7. *Studies in Fluctuations in the Heights of Yearly Mean Sea-levels.*

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1. INTRODUCTION. Usually, the mean sea-level for a sufficiently long period at a mareograph station is taken as the standard datum of the height of land. The height of the sea-level, which is measured with reference to a rigidly fixed point on the land, varies secularly and periodically with time. The factors that cause the changes in the height of the sea-level are (i) astronomical disturbances, (ii) meteorological disturbances, and (iii) land deformations.

These factors that cause changes in the height of sea-level, especially, the astronomical and meteorological disturbances, are mostly periodic, i. e., the astronomical disturbances vary mostly with diurnal, semidiurnal, and fortnightly periods, and the meteorological disturbances with an approximate annual period. The effects of these regularly varying disturbances can therefore be eliminated by taking the yearly mean value of the sea-level.

The changes in the height of the yearly mean sea-level are therefore usually regarded as indicating the secular variation in the height of the land, i. e., crustal deformations.

From actual data, however, the yearly mean sea-level varies rather irregularly, so that it cannot be regarded as being due merely to upheaval and subsidence of the earth's crust.

On the other hand, astronomical disturbances of longer periods which may cause changes in the height of the yearly mean sea-level, for instance, those with periods of 9 years and 18·6 years, cannot be regarded as varying uniformly throughout the whole period; in other words, although the disturbing force due to astronomical bodies become equal after the lapse of 9 or 18.6 years, the variations in these disturbing forces cannot be expressed by simple sinusoidal functions of time. The same may also be said of meteorological disturbances; cyclones and other meteorological factors that affect the sea-level vary quite irregularly with time.[1)]

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1) T. TERADA and S. YAMAGUTI, *Bull. Imp, Earthq. Inv. Comm.*, 9 (1928), 113.

Thus the yearly mean sea-level is disturbed in a marked manner by irregularly varying disturbances, so that these irregular variations in the height of the yearly mean sea-level cannot be attributed merely to crustal deformations.

In the present study, irregular variation in the heights of the yearly mean sea-level is believed to be due to irregular disturbances, both astronomical and meteorological. The effect of these irregular disturbance can be reduced by taking overlapping mean values over a number of years.

Of course, the crustal deformation may not only be chronic, but also acute, i. e., there may be quasi-periodic rise and subsidence of a part of the earth's crust with shorter periods, namely, periods of several years. Such fluctuations in land deformation cannot be separated from the irregular variations due to astronomical and meteorological disturbances. They can be discussed only when some accessory evidences are at hand, as will be dealt with later.

2. DATA OF YEARLY MEAN SEA-LEVEL. This paper deals with data of the yearly mean sea-level for the following stations, for the periods mentioned below.

	Stations	Time intervals
(i)	Hosozima, Japan	1900~1936[2]
(ii)	Frederikshavn, Denmark	1893~1931[3]
(iii)	Fort Hamilton, New York, U. S. A.	1893~1932[4]
(iv)	San Francisco, U. S. A.	1898~1935[4]
(v)	Seattle, U. S. A.	1899~1935[4]

Hosozima is in Kyûsyû, Japan, on the Pacific coast. The general course of the curve representing variations in the height of yearly mean sea-level at this station may be expressed approximately by

$$H_t = a + bt + ct^2,$$

where t is the number of years since 1900. The constants a, b, c in the above expression are determined by the method of least squares, and the deviations in actual height, H_a, from the calculated one, H_t, are obtained by

$$H_a - H_t = H_a - \{-35{\cdot}0 - 42{\cdot}3t - 9{\cdot}4t^2\} \quad \text{(mm)}.$$

Fredirikshavn is in Denmark. Egedar, in his paper,[3] gives the

2) Reports issued by the Military Land Survey.
3) J. Egedar, "*On the Determination of the normal height of the sea-level round the Danish coasts*", Copenhagen, 1933.
4) W. BOWIE. *Spec. Publ. Coast and Geodetic Survey, U. S. A.* No. 207 (1936).

fluctuations in the heights of the yearly mean sea-levels for several other stations. As the curves of fluctuations in the heights of the yearly mean sea-levels of various stations in Denmark are very similar to one another, we take, for example, the data of Frederikshavn. These fluctuations are deviations from D. N. N. given in mm. The variation in the yearly mean sea-level at Frederikshavn may be given approximately by

$$H_t = a + bt.$$

In cases like the foregoing, in which the variation in the yearly mean sea-level can as a rule be expressed by a linear function of time, the constants can be determined by using the overlapping mean values, because this process does not affect the value of the constants a and b. The constants a and b thus determined are 2·5 mm and $-0·16$ mm/year respectively for Frederikshavn.

