26. *Crustal Deformations in Central Taiwan.* (*Part 1*)

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I. Introduction.

The destructive earthquake that shook the central part of Taiwan on April 21, 1935, was accompanied by marked crustal deformations. Associated with this earthquake, there appeared two faults, the Siko fault and the Tonsikyaku fault, their locations being shown in Fig. 1. After the earthquake, extensive relevellings and re-triangulations were made by the Military Land Survey over such regions as where the earth's crust was supposed to have been disturbed by the earthquake. These re-surveys showed that horizontal and vertical displacements of triangulation points and bench-marks had occurred since the last surveys were made. It is believed that they are mostly associated with the earthquake just mentioned.[1] The horizontal displacements of the triangulation points and the vertical displacements of the bench-marks thus obtained are shown respectively by arraws and numerals against the corresponding points in Figs. 1 and 2.

In this paper, the writers discuss the crustal deformations on the basis of the above-mentioned data.

II. Horizontal Deformations.

1. Method of calculating the horizontal strains.

In calculating the horizontal strains, the following method was used. If we let u and v be the x- and y-components of the horizontal displacement of a triangulation point, referred to certain rectangu-

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* Comm. by N. Miyabe.

1) The data of horizontal displacements of triangulation points and the vertical displacements of the bench-marks are given in Bull. Earthq. Res. Inst., Suppl. Vol. 3 (1936), 216~227.

lar coordinates x and y, then the components of horizontal strain are

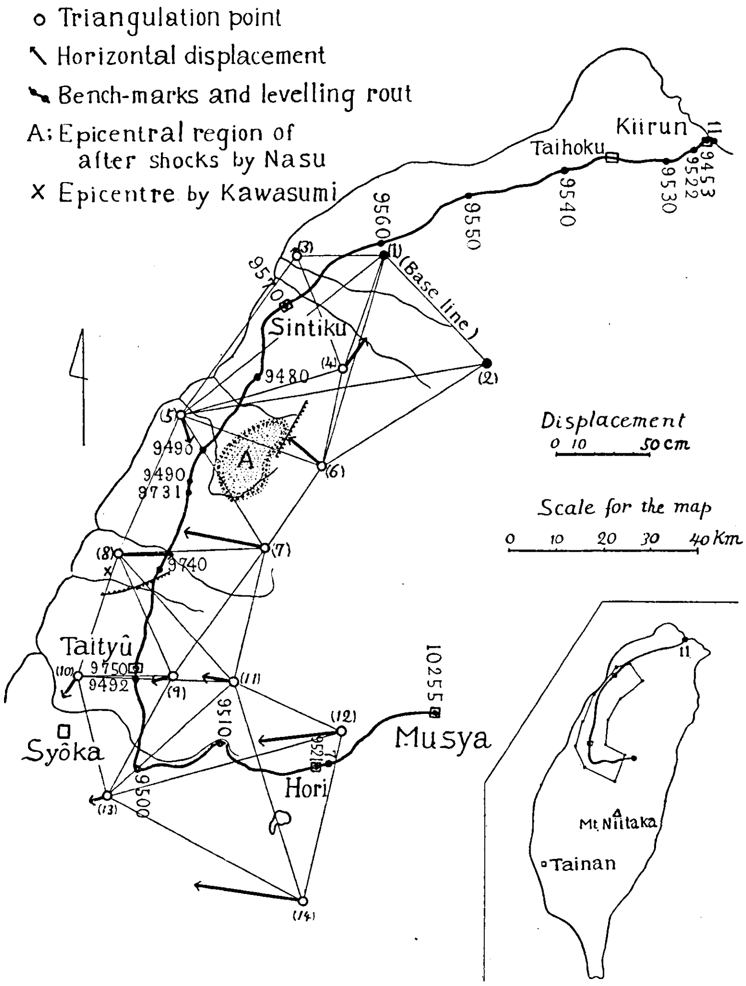

Fig. 1.

given by proper combinations of the values of $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, and $\frac{\partial v}{\partial y}$. In calculating the values of $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, etc., by the method proposed by T. Terada and N. Miyabe,[2] u and v are assumed to be linear functions of x and y, given in the form

2) T. TERADA and N. MIYABE, *Bull. Earthq. Res. Inst.*, **7** (1929), 223.

$$u = ax + by + c,$$
$$v = a'x + b'y + c',$$

within each triangle formed by adjoining triangulation points. Consequently we have

