac{d\theta_{x_1}}{dx_1}=0, \tag{90}$$

$$E'I'\frac{d^2u_{y_2}}{dy_2^2}-E'I'\frac{d^2u_{x_1}}{dx_1^2}+GJ\frac{d\theta_{z_4}}{dz_4}=0, \tag{91}$$

$$E'I'\frac{d^2v_{x_1}}{dx_1^2}-EI\frac{d^2u_{z_4}}{dz_4^2}+G'J'\frac{d\theta_{y_2}}{dy_2}=0, \tag{92}$$

$$z_1=z_4=l,\ y_1=y_2=0;\qquad u_{z_1}=u_{z_4}, \tag{93}$$

$$-E_0I_0\frac{d^3u_{z_1}}{dz_1^3}-EI\frac{d^3u_{z_4}}{dz_4^3}+E'I'\frac{d^3u_{y_1}}{dy_1^3}+E'I'\frac{d^3u_{y_2}}{dy_2^3}=m_0p^2u_{z_1}+mp^2u_{z_4}, \tag{94}$$

$$z_2=z_3=l,\ y_1=y_2=l;\qquad u_{z_2}=u_{z_3}, \tag{95}$$

$$-EI\frac{d^3u_{z_2}}{dz_2^3}-EI\frac{d^3u_{z_3}}{dz_3^3}-E'I'\frac{d^3u_{y_1}}{dy_1^3}-E'I'\frac{d^3u_{y_2}}{dy_2^3}=mp^2u_{z_2}+mp^2u_{z_3}, \tag{96}$$

$$z_1=z_2=l,\ x_1=x_2=0;\qquad w_{z_1}=w_{z_2}, \tag{97}$$

$$-E_0I_0\frac{d^3w_{z_1}}{dz_1^3}-EI\frac{d^3w_{z_2}}{dz_2^3}+E'I'\frac{d^3u_{x_1}}{dx_1^3}+E'I'\frac{d^3u_{x_2}}{dx_2^3}=m_0p^2w_{z_1}+mp^2w_{z_2}, \tag{98}$$

$$z_3=z_4=l,\ x_1=x_2=l;\qquad w_{z_3}=w_{z_4}, \tag{99}$$

$$-EI\frac{d^3w_{z_3}}{dz_3^3}-EI\frac{d^3w_{z_4}}{dz_4^3}-E'I'\frac{d^3u_{x_1}}{dx_1^3}-E'I'\frac{d^3u_{x_2}}{dx_2^3}=mp^2w_{z_3}+mp^2w_{z_4}. \tag{100}$$

Substituting (49) ~ (100) in (46) ~ (48), and eliminating constants, it is possible to get the frequency equations, and then to find the expressions of the displacements of every member. There are, however, 80 constants to be determined. We shall eliminate 63 of these constants, when the following 17 relations then hold. Although it is possible to eliminate more constants and thus reduce the number of the relations even down to 4, since the resulting relations take very complex forms, nothing is gained by doing so.

$$(6\eta+2\zeta_1)X-2\eta Y+2\eta l'^3D_{x_2}+(6\eta+\zeta_1)l'^3D_{y_2}-(4\eta+\zeta_1)l'B_{y_2}=0, \tag{101}$$

$$2\eta X-2(3\eta+\zeta_0)Y-(6\eta+\zeta_0)l'^3D_{x_1}-2\eta l'^3D_{y_1}+(4\eta+\zeta_0)l'B_{y_2}=0, \tag{102}$$

$$2\eta X+2(\eta+\zeta_0)Y+(5\eta+\zeta_0)l'^3D_{x_1}-\eta l'^3D_{x_2}+3\eta l'^3D_{y_1}$$

$$+\eta l'^3 D_{y_1} - (4\eta+\zeta_0) l' B_{y_2} = 0, \quad (103)$$

$$2(2\eta+\zeta_1)X - 2(2\eta+\zeta_0+\zeta_1)Y - (4\eta+\zeta_0)l'^3 D_{x_1} - (4\eta+\zeta_1)l'^3 D_{x_2}$$
$$-6\eta l'^3 D_{y_1} + (2\eta+\zeta_1)l'^3 D_{y_2} + (\zeta_0-\zeta_1)l' B_{y_2} = 0, \quad (104)$$

$$2\eta X + 2\eta Y - 2\eta l'^3 D_{x_1} - 2\eta l'^3 D_{y_2} - (4\eta+\zeta_1) l' B_{y_2} = 0, \quad (105)$$

where

$$X = l^2 C_{z_3} + l^3 D_{z_3} - l^2 C_{z_4} - l^3 D_{z_4}, \quad Y = l^2 G_{z_3} + l^3 H_{z_3} - l^2 G_{z_2} - l^3 H_{z_2}, \quad (106)$$

$$(4\gamma_0+4\eta+\zeta) l D_{z_1} - \zeta l D_{z_2} - 2\zeta C_{z_3} - 2\zeta l D_{z_3} + 2(\gamma_0+6\eta+\zeta) C_{z_4}$$
$$-2(\gamma_0+7\eta+\zeta) l D_{z_4} = 0, \quad (107)$$