Similarly, for the secular variations in the heights of the yearly mean sea-levels at Fort Hamilton and San Francisco, the linear functions of t are

For Fort Hamilton	$H_t = -7{\cdot}0 + 0{\cdot}4\,t$	(1/100 foot)
For San Francisco	$H_t = -3{\cdot}5 + 0{\cdot}2\,t$	(1/100 foot)

As to the yearly mean sea-level of Seattle, no such secular variation as could be represented by a linear function of time is noticable.[5]

In determining therefore the constants a, b, etc. of the expressions for the secular variations in the yearly mean sea-level, we find that the values of b in the expressions for Japanese stations is much greater than those for European and American stations, which may be an indication that the crustal deformation is more marked in Japan than in Europe and America. We shall deal with this matter in a future paper.

3. FLUCTUATIONS IN HEIGHT OF THE YEARLY MEAN SEA-LEVEL. After the terms of the secular variations referred in the preceding paragraph have been eliminated, the residuals are treated statistically.

The fluctuations in the heights of the yearly mean sea-levels of the various stations cited above are shown in Figs. 1~5. The curves in these figures will show that the yearly mean sea-levels appear to vary with periods amounting to several years. The mean period of the apparent periodic variation of the yearly mean sea-levels can be calculated by dividing the whole time interval, for which the yearly

5) The Data of fluctuations in the heights of the yearly mean sea-levels at various stations are given in a table in the appendix.

mean sea-levels are obtained, by the number of peaks in the respective

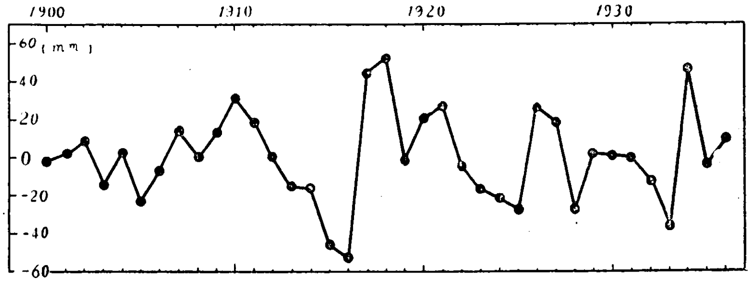

Fig. 1. Fluctuation in heights of yearly mean sea-level at Hosozima.

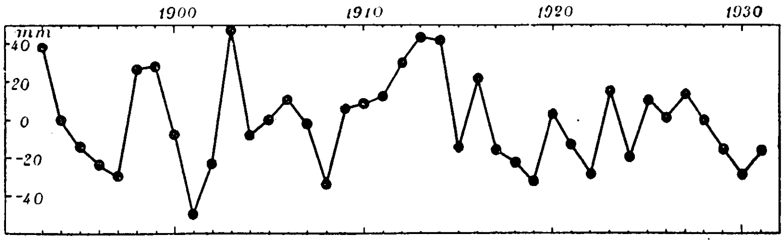

Fig. 2. Fluctuation in heights of yearly mean sea-level at Frederikshavn (Denmark).

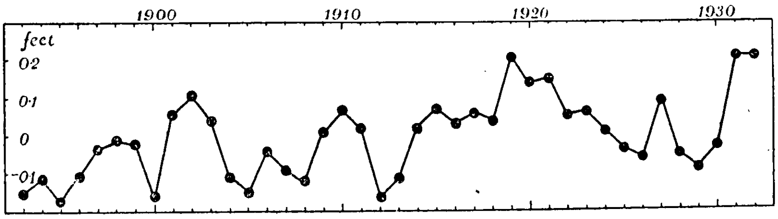

Fig. 3. Fluctuation in heights of yearly mean sea-level at Fort Hamilton (U. S. A.).

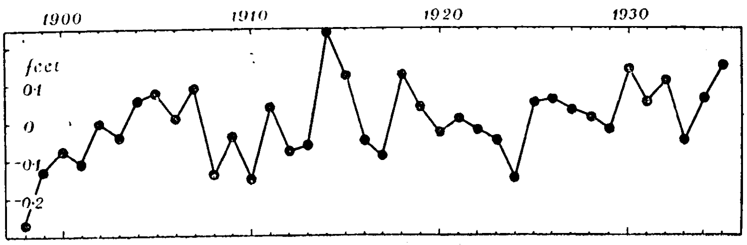

Fig. 4. Fluctuation in heights of yearly mean sea-level at San Francisco (U. S. A.).

curves. The results give the values of „Wiederkehrzeit".[6] The mean

6) Fürth. „*Schwankungserscheinungen in der Physik*". Leipzig, 1923.

periods thus calculated range, as shown in Table I, from 3 to 4, which

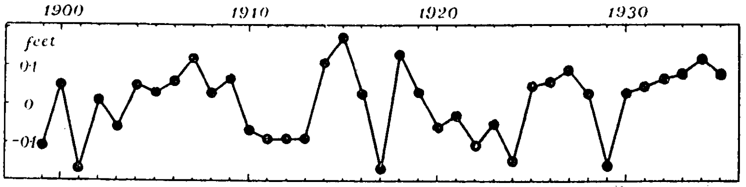

Fig. 5. Fluctuation in height of yearly mean sea-level at Seattle (U. S. A.).

is probably to be expected, seeing that the fluctuation in the heights of the yearly mean sea-levels is very irregular.[7]

TABLE I.