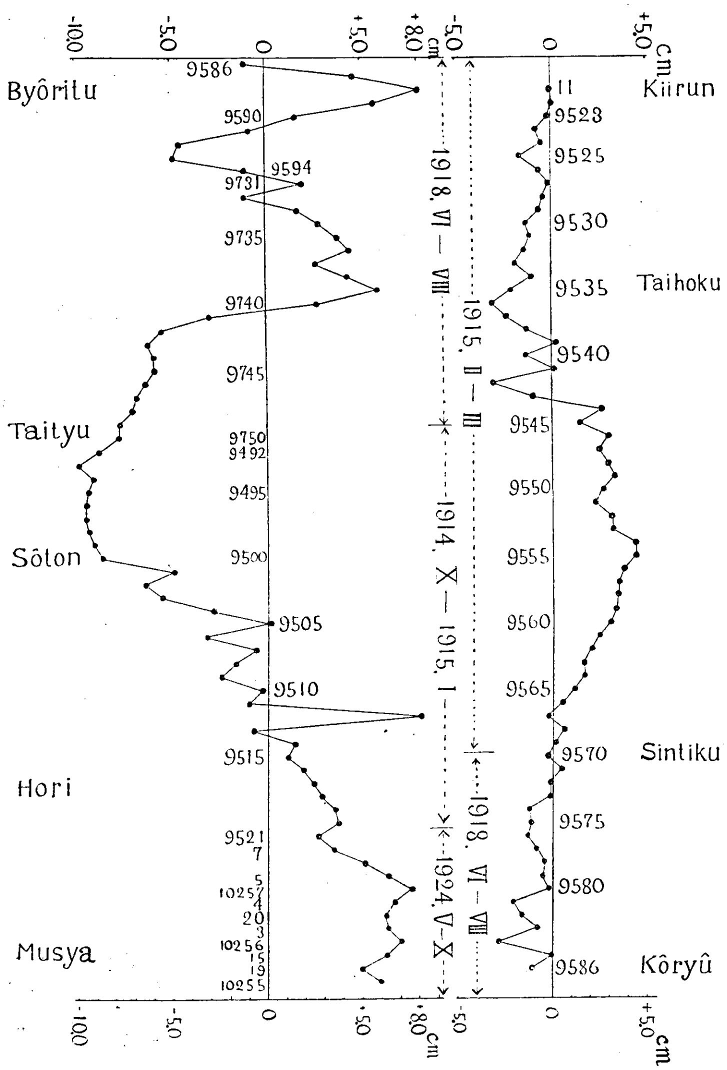

Fig. 2. Vertical displacements of bench-marks along the levelling routes.

$$\left.\begin{array}{l}\dfrac{\partial u}{\partial x}=a\\[2ex]\dfrac{\partial u}{\partial y}=b\end{array}\right\}\qquad\left.\begin{array}{l}\dfrac{\partial v}{\partial x}=a'\\[2ex]\dfrac{\partial v}{\partial y}=b'\end{array}\right\}$$

The four constants, a's and b's, are given by

$$\left.\begin{array}{l}a=\dfrac{|uy|}{|xy|}\\[2ex]b=\dfrac{|xu|}{|xy|}\end{array}\right\}\qquad\left.\begin{array}{l}a'=\dfrac{|vy|}{xy|}\\[2ex]b'=\dfrac{|xv|}{|xy|}\end{array}\right\}$$

in which $|uy|$ denotes the determinant $\begin{vmatrix}u_2-u_1 & y_2-y_1\\ u_3-u_1 & y_3-y_1\end{vmatrix}$ and so on, and x_1, y_1; x_2, y_2; x_3, y_3; and u_1, v_1; u_2, v_2; u_3, v_3; are respectively the coordinates and components of horizontal displacements of triangulation points I, II, and III, forming a triangle. In the present study, the values of the a's and b's are thus calculated by solving a simultaneous equation. For calculating the values of a's and b's, a graphical method was also proposed, by means of which the values of a's and b's obtained by the former method are checked. The graphical method is as follows:

Assuming u to be a linear function of x and y in the triangle having as vertices the triangulation points I, II, III (See Fig. 3a), we have straight lines of equal u denoted by A_0B_0, A_1B_1, A_2B_2, as shown in Fig. 3*a*, where the A's and B's are corresponding to the values of the components of the displacements obtained by interpolation (See Fig. 3b). Since the A's and B's are taken with equal intervals, the contour lines are parallel to one another. The length of a part of a line parallel to the x-axis, obtained between the successive contor lines, say α, and that of a line parallel to the y-axis, say β, are inversely proportional to a and b respectively. If the values of α and β are measured in km on the map and the contour lines are drawn with intervals of 1 cm, we have

$$\begin{array}{ll}\alpha=\dfrac{1}{a}\times10^{-5} & a=\dfrac{\partial u}{\partial x}=\dfrac{1}{\alpha}\times10^{-5}\\[2ex]\beta=\dfrac{1}{b}\times10^{-5} & b=\dfrac{\partial u}{\partial y}=\dfrac{1}{\beta}\times10^{-5}\end{array}\Bigg\}.$$

The values op a' and b' may also be obtained in a similar way by using the data of v's. The theoretical basis of this graphical method of calculation however is the same as that of the analytical method previously mentioned.

The values of a's and b's are thus calculated and, by combining

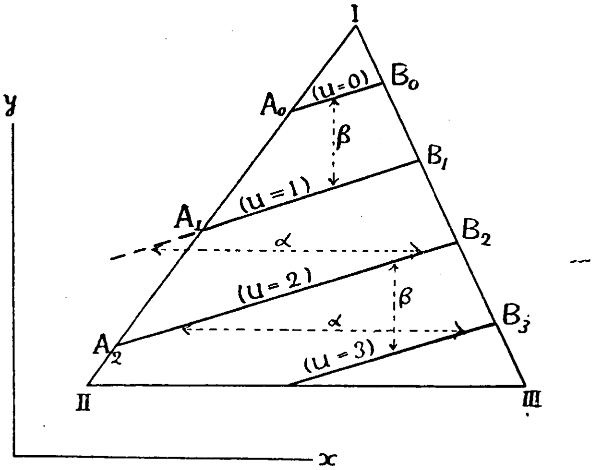

Fig. 3a.

