

the crater of Volcano Mihara and from similar observations carried out by T. Fukutomi⁷⁾ around the crater of Volcano Asama.

2. Method of observation

A compass transit was used in the present survey. Although this instrument is not accurate, errors of several minutes in observation of the magnetic meridian being inevitable, it suited our purposes, seeing that the magnitude of the anomalies in question for both volcanoes amounted to several degrees. The observations were made as follows: Upon measuring from station *A* angle α between the magnetic meridian and the direction of straight line *AB*, and then conversely angle β from station *B*, as shown in Fig. 1, the difference between α and β is that of the magnetic declinations at *A* and *B*. That is,

$$\alpha - \beta = \delta_A - \delta_B,$$

where δ_A and δ_B are declinations at *A* and *B* respectively. The positions of the stations were determined trigonometrically, the same transit being used. Repeating the operation successively at 20 stations, we traversed around the crater. To attain maximum accuracy, the traverses were conducted not only between adjoining stations, but also with as varying a combination of stations as possible. In determining α or β , we took the mean value of eight readings of an observed value at one station, the mean errors of one reading of the azimuth of the magnetic compass and also of the transit being 5' and 30'' respectively. The closing errors were 15' and 2.0' in the magnetic and trigonometric traverses respectively; which errors are negligible considering the observation mentioned in the next paragraph.

3. Results of observations

The distribution of the twenty stations and the results of observations are shown in Fig. 2 and in Table I. In this table, the numerals in the column entitled "Anomaly of declination" are deviations from the mean value of the observed declinations at twenty stations, the (+) and (-) signs referring to the westerly and easterly deviations respectively. As shown in Fig. 2, the anomalies in magnetic declina-

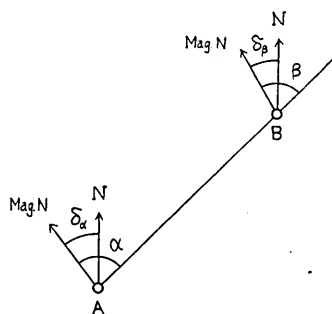


Fig. 1.

7) T. FUKUTOMI, *Zisin*, 2 (1930), 641 (In Japanese).

Table I.

Azimuth φ	Anomaly in Declination δ	φ	δ
12°	-1° 23'	186°	+2° 35'
40	0 00	200	-0 20
58	+5 20	232	+2 20
75	+5 00	255	+0 40
92	+1 40	277	-1 20
112	-1 10	290	-1 30
123	-0 20	310	-7 20
135	-1 30	328	-3 40
152	-1 40	348	-2 15
170	+3 10		

tion around the crater are distributed in quadrants, divided by the magnetic meridian and the magnetic E-W line through the centre of the crater; i.e. the anomalies have a plus (westerly) sign in the SW and NE quadrants, and a minus (easterly) sign in the SE and NW quadrants. Very similar results were obtained in the case of Volcano Asama in the course of observations carried out by T. Fukutomi in 1930, as shown in Fig. 3. The relations between the magnitude of the anomaly and the azimuths of stations from the centre of the crater are shown in Figs. 4, 5.

Comparing these two figures, we see that, although there are certain differences in the details, the general tendencies are very similar in the two cases.

In both cases, the magnitude of the anomaly on the magnetic southern side is larger than that on the northern side, i.e. the anomaly around the crater may be expressed not only with second harmonics but also with a superposed curve of the first and second harmonics of the azimuthal angle from the centre of the crater.

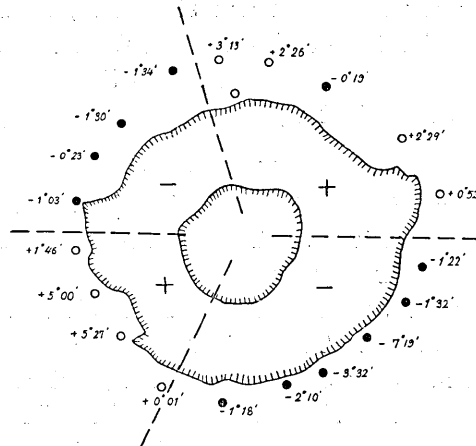


Fig. 2. Distribution of anomaly in declination around the crater of Mt. Mihara.