Station	"Wiederkehrzeit"	($\overline{\lvert\delta\rvert}$) cal	($\overline{\lvert\delta\rvert}$) obs
Hosozima	3·7	0·54	0·99
Frederikshavn	4·3	0·47	0·39
Fort Hamilton	3·7	0·54	0·52
San Francisco	3·0	0·67	0·37
Seattle	3·4	0·59	0·68

Moreover, when the number of occurrences N, which corresponds to the number of years in the present case, is very large, the value of „Schwankungsgrösse" $\overline{|\delta|}$ is given by

$$\overline{|\delta|}=\frac{1}{N}\sum\frac{\varDelta_n-\bar{\varDelta}}{\bar{\varDelta}}=2W(\delta)=\frac{2}{\text{„Wiederkehrzeit"}}.$$ [8]

This relation is to some extent confirmed by actual data, as shown in Table I.

These facts agree with the conclusion that the fluctuations in the heights of the yearly mean sea-levels are very irregular.

4. ON MEAN DEVIATIONS. In order to study in further detail the nature of the fluctuations in the heights of the yearly mean sea-levels, the actual curves showing the fluctuations are smoothed. In smoothing the curves, the overlapping mean values are calculated with respect to different time intervals, i. e., the values of

7) The fluctuation in the heights of the yearly mean sea-levels may not be „Schwankungen mit Nachwirkung". The causes of the fluctuations, such as the number of cyclones per year, may be quite fortuitous.

8) In calculating „Schwankungsgrösse" from actual data, the deviations $\varDelta_n$ are measured from the least value of the height of the yearly mean sea-level. The symbol $\bar{\varDelta}$ designates the mean value of $\varDelta_n$.

$$\eta_i=\frac{\xi_{i-m}+\xi_{i-m+1}+\cdots+\xi_i+\xi_{i+1}+\cdots\cdots+\xi_{i+m}}{2m+1} \qquad \ldots\ldots\ldots\ldots(1)$$

are calculated, where ξ's are the amounts of deviations of the heights of the yearly mean sea-levels. The values of η_i's are calculated for cases of $2m+1=3$, 4, 5, 6, 7, 8, 9 and 10.

For each smoothed curve, the mean deviation, i. e., the value of $|\overline{\eta}|$ are calculated and plotted against τ,[9] the time interval over which the overlapping mean values are calculated in smoothing the actual fluctuations, as shown in Figs. 8∼12. As will be seen in these figures, the values of $|\overline{\eta}|$ decrease with increasing τ, which fact may be understood by studying the nature of the mean deviation.

Firstly, we consider the case in which the fluctuation ξ is given by the sum of the periodic functions, i. e..

$$\xi=\sum a_n \sin(nt+q_n)$$

Then the value of η is given by

$$\eta=\frac{1}{\tau}\int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}}\sum a_n \sin(nt+q_n)\,dt.$$

Taking one of the components of η, we have

$$\eta_n=\frac{1}{\tau}\int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} a_n \sin(nt+q_n)\,dt.$$

$$=a_n\frac{\sin n\frac{\tau}{2}}{n\frac{\tau}{2}}\sin(nt+q_n)$$

Hence, the mean deviation is given by

$$|\overline{\eta_n}|=\frac{2}{\pi}a_n\left|\frac{\sin n\frac{\tau}{2}}{n\frac{\tau}{2}}\right|(\cos q_n-\sin q_n).$$

If we could put $q_n=0$,

9) The time interval τ corresponds to $(2m+1)$ in equation (1).

$$\overline{|\eta_n|} = \frac{2}{\pi} a_n \left| \frac{\sin n\frac{\tau}{2}}{n\frac{\tau}{2}} \right| .$$

If however the value of η consists of several components of periodic terms with different periods, it may be more convenient for the present discussion to take the standard deviations, i. e.,

$$\sqrt{\overline{\eta^2}} = \frac{1}{2}\sqrt{\sum a_n^2 \frac{1}{n}\left(\frac{\sin\frac{n\tau}{2}}{\frac{n\tau}{2}}\right)^2} .$$

In Fig. 6, the variations in the values of $\left(\frac{\sin\frac{n\tau}{2}}{\frac{n\tau}{2}}\right)$ are plotted against $n\tau$ or $\frac{\tau}{T_n}$ where $n=\frac{2\pi}{T_n}$, from which it may be anticipated