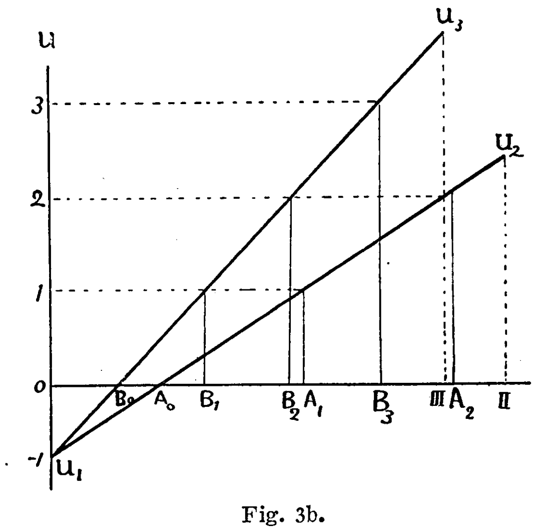

Fig. 3b.

these values in various ways, the following strain components are obtained:

rotation	$\omega = \frac{1}{2}(b - a')$
dilatation	$\Delta = a + b'$
shear	$S = \frac{1}{2}(a' + b)$
maximum shear	$S_m = \sqrt{(a' + b)^2 + (a - b')^2}$

principal axes of strains $\gamma_{\frac{1}{2}} = \Delta \pm S_m$

their directions $\tan 2\theta_{\frac{1}{2}} = \dfrac{a'+b}{a-b'}$

In this study, the strain components are calculated by using the the data of horizontal displacements of the primary triangulation points reproduced in Table I, the results being given in Table II. The distribution of dilatation, rotation, shear, and principal strains are shown in Figs. 4∼7, in which the numerical values of the strain components are set down at the approximate centres of the corresponding triangles.

Table I.

Triangulation point	u eastward displacement (m)	v northward displacement (m)	Triangulation point	u eastward displacement (m)	v northward displacement (m)
1	0·00	0·00	8	+0·26	+0·01
2	0·00	0·00	9	−0·13	−0·02
3	−0·01	+0·02	10	−0·07	−0·12
4	+0·13	+0·17	11	−0·17	+0·03
5	+0·04	−0·14	12	−0·45	−0·05
6	−0·26	+0·20	13	−0·10	−0·04
7	−0·43	+0·07	14	−0·57	+0·08

Table II.

		ω in 10^{-6} +, counter-clockwise	Δ in 10^{-6}	S_m in 10^{-6}	γ_1 in 10^{-6}	γ_2 in 10^{-6}	θ_1, referred to E., measured counter-clockwise
(1)	1—2—5	+ 1·4	+ 1·4	3·1	+ 4·5	− 1·7	73° 40′
(2)	1—2—6	− 4·6	+ 1·8	9·3	+11·1	− 7·5	1 45
(3)	1—3—4	+ 2·6	− 6·4	10·9	+ 4·5	−17·3	−20 50
(4)	1—3—5	+ 0·3	+ 7·0	6·6	+13·6	+ 0·4	16 30
(5)	1—4—5	+11·1	− 7·1	19·2	+12·1	−26·3	20 50
(6)	1—5—6	+ 0·9	−16·2	19·1	+ 2·9	−35·3	46 10
(7)	2—5—6	− 6·6	−23·1	29·1	+ 6·0	−52·2	30 10
(8)	3—4—5	+ 8·1	+ 2·1	9·8	+11·9	− 7·7	21 50
(9)	4—5—6	− 4·8	− 7·4	33·4	+26·0	−40·8	44 20
(10)	5—6—7	− 0·6	− 6·4	28·6	+22·2	−35·0	50 55
(11)	5—7—8	− 0·1	−33·1	20·6	−12·5	−53·7	72 35
(12)	7—8—9	− 1·5	−24·4	29·6	+ 5·2	−54·0	82 40
(13)	7—8—11	+ 3·3	−25·1	27·2	+ 2·1	−52·3	−87 50
(14)	8—9—10	− 4·4	+ 0·1	21·8	+21·9	−21·7	54 50
(15)	8—9—13	− 4·0	−17·8	21·8	+ 4·0	−39·6	78 5
(16)	8—10—11	− 4·5	+ 0·6	21·2	+21·8	−20·6	54 30
(17)	8—11—13	− 3·0	−10·0	15·7	+ 5·7	−25·7	69 50
(18)	9—10—13	+ 2·7	− 5·7	6·5	+ 0·8	−12·2	50 55
(19)	10—11—13	+ 2·4	− 5·9	5·9	+ 0·1	−11·8	51 20
(20)	11—12—13	− 4.8	− 5·8	16·7	+10·9	−22·5	79 30
(21)	11—12—14	− 5·7	−14·3	8·8	− 5·5	−23·1	88 0
(22)	11—13—14	− 1·7	− 9·8	13·6	+ 3·8	−23·4	66 40
(23)	12—13—14	− 2·3	−14·1	9·0	− 5·1	−23·1	65 40

(2) Geographical distributions of horizontal strains.

From the geographical distributions given in Figs. 4~7, we notice the following points.