- Westerly anomaly
- Easterly "

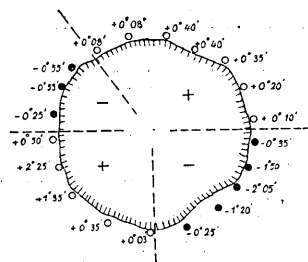


Fig 3. Distribution of anomaly in declination around the crater of Mt. Asama (Observed by T. Fukutomi).

If now we let the direction of magnetic south from the centre of the crater be the zero line, and let the azimuthal angle as measured clockwise from the zero line be φ , then the anomaly in declinations δ and φ may be approximately expressed by the equation,

$$\text{tg } \delta = A \sin \varphi + B \sin 2\varphi + C \quad (1).$$

Assuming that relation (1) holds for both volcanoes, Asama and Mihara, the constant, A , B , C are determined from the observed values with the aid of least squares, as follows,

$$\left. \begin{aligned} (\text{tg } \delta)_{\text{Asama}} &= 0.0133 \sin \varphi + 0.0221 \sin 2\varphi + 0.0017 \\ (\text{tg } \delta)_{\text{Mihara}} &= 0.0283 \sin \varphi + 0.0468 \sin 2\varphi + 0.0002 \end{aligned} \right\} \quad (2).$$

The relations expressed by equations (2) are graphed in Figs. 4 and 5 in full lines.

These anomalies just mentioned are believed to be due to singularities in the topography of the crater. Considering that the distributions of these anomalies are nearly symmetrical with the magnetic meridian, they are probably due to induced magnetism in the rocks with large magnetic susceptibility around the crater. If this assumption holds in our case, the susceptibility may be calculated from our observed data. Here we shall estimate the extent of the susceptibility by roughly approximating the shape of the crater. We could assume that the crater is a circular hole of cylindrical form, of depth d and radius l , on the plane surface of a semi-infinite body of susceptibility K , as shown in Fig. 6. The magnetic anomaly due to the presence of

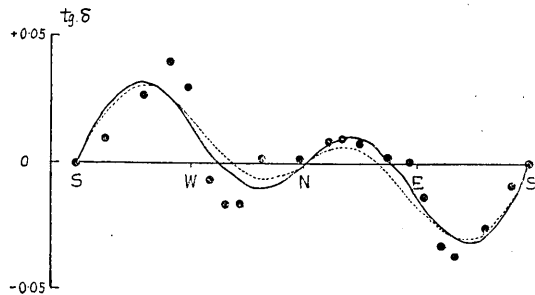


Fig. 4. The observed and theoretical distribution of anomaly in declination around the crater of Mt. Asama.

Full line=observed. Dotted line=theoretical.

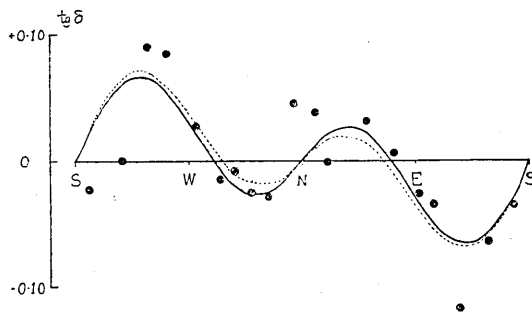


Fig. 5. The observed and theoretical distributions of anomaly in declination around the crater of Mt. Mihara.

Full line=observed. Dotted line=theoretical.

as shown in Fig. 6. The magnetic anomaly due to the presence of

this hole is exactly the same as that due to a body having the same dimensions and the same position as this hole, and having the same susceptibility as the surrounding medium, but magnetized inversely to the earth's normal magnetic field in that region, for if there were no hole, the magnetic field on the surface should be quite uniform.