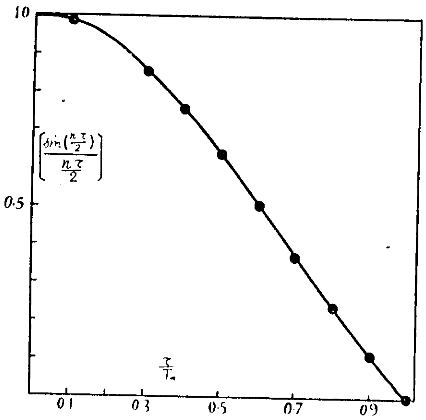

Fig. 6. Values of $\frac{\sin\frac{n\tau}{2}}{\frac{n\tau}{2}}$ plotted against $n\tau$, or $\frac{\tau}{T_n}$

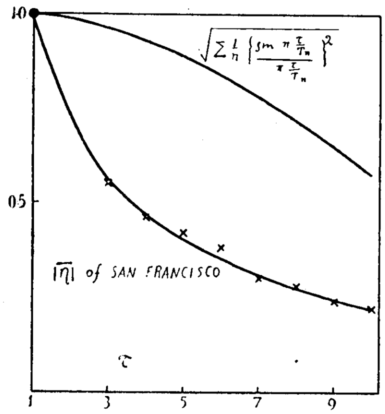

Fig. 7. The values of $\sqrt{\sum\frac{1}{n}\left\{\frac{\sin\frac{\pi\tau}{T_n}}{\frac{\pi\tau}{T_n}}\right\}^2}$ is compared with actual value of $\overline{|\eta|}$ of San Francisco.

that the variation in $\overline{|\eta_n|}$ with respect to $n\tau$ or $\frac{\tau}{T_n}$ cannot agree with the actual variation of $\overline{|\eta|}$ with respect to τ. In Fig. 7, the curve of variation of an actual $\overline{|\eta|}$ is compared with that of

$$\sqrt{\frac{1}{n}\left(\frac{\sin\frac{n\tau}{2}}{\frac{n\tau}{2}}\right)^2}, \quad T_n \text{ being 12, 15, and 18 years, where } T_n=2\frac{\pi}{n}.$$

This discordance between the actual and the calculated mean deviations, based on the assumption that $\xi=\sum a_n \sin(nt+q_n)$, may become smaller if a sufficiently large number of terms are taken into consideration, in which case however the problem may be similar to that treated statistically. Since, on the other hand, the external disturbances are regarded as being quite irregular as already mentioned, the statistical treatment of the proposed problem may be considered reasonable, and the above mentioned deviations are therefore discussed from a statistical point of view.

Let the deviations ξ_i be

$$\xi_i=x_i-X, \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

where the x_i's are the actual values of the yearly mean sea-level, and X the mean value of an infinitely large number of x_i's.

Assuming that the frequency distribution of ξ is given by Gauss's law, the probability that the mean value

$$s=\frac{1}{n+1}(\xi_1+\xi_2+\cdots\cdots+\xi_{n+1})$$

lies between s and $s+ds$ will be

$$Uds=(n+1)ds\left(\frac{K}{\pi}\right)^{\frac{n+1}{2}}\int_{-\infty}^{\infty} e^{-K\xi_1^2}e^{-K\xi_2^2}\cdots\cdots e^{-K\xi^2_{n+1}}d\xi_1,\ d\xi_2\cdots\cdots d\xi_n.$$

Substituting ξ_{n+1} with $(n+1)s-\xi_1-\xi_2-\xi_3-\cdots\cdots-\xi_n$, we have

$$I=\int_{-\infty}^{\infty} e^{-K\xi_1^2}\cdots\cdots d\xi_1\cdots\cdots$$

$$=\int_{-\infty}^{\infty} e^{-K\xi_n^2}d\xi_n\int_{-\infty}^{\infty} e^{-K\xi^2_{n-1}}d\xi_{n-1}\cdots\cdots\int_{-\infty}^{\infty} e^{-K\xi_1^2-K\{(n+1)s-\xi_1-\xi_2\cdots\cdots\}^2}d\xi_1.$$

Therefore, the probability is given by

$$U=\sqrt{n+1}\sqrt{\frac{K}{\pi}}e^{-K(n+1)s^2}.$$

Next, we consider the summation

$$S=\sum_{i=0}^{\infty}\left\{\frac{x_i+x_{i+1}+\cdots\cdots+x_{i+n}}{n+1}-X\right\}$$

$$=\frac{1}{n+1}\sum_{i=0}^{\infty}\left\{\xi_i+\xi_{i+1}+\cdots\cdots+\xi_{i+n}\right\} .$$

