4. *Studies on the Seismic Vibration of a Gozyûnotô. III.*

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1. *Introduction.*

In the previous paper[1] we concluded that the aseismic properties of a gozyûnotô (pagoda) were mainly the result of Coulomb's damping in every columnar part between the roof truss parts. Since the structural members in the columnar part are connected by joggle joints, and not by simple slip joints, Coulomb's damping of vibration increases with increase in relative displacement, so that it is possible to put the damping force in the form

$$Fi(y_n-y_{n-1}), \tag{1}$$

but not

$$Fpi(y_n-y_{n-1}) \tag{2}$$

as in the case of viscous damping, nor

$$Fi \tag{3}$$

as in the case of constant damping for any amplitude including the motionless state. In the expressions (1) and (2), y_n, y_{n-1} represent the displacements of two adjacent roof truss parts, and p the vibrational frequency.

Although in the previous case, we did not specially refer to the effects of the inertia mass nor to the natural period of every part of the roof truss, three elements, namely, the elastic force, damping resistance, and the inertia force in every vibrational part, contribute, as a matter of fact, to the damping phenomena of the structure in seismic vibration, the reason of which is apparent from our investigation of the dynamic damper of structural vibrations. Since the case treated in the previous paper was one in which the mass, stiffness, and damping resistance are respectively the same for any part of the roof-truss, the damping condition of the gozyûnotô was naturally satisfied. Such

1) K. SEZAWA and K. KANAI, "Further Studies on the Seismic Vibrations of a Gozyûnotô", *Bull. Earthq. Res. Inst.*, **15** (1937), 33~40.

resonance curves as those in Fig. 3 in the last paper[2] are scarcely obtainable in the usual structure, even were an artificial damper fitted.

2. *Analysis of the quality of vibration damping in a gozyûnotô.*

It is extremely difficult to analyse the quality of damping in a gozyûnotô, assuming that every part of the roof truss is a special damping mass. Although it is possible for the solutions (7)$\sim$(12) in the previous paper[3] to be analytically correct, since those solutions are too complex to be of help in obtaining a comprehensive insight into the mechanism of the damping problem, we shall now suppose that only a mere part of the roof truss acts as a damper to the vibrations of all the remaining parts, making the problem a very idealized one.

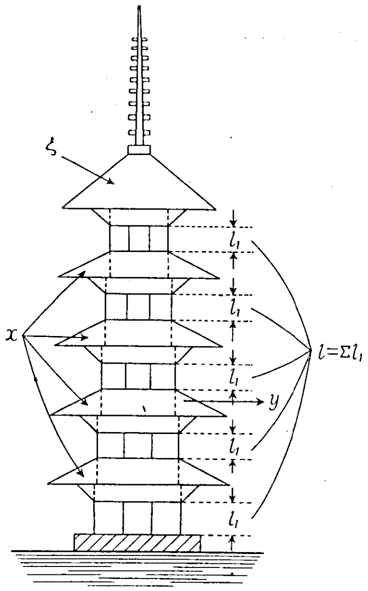

Fig. 1.

Let us assume that the gozyûnotô vibrates as a whole with a shearing type of vibration, the equation of motion of the roof truss part being therefore

$$\frac{m}{l_1}\frac{\partial^2 y}{\partial t^2}=Ga\frac{\partial^2 y}{\partial x^2}, \tag{4}$$

where y is the lateral deflection at the roof truss part x, a the area of columns of length l_1 at a section, m the mass to be concentrated at each part of the roof truss, and G the effective shear modulus. Since each elementary part of the structure never moves with a vibration of shearing type, but rather in flexural vibration type, we have, from our previous result,[4]

$$G=12{\cdot}4\frac{Ek^2}{l_1^2}. \tag{5}$$

Writing $m/al_1=\rho$, (4) reduces to

$$\rho\frac{\partial^2 y}{\partial t^2}=G\frac{\partial^2 y}{\partial x^2}, \tag{6}$$

the solution of which is

2), 3) *loc. cit.* 1).
4) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **12** (1934), 819.

