5. *Gravity Anomalies and Deviations of the Vertical in Izu-Osima.*

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During the period between Jan. 22 and March 2, 1937, the Military Land Survey conducted astronomical observations at Habu and Okada-mura, in Izu-Osima, as the result of which the meridian and prime vertical components of deviations of the verticals $\xi_{(G-A)}$ and $\eta_{(G-A)}$ at these places were found, as follows;

	$\xi_{(G-A)}$	$\eta_{(G-A)}$		$\xi_{(G-A)}$	$\eta_{(G-A)}$
Habu	$-11{\cdot}4''$	$-3{\cdot}9''$	Okada	$-28{\cdot}6''$	$+0{\cdot}8''$

The geodetic positions of these points were determined by tertiary triangulation with the standard datum of Tokyo as reference, the deviation of the vertical of which was assumed to be zero.

The astronomical longitudes and latitudes of the above mentioned places were determined by "the method of equal altitudes of different stars." The instrument used was Carl Bamberg's 21 cm Universal theodolite. The sensibility of the Talcott level is $1{\cdot}8''$ per 2 mm run, and that of the striding level $4{\cdot}5''$. For determining the time, the time signal of the Hunabasi Wireless Station was recorded by chronograph, and the pointing of the stars done by the key method.

The probable errors of astronomical observations are

at Habu	for longitude	$\pm 0{\cdot}240''$	at Okada	for longitude	$\pm 0{\cdot}195''$
	for latitude	$\pm 0{\cdot}117$		for latitude	$\pm 0{\cdot}174$

Position errors of observation points due to the triangulation may be ruled out, since it is very small (a few decimetres at most) compared with the astronomical results just mentioned. The deviations of the verticals obtained at Izu-Osima may be regarded as accurate, so far as precision of the astronomical work is concerned. The magnitude of deviation of the vertical measured in Izu-Osima is larger than those of other points in the inland. Large deviations of the verticals have also been noticed in Hawaii by the U. S. Coast and Geodetic Survey, their measurements being as follows.[1]

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* Comm. by N. MIYABE.

1) Coast and Geodetic Survey, U. S. A., *Spec. Publ. No.* 156 (1930).

Latitude stations

Kauai Island		Pakaoao	$-10{\cdot}87''$
Waimea	$-35{\cdot}58''$	Kaupo	$-40{\cdot}23$
Koloa	$-44{\cdot}28$	Hana	$+11{\cdot}80$
Hanalei	$+24{\cdot}64$	Hawaii Island	
Oahu Island		Kohala	$+31{\cdot}12''$
Kahuku	$+24{\cdot}52''$	Kawaihae	$+\ 0{\cdot}20$
Honolulu	$-25{\cdot}55$	Mauna Kea	$-\ 0{\cdot}13$
Waikiki	$-27{\cdot}84$	Kalaieha	$-12{\cdot}17$
Maui Island		Hilo	$-\ 0{\cdot}35$
Lahaina	$-\ 9{\cdot}81''$	Kailua	$-23{\cdot}55$
Haiku	$+18{\cdot}79$	Ka Lae	$-67{\cdot}64$

Longitude stations

Oahu Island			
Honolulu	$+\ 0{\cdot}50''$	Niu	$-15{\cdot}06''$

On comparing the above mentioned values with those of Izu-Osima, the values of deviations of the verticals which we measured in Izu-Osima may be regarded as not being exceptionally large. A very remarkable point in which they differ from the Hawaiian values is that the sign of the ξ components of Izu-Osima is the same for both points, notwithstanding that Habu and Okada-mura are situated respectively at the southern and northern ends of the island, the great mass of Mihara Volcano lying between these stations.

Table I.

Station	Long. (E)	Lat. (N)	Height or Depth (in m)	g_0 (in gal)	$g_0-\gamma_0$ (in gal)
Hatizyōzima	139°50′	33°66′	64	979·765	+0·194
Kamakura	139 34	35 19	13	·783	+0·026
Osima	139 22	34 45	24	·862	+0·153
Odawara	139 9	35 15	65	·796	+0·044
Rendaizi	138 57	34 42	14	·812	+0·108
Numazu	138 52	35 5	7	·789	+0·051
Sizuoka	138 23	34 58	23	·760	+0·032
A	140 00·0	34 10·0	−1200	·800	+0·140
(1)	139 30·4	35 07·8	− 700	·752	+0·011
(3)	139 30	34 57	−1250	·758?	+0·032?
(2)	139 23·2	34 56·1	−1470	·749	+0·024
Z	139 14·7	33 38·2	− 790	·691	+0·076
B	138 45·8	33 00·6	−2500	·660	+0·097
P	138 30·2	34 15·2	−1570	·668	+0·001
S	137 56·8	33 36·0	−3700	·615	+0·003
T	137 27·7	33 00·8	−4100	·629	+0·066

In connection with the deviation of the vertical, gravity was measured at Izu-Osima and on the sea in the neighbourhood of Izu-Osima and other islands.

The values of the gravity anomaly $g_0 - \gamma_0$ in the region under consideration are reproduced in Table I from Matuyama's report[2] and Tsuboi's book.[3]

In Table I, the values of the actual gravity g_0 are those reduced from observed values by applying the "free air" correction, while those of normal gravity γ_0 are those computed by Helmert's formula of 1901. In order to show the distribution of gravity anomalies, isanomaly lines