The sum S is regarded as the sum of m terms of s, where $(n+1)s=\left|\sum_{K=0}^{\infty}\xi_{i+K}\right|$, i. e,

$$S=m.|s|.$$

The value of $\overline{|\eta|}$, which is the subject of our present investigation, is the given by

$$\overline{|\eta|}=\lim_{N\to\infty}\frac{S}{N}=\lim_{N=\infty}\frac{m|S|}{N} .$$

Since

$$\lim_{N\to\infty}\frac{m}{N}=U ,$$

the value of $\overline{|\eta|}$ becomes

$$\overline{|\eta|}=2\sqrt{n+1}\sqrt{\frac{K}{\pi}}\int_0^{\infty}se^{-K(n+1)s^2}ds=\frac{1}{\sqrt{\pi}}\frac{1}{\sqrt{K(n+1)}} \quad \cdots\cdots\cdots(3)$$

in which $(n+1)$ corresponds to τ, the time interval of overlapping mean for smoothing the fluctuation curves.

The mean deviations $\overline{|\eta|}$ are thus shown as being dependent on the interval of time for the overlapping τ. The variations in the mean deviations $\overline{|\eta|}$ with respect to τ are as shown in Figs. 8∼12. The curves showing the variations in $\overline{|\eta|}$ are very similar to those which may be expected from equation (3). This fact may suggest that the disturbances that cause variations in the heights of the yearly mean sea-levels are not periodic, and occur fortuitously.

The mean error for the single value of the yearly mean sea-level is then given theoretically by

$$\text{Mean Error}=\frac{1}{\sqrt{2}}\cdot\frac{1}{K} ,$$

where K is the precision. Since K is connected with $\overline{|\eta|}$, the mean deviation, by the equation (3), the mean error will be

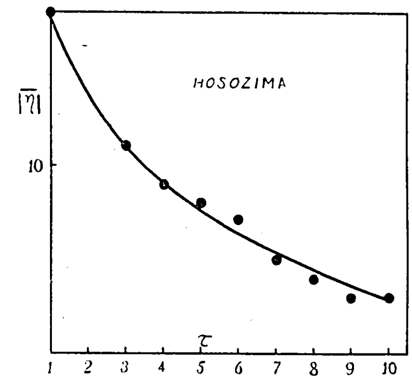

Fig. 8. Variation in $|\overline{\eta}|$ with respect to τ. (Hosozima.).

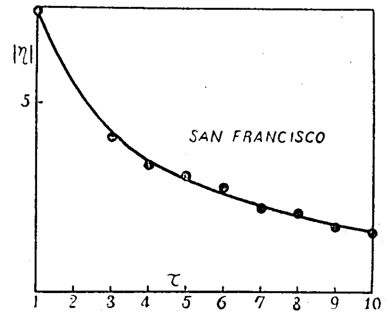

Fig. 11. Variation in $|\overline{\eta}|$ with respect to τ. (San Francisco.).

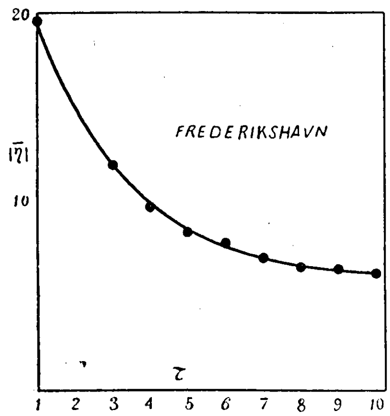

Fig. 9. Variation in $|\overline{\eta}|$ with respect to τ. (Frederikshavn.).

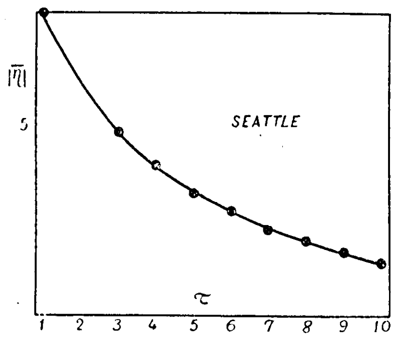

Fig. 12. Variation in $|\overline{\eta}|$ with respect to τ. (Seattle.).

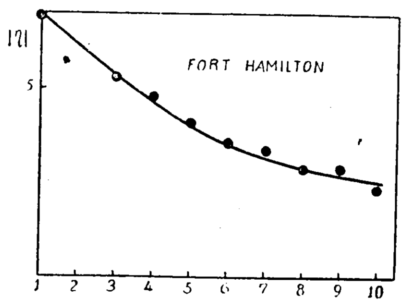

Fig. 10. Variation in $|\overline{\eta}|$ with respect to τ. (Fort Hamilton.).

$$\text{Mean Error} = \sqrt{\frac{\pi}{2}} \cdot |\overline{\eta}|.$$

Since the values of the mean deviations $|\overline{\eta}|$ for the case $\tau=1$ are about 20 mm, the yearly mean sea-level may be regarded as being variable within about 20 mm.