$$y=e^{ipt}\{Ae^{ifx}+Be^{-ifx}\}, \tag{7}$$

where

$$f=\sqrt{\rho p^2/G}. \tag{8}$$

Assuming that the restitutive force and Coulomb's damping of a special roof truss part (uppermost roof truss part including Kurin, that is, a central nine-ringed pinnacle, in the present case) for relative displacement ξ are

$$c\xi(=c\xi_0 e^{ipt}), \qquad \frac{\mu'}{p}\frac{\partial\xi}{\partial t}=(Fi\xi) \tag{9), (10}$$

respectively, we find the boundary conditions to be

$$x=0; \qquad y=be^{ipt}, \tag{11}$$

$$x=l; \qquad Ga\frac{\partial y}{\partial x}-c\xi-\frac{\mu'}{p}\frac{\partial\xi}{\partial t}=0, \tag{12}$$

$$M\left(\frac{\partial^2\xi}{\partial t^2}+\frac{\partial^2 y}{\partial t^2}\right)+c\xi+\frac{\mu'}{p}\frac{\partial\xi}{\partial t}=0, \tag{13}$$

corresponding to the seismic vibration of the ground,

$$be^{ipt}, \tag{11'}$$

where l is the effective height of the gozyûnotô.

Substituting (7) in (11)$\sim$(13), we get

$$A\Phi=be^{-ifl}\left[\left\{\frac{Ga}{lc}(fl)^2-\frac{\mu'}{c}fl-\frac{\rho al}{M}\right\}+i\left\{fl-\frac{\rho al}{M}\frac{\mu'}{c}\right\}\right], \tag{14}$$

$$B\Phi=be^{ifl}\left[\left\{\frac{Ga}{lc}(fl)^2+\frac{\mu'}{c}fl-\frac{\rho al}{M}\right\}-i\left\{fl+\frac{\rho al}{M}\frac{\mu'}{c}\right\}\right], \tag{15}$$

$$\xi_0\Phi=-2b\frac{Ga}{lc}(fl)^2, \tag{16}$$

$$\Phi=2\left[\left\{\frac{\rho al}{M}\left(\frac{M}{\rho al}\frac{Ga}{lc}(fl)^2-1\right)\cos fl+fl\sin fl\right\}\right.$$
$$\left.+i\frac{\mu'}{c}\left\{fl\sin fl-\frac{\rho al}{M}\cos fl\right\}\right], \tag{17}$$

so that the final solutions are such that

$$y_{x=l}=b\sqrt{\frac{R^2+S^2}{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}-\tan^{-1}\frac{S}{R}\right), \tag{18}$$

$$\xi=-b\frac{\frac{Ga}{lc}(fl)^2}{\sqrt{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}\right), \tag{19}$$

where

$$R=\frac{Ga}{lc}(fl)^2-\frac{\rho al}{M}, \qquad S=\frac{\rho al}{M}\frac{\mu'}{c}, \tag{20), (21}$$

$$P=\left\{\frac{Ga}{lc}(fl)^2-\frac{\rho al}{M}\right\}\cos fl+fl\sin fl, \tag{22}$$

$$Q=\frac{\mu'}{c}\left\{fl\sin fl-\frac{\rho al}{M}\cos fl\right\}. \tag{23}$$

It will be seen that the vibration amplitudes of a gozyûnotô are functions of Ga/lc, $M/\rho al$, μ'/c, besides the frequency condition fl. Since

$$\frac{M}{\rho al}=\frac{\text{Damper mass}}{\text{Mass of main part}}, \tag{24}$$

$$\frac{\pi}{2}\sqrt{\frac{Ga}{lc}}\sqrt{\frac{M}{\rho al}}=\frac{\pi}{2}\sqrt{\frac{M}{c}}\sqrt{\frac{G}{\rho l^2}}=\frac{T_2}{T_1}$$

$$=\frac{\text{Natural period of damper part}}{\text{Natural period of main part}}, \tag{25}$$

it is possible to conclude that the ratio of the natural period of the so-called damper part (the uppermost roof truss part) to that of the main part and also the ratio of masses of both parts in question greatly participate in the vibration of the gozyûnotô.