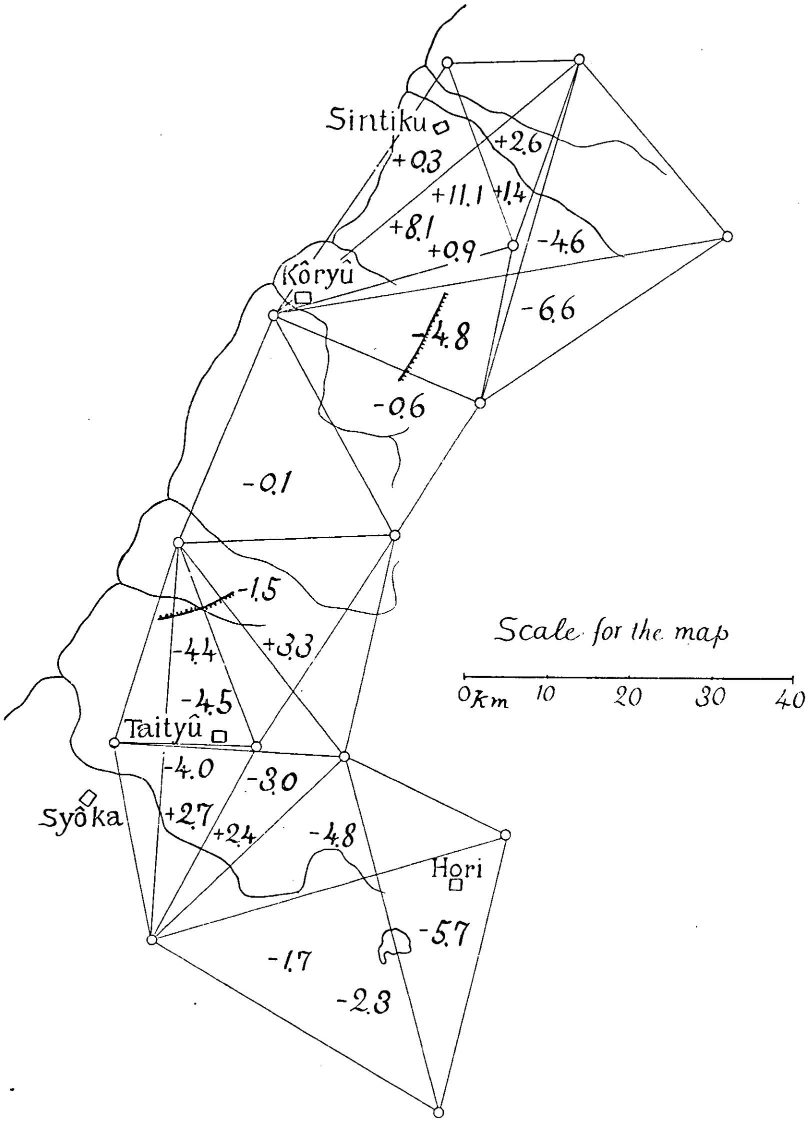

Fig. 4. Distribution of rotation.

a) From the distribution of rotational deformation shown in Fig. 4, it is noticed that, with minor exceptions, the rotational deformation is negative, i.e. the sense of rotation is clockwise, its magnitude being of the order of 10^{-6}.

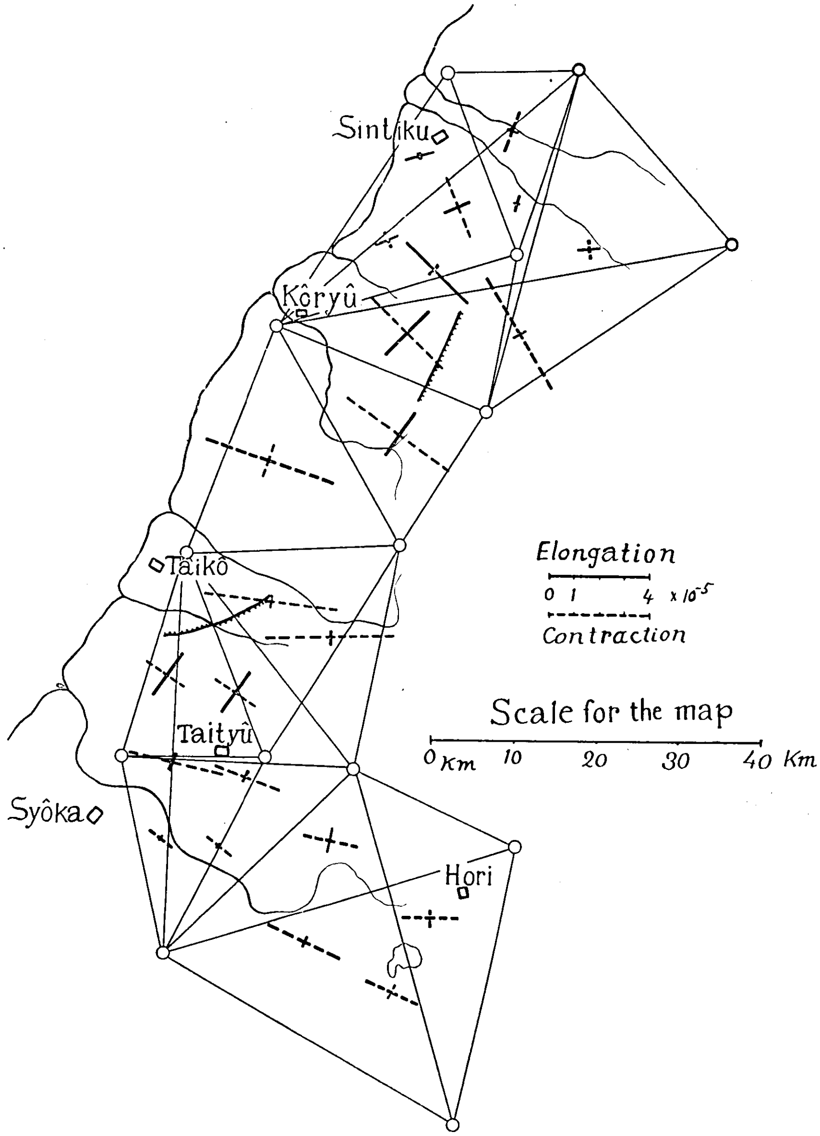

Fig. 5. Distribution of principal strains.

b) From Fig. 5, in which the distribution of the principal axes of strain ellipses are shown, it may be suggested that the region under consideration could be divided tentatively into three parts, namely, the middle part where crustal deformation was most conspicuous, and the northern and the southern parts where the earth's crust was not so disturbed as in the middle part.