For the data of Japanese stations, however, the fluctuations in the height of the yearly mean sea-level with shorter periods may not be due merely to irregular meteorological and astronomical disturbances, seeing that earth's crust in Japan at present is now moving rather markedly in vertical directions.

The fluctuations in the height of the vearly mean sea-level in Japan are, therefore, in most cases, regarded as the sum of those due to astronomical and meteorological disturbances, and crustal deformations.

5. THE 18·6 YEAR'S PERIOD VARIATION IN THE YEARRLY MEAN SEA-LEVEL. As already mentioned in § 1, the variations in the heights of the yearly mean sea-levels do not clearly show an 18·6 years' period. This point will now be considered in greater detail.

As the variations in the heights of the yearly mean sea-level at Seattle seem to be somewhat periodic,[10] as will be seen from Fig. 5, the amplitude of the periodic variations with a period of 18·6 years was determined by using the data of Seattle by the method of least squares. The amplitude of the 18·6 years' period variation thus determined is about 2·3 units (1/100 foot) or about 7 mm which is much smaller than the value of the mean deviation already obtained.

Since, actually, the periodic fluctuation, if any, may be superposed by irregular fluctuations, it may perhaps be worth while to refer to the case in which the actual deviation ξ is composed of a term due to variation with a period of 18·6 years and that due to irregular fluctuations.

Putting

$$\xi_i = a\sin\left(\frac{2\pi}{T}t_i + q\right) + \varepsilon_i\,,$$

where ε_i on the right-hand side of the equation is the term due to the irregular variations.

We calculate the mean value of

$$\xi_i^2 = a^2\sin^2\left(\frac{2\pi}{T}t_i + q\right) + \varepsilon_i^2 + 2a\varepsilon_i\sin\left(\frac{2\pi}{T}t_i + q\right),$$

i. e.,

$$\overline{\xi^2} = \frac{\Sigma\xi_i^2}{N} = a^2\frac{\sin^2\left(\frac{2\pi}{T}t_i + q\right)}{N} + \frac{\varepsilon_i^2}{N} + 2a\frac{\Sigma\varepsilon_i\sin\left(\frac{2\pi}{T}t_i + q\right)}{N}.$$

10) Variations in the heights of the yearly mean sea-levels with long periods are not noticed in the data of other stations, which may be one of the evidences for the conclusion that variations in the heights of the yearly mean sea-levels with a period of 18·6 years are practically negligible.

When N is very large, the expression $\dfrac{\Sigma \sin^2\left(\frac{2\pi}{N}t_i+q\right)}{N}$ may be substituted with an integral

$$\frac{4}{T}\int_0^{\frac{T}{4}} \sin^2\left(\frac{2\pi}{T}t+q\right)dt.$$

Since

$$\frac{4}{T}\int_0^{\frac{T}{4}} \sin^2\left(\frac{2\pi}{T}t+q\right)dt=\frac{1}{2},$$

and

$$\left|\frac{\sum \varepsilon_i \sin\left(\frac{2\pi}{T}t+q\right)}{N}\right| \xrightarrow[N\to\infty]{} 0,$$

we have

$$\overline{\xi^2}=\frac{a^2}{2}+(\text{Standard deviation})^2$$

The actual values of $\sqrt{\overline{\xi^2}}$ for the various mareograph stations under consideration are shown in Table II.

TABLE II.

Station	Hosozima	Frederikshavn	Fort Hamilton	San Francisco	Seattle
$\sqrt{\overline{\xi^2}}_{,\tau=3}$	14 (mm)	14·6 (mm)	6·38 ($\frac{1}{100}'$)	5·26 ($\frac{1}{100}'$)	5·84 ($\frac{1}{100}'$)

The amplitude of variations in the heights of the yearly mean sea-level with a period of 18·6 years is, if any, of the order of 7 mm at most, as obtained above from the data for Seattle. Since, from the data of most stations, the order of magnitude of the actual $\sqrt{\overline{\xi^2}}$ is about 20 mm, the periodic variations, i. e., variation in the heights of the yearly mean sea-level with a period of 18·6 years, can affect the value of $\overline{\xi^2}$ but slightly.

The results given in Table III below, suggests likewise. This Table gives the values of the ratios $\dfrac{\overline{|\xi|}}{\sqrt{\overline{\xi^2}}}$ for the various stations, which must be $\sqrt{\dfrac{2}{\pi}}=0{\cdot}8$, so that the fluctuations are quite fortuitous.

TABLE III.

Station	Hosozima	Frederiks-havn	Fort Hamilton	San Francisco	Seattle
$\lvert\xi\rvert/\sqrt{\xi^2}$	0·78	0·81	0·78	0·83	0·84

6. CHANGES IN THE HEIGHT OF THE YEARLY MEAN SEA-LEVEL AT ABURATUBO. The Aburatubo mareograph station was levelled a number fo times since 1923 with reference to the standard datum point at Tokyo. An important point in our present study is that the changes in the height of the Aburatubo bench-mark do not agree well with those deduced from the data of the yearly mean sea-level for the same place.