The free vibration period of the main part of the gozyûnotô is obtained by putting the solution (7) in

$$x=0; \qquad y=0, \tag{26}$$

$$x=l; \qquad \frac{\partial y}{\partial x}=0, \tag{27}$$

from which the period of the free vibration becomes

$$T_1=\frac{2\pi}{p}=4\sqrt{\frac{\rho l^2}{G}}. \tag{28}$$

The free vibration of the uppermost roof truss part that is to be a damper is defined by

$$M\frac{d^2\xi}{dt^2}+\frac{\mu'}{p}\frac{d\xi}{dt}+c\xi=0, \tag{29}$$

from which the period of its natural vibration and the damping coefficient assume the forms

$$\frac{2\pi}{p}=2\pi\Big/\left\{\left(\frac{c}{M}\right)^2+\left(\frac{\mu'}{M}\right)^2\right\}^{\frac{1}{4}}\cos\left(\frac{1}{2}\tan^{-1}\frac{\mu'}{c}\right), \tag{30}$$

$$f=\left\{\left(\frac{c}{M}\right)^2+\left(\frac{\mu'}{M}\right)^2\right\}^{\frac{1}{4}}\sin\left(\frac{1}{2}\tan^{-1}\frac{\mu'}{c}\right). \tag{31}$$

When, specially, the period of the main structure is equal to that of the damper, we get

$$\frac{Ga}{lc}=\left(\frac{2}{\pi}\right)^2\frac{\rho al}{M}, \tag{32}$$

which is one of the efficient damping condition of the structure.

By using the above equations, it is possible to understand, mathematically, why a gozyûnotô resists seismic vibration.

3. *Some numerical examples, and the interpretation of the problem.*

It is uncertain whether or not the roof parts in an actual gozyûnotô are in the condition of the most favourable dynamic damper. Even an accurate observation of the seismic vibrations of a gozyûnotô would not give us the information.

From Omori's report,[5] the natural vibration period of a gozyûnotô is more than 1 sec, and the amplitude of the free vibration diminishes to about one-half its initial value after three or four cycles of vibrations. Although the damping coefficient of the free vibration of the whole gozyûnotô can be obtained by solving (17), since its treatment is extremely difficult, we performed a number of tentative calculation from Omori's data with repect to the free and forced vibration experiments of gozyûnotôs, after which we found that the following two examples represent the most probable condition of the actual towers, or at least a condition that is more favourable than what is actual.

(i) $$\frac{\rho al}{M}=5,\qquad \frac{T_2}{T_1}=1,\qquad \frac{\mu_1}{c}=0{\cdot}45; \tag{31}$$

5) F. OMORI, *Bull. Earthq. Inst. Comm.*, **9** (1918~21), 110~150.

(ii) $$\frac{\rho al}{M}=5, \qquad \frac{T_2}{T_1}=1{\cdot}370, \qquad \frac{\mu'}{c}=0{\cdot}45. \tag{32}$$

The resonance curves for the two cases are shown in Figs. 2, 3. The full and broken lines in these figures represent the displacement ratios of the uppermost point of the main part and the damper part to the ground vibration, namely $y_{x=l}/b$, ξ/b respectively, whereas the dotted lines indicate the phase difference between the displacement y at $x=l$ and the displacement ξ.

The solution (18) shows that the displacement y is independent of μ', provided

$$\left|\frac{\frac{Ga}{lc}(fl)^2-\frac{\rho al}{M}}{\left\{\frac{Ga}{lc}(fl)^2-\frac{\rho al}{M}\right\}\cos fl+fl\sin fl}\right|=\left|\frac{\frac{\rho al}{M}}{fl\sin fl-\frac{\rho al}{M}\cos fl}\right|. \tag{33}$$

This shows that the ordinates corresponding to the abscissae, for which (33) holds, are constant for any damping resistance of the damper part, unless the value Ga/lc, showing the frequency ratio for both parts, and $\rho al/M$, showing the mass ratio for the same parts, were not changed. There are an infinite number of such points for a wide