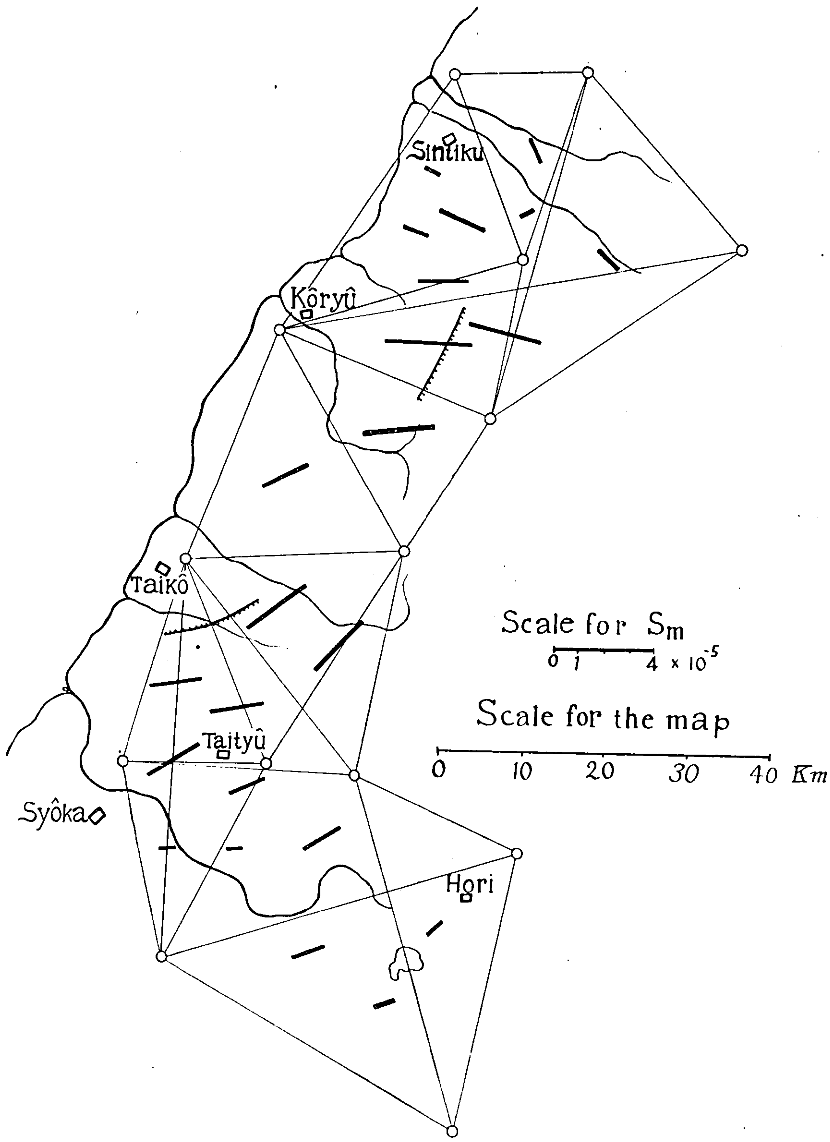

Fig. 6. Distribution of maximum shear.

In the middle part, the directions of the axes of contraction of the principal strains are approximately perpendicular to the general trend of the Siko fault, its magnitude being of the order of 10^{-5}. In the other parts, the magnitudes of elongation and contraction are decidedly small compared with those of the middle part.

c) It will be noticed from Fig. 7, in which the distribution of dilatation is shown, that throughout the region under consideration, the sign