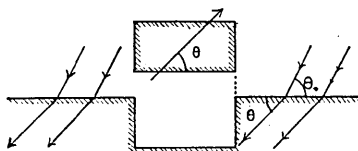


Fig. 6.

We shall, therefore, estimate the value of the magnetic anomaly due to the body just mentioned. For simplifying the mathematics, it is assumed that the body is a rotational ellipsoid, the axis of which is vertical, and satisfying the condition

$$\frac{4\pi}{3}ab^2 = V_0 = \pi l^2 d,$$

where a and b are the shorter (vertical) and the longer radii respectively. This approximation is sufficient for the present in order to get a general idea of the anomaly.

The magnetic field due to the magnetism induced by the earth's natural magnetic field into a rotational ellipsoid, expressed by the equation $\frac{x^2}{b^2} + \frac{z^2}{(a-h)^2} = 1$ in Cartesian coordinates, where the x , y , z axes are drawn northward, eastward, and downward respectively, has been calculated by J. Koenigsberger⁸⁾, according to whom, the magnitude of declination of the magnetic anomaly on the earth's surface ($z=0$ plane) is expressed by the equation,

$$\operatorname{tg} \delta = \frac{3V_0 K}{1-LK} \operatorname{tg} \theta \frac{yh}{m^2(b^2+n^2)(a^2+n^2)^{\frac{1}{2}}} - \frac{4V_0 K}{1-MK} \frac{xy(a^2+n^2)^{\frac{1}{2}}}{m^2(b^2+n^2)^2} \quad (3),$$

where

K = magnetic susceptibility,

θ = dip angle of the earth's field,

V_0 = volume of the rotational ellipsoid,

and

$$L = -4\pi \frac{1+u^2}{u^3} (u - \operatorname{arctg} u), \quad M = -2\pi \frac{1+u^2}{u^3} \operatorname{arctg} \left(u - \frac{u}{1+u^2} \right),$$

$$u = \frac{\sqrt{b^2 - a^2}}{a},$$

8) J. Königsberger, *Gerl. Beitr. Geophys.*, 19(1928), 241.

$$m^4 = (h^2 + x^2 + y^2)^2 + (b^2 - a^2) \left[(b^2 - a^2) - 2(x^2 + y^2 - h^2) \right],$$

$$n^2 = \frac{1}{2} (h^2 + x^2 + y^2 - b^2 - a^2 + m^2).$$

Assuming that the formula (3) is applicable to our cases, we shall estimate the susceptibility of the rocks composing the volcanoes Asama and Mihara.

(i) Asama

In the case of Volcano Asama, the magnitudes of a , b , h are taken as follows:

$$a = h = 100 \text{ m}$$

$$b = 170 \text{ m}.$$

As our stations are distributed almost in a circle with its centre on the centre of the crater, as shown in Fig. 3, we take

$$x = -r \cos \varphi, \quad y = -r \sin \varphi \quad (4),$$

where the positive sense of φ is taken clockwise, when equation (3) becomes

$$-\text{tg } \delta = \frac{3V_0 K}{1 - KL} \text{tg } \theta \frac{ar}{m^2 (b^2 + n^2) (a^2 + n^2)^{\frac{1}{2}}} \sin \varphi$$

$$+ \frac{2V_0 K}{1 - KM} \frac{(a^2 + n^2)^{\frac{1}{2}} r^2}{m^2 (b^2 + n^2)^2} \sin 2\varphi \quad (5).$$

If we take approximately $r = 200 \text{ m}$, we get

$$-\text{tg } \delta = K \left(\frac{1.59}{1 + 4.7K} \text{tg } \theta \sin \varphi + \frac{1.40}{1 + 5.0K} \sin 2\varphi \right) \quad (6).$$

It is not possible to get the magnetic susceptibility K direct from equation (6), seeing that the dip angle θ is not equal to the dip θ_0 on the earth's surface, θ and θ_0 being related as in

$$\cot \theta_0 = \frac{\cot \theta}{1 + 4\pi K} \quad (7).$$

We shall accordingly estimate the magnitude of K successively. First, instead of (6), we take

$$-\text{tg } \delta = K' (1.6 \text{tg } \theta_0 \sin \varphi + 1.4 \sin 2\varphi) \quad (8).$$