$$2\zeta l D_{z_1} - 2(3\eta+\zeta+4) l D_{z_2} - 2(6\eta+\zeta+1) C_{z_3} - (12\eta+\zeta-2) l D_{z_3}$$
$$+2\zeta C_{z_4} + \zeta l D_{z_4} = 0, \quad (108)$$

$$(4\gamma_0+3\eta) l D_{z_1} + (3\eta+4) l D_{z_2} + (6\eta+1) C_{z_3} + (6\eta-1) l D_{z_3}$$
$$+ (2\gamma_0+6\eta-1) C_{z_4} + (2\gamma_0+6\eta-3) l D_{z_4} = 0, \quad (109)$$

$$\zeta l D_{z_1} - (4\eta+\zeta+4) l D_{z_2} - 2(6\eta+\zeta+1) C_{z_3} - 2(7\eta+\zeta+1) l D_{z_3}$$
$$+2\zeta C_{z_4} + 2\zeta l D_{z_4} = 0, \quad (110)$$

$$6\gamma_0 l^3 D_{z_1} + 6 l^3 D_{z_2} + 2\gamma l^2 C_{z_3} + 2(\gamma+3) l^3 D_{z_3} + (\gamma_0+\gamma) l^2 C_{z_4}$$
$$+ (\gamma_0+\gamma+6) l^3 D_{z_4} = -a(3\gamma+\gamma_0), \quad (111)$$

$$3 l^3 D_{z_2} + \gamma l^2 C_{z_3} + (\gamma+3) l^3 D_{z_3} + 3\eta\xi^2 l'^3 D_{y_1} + 3\eta\xi^2 l'^3 D_{y_2} = -\gamma a, \quad (112)$$

$$3 l^3 H_{z_4} + \gamma l^2 G_{z_3} + (\gamma+3) l^3 H_{z_3} + 3\eta\xi^2 l'^3 D_{x_1} + 3\eta\xi^2 l'^3 D_{x_2} = -\gamma b, \quad (113)$$

where

$$\left.\begin{aligned} &\eta = \frac{E'I'l}{EIl'}, \quad \eta_0 = \frac{E_0 I_0}{EI}, \quad \zeta = \frac{G'J'l}{EIl'}, \quad \zeta_0 = \frac{G_0 J_0}{EI}, \quad \zeta_1 = \frac{GJ}{EI}, \\ &\xi = \frac{l}{l'}, \quad \gamma = \frac{mp^2 l^3}{EI}, \quad \gamma_0 = \frac{m_0 p^2 l^3}{EI}. \end{aligned}\right\} \quad (114)$$

(I) Now, solving (101) ~ (105), it is possible to determine D_{x_1}, D_{x_2}, D_{y_1}, D_{y_2}, as functions of C_{z_3}, D_{z_3}, C_{z_4}, D_{z_4}, G_{z_3}, H_{z_3}, G_{z_2}, H_{z_2}.

(II) On the other hand, (107) ~ (111) render six relations representing D_{z_2}, C_{z_3}, D_{z_3}, C_{z_4}, D_{z_4} in terms of D_{z_1}, a.

(III) Judging from some of boundary conditions, it is possible to understand that the mutual relations between H_{z_4}, G_{z_3}, H_{z_3}, G_{z_2}, H_{z_2}, H_{z_1}, b are quite similar to those between D_{z_2}, C_{z_3}, D_{z_3}, C_{z_4}, D_{z_4}, D_{z_1}, a; from which it is very simple to represent H_{z_4}, G_{z_3}, H_{z_3}, G_{z_2}, H_{z_2} in terms of H_{z_1}, b.

Substituting three kinds of relations, (I), (II), (III), in (112), (113), we finally get mere two relations representing D_{z1}, H_{z1} in terms of a, b. The denominator of either one of D_{z1} or H_{z1} then determined corresponds to the frequency equation of the natural vibration of the structure.

Other constants can be determined from above results in successive calculations.

We shall calculate six cases, of E_0I_0/EI, namely, $E_0I_0/EI=0$, 1, 3, 10, 20, 100; the relations $m_0=m$, $l=l'$, $E'I'=EI=G'J'=GJ$, $E_0I_0=G_0J_0$ being kept costant. The frequency equations then become

(i) $E_0I_0/EI=0$; $43008\gamma^2-1567744\gamma+7709184=0$,
$43008\gamma^2-594016\gamma+1762488=0$,

(ii) $E_0I_0/EI=1$; $(13\gamma-285)^2-144^2=0$, $(5\gamma-42)^2=0$,

(iii) $E_0I_0/EI=3$; $228072\gamma^2-10549792\gamma+85605948=0$,
$228072\gamma^2-5652640\gamma+33055740=0$,

(iv) $E_0I_0/EI=10$; $(19412\gamma^2-928170\gamma+8706429)^2 -(197496\gamma-3265002)^2=0$,

(v) $E_0I_0/EI=20$; $(2372316\gamma^2-150533310\gamma+1649095272)^2 -(23429628\gamma-566649351)^2=0$,

(vi) $E_0I_0/EI=100$; $(1781136\gamma^2-326221506\gamma+5161564296)^2 -(16916688\gamma-1566296109)^2=0$.