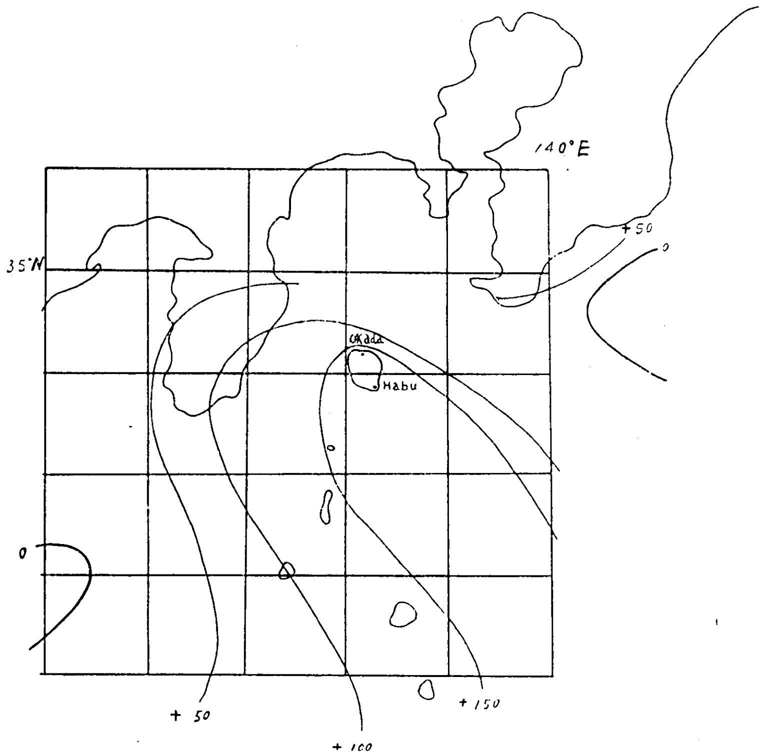

Fig. 1. Distribution of gravity anomalies in the neighbourfood of Izu-Osima.

are drawn from the data given in Table I, as shown in Fig. 1, the numerals affixed to the isanomaly lines designating the value of the anomaly in milligals.

In this paper the writer deals with the results of his studies with

2) M. MATUYAMA, "Gravity Survey by the Japanese Geodetic Commission since 1932."

3) C. TSUBOI, "Zyūryoku" (Gravity), Tokyo, 1936.

the object of ascertaining, under a certain likely assumption, the distribution of excess mass in the earth's crust (assuming that the earth's crust is in general homogeneous and of uniform density) that might admit of being estimated from the distribution of gravity anomaly on the surface of the earth. In doing so, the deviations of the verticals estimated from the mass distribution thus determined are compared with the actual values recently measured.

THE HORIZONTAL AND VERTICAL COMPONENTS OF THE ATTRACTION DUE TO THE EXCESS MASS OF THE EARTH'S CRUST.

We shall suppose the acceleration due to the attraction from the excess mass to be α. The deviation of the vertical $\delta\varphi$ and the gravity anomaly δg are then expressed respectively by

$$\delta\varphi=\frac{\alpha_x}{g}=\frac{1}{g}\frac{\partial}{\partial x}\delta w, \qquad \delta g=\alpha_z=\frac{\partial}{\partial z}\delta w,$$

where δw is the gravitational potential due to the excess mass. If, therefore, the distribution of the disturbing mass is given, the deviation of the vertical and the gravity anomaly at any point can be computed.

Generally speaking, if the following equation can be solved, the underground distribution of the excess mass can be determined from the data of gravity anomalies on the surface of the earth's crust.

The equation to be solved is then

$$A(x_1,\ y_1)=k^2\int_V \frac{\rho'(x,y,z)zdV}{r^3}, \tag{1}$$

where A: gravity anomaly
k^2: gravitational constant
$\rho'(x,y,z)$: excess or defect density in the earth's crust, the last mentioned being assumed to be homogeneous.

$$r=\sqrt{(x-x_1)^2+(y-y_1)^2+z^2}.$$

In the above equation, the origin of the co-ordinates is taken at a point on the surface of the earth, and the positive direction of z is taken downwards. To solve mathematically the above equation is not easy.

If the distribution of the disturbing mass were estimated from geological investigations, and geometrically it is of simple form, the above equation could be easily solved and the density distribution responsible for the gravity anomalies determined.

The object of the present study is, however, to determine the density distribution that causes the gravity anomalies as deduced from the distribution of gravity anomalies on the surface of the earth, provided it is not so complex as to render virtually impossible a ready solution of the above equation.

METHOD OF DETERMINING THE DISTRIBUTION OF EXCESS MASS IN THE EARTH'S CRUST.

In order to simplify the problem to be treated here, the following method was used. The earth's crust was divided into a number of cubes of equal volume. Let $\alpha\rho' = k^2\int\frac{\rho' z}{r^3}dV$ be the vertical component of attraction at a certain point on the surface of the earth caused by a cube having an excess density of ρ'. The total gravity anomaly A_p at point p due to the cubes under consideration is approximately given by

$$A_p = \alpha_{p.1}\rho'_1 + \alpha_{p.2}\rho'_2 + \alpha_{p.3}\rho'_3 + \dots\dots\dots\dots + \alpha_{p.i}\rho'_i. \qquad (2)$$

When the earth's crust in the region in which the surface distribution of the gravity anomalies are given, is divided into i cubic blocks, and, in each of these cubes the density is equal, the gravity anomaly at a point situated at the centre of the surface of each cube in the uppermost layer is given by (2). Hence, when $i=p$, simultaneous equations of the type (2) are obtained, and by solving them, the distribution of the excess density can be obtained. It is however still very difficult to solve them.

The region in question, in which the disturbing mass is supposed to be distributed, lies within the area enclosed by latitudes 35.3° N, 33·8° N and longitudes 138.5° E, 140° E, and to a depth of 90 km. The whole mass of the earth's crust is divided into 75 square blocks as shown in Figs. 1 and 2, the volume of each being $30\times30\times30\ \overline{\text{km}}^3$. The density of each cube is assumed to be 0 or some constant which is common to all cubes.

To calculate numerically the vertical component of the attraction at a certain point (at the centre of the surface of the cube in the uppermost layer), each cube was subdivided into 27 smaller ones, all of the same volume, and the excess mass in each small cube supposed to be concentrated at its centre. The vertical components of the attraction due to each cubic mass for a point is then easily computed. In comparing these two calculated values of the vertical components of attraction, namely, the one that is calculated on the assumption that the excess mass is concentrated at the centre of each cubic block, and the other that is calculated on the assumption that the excess mass is distributed at the centre of the 27 smaller cubes composing the cubic block, we notice that the former is larger by several percent than the latter, although the difference is negligible within the limits of errors in the present discussion.