Since, according to T. Minakami⁹⁾, the mean value of the dip in

9) T. MINAKAMI, *Bull. Earthq. Res. Inst.*, 16 (1938), 100.

the vicinity of the crater of Volcano Asama is 50° , we take

$$\operatorname{tg} \theta_0 = \operatorname{tg} 50^\circ = 1.2.$$

Comparing eq (8) with eq (2), we have

$$K' = 0.012 \quad (9).$$

In this comparison, the sign of the right side of eq (8) should be reversed since the direction of magnetization of our model is exactly opposite to that of the natural field, as mentioned in the beginning of this paragraph. From (7) and (9) we get $\operatorname{tg} \theta = 1.0$,

$$\text{whence} \quad \operatorname{tg} \delta = K \left(\frac{1.6}{1+4.7K} \sin \varphi + \frac{1.4}{1+5.0K} \sin 24\varphi \right) \quad (10).$$

Comparing eq (9) with eq (2), we have

$$K = 0.011$$

$$(\operatorname{tg} \delta)_{\text{Asama}} = 0.019 \sin \varphi + 0.017 \sin 2\varphi.$$

The relation between δ and φ shown in eq (11) is graphed with dotted lines in Fig. 4.

(ii) Mihara

In this case, taking the constants a , b , h , r to be

$$a = h = 35 \text{ m}$$

$$b = r = 500 \text{ m},$$

we get

$$\begin{aligned} -\operatorname{tg} \delta &= \frac{3V_0K}{1-KL} \operatorname{tg} \theta \frac{\sqrt{a^2+ab}}{2ab(a+b)^2} \sin \varphi + \frac{V_0K}{1-KM} \frac{\sqrt{a^2+ab}}{ab(a+b)^2} \sin 2\varphi \\ &= K \left(\frac{1.5}{1+11.0K} \operatorname{tg} \theta \sin \varphi + \frac{1.0}{1+0.7K} \sin 2\varphi \right) \end{aligned} \quad (12).$$

Taking $\theta_0 = 50^\circ$, we get eq (13) from eq (12), with the same successive method used in the case of Asama,

$$-\operatorname{tg} \delta = \frac{1.3K}{1+11.0K} \sin \varphi + \frac{K}{1+0.7K} \sin 2\varphi.$$

Comparing eq (13) with eq (2), we have

$$\left. \begin{aligned} K &= 0.038 \\ \operatorname{tg} \delta &= 0.036 \sin \varphi + 0.038 \sin 2\varphi \end{aligned} \right\} \quad (14).$$

(iii) Effect of the present pit in the case of Mihara

10) R. TAKAHASHI and T. NAGATA, *loc. cit.*

In the preceding calculations the effect of the presence of the pit was neglected. We, now, estimate approximately the extent of the effect, assuming that it is almost the same as that due to a sphere having suitable dimensions and position. The declination on the plane surface $z=0$ due to the induced magnetism of a sphere is expressed by the relation,

$$\left. \begin{aligned} \operatorname{tg} \delta' &= \frac{3V_0 K}{1 + \frac{4\pi}{3} K} \frac{y}{R^5} (h \operatorname{tg} \theta - x) \\ R^2 &= x^2 + y^2 + h^2 = r^2 + h^2 \end{aligned} \right\} \quad (14),$$

in the same coordinates and notations as used in cases (i) and (ii). Modifying eq (14) in exactly the same way as in the above case, we get

$$-\operatorname{tg} \delta' = \frac{3V_0 K}{1 + \frac{4\pi}{3} K} \frac{r}{R^5} \left(h \operatorname{tg} \theta \sin \varphi + \frac{r}{2} \sin 2\varphi \right) \quad (15).$$

Taking the constants R , h , and the radius of the sphere ρ as

$$R = 500 \text{ m}, \quad h = 200 \text{ m}, \quad \rho = 120 \text{ m},$$

we get

$$-\operatorname{tg} \delta' = \frac{K}{1 + 4.2K} (0.048 \operatorname{tg} \theta \sin \varphi + 0.061 \sin 2\varphi) \quad (16).$$