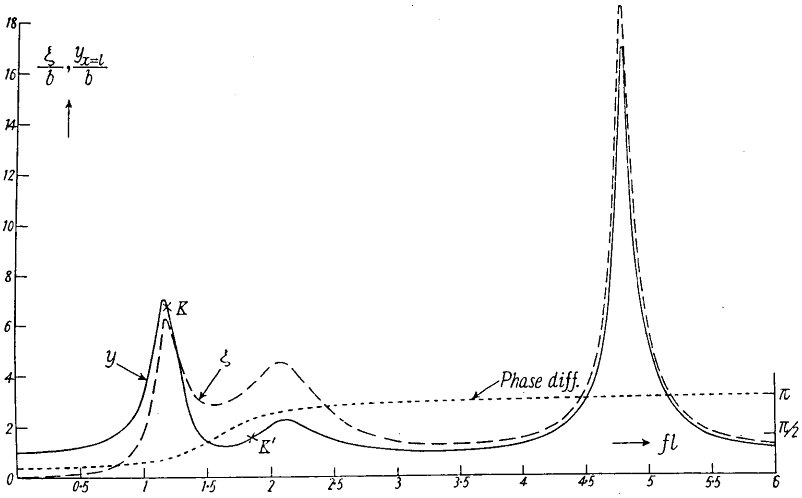

Fig. 2. Case (i), $\rho al/M=5$, $\mu'/c=0{\cdot}45$, $T_2/T_1=1$; Full, broken, and dotted lines represent $y_{x=l}$, ξ, and phase difference respectively.

range of the vibrational frequencies. The same points for low vibrational frequencies are shown by the symbols K, K' in the figures ($fl=1{\cdot}191$, $1{\cdot}8523$, ..., in Fig. 2; $fl=1{\cdot}0241$, $1{\cdot}5859$, ..., in Fig. 3).

When the damping force μ' is infinitely large, the damper part is in a condition that is rigidly fixed to the main part, so that resonance curve that passes through K, K' has a peak, of infinite height, at $fl=$

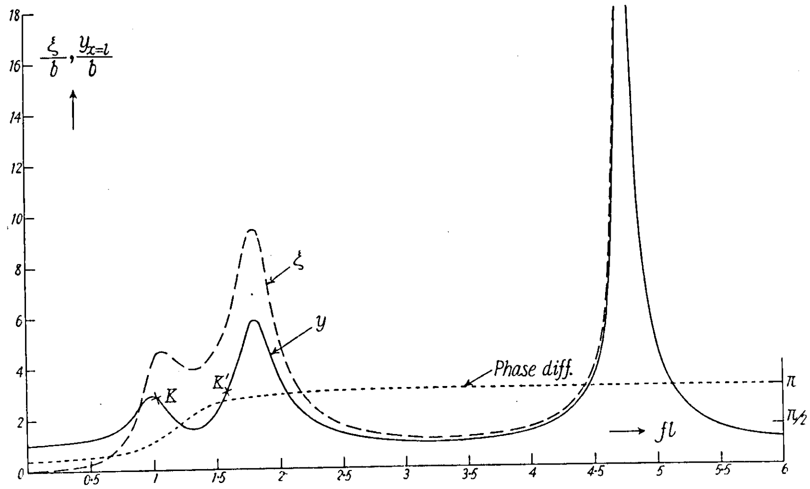

Fig. 3. Case (ii), $\rho al/M=5$, $\mu'/c=0{\cdot}45$ $T_2/T_1=1{\cdot}370$; Full, broken and dotted lines represent $y_{x=l}$, ξ, and phase difference respectively.

$\pi/2$; whereas when the damping force is zero, the damper part is connected to the main part only elastically, so that the resonance curve that passes through K, K' then has two peaks, of infinite height, nearly at the same abscissae corresponding to the peaks (left-hand two peaks) in the figures.

Whatever case may be, it is impossible to get resonance curves, the peaks of which are lower than the ordinates at K, K'.

Higher resonance conditions occur at the frequencies $fl=(n+1/2)\pi$. In Figs. 2, 3 the second resonance corresponding to $fl=3\pi/2$ is given. Since the action of the damper part on the main structure is effective only near the first resonance condition $fl=\pi/2$, the peak at $fl=3\pi/2$ is extremely sharp. It has often been remarked that mathematical sharp peaks at higher resonance are not of practical importance. In the actual condition of a gozyûnotô, since there are five damper parts, the peaks of higher resonance are never very marked, as will be seen from Fig. 3 in the preceding paper.[6]

6) *loc. cit.* 1). It should be borne in mind that the condition of the problem in the present paper differs somewhat from that in the previous one. While in the previous case there was damping force in every columnar part, in the present case, on the other hand, no such force exists excepting in the part between the fourth and fifth roof truss parts, the result being such that the damping condition of the present case is effective for a moderate value of μ'/c.