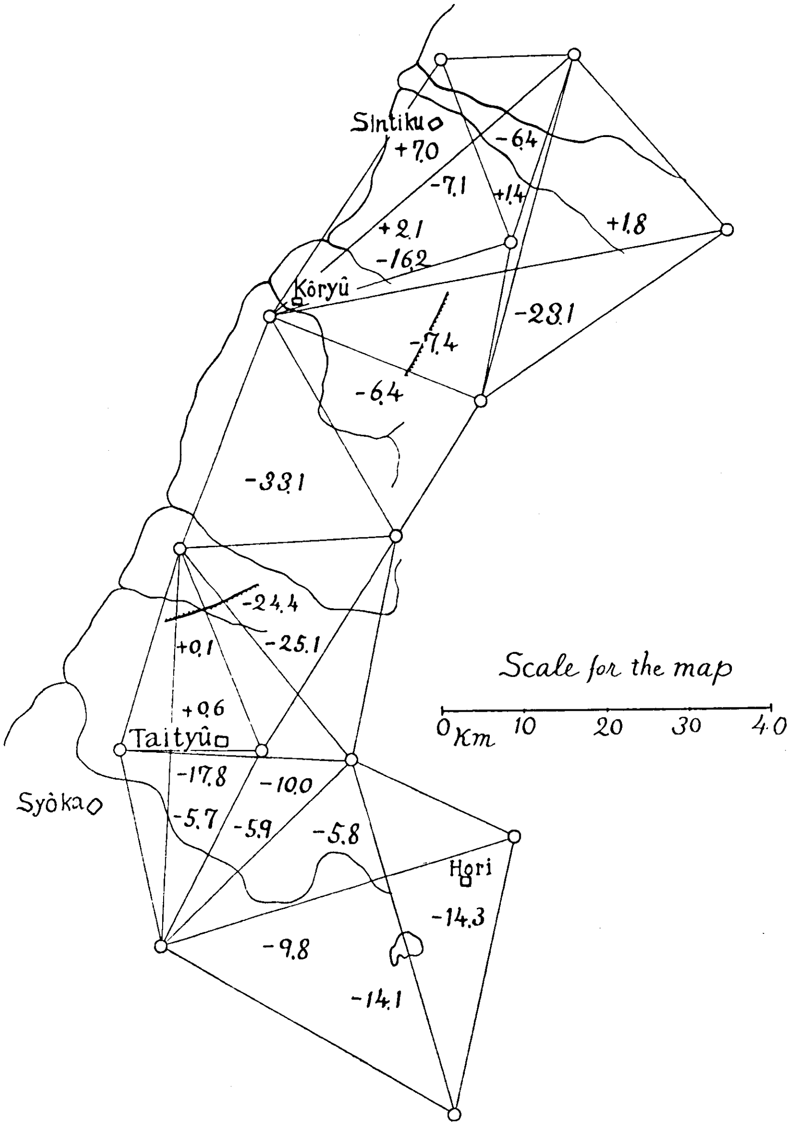

Fig. 7. Distribution of dilatation.

of dilatation is on the whole negative, its magnitude being of the order of 10^{-6} -10^{-5}.

d) In the above calculation it was assumed that in each of the triangular areas, the horizontal displacements are linear and are continuous functions of x and y. It may however be questioned whether this assumption is valid, especially in discussing crustal deformation in regions traversed by active faults.

In connection with this question, the following point may perhaps be worthy of note. The active Siko fault is a thrust fault, the amount of vertical dislocation of which is mostly from 1 to 2 m, although in a number of places it reaches 4 m. The inclination of the thrust plane to the vertical was found to be 15°.[3] If, for simplicity, we assume that the upper thrust bed overlies the lower bed without any gap between them, as illustrated in Fig. 8, the amount of horizontal dislocation, namely, the heave, BH, is given by

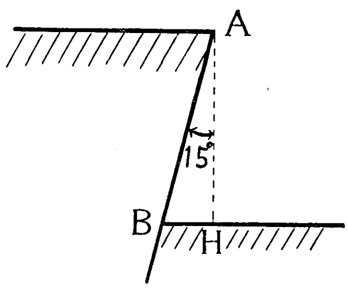

Fig. 8.

$$BH = AH \cdot \tan 15° = \begin{cases} 0{\cdot}4\ \text{m for } AH = 1{\cdot}5\ \text{m} \\ 0{\cdot}5\ \text{m for } AH = 2{\cdot}0\ \text{m} \end{cases}$$

It may therefore be concluded that, even had no horizontal contraction occurred on both sides of the fault line, the amount of relative horizontal displacement between the two triangulation points situated on opposite sides of the fault line becomes as much as 0·5 m, which is approximately equal to the amount of the relative displacement between the triangulation points 5 and 6, thd actual amount of the relative displacement being 0·4 m. as shown in Fig. 1. This fact suggests making a restudy of the subject based on another assumption that the crustal deformation is discontinuous in the zone that is traversed by the active fault. This will be done by using the data of the horizontal displacements of secondary and tertiary triangulation points that are distributed more densely in the same region.[4] A study of the crustal deformation in this direction is now in progress, the results of which will be published in due course.

e) In order to avoid any ambiguities in what was just mentioned, the components of horizontal displacements parallel to the fault lines are dealt with. Figs. 9～10 show the distributions of the components of horizontal displacements parallel to the trends of the Siko and the Tonsikyaku faults.

A glance at Fig. 9 will show that the amount of the component of displacement at any triangulation point on the west side of the Siko

3) Y. OTUKA, *Bull. Eartq. Res. Inst., Suppl.* Vol. 3 (1936). 22～74 (Japanese)

4) The data are published by the Military Land Survey (1937).

fault is equal to that of the triangulation points situated on the east side of the fault. In Fig. 10 it is pointed out that, in the eastern part of the Tonsikyaku fault, the components of horizontal displacements parallel to the fault line are independent of the distance from the fault, so that the general trend of the contour lines of displacement components is in a N-S direction.

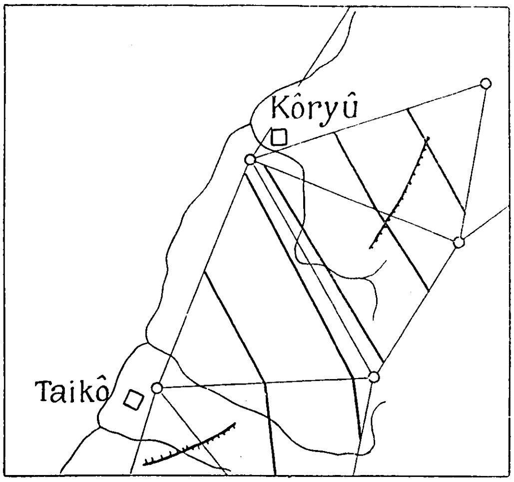

Fig. 9. Distribution of components of horizontal displacements parallel to the trend of Siko fault.

III. Vertical Displacements.

The changes in the heights of bench-marks on the line of levels from Kiirun to Musya are shown in Fig. 2. The vertical displacements are measured with reference to B. M. 11, situated at Kiirun, whose height is assumed unchanged during the period from 1915 to 1936. As will be seen from the diagram, there are two elevated bulges near Byōritu and Toyohara, where the intensity of the earthquake was also reported as severest, the maximum elevation in these regions amounting to about 80 cm.

