11. *Notes on the Origin of Earthquakes. (Fifth paper.)*

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1. Introduction.

In his previous paper[1] the writer discussed the conditions supposed to prevail within the seismic focus as based on observations so far made in connexion with the initial phase of seismic waves, and arrived at the conclusion that the maximum shear stresses, which might be constant within the seismic focus, reaches the limit of ultimate strength of the material immediately preceeding the earthquake.

In that paper, the writer treated the problem without taking into consideration the external and internal forces that bring about the earthquake.

In the following paragraphs, he attempts to make clear the forces that bring about the condition that the maximum shear stresses within the seismic focus are constant.

2. The Conical Type.

(a) It is supposed that the elastic body, which is stressed in one direction, has a spherical grain imbedded in it.

The problem has already been solved by Prof. K. SEZAWA and Dr. G. Nishimura.[2]

The boundary conditions, assuming the radius of the grain to be a, are

at $$r=\infty\,; \qquad \widehat{rr}=T\cos^2\theta, \qquad \widehat{\theta\theta}=T\sin^2\theta,$$

where T may be either uniform tension or uniform compression; and at $r=a$ the stresses and displacements are continuous.

The stresses in the internal portion are then expressed by

$r\leqq a$:

$$\widehat{rr'}=T\left[\frac{(\lambda+2\mu)}{(3\lambda+2\mu)}\,\frac{(3\lambda'+2\mu')}{(3\lambda'+2\mu'+4\mu)}+4\mu'\frac{D_1}{D}P_2(\cos\theta)\right],$$

1) Win INOUYE, *Bull. Earthq. Res. Inst.*, **15** (1937), 686.

2) K. SEZAWA and G. NISHIMURA, *Bull. Earthq. Res, Inst.*, **7** (1929), 389.

$$\widehat{\theta\theta'}=T\left[\frac{(\lambda+2\mu)}{(3\lambda+2\mu)}\frac{(3\lambda'+2\mu')}{(3\lambda'+2\mu'+4\mu)}+2\mu'\frac{D_1}{D}-4\mu'\frac{D_1}{D}P_2(\cos\theta)\right],$$

$$\widehat{\phi\phi'}=T\left[\frac{(\lambda+2\mu)}{(3\lambda+2\mu)}\frac{(3\lambda'+2\mu')}{(3\lambda'+2\mu'+4\mu)}+2\mu'\frac{D_1}{D}\right],$$

$$\widehat{r\theta'}=T\left[2\mu'\frac{D_1}{D}\frac{dP_2(\cos\theta)}{d\theta}\right],$$

where λ, μ are the elastic constants of the external medium and λ', μ' are those of the internal grain, while

$$D=\begin{vmatrix} \frac{\lambda'}{7}, & 4\mu', & -\frac{9\lambda+10\mu}{3}, & -24\mu \\ -\frac{8\lambda'+7\mu'}{21}, & 2\mu', & \frac{3\lambda+2\mu}{6}, & 8\mu \\ -\frac{\lambda'}{7\mu'}, & 2, & \frac{3\lambda+5\mu}{6\mu}, & 3 \\ -\frac{5\lambda'+7\mu'}{42\mu'}, & 1, & \frac{1}{6}, & -1 \end{vmatrix},$$

$$D_1=\begin{vmatrix} \frac{\lambda'}{7}, & \frac{2}{3}, & -\frac{9\lambda+10\mu}{3}, & -24\mu \\ -\frac{8\lambda'+7\mu'}{21}, & \frac{1}{3}, & \frac{3\lambda+2\mu}{6}, & 8\mu \\ -\frac{\lambda'}{7\mu'}, & \frac{1}{3\mu}, & \frac{3\lambda+5\mu}{6\mu}, & 3 \\ -\frac{5\lambda'+7\mu'}{42\mu'}, & \frac{1}{6\mu}, & \frac{1}{6}, & -1 \end{vmatrix}.$$

The maximum shear stress at any point is given by

$$\widehat{r\theta'}^2+\left(\frac{\widehat{rr'}-\widehat{\theta\theta'}}{2}\right)^2=T^2\left(3\mu'\frac{D_1}{D}\right)^2=K^2.$$

Thus, in the spherical grain, the condition that the maximum shear stresses are constant is fulfilled.

As will be seen in the expressions of the stresses, there are two terms; the one which has no azimuthal differences, and the other which has azimuthal differences in the form $P_2(\cos\theta)$.

The former term, that is the $P_0(\cos\theta)$ term, makes the maximum shear stress zero. In other words, the $P_0(\cos\theta)$ term has no part in the value of the maximum shear stress at any point in the spherical grain.

The latter term, that is the $P_2(\cos\theta)$ term, is derived from the displacements, independent of dilatation (Δ) and rotation (ϖ); and it only satisfies the condition of the maximum shear stress. These facts were already noted in the previous paper.