Comparing the full lines with the chain or dotted lines in the resonance curves in Figs. 2, 3, it will be seen that the damping effect of the damper parts in a gozyûnotô is enormous.

It should be borne in mind that, as we have stated in the beginning of the last section, the damping condition here given is merely an idealized one. The result of the damping effect of the actual gozyunotô would be of the type shown in Fig. 3 in our previous paper.[7] Since in the previous case $F/Mg \ (=\mu'/Mg) = 0{\cdot}2487\ \mathrm{cm}^{-1}$ and $M/c = 0{\cdot}002052$ sec^2, it follows that $\mu'/c\{=(\mu'/Mg)(M/c)g\} = 0{\cdot}2487.0{\cdot}002052.980 \approx 0{\cdot}50$, which is approximately the same as the value of $\mu'/c(=0{\cdot}450)$ in the present case. Although the ordinate at the peak in the resonance curve in the previous case was less than 2·5, the same ordinate in the present case is larger than 5, which shows that the damping condition in the actual gozyûnotô should be much more effective than that in the present idealized one.

In conclusion, we wish to express our thanks to the Council of the Japan Society of the Promotion of Scientific Research, with whose aid progress in the present investigation was considerably furthered.

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4. 五重塔の耐震性（第3報）

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五重塔の耐震性は各軸部に働く Coulomb 摩擦が大いに與ることをこの前述べて置いたが，その場合に僅かの摩擦力によつても可なりの効果があつたのである．之はよく考へて見るとその計算に於て順々の屋根及斗組の質量を一定にし且つ軸部の彈性及び摩擦力が夫々一定にしてあつたのであるから，しばしば述べたところの構造物の力學的制振器が幾つも組合つてをることを示すのである．殊に制振器はその質量が大きい程，且つその自己振動周期が主要構造部の周期に近い程効力が多いのであるから，この前の研究は單に摩擦力のみでなく，制振器の研究をなしてをつたことになるのである．

計算が相當複雜であつたから，力學的計算結果の何れの部分が制振器の作用をなしたのか判斷に苦しむのである．それで玆に一つの層の屋根例へば第5層（九輪を含み）を制振器と考へ，他の層は全部本構造と見做して計算を試みた．即ち相當抽象的であつてこの前の計算よりも一層實際から離れてをるのである．

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7) *loc. cit.* 1).

その結果を見ると摩擦力がこの前の場合と殆ど同じであるのにも拘らず，効果は前のものよりも大分少な氣味である．制振器の質量は構造全體の質量の 1/5 位しかなく，從て効果が前のものよりも劣るのは當然な話である．それにしても共振曲線の峯は制振作用のないときから見て非常に低くなつてをる．尙，構造物の第 2 次以上の共振に對しては殆ど制振作用を與へない．しかし高次の共振には種々の制振作用が働くから之は始めから問題とするに足らぬものである．このやうな高次のものでも，この前のやうに各屋根及斗組部分が制振器の作用を持つ場合にはやはり共振の峯がなくなるものである．

摩擦力の大きさは單に假定したのでなく，大森博士が試みた五重塔の振動實驗の自由振動及び強制振動の減衰率から推定したのである．復原力も振動周期から定めた．實際の五重塔に於ては，この前の報告や只今の研究結果程制振作用があるかどうかは各部の振動周期が同じかどうかわからぬ（之は實驗でも出し難い）だけそれだけ多少疑しいものである．しかし筆者等をして新に五重塔を設計せしめれば昔のものよりも一層耐震的の五重塔を作り得るといふ考を持つ次第である．

Corrigenda to the authors's paper, "Vibrations of a Single-storied Framed Structure", published in the Bull. X (1932), Part 3, p. 767.

From the nature of longitudinal thrust in floor beams, the conditions (33), (44), (52) in the paper should be replaced by

$$\left.\begin{aligned} -E_1 a_1 k_1^2\left(\frac{d^3u_1}{dx_1^3}+\frac{d_3u_1'}{dx_1'^3}+\frac{d^3u_1''}{dx_1''^3}\right)&=2\rho_2 a_2 l_2 p^2 u_1, \\ u_1=u_1'&=u_1''. \end{aligned}\right\}$$

Since, nevertheless, the thrust in question is not large in such a particular problem as in the same paper, the numerical result obtained had no much error.