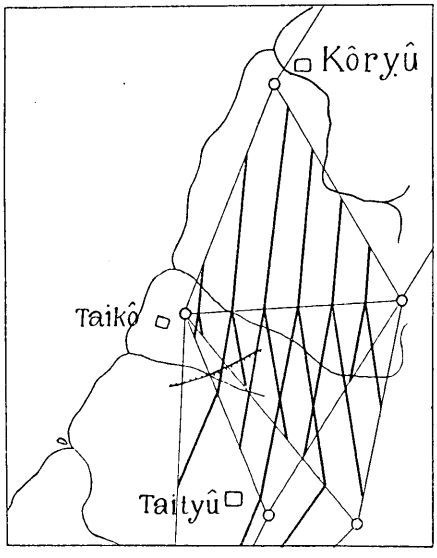

Fig. 10. Distribution of components of horizontal displacements parallel to the trend of the Tonsikyaku fault.

As reported by Nasu,[5] most of the epicentres of after shocks are located within certain limited areas, as shown Fig. 1. It will be noticed from this figure that the region where the epicentres of the after-shocks are concentrated is very close to the elevated region, a fact that agrees with the general tendency pointed out by Ishimoto[6] that

5) N. Nasu, *Bull. Earthq. Res. Inst., Suppl.* Vol. **3** (1936), 75. (in Japanese)
6) M. Ishimoto, *Zisin,* Vol. **9** (1937), 108. (in Japanese)

the epicentres of after–shocks distribute themselves mostly in regions where the earth's crust has been elevated.

It will also be seen from Fig. 2 that there are remarkable discontinuities in the vertical displacements at the northern and southern ends of the region of up-bulge near Byōritu, namely, between B.M.'s 9586 and 9987, and between B.M.'s 9589 and 9592. Discontinuities in the vertical displacements are also noticed near Naiho-syō, between B. M.'s 9740 and 9742. Although the amount of dislocation at these discontinuities are approximately the same, an active fault was found only between B.M.'s 9740 and 9742.

IV. Relation between Horizontal and Vertical Displacements.

We have so far treated the horizontal and vertical displacements of the earth's crust separately. We shall now deal with the relation between the two components, the horizontal and vertical earth movements. We first introduce the quantity "local disturbance of vertical displacements", or, shortly, vertical disturbances, defined as follows. Let w_n be the change in the height of the n-th bench-mark, and take the quantity

$$\delta_n = w_n - \overline{w}_n,$$

where

$$\overline{w}_n = \frac{1}{4}\left\{w_{n-1} + 2w_n + w_{n+1}\right\}.$$

The vertical disturbance in a certain region is then expressed by

$$\frac{1}{N}\left|\sum \delta_n\right|,$$

where N is the total number of bench-marks in the region in queastion and $\sum$ the summation over N bench-marks. The physical meaning of this quantity will be explained later.

Second, we take as the "horizontal disturbance" between any two triangulation points, the amount of elongation or contraction in length in the direction of the line joining the two triangulation points, namely, the quantity denoted by $|\delta l|/l$, where l is the length of the line joining the triangulation points and δl the variation in its direction.

In comparing the vertical disturbance with the horizontal disturbance just defined, the values of $\frac{1}{N}\sum|\delta_n|$ are calculated for parts of the line of levels approximately parallel to the line joining the two triangulation points. The values of $\frac{1}{N}\sum|\delta_n|$ are thus calculated for six parts of the line of levels under consideration, the results being given in Table III, together with the values of the horizontal disturbances. In Fig. 11, the vertical disturbance for various parts of the line of levels thus calculated are plotted against the corresponding horizontal disturbance.

Table III.

No.	Triangulation points	l in km	Δl in cm	$\frac{\Delta l}{l}$ in 10^{-6}	Bench-marks	N	$\sum\|\delta\|$ in mm	$\frac{1}{N}\sum\|\delta\|$ in mm
I	1~ 3	15	+0	+0·0	9560~9567	8	8·5	11
II	3~ 5	32	+11	+3·4	9568~9586	19	87·0	44
III	5~ 8	27	−20	−7·4	9587~9739	16	108·5	68
IV	8~10	23	+22	+9·6	9740~9493	13	33·4	26
V	9~13	20	+0	+0·0	9494~9500	7	4·2	6
VI	13~12	40	−35	−8·8	9501~4	25	192·3	77

As will be seen from this figure, the horizontal disturbances and the vertical disturbances are in approximate linear relation, and the former is large when the latter is large. There is a point (IV) in the diagram of Fig. 11 which deviates considerably from the approximate linear relation. This shows the relation between the two sorts of disturbances for the part of the line that crosses the Siko fault.

It is of course a question whether the linear relation mentioned above has really any physical significace. However, if the linear relation mentioned above really exists, the following may be one of the explanations for it.

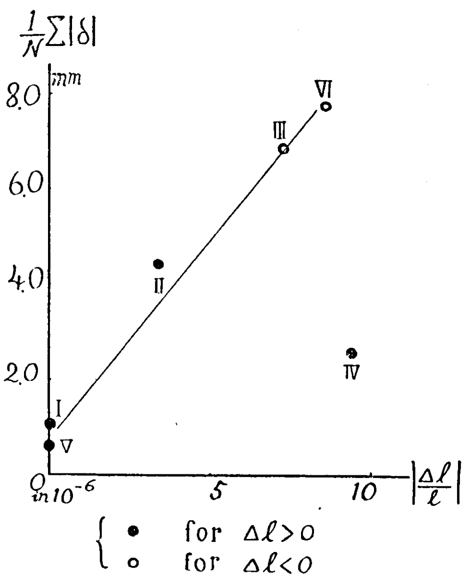

Fig. 11. Relation between horizontal and vertical disturbances.