Inserting the values of K and $\operatorname{tg} \theta$ obtained in case (ii) in eq (16) we get,

$$-\operatorname{tg} \delta' = 0.0012 \sin \varphi + 0.0020 \sin 2\varphi \quad (17).$$

Adding eq (17) to eq (14), as $\delta \ll 1$, and $\delta' \ll 1$,

$$\left. \begin{aligned} (\operatorname{tg} \delta_0)_{\text{Mthara}} &= \operatorname{tg} \delta + \operatorname{tg} \delta' = 0.037 \sin \varphi + 0.040 \sin 2\varphi \\ K &= 0.038 \end{aligned} \right\} \quad (18).$$

The relation between δ_0 and φ shown in eq (18) is graphed with dotted lines in Fig. 5.

4. Discussion of results

From Figs. 6, 7, we see that, barring the slight discrepancy in them, the general tendencies in both the theoretical and the analysis curves, are similar. This shows that the anomalous distribution of

magnetic declinations is mainly due to the anomalous topography of the crater, and that the earth's magnetic field induces magnetism in the rocks around the crater; at any rate the direction of its magnetization is parallel to the present magnetic field. Upon going strictly into details, however, we notice that there is some discrepancy in the ratio of the coefficient of $\sin\varphi$ to that of $\sin 2\varphi$ in both the theoretical and analysis curves; that is, the ratio in the theoretical result is larger than that in the results obtained by analysing the observed data. Seeing that the coefficient of $\sin\varphi$ in the theoretical formula contains $\text{tg}\theta$, and assuming that the discrepancy is due to the difference between the dip-angles of magnetization of the rocks and of the earth's magnetic field, we get the following relation from (2) and (6) and from (2) and (12),

$$\begin{aligned} \text{tg}\theta &= 0.54, \quad \theta = 28.5^\circ \quad \text{for Asama,} \\ \text{tg}\theta &= 0.46, \quad \theta = 24.5^\circ \quad \text{for Mihara.} \end{aligned}$$

Inserting the values of this $\text{tg}\theta$ in formulae (6), (12), we get the same results as the observed values. The discrepancy, however, could be also due to our assuming the shape of the crater to be a rotational ellipsoid, since only a slight difference between the shape of the actual crater and the model affects the result not a little, particularly as our observations were made near the edge of the crater.

The magnetic susceptibilities of the andesite of Asama and the basalt of Mihara seem to be somewhat larger than the values determined experimentally by a number of investigators¹¹⁾. These problems will be discussed after the completion of our experiments on the magnetic properties of the rocks of Volcanoes Asama and Mihara.

5. Appendix

In the above calculations, the form of the crater was assumed to be a rotational ellipsoid, under which assumption, however, there seem to be some discrepancies in the details between the theoretical results and the observed data.

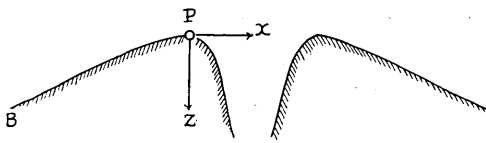


Fig. 7.

We shall now take more suitable assumption for the form of crater, although the consequent calculations will not be mathematically elegant.

11) For example, *Handb. d. Exp. phys.*, 3 Teil. (1930), pp. 28~31.

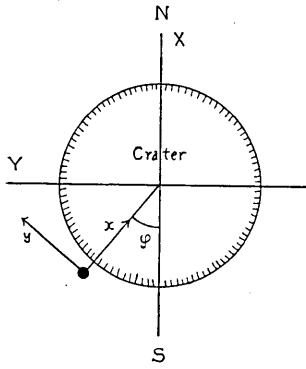


Fig. 8.