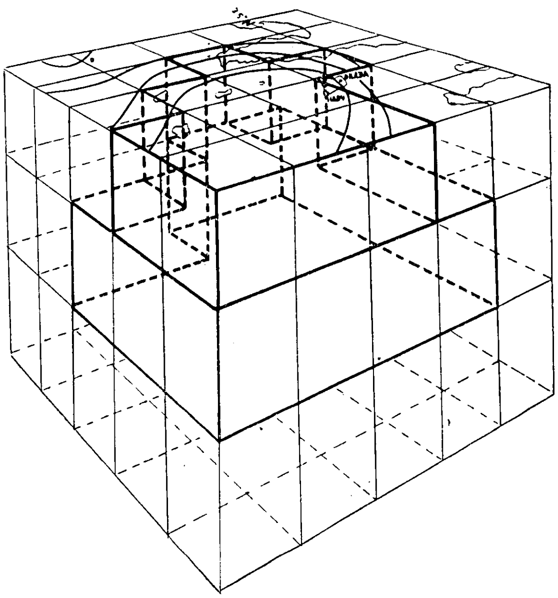

Fig. 2.

It may therefore be worth while to try to ascertain the extent to which the value of attraction at a point on the surface (calculated on the assumption that the excess mass is concentrated at the centre of the cubic block, just underneath the said point), deviates from that calculated on the assumption that the excess mass is distributed uniformly within the cubic block. For this purpose, we have used the following method.

Firstly, the vertical component of attraction at point p in Fig. 3 due to a cubic block and that of the same point due to the 27 smaller blocks are compared, the latter smaller blocks being formed by subdividing each side of the former block into 3 parts.

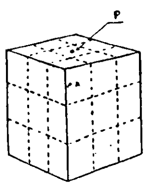

Fig. 3.

In calculating, the excess mass is treated as being concentrated at the centre of each of the cubic blocks. Let the length of a side and the excess density of the smaller block be $2a$ and ρ respectively. Then the gravity due to the mass of the larger block is $k^2 \times \rho \times 24a$, and that due to a cube consisting of 27 smaller blocks is $k^2 \times \rho \times 18{\cdot}584a$. The difference in gravity is $k^2 \times \rho \times 5{\cdot}416 \times a$. Similarly, the smaller cube just beneath the "point" mentioned, above the gravity at which is being sought, is subdivided into 27 smaller square blocks, when the gravity thus calculated is found to be smaller by $k^2 \times \rho \times 5{\cdot}416a'$, where $a' = \frac{a}{3}$, than the larger block.

Since, in this case, the excess mass is 1/27 times that of the former case, the gravity thus calculated is smaller by $k^2 \times \rho \times 5{\cdot}416 \times a \times \frac{1}{3}$ than that calculated on the assumption that excess mass of the cube of $(6a)^3$ is concentrated at the center.

Thus by dividing the square block into a number of very small blocks, the value of gravity calculated approaches that which is obtained by integrating throughout the volume of the larger block. The correction for the gravity value calculated by the most simple method is therefore

$$C = \delta\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\dots\dots\right) = \delta \times \frac{3}{2}.$$

Since, in the present case, $\rho = 0{\cdot}1$ and $a = 5$ km, we have

$$\delta = 18{\cdot}1 \text{ (milligal)}.$$

The vertical components of attraction due to each cubic block at a point on the surface is shown in Table II, in which ρ' is taken as 1 for all cubes.

Table II.

Z in km	$X(Y)$ in km / $Y(X)$ in km	0	30	60	90	120	150
		$a\rho'$ in milligal					
$Z=15$	0	528	72	11	4	2	1
	30		30	8	3	1	1
	60			4	2	1	1
	90				1	1	0
	120					1	0
	150						0
$Z=45$	0	89	51	19	8	4	2
	30		34	15	7	4	2
	60			9	5	3	2
	90				3	2	1
	120					1	1
	150						1
$Z=75$	0	32	26	15	8	5	3
	30		21	13	8	4	3
	60			9	6	4	3
	90				4	3	2
	120					2	2
	150						1

For convenience, the square blocks are designated by numbers as shown in Table III.

Table III.

upper layer

55	56	57	58	59
54	63	64	65	66
53	62	69	70	71
52	61	68	73	74
51	60	67	72	75

middle layer

30	31	32	33	34
29	38	39	40	41
28	37	44	45	46
27	36	43	48	49
26	35	42	47	50

lower layer

5	6	7	8	9
4	13	14	15	16
3	12	19	20	21
2	11	18	23	24
1	10	17	22	25

The gravity anomalies at the centres of the upper surfaces of cubes, numbers 61 to 65, 68 to 70 and 73 in Fig. 1, have been estimated, and the anomalies calculated for the above "points" with the aid of equation (2) and Table II, as shown in Table IV, in which ρ' is taken as 1, and in which the numerals in compartments show the values of $\alpha\rho'$ of the respective cubes for the point under consideration, the compartments being arranged as shown in Table III. The gravity anomalies were not calculated for points lying near the borders of the region under consideration, for which reason the value of attraction may not be correct as may be expected.

Taking the assumed value for the anomalies in density as being likely, the distribution of the excess mass, that satisfies the condition given by a series of equations, may be found by inspection, but in this

Table IV.