In discussing fluctuations in the heights of the yearly mean sea-level at Aburatubo, data obtained before the earthquake of 1923 are excluded, because the land level underwent marked elevation at the time of the earthquake with the result that the reference surface for the sea-level has changed.

The data of the yearly mean sea-level and the vertical displacements of the bench-mark measured with reference to the standard datum point of Tokyo in 1930, 1934, and 1937 are given in Table IV.

TABLE IV.

Year	Y. M. Sea-level	Level (ref. Tokyo)	Year	Y. M. Sea-level	Level (ref. Tokyo)
1925	0 mm	0·0 mm	1931	− 7 mm	
1926	26		1932	−21	
1927	29		1933	−43	
1928	7		1934	−13	−81·4
1929	−10		1935	−62	
1930	−21	+ 1·7	1936	−53	−17·5

In Fig. 13, the deviations in the heights of the yearly mean sea-level, as measured from the yearl mean sea-level in 1925, together with the vertical displacements compared with the height measured in 1925 are plotted against the time by symbols ● and × respectively.

It is of course difficult to separate the term for fluctuation due to crustal deformation from that of fluctuations due to astronomical and meteorological disturbances in the data given above.

For trial, we shall assume that a smooth curve, joining the points representing the vertical displacements in Fig. 13, represents the change in the height of the land level at Aburatubo with respect to time, the reference being the standard datum point of Tokyo. Then the diffe-

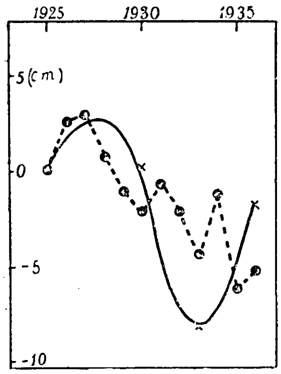

Fig. 13. Vertical displacement of the land at Aburatubo.
● : Deduced from mareogram data. × : Deduced from result of precise levels, referring to the standard datum point at Tokyo.

rences between the actual values of the yearly mean sea-level and those estimated from the hypothetical curve of the vertical displacements may be regarded as fluctuations due to astronomical and meteorological disturbances. In the case, the changes in the heights of the standard datum point, or those in the heights of the yearly mean sea-level at Tokyo may seriously affect the amplitude of the fluctuations.

The yearly mean sea-levels of Tokyo, the data of which are given in Table V together with the fluctuations in the heights of the yearly mean sea-level at Aburatubo, do not show any marked systematic variation.[11]

TABLE V.
(Y. M. Sea-level at Tokyo)

Year	Y. M. Sea-level	Year	Y. M. Sea-level
	m		m
1924	1·133	1930	1·132
1925	1·140	1831	1·143
1926	1·125	1932	1·139
1927	1·135	1933	1·147
1928	1·138	1934	1·170
1929	1·123	1935	1·208

TABLE VI.

Year	Fluctuations	Year	Fluctuations
1913	− 9 mm	1925	0 mm
1914	−13	1926	+ 8
1915	0	1927	+11
1916	−15	1928	−15
1917	25	1929	−27
1918	13	1930	−21
1919	8	1931	+20
1920	−36	1932	+55
1921	−17	1933	+40
1922	10	1934	+60
1923	12	1935	−12
1924	?	1936	−32

11) The upheaval of the mean sea-level in 1934 and 1935 was somewhat remarkable, which however cannot be regarded as being due merely to depression of the land.

The curve in Fig. 13, showing the hypothetical earth movements at Aburatubo, may therefore show approximately the crustal deformation, although the fluctuations of shorter periods, if any, are smoothed out.

The fluctuations in the heights of the yearly mean sea-levels during 1925–1936 are slightly larger than those during 1900∼1923, the data of which are given in Table VI.

The values given in the table above are the differences between the actual and the calculated values of the heights of the yearly mean sea-level. The general trend of the curve showing the chronic changes in the heights of the yearly mean sea-level during 1900∼1923 is expressed by

$$H_t = 91{\cdot}3 - 136{\cdot}4t + 38{\cdot}5t^2 \text{[12]} \quad \text{(mm)}.$$

from which the heights of the yearly mean sea-level were calculated and the results compared with the actual values.

The mean deviation in the heights of the yearly mean sea-level of Aburatubo during 1925∼1936, given in Table V, is about 25 mm, which is slightly larger than those obtained for the foreign mareograph stations. This may partly be due to the assumption that the smooth curve in Fig. 13 represents the actual crustal deformation. If we were to take a more complex curve to represent the crustal deformation, the mean deviation could be made smaller so that it may more closely approach the ordinary value obtained above.