Fig. 7 shows the vertical section through the centre of the crater, the topography around which is assumed to be quite symmetric. In Fig. 7, we assume that magnetic anomaly at every observing station P is due to the presence of a mass $A-P-B$, which continues infinitely in a direction perpendicular to the paper. In Fig. 8, the circle represents the boundary of the crater. Coordinate systems (XYZ) and (xyz) are taken as shown in the figure. If H_0 and Z_0 denote the horizontal and

vertical components of the normal earth's magnetic field respectively, then the x , y , and z -components of the field x_0 , y_0 , and z_0 are

$$x_0 = H_0 \cos \varphi, \quad y_0 = H_0 \sin \varphi, \quad z_0 = Z_0 \quad (1).$$

And the three components of the magnetic anomaly due to the presence of mass $A-P-B$ at observing station P are

$$\left. \begin{aligned} \Delta x &= KH_0 \left(\cos \varphi \frac{\partial^2 W}{\partial x^2} + \frac{Z_0}{H_0} \frac{\partial^2 W}{\partial x \partial z} \right) \\ \Delta y &= 0 \\ \Delta z &= KH_0 \left(\cos \varphi \frac{\partial^2 W}{\partial x \partial z} - \frac{Z_0}{H_0} \frac{\partial^2 W}{\partial x^2} \right) \end{aligned} \right\} \quad (2),$$

where K is the magnetic susceptibility of the rocks composing the mass, and W the gravitational potential due to the mass. The three components of the total magnetic field at station P , therefore, are

$$\left. \begin{aligned} x &= x_0 + \Delta x = H_0 \cos \varphi + KH_0 \left(\cos \varphi \frac{\partial^2 W}{\partial x^2} + \frac{Z_0}{H_0} \frac{\partial^2 W}{\partial x \partial z} \right) \\ y &= y_0 = H_0 \sin \varphi \\ z &= z_0 + \Delta z = Z_0 + KH_0 \left(\cos \varphi \frac{\partial^2 W}{\partial x \partial z} - \frac{Z_0}{H_0} \frac{\partial^2 W}{\partial x^2} \right) \end{aligned} \right\} \quad (3).$$

Since transformations of these values from the (xyz) coordinate system to the (XYZ) coordinate system give the relations,

$$\left. \begin{aligned} X &= x \cos \varphi + y \sin \varphi \\ Y &= -x \sin \varphi + y \cos \varphi \\ Z &= z \end{aligned} \right\} \quad (4),$$

the north and west components of the total magnetic intensity at a station become

$$\left. \begin{aligned} X &= H_0 + H_0 K \left(\cos \varphi \frac{\partial^2 W}{\partial x^2} + \operatorname{tg} \theta \frac{\partial^2 W}{\partial x \partial z} \right) \cos \varphi \\ Y &= -KH_0 \left(\cos \varphi \frac{\partial^2 W}{\partial x^2} + \operatorname{tg} \theta \frac{\partial^2 W}{\partial x \partial z} \right) \sin \varphi \end{aligned} \right\} \quad (5),$$

where θ is the dip angle, i.e. $\operatorname{tg} \theta = Z_0/H_0$.

If δ denotes the anomaly in declination at a station, we get,

$$\begin{aligned} \operatorname{tg} \delta = Y/X &= \frac{-KH_0 \left(\cos \varphi \frac{\partial^2 W}{\partial x^2} + \operatorname{tg} \theta \frac{\partial^2 W}{\partial x \partial z} \right) \sin \varphi}{H_0 + H_0 K \left(\cos \varphi \frac{\partial^2 W}{\partial x^2} + \operatorname{tg} \theta \frac{\partial^2 W}{\partial x \partial z} \right) \cos \varphi} \\ &\doteq -K \left(\frac{\partial^2 W}{\partial x \partial z} \operatorname{tg} \theta \cdot \sin \varphi + \frac{1}{2} \frac{\partial^2 W}{\partial x^2} \sin 2\varphi \right) \end{aligned} \quad (6).$$