$A_{69}=140$ mgal (upper)

4	8	11	8	4
8	30	72	30	8
11	72	528	72	11
8	30	72	30	8
4	8	11	8	4

$A_{61}=50$ mgal (upper)

3	4	3	2	1
8	11	8	4	2
30	72	30	8	3
72	523	72	11	4
30	72	30	8	3

$A_{69}=140$ mgal (middle)

9	15	19	15	9
15	34	51	34	15
19	51	89	51	19
15	34	51	34	15
9	15	19	15	9

(lower)

9	13	15	13	9
13	21	26	21	13
15	26	32	26	15
13	21	26	21	13
9	13	15	13	9

$A_{61}=50$ mgal (middle)

7	8	7	5	3
15	19	15	9	5
34	51	34	15	7
51	89	51	19	8
34	51	34	15	7

(lower)

8	8	8	6	4
13	15	13	9	6
21	26	21	13	8
26	32	26	15	8
21	26	21	13	8

$A_{62}=90$ mgal (upper)

8	11	8	4	2
30	72	30	8	3
72	528	72	11	4
30	72	30	8	3
8	11	8	4	2

(middle)

15	19	15	9	5
34	51	34	15	7
51	89	51	19	8
34	51	34	15	7
15	19	15	9	5

(lower)

13	15	13	9	6
21	26	21	13	8
26	32	26	15	8
21	26	21	13	8
13	15	13	9	6

$A_{63}=60$ mgal (upper)

30	72	30	8	3
72	528	72	11	4
30	72	30	8	3
8	11	8	4	2
3	4	3	2	1

(middle)

34	51	34	15	7
51	89	51	19	8
34	51	34	15	7
15	19	15	9	5
7	8	7	5	3

(lower)

21	26	21	13	8
26	32	26	15	8
21	26	21	13	8
13	15	13	9	6
8	8	8	6	4

A_{63}=120 mgal (upper)

2	3	4	3	2
4	8	11	8	4
8	30	72	30	8
11	72	528	72	11
8	30	72	30	8

(middle)

5	7	8	7	5
9	15	19	15	9
15	34	51	34	15
19	51	89	51	19
15	34	51	34	15

(lower)

6	8	8	8	6
9	13	15	13	9
13	21	26	21	13
15	26	32	26	15
13	21	26	21	13

A_{64}=100 mgal (upper)

8	30	72	30	8
11	72	528	72	11
8	30	72	30	8
4	8	11	8	4
2	3	4	3	2

(middle)

15	34	51	34	15
19	51	89	51	19
15	34	51	34	15
9	15	19	15	9
5	7	8	7	5

(lower)

13	21	26	21	13
15	26	32	26	15
13	21	26	21	13
9	13	15	13	9
6	8	8	8	5

A_{73}=150 mgal (upper)

1	2	3	4	3
2	4	8	11	8
3	8	30	72	30
4	11	72	528	72
3	8	30	72	30

(middle)

3	5	7	8	7
5	9	15	19	15
7	15	34	51	34
8	19	51	89	51
7	15	34	51	34

A_{70}=150 mgal (upper)

2	4	8	11	8
3	8	30	72	30
4	11	72	528	72
3	8	30	72	30
2	4	8	11	8

(middle)

5	9	15	19	15
7	15	34	51	34
8	19	51	89	51
7	15	34	51	34
5	9	15	19	15

A_{73}=150 mgal (lower)

4	6	8	8	8
6	9	13	15	13
7	13	21	26	21
8	15	26	32	26
8	13	21	26	21

A_{70}=150 mgal (lower)

6	9	13	15	13
8	13	21	26	21
8	15	26	32	26
8	13	21	26	21
6	9	13	15	13

A_{65}=70 mgal

(upper)

3	8	30	72	30
4	11	72	528	72
3	8	30	72	30
2	4	8	11	8
1	2	3	4	3

(lower)

8	13	21	26	21
8	15	26	32	26
8	13	21	26	21
6	9	13	15	13
4	6	8	8	8

(middle)

7	15	34	51	34
8	19	51	89	51
7	15	34	51	34
5	9	15	19	15
3	5	7	8	7

paper the following method was used to ascertain the distribution of the excess mass.

The terms on the left-hand side of the equations (2), namely, A in Table IV, are reduced to a value common to all the equations, and the sum of $\alpha\rho'$ of successive compartments plotted against the number of cubes as abscissa, when all the values of $\sum\alpha\rho'$ corresponding to a certain point on the abscissa should either become equal to or $\frac{1}{\rho'}$ of the value on the left-hand side of the equations, ρ' being the same for every cubic block, as already assumed.

In the present problem, the numerical value on the left-hand side of the equation was reduced to 100 milligals and the value of ρ' taken as 0·1. The results of the reduction are given in Table V, and the values of $\sum\alpha\rho'$ plotted as shown in Fig. 4.

Of the cubic blocks in the uppermost layer in Fig. 4. only the values of $\alpha\rho'$ due to the cubic blocks Nos. 62, 64 and 68 to 75 have been taken into consideration, because the values of $\alpha\rho'$ due to the other cubic blocks may obviously be rejected by inspection. For example, the observed gravity anomaly at the middle of the surface of block No.

60 is about 40 milligals, whereas the attraction at the same point due to cube No. 60 alone amounts to 53 milligals. In the writer's opinion