(b) We next consider the stress distribution in the interior of a spherical inclusion in a gravitating semi-infinite elastic solid, assuming that the state of the gravitating semi-infinite medium is one of plane strain.

This problem has already been solved by Dr. G. Nishimura and Mr. T. Takayama.[3)]

Let u, v be the components of displacement in the directions of radius r, co-latitude θ, and $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{\phi\phi}$ the normal components of traction, $\widehat{r\theta}$ the shearing component of stress in the outer medium. And let u', v' be the radial and co-latitudinal components of displacement, and $\widehat{rr}'$, $\widehat{\theta\theta}'$, $\widehat{\phi\phi}'$ the normal components of stress, $\widehat{r\theta}'$ the shearing component of stress in the spherical inclusion, the radius of which is a.

The boundary conditions of the present problem are:

At $$r=a;\quad u=u',\quad v=v',\quad \widehat{rr}=\widehat{rr}',\quad \widehat{r\theta}=\widehat{r\theta}'.$$

Secondly, the displacement and the stress in the whole of the space in the medium far from that point where spherical inclusion is, are the same as those found in solving the problem relating to a gravitating semi-infinite elastic solid without any heterogeneous matter within it.

If we assume $\xi\gg a$, where ξ is the depth of the centre of the inclusion from the free surface, the stresses in the inclusion are given by

$r\leqq a$:—

$$\widehat{rr}'=\left(\lambda'+\frac{2\mu'}{3}\right)D_0P_0(\cos\theta)+4\mu'E_2P_2(\cos\theta)\,,$$

$$\widehat{\theta\theta}'=\left(\lambda'+\frac{2\mu'}{3}\right)D_0P_0(\cos\theta)+4\mu'E_2P_2(\cos\theta)+2\mu'E_2\frac{\partial^2P_2(\cos\theta)}{\partial\theta^2}\,,$$

$$\widehat{\phi\phi}'=\left(\lambda'+\frac{2\mu'}{3}\right)D_0P_0(\cos\theta)+4\mu'E_2P_2(\cos\theta)+2\mu'E_2\cot\theta\frac{\partial P_2(\cos\theta)}{\partial\theta}\,,$$

$$\widehat{r\theta}'=2\mu'E_2\frac{\partial P_2(\cos\theta)}{\partial\theta}\,,$$

3) G. NISHIMURA and T. TAKAYAMA, *Bull. Earthq. Res. Inst.*, **11** (1933), 196.

where

$$D_0 = -\frac{3\rho g\xi}{(4\mu+3\lambda'+2\mu')},$$

$$E_2 = -\frac{5\rho g\xi\mu}{\{3\lambda(3\mu+2\mu')+2\mu(7\mu+8\mu')\}},$$

while λ, μ are the Lame's elastic constants in the outer medium, and λ', μ' those in the spherical inclusion,

ρ is the density of the outer medium, and

g is the acceleration due to gravity.

The $P_0(\cos\theta)$ term in the expressions of the stresses has no part in the value of the maximum shear stress at any point in the inclusion; while the $P_2(\cos\theta)$ term, which is derived from the displacements, independent of dilatation and rotation, satisfies the condition that the maximum shear stresses are constant throughout the inclusion.

Therefore, the maximum shear stress at any point in the spherical inclusion is given by

$$\widehat{r\theta}'^2 + \left(\frac{\widehat{rr}'-\widehat{\theta\theta}'}{2}\right)^2 = (\rho g\xi)^2\left(\frac{15\mu\mu'}{3\lambda(3\mu+2\mu')+2\mu(7\mu+8\mu')}\right)^2 = K^2.$$

3. The Quadrand Type.

Now, we shall study the case where the elastic body that contains a spherical inclusion is subjected to a uniform simple shear.

The problem has already been solved by Dr. G. Nishimura.[4]

Let a be the radius of the spherical inclusion, λ', μ' the Lame's elastic constants, u', v', w' the components of displacement, and $\widehat{rr}'$, $\widehat{\theta\theta}'$, $\widehat{\phi\phi}'$, $\widehat{r\theta}'$, $\widehat{r\phi}'$, $\widehat{\theta\phi}'$ the components of stress in the inclusion, and let the symbols without dash represent those in the outer medium.

The boundary conditions are then as follows.