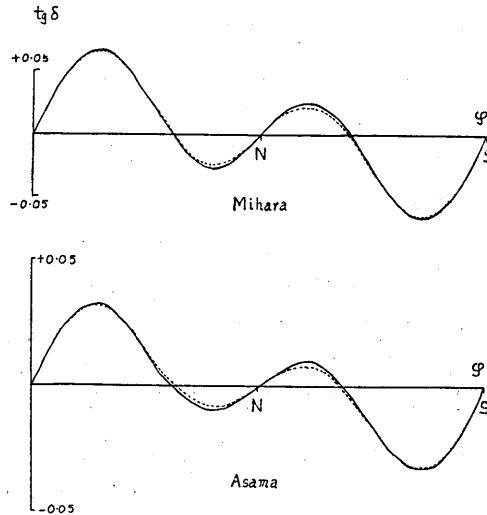


Fig. 9. The observed and theoretical distributions of anomaly in declination around the craters.
Full lines=observed. Dotted lines=theoretical (§ 5).

This relation agrees with the empirical formula (2) in § 3. Calculating with the aid of numerical integration, the values of $\frac{\partial^2 W}{\partial x^2}$ and $\frac{\partial^2 W}{\partial x \partial z}$ due to mass $A-P-B$, the form of which in the cases of Volcanoes Asama and Mihara, is the mean value of the topography around the crater, we obtain

$$\operatorname{tg} \delta = K(2.41 \operatorname{tg} \theta \sin \varphi + 3.26 \sin 2\varphi) \dots \dots \dots \text{Mihara,}$$

$$\operatorname{tg} \delta = K(2.54 \operatorname{tg} \theta \sin \varphi + 3.05 \sin 2\varphi) \dots \dots \dots \text{Asama.}$$

Comparing these relations with eq (2) in § 3 by means of successive approximations, we get

$$\left. \begin{aligned} \operatorname{tg} \delta &= 0.031 \sin \varphi + 0.043 \sin 2\varphi. \dots \dots \dots \text{Mihara,} \\ K &= 0.013. \\ \operatorname{tg} \delta &= 0.015 \sin \varphi + 0.017 \sin 2\varphi. \dots \dots \dots \text{Asama} \\ K &= 0.0055. \end{aligned} \right\} (7).$$

The relations between δ and φ shown in eq (7) are graphed with dotted lines in Fig. 9.

These results seem to be more reliable than the values obtained by calculating with the assumption of the presence of a negative mass having the form of a rotational-ellipsoid.

In conclusion, the writer wishes to express his sincere thanks to Prof. M. Ishimoto and Dr. R. Takahasi for their encouragement in the course of this study. His cordial thanks are also due to Dr. Ch. Tsuboi for his interest in the present work. The writer also wishes to express his hearty thanks to the Hattori-Hokokai, with the aid of whose grant the present study was made.

25. 火口周邊に於ける地磁氣偏角の異常分布

地震研究所 永 田 武

三原火山の火口周邊に於ける地磁氣偏角の異常量の分布を測定した結果、火口附近の特異な地形の影響によつて、一つの規則だつた分布をしてゐる事を知つた。此の測定結果は、福富理學士等による淺間火山の火口周邊に於ける同様の測定の結果と全く一致する。偏角異常量を δ とし、各測點の火口中心より、磁氣南を 0 にする方位角を φ とすれば δ の分布は次式で表はされる。

$$\operatorname{tg} \delta = A \sin \varphi + B \sin 2\varphi.$$

此の分布は火口附近の岩石に Induce された磁氣に因るを考へて説明出来る事を示し、且つ三原及び淺間兩火山の場合に於ける平均帯磁率を求めた。火口の地形に對しては、二つの理想的な場合、即ち一つは火口の形を迴轉橢圓體で近似した場合、他の一つは、各測點に於いては地形を二次元的分布として取扱ふ場合に就いて行つた。何れによつても、分布の形式は説明出来るし、又三原山の玄武岩の帯磁率は淺間山の安山岩の 2.5 倍乃至 3 倍位になる。然し後の場合の近似の方が實際により近い結果を得たと思はれる。

此の研究は、服部報公會より高橋助教授及び筆者に與へられた三原火山研究費の一部で行はれたものであつて、此處に同會に對して厚く感謝の意を表する。