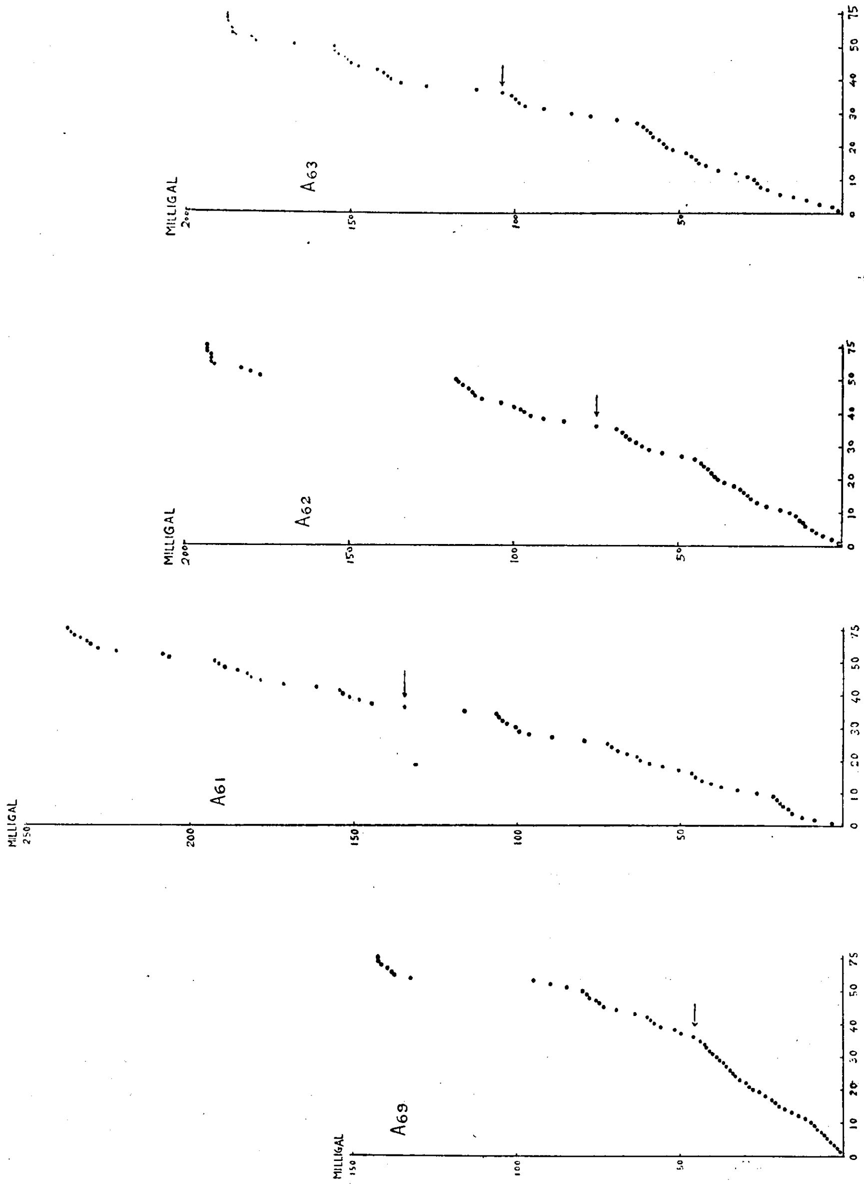

Fig. 4, 1.

this may show that there is no excess mass in the cubic block to cause any gravity anomaly on the surface of the earth.

It will be noticed that the sum of the $\alpha\rho'$ corresponding to cubes from No. 37 to No. 75 is almost the same in all the graphs. These

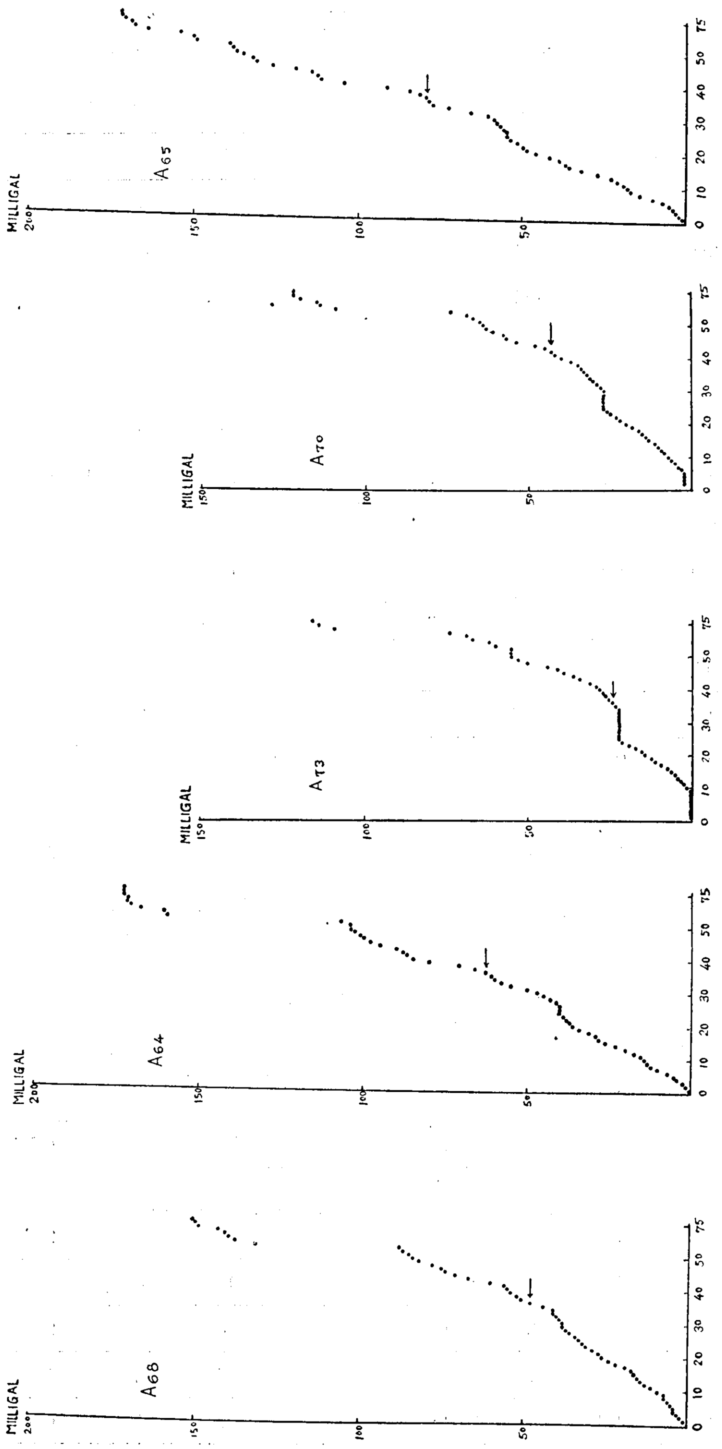
MILLIGAL
A65
MILLIGAL
A70
MILLIGAL
A73
MILLIGAL
A64
MILLIGAL
A68

Fig. 4, 2.

Table V.