At $r=a$;

$$u=u', \qquad v=v', \qquad w=w',$$
$$\widehat{rr}=\widehat{rr}', \quad \widehat{r\theta}=\widehat{r\theta}', \quad \widehat{r\phi}=\widehat{r\phi}',$$

and at $r=\infty$;

$$\widehat{rr}=S\sin^2\theta\sin2\phi, \qquad \widehat{r\theta}=\frac{S}{2}\sin2\theta\sin2\phi,$$

$$\widehat{\theta\theta}=S\cos^2\theta\sin2\phi, \qquad \widehat{r\phi}=S\sin\theta\cos2\phi,$$

$$\widehat{\phi\phi}=-S\sin2\phi, \qquad \widehat{\theta\phi}=S\cos\theta\cos2\phi,$$

4) G. NISHIMURA, *Kaheigakkaisi*, **25** (1931), 191.

where S is the shearing stress applied to the elastic body.

The stresses in the spherical inclusion are given by[5)]

$r \leqq a$:—

$$\widehat{rr}' = 12\mu' E \sin^2\theta \sin 2\phi \,,$$
$$\widehat{\theta\theta}' = 12\mu' E \cos^2\theta \sin 2\phi \,,$$
$$\widehat{\phi\phi}' = -12\mu' E \sin 2\phi \,,$$
$$\widehat{r\theta}' = 6\mu' E \sin 2\theta \sin 2\phi \,,$$
$$\widehat{r\phi}' = 12\mu' E \sin\theta \cos 2\phi \,,$$
$$\widehat{\theta\phi}' = 12\mu' E \cos\theta \cos 2\phi \,,$$

where

$$E = \frac{5}{4} \cdot \frac{(\lambda + 2\mu) S}{\{\mu(9\lambda + 14\mu) + 2\mu'(3\lambda + 8\mu)\}} \,.$$

The stresses shown above are derived from the displacement that satisfies $\Delta = 0$, $\varpi_r = 0$, $\varpi_\theta = 0$, and $\varpi_\phi = 0$.

The maximum shearing stress at any point in the spherical inclusion is given by

$$\frac{1}{2}\left[\left(\frac{\widehat{rr}' - \widehat{\theta\theta}'}{2}\right)^2 + \left(\frac{\widehat{\theta\theta}' - \widehat{\phi\phi}'}{2}\right)^2 + \left(\frac{\widehat{\phi\phi}' - \widehat{rr}'}{2}\right)^2\right]$$
$$+ \frac{3}{4}(\widehat{r\theta}'^2 + \widehat{\theta\phi}'^2 + \widehat{\phi r}'^2) = 108\mu'^2 E^2 \equiv K^2 \,.$$

The condition that the maximum shearing stress is constant throughout the spherical inclusion is now fulfilled.

4. Remarks.

In the preceeding two paragraphs, the writer studied the forces that bring about an earthquake and treated the problem according to the ordinary theory of elasticity. Nevertheless, the condition that the maximum shearing stress is constant throughout the seismic focus suggests that the seismic focus may be in a plastic state.

Thus, it seems to the writer that seismic waves may be generated as the result of changes in stress in the elastic medium about the seismic focus due to plastic yielding within the said seismic focus through the action of external forces.

As the external forces that are operating in the earth's crust, we

5) The writer made some corrections in the calculations carried out by Dr. G. Nishimura.

may take the compressing stresses prevailing in the orogenic events, the forces due to gravity, and the forces operating from the magma reservoirs.

The variations in temperature and the changes in the state of those substances that are within the seismic focus may play an important part in the plastic yielding within the seismic focus.

5. Summary.

The writer, after a study of the forces that bring about an earthquake, concludes that seismic waves may be generated as the result of changes in stress in the elastic medium about the seismic focus due to plastic yielding within the said focus under the action of certain external forces.

In concluding this short note, the writer's cordial thanks are due to Prof. Ch. Tsuboi and Dr. G. Nishimura for their encouragement and advices.

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11. 發震機構に就いて (第5報)

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第4報に於て初動の“壓し波”“引き波”の地理的分布に二種類ある事から，地震の發生する直前に於ては震源の全域に亘り，最大剪應力が彈性の破壞限度に達してゐるのではないかと云ふ事を論じて居いた.

其の際には，所謂境界條件を問題にしなかつたので此の報告では外力を考へ境界條件を合してみた.

其の結果球狀の異物體を含む一樣な彈性體內に重力の作用して居る場合に於ても，一樣な壓縮力或は伸延力が作用した場合に於ても，或は又單純な剪應力が作用した場合に於ても其の異物體內では各點の最大剪應力が一定の値を取る事を知つた.

從つて此等の力が或る限度を越へ從つて異物體內の最大剪應力が彈性の破壞限度に達するか，或は又此等の力の作用下に於て異物體內の物理化學的條件の變化に依つて物質の彈性が減じ此の部が所謂可塑性變形をするならば，其の周圍の彈性體內の彈性應力が急激に變化して彈性波を生じ得るものと考へられる.

此の場合の異物體の占める部分が震源に相當しその附近より生ずる彈性波を地震波と考へて差支へない樣である.