There are four vibrational frequencies in every case. Although it is possible for four vibrational frequancies to exist even in Case (ii), owing to the nature of the symmetry, mere three frequencies exist in this case.

The vibrational frequencies thus found are

(i); 4·322, 5·865, 9·5, 30·63,
(ii); $\gamma=8{\cdot}4$, 8·4, 10·846, 33,
(iii); $\gamma=9{\cdot}5$, 10·5, 15·3, 35·77,
(iv); $\gamma=10{\cdot}19$, 14·03, 27·45, 43·8,
(v); $\gamma=10{\cdot}63$, 16·46, 42·97, 56·9,
(vi); $\gamma=12{\cdot}58$, 22·13, 161·1, 170·5.

The first and second frequencies of case (i), (ii), or (iii) correspond to the mode of vibration parallel to every diagonal of the square formed by the horizontal members, the third frequency to the mode of rotary vibration, and the fourth frequency to the vibration, according to which every diagonal of the square elongates and contracts alternately. In other words, for the first, second and third frequencies, the square just mentioned remains nearly in its original form, whereas for the fourth frequency at any instant is deformed into a rhombic shape.

The third and the fourth frequencies of cases (iv), (v), or (vi) roughly correspond to the third and the fourth frequencies of the first and the second of cases (i), (ii), or (iii); but, other two vibrations of cases (iv), (v), (vi) are in much more coupled conditions.

It will be seen that in the present case, the first and second frequencies are very low, regardless of whether the thick column (daikoku-basira) is very stiff or not, which arises from the condition that there is no diagonal member in the plane of the square formed by the horizontal members. It is evident that some diagonal members should be fitted within the square formed by the horizontal members to render the condition of the thick column effective. Since, in an actual problem, however, it is by no means difficult to place diagonal members in the plane of any floor, the use of the thick column in palace or hotel types of buildings is quite practicable. The mathematical solution of such a case is reserved for next study.

It is obvious that if the thick column is near the centre of the building, its aseismic properties are almost the same as those in the case of a two-dimensional problem. It appears that even in the event that the thick column is at some distance from the centre of the building, its aseismic properties are still fairly satisfactory, details of which, however, will be shown in a forthcoming paper.

5. *Concluding remarks.*

From mathematical calculations we found that the use of a principal column, that is, a daikoku-basira (extra thick column) is to be recommended in Japanese style structures of a palace or hotel style, provided that the floor and the roof are made rigid, particularly in their diagonal directions. The column need not necessarily be of wood, but may, instead, be of built-up form in steel or of reinforced concrete, the building in which case, could be as nearly wall-less.

The present investigation was made at Professor SEZAWA's suggestion in connection with his other research work as member of the Investigation Committee for Earthquake-proof Construction, of the Japan Society of the Promotion of Scientific Research, to whom I wish to express my sincerest thank for his kind advices.

Note. While the importance of this problem in connection with Japanese style buildings is admittedly great, its solution is so difficult that as many as 80 constants have to be eliminated from a like number of conditions. Dr. Kanai, in my opinion, being the person most fitted for attacking the problem, I requested him to do so. The foregoing result, probably, is only preliminary, in which case it is hoped that the work will be continued with success.

Finally, I wish to express my warmest thanks to the Council of the Japan Society for the Promotion of Scientific Research for aid received in a series of investigation of which this is a part.

K. SEZAWA.

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22. 家屋の大黒柱又は類似部の耐震的効果

地震研究所 金 井 清

日本式の御殿風の家や旅館等では壁をなるべく用ひない方が望ましい。古い家屋に用ひられた大黒柱の考を今少しく活用する道はないものかと，この研究を試みたのである。數理的計算を試みたところが，家屋の斜の方向の剛性をよくしさへすれば，大黒柱の効果が發揮することがわかつた。但し効果といふのは材料を經濟的に考へた場合の効果であつて，全柱をそれぞれ大黒柱の如くすれば尙更よいのにきまつてをる。實際問題として大黒柱は必しも木材でなくてもよろしく，場合によつては鐵組立材又は鐵筋コンクリートで作れば一層効果がある譯である。尙この考は構造物の材料をなるべく均等に分布しようといふ工學的常識とはむしろ逆の傾向にあるのである。床面に斜材のないときの立體架構の場合を計算して見ると，大黒柱が家の隅にあり且つ斜材がない爲に極めて低い振動數の自己振動のあることがわかつた。卽ち斜材の必要な事がよく知られる。この立體架構の問題は面白いけれども甚だ複雜であり，今後も研究を續ける積りである。

註——家屋の大黒柱の問題は日本式の家屋に極めて大切なのにも拘らず，その一般的の理論計算は甚だ複雜である。之の解決には金井氏が最適任と考へて之を取扱つて貰つたのである。

只今の問題に限らずこの連續の研究には日本學術振興會の御援助に負ふ所が多く玆に厚く感謝する次第である(妹澤克惟)