A_{60}=100 mgal (upper)

0·3	0·6	0·8	0·6	0·3
0·6	2·1	5·1	2·1	0·6
0·8	5·1	37·7	5·1	0·8
0·6	2·1	5·1	2·1	0·6
0·3	0·6	0·8	0·6	0·3

(middle)

0·6	1·1	1·4	1·1	0·6
1·1	2·4	3·6	2·4	1·1
1·4	3·6	6·4	3·6	1·4
1·1	2·4	3·6	2·4	1·1
0·6	1·1	1·4	1·1	0·6

(lower)

0·6	0·9	1·1	0·9	0·6
0·9	1·5	1·9	1·5	0·9
1·1	1·9	2·3	1·9	1·1
0·9	1·5	1·9	1·5	0·9
0·6	0·9	1·1	0·9	0·6

A_{61}=100 mgal (upper)

0·6	0·8	0·6	0·4	0·2
1·6	2·2	1·6	0·8	0·4
6·0	14·4	6·0	1·6	0·6
14·4	105·6	14·4	2·2	0·8
6·0	14·4	6·0	1·6	0·6

(middle)

1·4	1·6	1·4	1·0	0·6
3·0	3·8	3·0	1·8	1·0
6·8	10·2	6·8	3·0	1·4
10·2	17·8	10·2	3·8	1·6
6·8	10·2	6·8	3·0	1·4

(lower)

1·6	1·6	1·6	1·2	0·8
2·6	3·0	2·6	1·8	1·2
4·2	5·2	4·2	2·6	1·4
5·2	6·4	5·2	3·0	1·6
4·2	5·2	4·2	2·6	1·4

A_{62}=100 mgal (upper)

0·9	1·2	0·9	0·4	0·2
3·3	8·0	3·3	0·9	0·3
8·0	58·7	8·0	1·2	0·4
3·3	8·0	3·3	0·9	0·3
0·9	1·2	0·9	0·4	0·2

(middle)

1·7	2·1	1·7	1·0	0 6
3·8	5·7	3·8	1·7	0·8
5·7	9·9	5·7	2·1	0·9
3·8	5·7	3·8	1·7	0·8
1·7	2·1	1·7	1·0	0·6

A_{63}=100 mgal (upper)

5·0	12·0	5·0	1·3	0·5
12·0	8·8	12·0	1·8	0·7
5·0	12·0	5·0	1·3	0·5
1·3	1·8	1·3	0·7	0·3
0·5	0·7	0·5	0·3	0·2

(middle)

5·7	8·5	5·7	2·5	1·2
8·5	14·8	8·5	3·2	1 3
5·7	8·5	5·7	2·5	1·2
2·5	3·2	2·5	1·5	0·8
1·2	1·3	1·2	0·8	0·5

A_{62}=100 mgal (lower)

1·4	1·7	1·4	1·0	0·7
2·3	2·9	2·3	1·4	0·9
2·9	3·6	2·9	1·7	0·9
2·3	2·9	2·3	1·4	0·9
1·4	1·7	1·4	1·0	0·7

A_{63}=100 mgal (lower)

3·5	4·3	3·5	2·2	1·3
4·3	5·3	4·3	2·5	1·3
3·5	4·3	3·5	2·2	1·3
2·2	2·5	2·2	1·5	1·0
1·3	1·3	1·3	1·0	0·7

A_{63}=100 mgal (upper)

0·2	0·3	0·3	0·3	0 2
0·3	0·7	0·9	0·7	0·3
0·7	2·5	6·0	2·5	0·7
0·9	6·0	44·0	6·0	0·9
0·7	2·5	6·0	2·5	0·7

(middle)

0·4	0·6	0·7	0·6	0·4
0·8	1·3	1·6	1·3	0·8
1 3	2·8	4·3	2·8	1·3
1·6	4·3	7·4	4·3	1·6
1·3	2·8	4·3	2·8	1·3

(lower)

0·5	0·7	0·7	0·7	0·5
0·8	1·1	1·3	1·1	0·8
1·1	1·8	2·2	1·8	1·1
1·3	2·2	2·7	2·2	1·3
1·1	1·8	2·2	1·8	1·1

A_{64}=100 mgal (upper)

0·8	3·0	7·2	3·0	0·8
1·1	7·2	52·8	7·2	1·1
0·8	3·0	7·2	3·0	0·8
0·4	0·8	1·1	0·8	0·4
0·2	0·3	0·4	0·3	0·2

(middle)

1·5	3·4	5·1	3·4	1·5
1·9	5·1	8·9	5·1	1·9
1·5	3·4	5·1	3·4	1·5
0·9	1·5	1·9	1·5	0·9
0·5	0·7	0·8	0·7	0·5

(lower)

1·3	2·1	2·6	2·1	1·3
1·5	2·6	3·2	2·6	1·5
1·3	2·1	2·6	2·1	1·3
0·9	1·3	1·5	1·3	0·9
0·6	0·8	0·8	0·8	0·6

A_{73}=100 mgal (upper)

0·1	0·1	0·2	0·3	0·2
0·1	0·3	0·5	0·7	0·5
0·2	0·5	2·0	4·8	2.0
0·3	0·7	4·8	35·2	4·8
0·2	0·5	2·0	4·8	2·0

A_{70}=100 mgal (upper)

0·1	0·3	0·5	0·7	0·5
0·2	0·5	2·0	4·8	2·0
0·3	0·7	4·8	35·2	4·8
0·2	0·5	2·0	4·8	2·0
0·1	0·3	0·5	0·7	0·5

A_{73}=100 mgal (middle)

0·2	0·3	0·5	0·5	0·5
0·3	0·6	1·0	1·3	1·0
0·5	1·0	2·3	3·4	2·3
0·5	1·3	3·4	5·9	3·4
0·5	1·0	2·3	3·4	2·3

(lower)

0·3	0·4	0·5	0·5	0·5
0·4	0·6	0·9	1·0	0·9
0·5	0·9	1·4	1·7	1·4
0·5	1·0	1·7	2·1	1·7
0·5	0·9	1·4	1·7	1·4

A_{70}=100 mgal (middle)