The fact that the durations of time for which the yearly mean sea-level are given was small, may also explain why the value of the mean deviation is slightly larger.

As found above, the fluctuations in the heights of the yearly mean sea-levels is about 20 mm on the average. Hence the discordance, amounting to about 20 mm, between the vertical displacements obtained by precise levelling and those deduced from data of the yearly mean sea-level, may sometimes be attributed to irregular fluctuations in the astronomical and meteorological disturbances.

In conclusion, the writers wish to express their sincere thanks to Dr. Tosio Uno for his numerous valuable suggestions. Their thanks are also due to Miss C. Setoyama for her assistance in calculating the numerical results in the tables given in this paper.

12) The constants in this formula were determined by the method of least squares.

Appendix Table.

Heights of Yearly Mean Sea-level.

Year A. D.	(Japan) Hosozima	(Denmark) Frederikshavn D. N. N.	(U. S. A.) Fort Hamilton	(U. S. A.) San Francisco	(U. S. A.) Seattle
1893		42 mm.	4·94 feet		
1894		3	4·98		
1895		−11	4·91		
1896		−20	4·99		
1897		−27	5·06		
1898		29	5·08	8·30 feet	
1899	m	30	5·07	8·44	3·94 feet
1900	2·557	− 5	5·93	8·50	4·10
1901	2·565	−48	5·16	8·46	3·88
1902	2·576	−21	5·20	8·57	4·06
1903	2·556	50	5·13	8·53	3·99
1904	2·577	− 7	4·98	8·63	4·10
1905	2·555	1	4·94	8·65	4·08
1906	2·574	12	5·05	8·58	4·11
1907	2·598	− 1	5·00	8·66	4·17
1908	2·587	−33	4·97	8·43	4·08
1909	2·603	8	5·10	8·53	4·12
1910	2·623	9	5·16	8·42	3·98
1911	2·612	14	5·11	8·61	3·96
1912	2·596	30	4·93	8·49	3·96
1913	2·583	44	4·98	8·51	3·96
1914	2·584	42	5·11	8·81	4·16
1915	2·555	−15	5·16	8·69	4·23
1916	2·550	21	5·12	8·52	4·08
1917	2·648	−17	5·15	8·48	3·88
1918	2·657	−24	5·13	8·70	4·18
1919	2·604	−34	5·30	8·61	4·08
1920	2·626	2	5·23	8·54	3·99
1921	2·633	−14	5·24	8·58	4·02
1922	2·602	−31	5·14	8·55	3·94
1923	2·590	13	5·15	8·52	4·00
1924	2·584	−22	5·10	8·42	3·90
1925	2·578	8	5·05	8·62	4·10
1926	2·632	− 1	5·03	8·63	4·11
1927	2·620	12	5·18	8·60	4·14
1928	2·577	2	5·04	8·58	4·08
1929	2·605	−13	5·00	8·55	3·89
1930	2·602	−26	5·06	8·72	4·08
1931	2·594	−14	5·30	8·62	4·10
1932	2·575		5·30	8·68	4·12
1933	2·560			8·52	4·13
1934	2·642			8·63	4·17
1935	2·582			8·72	4·13

7. 年平均潮位のフラクチュエーション

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日本，アメリカ，デンマークの各地にある數個所の驗潮場につき，年平均潮位の變化を調べたその結果を概括すれば，

（i） 長期に亘る緩やかな變化——主として土地の變動と考へられる——の外に不規則な昇降運動が重つてゐる

（ii） この不規則な變動を起す原因として種々なものが考へられるが，其等を分離することは却々困難であるから，一先づ，一括して，時間的に不規則な分布をなす原因によつて生ずるものと考へる．何等か週期的な變化があるかにも思はれるので，色々の方法で檢したけれども，明瞭な週期的變化を取り出すことは出來なかつた．

（iii） 約 19 年を週期とする變動は，天文學的に豫期される所であるが，實際の資料から斯の如き週期を持つ變動を見出すことは困難である．

（iv） 油壺の驗潮記錄に表はれた所の土地の昇降と思はれるものと，東京を不動とした時の油壺の水準點の昇降との間には若干の不一致がある．これは，東京の基準點をそれだけ動かせば簡單に解消する．併し，不規則な原因による年平均潮位の變動があつて，20 mm 程度の範圍內で正しく年平均潮位を定めることは困難であるから，簡單に東京の基準點の高さを變更してよいか，否か再考の餘地がある．

尙，本文中，年平均潮位が單に不規則な原因によつて變化するとしたが，この點に關しては，野滿博士や，寺田，山口兩博士の研究等もある如く，幾分かは知られてゐる點もある．併し，實際の場合には，鹽分，溫度，海流，低氣壓等の 1～2 のもののみでは說明しきれないものがある．今後，尙，研究を進めるつもりである．