Consider an extreme case in which the vertical displacements are expressed by a circular function of the number of the bench-marks, n, namely,

$$w_n = A\sin\left(\frac{2\pi}{\lambda}\cdot n+\alpha\right), \qquad (\lambda \geqq 2, \quad n=1,\ 2,\cdots\cdots N)$$

we have

$$\overline{w}_n = \frac{1}{2}\left(1+\cos\frac{2\pi}{\lambda}\right)w_n,$$

$$\delta_n = \frac{1}{2}\left(1-\cos\frac{2\pi}{\lambda}\right)w_n,$$

and

$$\frac{1}{N}\sum|\delta_n| = \frac{1}{2N}\left(1-\cos\frac{2\pi}{\lambda}\right)\cdot\sum|w_n|.$$

And since

$$\sum|w_n| = A\cdot\sum_{n=1}^{N}\left|\sin\left(\frac{2\pi}{\lambda}n+\alpha\right)\right|$$

$$\doteqdot A\,\frac{2N}{\lambda}\int_0^{\frac{\pi}{2}}\sin x\,dx$$

$$= A\cdot\frac{2N}{\lambda},$$

the vertical disturbance for this case is given by

$$\frac{1}{N}\sum|\delta_n| = \frac{1}{\lambda}\left(1-\cos\frac{2\pi}{\lambda}\right)\cdot A.$$

The effect of λ on the vertical disturbance is therefore expressed by

$$\frac{1}{\lambda}\left(1-\cos\frac{2\pi}{\lambda}\right).$$

Table IV.

λ	$\frac{1}{\lambda}\left(1-\cos\frac{2\pi}{\lambda}\right).$
2	1·00
3	0 50
4	0·25
5	0·14
6	0·08
7	0·06
8	0·04

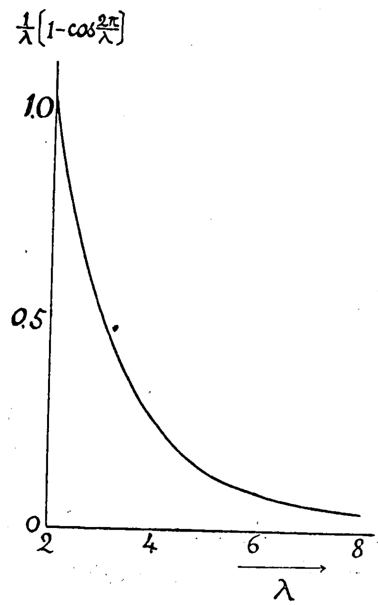

Fig. 12.

In Table IV, will be found the amount of $\frac{1}{\lambda}\left(1-\cos\frac{2\pi}{\lambda}\right)$ for various values of λ. The function

$\frac{1}{\lambda}\left(1-\cos\frac{2\pi}{\lambda}\right)$ is plotted against λ as shown in Fig. 12.

As shown in Fig. 12, the amount of $\frac{1}{\lambda}\left(1-\cos\frac{2V}{\lambda}\right)$, consequently $\frac{1}{N}\sum|\delta_n|$ in this extreme case, decreases rapidly with increasing λ.

Should the foregoing consideration for the extreme case be applied in explaining the actual deformation, it is possible to conclude from the linear relation between the horizontal and the vertical disturbances, as shown in Fig. 12, that, except for the region traversed by active faults, the greater the horizontal disturbances, the smaller the value of λ, as in the case of wave-length when the amplitude of fluctuation in vertical or horizontal disturbance is kept constant. Should, in this case, the wave-length of the assumed wavy deformation be constant, the amplitude would naturally become greater.

V. Summary.

The results of the present study are summarized as follows;

(1) From the results of re-triangulation of the primary triangulation points in the central part of Taiwan, where the destructive earthquake of 1935 occurred, the horizontal strain components are calculated for each triangle having three adjoining triangulation points at their vertices. The geographical distribution of rotation, dilatation, shear, maximum shear, as well as the principal strains, are shown.

(2) The mode of deformation is discussed on the basis of two different assumptions. The one that the deformation is continuous in the zone of active faults and the other that the deformation is discontinuous there.

(3) In that region where the epicentres of the after-shocks were concentrated, the earth's crust was elevated in a marked manner, as it had already done in other districts.

(3) A linear relation between the horizontal and the vertical disturbances was noticed, and its physical meaning considered.

In conclusion, the writers wish to express their sincere thanks to Professor Sakuhei Fujiwhara for his suggesting the subject for study, and to Doctor Naomi Miyabe for much information and kind guidance received throughout the course of this study.

26. 臺灣中部の地殼變形（第一報）

地震學教室 佐藤光之助
福永三郎

昭和 10 年 4 月 21 日の臺中新竹烈震後行はれた復舊測量に依つて地震前後數年間に於ける地殼變動が明らかにされた．本論文はそれを取扱つたものである．今回は一等三角點と一等水準點との變動に就いて論ずるに止めた．水平の歪の計算に當つては二つの別な方法を取つた．第一は各三角内で變移が連續であると假定して行つたもので，その結果は第4圖～第7圖に示してある．紙湖斷層附近では，それに垂直な縮みが大きく，屯子脚斷層附近ではそれに略平行な縮みが大きい．垂直變動については紙湖斷層の西側に二つの隆起部分が有り，而もそれが餘震の頻發地域に近接してゐる事が著しい．最後に垂直變動と水平變動とを併せ考へて見た．卽ち數學的に假りに定義した「水平の disturbance」と「垂直の disturbance」とを色々の地域に就いて比較した所，兩者の間に稍明瞭な關係を得たので之に對する物理的解釋を述べておいた．

終りに，本研究は筆者等後期學生の演習問題として藤原先生から戴き，專ら宮部先生の御懇切な御指導の下に行はれたものである事を記して兩先生に厚く御禮を申し上げます．