0·3	0·6	1·0	1·3	1·0
0·5	1·0	2·3	3·4	2·3
0·5	1·3	3·4	5·9	3·4
0·5	1·0	2·3	3·4	2·3
0·3	0·6	1·0	1·3	1·0

(lower)

0·4	0·6	0·9	1·0	0·9
0·5	0·9	1·4	1·7	1·4
0·5	1·0	1·7	2·1	1·7
0·5	0·9	1·4	1·7	1·4
0·4	0·6	0·9	1·0	0·9

A_{65}=100 mgal

(upper)

0·4	1·1	4·3	10·3	4·3
0·6	1·6	10·3	75·4	10·3
0·4	1·1	4·3	10·3	4·3
0·3	0·6	1·1	1·6	1·1
0·1	0·3	0·4	0·6	0·4

(middle)

1·0	2·1	4·9	7·3	4·9
1·1	2·7	7·3	12·7	7·3
1·0	2·1	4·9	7·3	4·9
0·7	1·3	2·1	2·7	2·1
0·4	0·7	1·0	1·1	1·0

(lower)

1·1	1·9	3·0	3·7	3·0
1·1	2·1	3·7	4·6	3·7
1·1	1·9	3·0	3·7	3·0
0·9	1·3	1·9	2·1	1·9
0·6	0·9	1·1	1·1	1·1

cubes are numbered in Table VI, and the values of $\sum\alpha\rho'$ for points, summed over the interval corresponding to that mentioned above satisfy the respective equations fairly well as shown in Table VII.

Table VI.

		64		
	62	69	70	71
		68	73	74
			72	75

	38	39	40	41
	37	44	45	46
		43	48	49
		42	47	50

Table VII.

No. of cubes	Values of A in mgal		Differences
	Observed	Computed	
69	140	136	+ 4
61	50	51	− 1
62	90	105	−15
63	60	52	+ 8
68	120	125	− 5
64	100	110	−10
73	150	142	+ 8
70	150	137	+13
65	70	66	+ 4

As already mentioned, the value of ρ' was assumed to be 0·1. Since the equations are well satisfied with this value of ρ', this order of magnitude of the value of ρ' may be regarded as probable. The space which may be occupied by the excess mass obtained above is shown in Fig. 2, the boundary of which is drawn with thick lines. The width of the space occupied by the excess mass reaches 120 km and the depth at most 60 km.

Of course the distribution of the excess mass obtained above is only one of solutions obtained on the basis of the assumed uniform density. Actually, however, the density may be distributed irregularly, so that there may be several other ways in which the excess density in the earth's crust may be distributed, and which may satisfy the given mode in distribution of gravity anomaly. If, however, the deviations of the vertical computed by using the given mode of distribution of excess mass agree with the observed deviations, the distribution of the excess mass obtained above may become more likely.

DEVIATIONS OF THE VERTICALS IN IZU-OSIMA, COMPUTED AND OBSERVED.

The meridian and the prime vertical components of deviation of the vertical are given respectively by

$$\left.\begin{aligned} \xi&=\frac{1}{g}\frac{\partial}{\partial x}\delta w=\frac{k^2}{g}\int\frac{\rho' x}{r^3}dV \\ \eta&=\frac{1}{g}\frac{\partial}{\partial y}\delta w=\frac{k^2}{g}\int\frac{\rho' y}{r^3}dV \end{aligned}\right\}. \qquad (4)$$

The values of $\xi_{(G-A)}$ and $\eta_{(G-A)}$ due to the excess mass, the distribution of which has been given above, are shown in the following table together with the observed values.

	$\xi_{(G-A)}$			$\eta_{(G-A)}$		
	Computed	Observed	Difference	Computed	Observed	Difference
Habu	− 9·5″	−11·4″	+ 1·9″	−2·4″	−3·9″	+1·5″
Okada	−12·4	−28·6	+16·2	−4·0	+0·8	−4·8

The computed values of ξ's and η's given in the above table were integrated numerically in the same way as that used in computing the values of gravity anomalies given in Table II. Since the topographic correction is not likely to greatly affect the result, in determining the distribution of the excess mass in the earth's crust, this correction was not taken into consideration in reducing the observed gravity at Izu-Osima. But in discussing the deviation of the vertical at Izu-Osima, topographic correction may play an important part, for it is possible that the deviations of the vertical may be the result, chiefly, of the lateral force due to the attraction of the mass of the island, i. e., the mass of Mihara Volcano.

The topographic corrections for the deviations of the vertical due to the mass of Izu-Osima above sea level are computed by formulae (see Fig. 5)

$$\delta\varphi = \frac{k^2\rho d}{g}(\sin\alpha_{n+1} - \sin\alpha_n)\left\{ln\frac{a+r}{t} - 1\right\}, \tag{5a}$$

or

$$\delta\varphi = \frac{k^2\rho d}{g}(\sin\alpha_{n+1} - \sin\alpha_n)\left\{ln\frac{a_{m+1}+r_{m+1}}{a_m+r_m}\right\}, \tag{5b}$$

the results being

	ξ	η
Habu	$+7{\cdot}5''$	$-5{\cdot}1''$
Okada	$-7{\cdot}2$	$+0{\cdot}8$

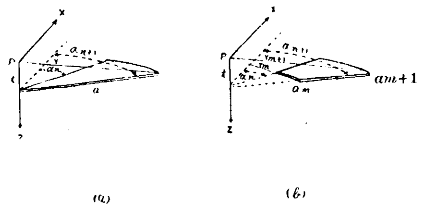

Fig. 5.

in which the mean density ρ of Izu-Osima is assumed to be 2·70. Applying this correction to the computed values of ξ's and η's, we have

	$\xi_{(G-A)}$			$\eta_{(G-A)}$		
	Computed	Observed	Difference	Computed	Observed	Difference
Habu	$-2{\cdot}0''$	$-11{\cdot}4''$	$+9{\cdot}4''$	$-7{\cdot}5''$	$-3{\cdot}9''$	$-3{\cdot}6''$
Okada	$-19{\cdot}6$	$-28{\cdot}6$	$+9{\cdot}0$	$-3{\cdot}2$	$+0{\cdot}8$	$-4{\cdot}0$

The meridian components of the deviations of the verticals computed differ by about 9″, and the prime vertical components by −4″. Many reasons could be invoked to explain the difference between the values computed and those observed, the most probable of which are

(i) The smallness of the width of the region under consideration.

(ii) The deviation of the vertical at the standard datum point at Tokyo.

(iii) The dimensions of the cubic blocks into which the earth's crust of the region under consideration were divided for convenience of calculation.

(iv) Unsuitableness of the formula for normal gravity in this region.

Cause (i) may be considered from a number of view points. For instance, the configuration of the isanomaly lines of gravity suggests that the excess mass may be present in that region which extends southward of the one under consideration. The fact that the computed gravity anomalies at points on the surfaces of cubic blocks No. 70 and 73 are smaller than those observed, suggest likewise.

As to cause (ii), it may be inferred that the deviations of the vertical at the standard datum point at Tokyo amounts to $-10{\cdot}0''$ in ξ and $6{\cdot}7''$, in η.[4]

The fact that the signs and approximate magnitudes of the value of (Obs.-Comp.) for Habu and Okada are equal in both ξ and η components, does not militate against the above inference.

We often hear that the distribution of g in Japan does not agree well with normal gravity. If we use another formula for normal gravity fitting well with the values of g in Japan,[5] the values of gravity anomalies will differ. The distribution of excess mass in the earth's crust, however, may not be much affected, seeing that it is determined from the distribution of gravity anomaly on the surface and from the configuration of the iso-Δg curves. The mode of distribution of Δg can not be greatly affected, even were the well fitting formula for the distribution of gravity in Japan used.

It may be worthy of note that, in drawing the isanomaly curves in the region under consideration, the value of gravity anomaly at Osima was not corrected. As a general rule, the value of gravity anomaly at a station in an oceanic island is very large, and this is the case with Osima. This fact is explained by considering the mass of the island to be the excess that is resting on the ocean bed, the last named consisting of sub-crustal rocks. So long as the ocean bed is considered to be in gravitational equilibrium, the excess mass of the island should cause an extraordinary anomaly in gravity in the region in the neighbourhood of Izu-Osima. As a matter of fact the value of gravity anomaly in Osima is very large, while on the ocean the value of gravity is also very large, it being of the same order as that of Osima. The large value of the gravity anomaly in oceanic islands was

4) Y. KAWAHATA, *Geopys. Mag.*, **10** (1936), 94.

5) C. TSUBOI, *Bull. Earthq. Res. Inst.*, **13** (1935), 554~557.

hitherto believed to be the effect of the excess mass of the island, but in the opinion of the writer, the large value of the gravity anomaly of the island of Osima is due not only to the excess mass of Mihara volcano alone, but is due largely to the effects of a still greater mass.

We also know that many earthquakes occur in the neighbourhood of this region, though not so frequently as in other regions of Japan. This may suggest that the earth's crust in this neighbourhood is not yet in (isostatic) equilibrium. From these facts, it is probable that the ocean bed in this neighbourhood is not in equilibrium, and that the mass of Osima is only an excess mass. This explains why, against usual practice, the value of gravity anomaly at Osima was not corrected.

SUMMARY

The result of the present study is summarized as follows:

The distribution of excess mass in the earth's crust in the region near Izu-Osima was determined by using the data of gravity anomalies on the surface of the earth.

In determining the distribution of the excess mass, we assumed, for convenience in calculation, that

(i) The earth's crust in the region under consideration is divided into 75 blocks or cubes; and that the excess mass that is responsible for the gravity anomalies on the surface of the earth is concentrated at the centre of each of such cubes.

(ii) The density of the excess mass ρ' was assumed to be 0·1 or zero.

After the distribution of excess mass was thus determined, the deviations of the verticals for the stations, Okada and Habu, were calculated, and the results compared with actual data, with satisfactory results.

It should be added, however, that the mode of distribution of the excess mass is not a unique solution.

5. 大島に於ける重力異常と鉛直線偏倚

陸地測量部 武藤勝彥

最近伊豆大島に於いて鉛直線偏倚を測つてみた結果.

波浮にて	南北成分 (G-A)	$-11{\cdot}4''$	東西成分 (G-A)	$-3{\cdot}9''$
岡田にて	〃	$-28{\cdot}6$	〃	$+0{\cdot}8$

なる値を得た．この程度の大いさの偏倚は特に重大視する程のものではないが，こゝに特異とすべき點は，大島の北端岡田に於ても南端の波浮に於いても南北成分が共に符號が同一なる點である．

これは，大島及びその南方海中に於ける重力異常の大なる事實と關聯してゐることと考へられる．

そこで，大島を中心とする 150 km 平方の地域の地殼內に於ける過大質量分布の有樣をこの地域の地表面に於ける重力異常分布の狀況から決定することを試みた．

この問題は，純數理的には解法を得ることが困難であるので，數值計算法によることにした．それ故に求め得た一つの過大質量の分布樣式を決定し得たとしても，それが唯一解なりや否やを判定することは困難である．

筆者の採用した方法は次の如きものである．問題の地域の地殼を若干個の正立方體部分に分け，その各立方體の中心に，その立方體內の過大質量が集中してゐるとし，且つ，少くとも立方體內では質量は一樣とする．その場合には一つの立方體の表面の中心點に於ける重力異常 Ap は，

$$Ap=\sum_i \alpha_{p.i}\rho_i$$

となる．こゝで更に ρ_i は一定の常數か 0 かであるとして，どれだけの立方體に過大質量があれば所與の重力異常分布を得るかを圖表計算的に求めたのである．

斯くして，兎も角も，求め得た結果は，大體に於いて尤もらしいものである．卽ちそれから計算される鉛直線偏倚と實測された鉛直線偏倚と比較しても大體合理的と解せられ，その他の諸種の事情環境とも排反しない．