55. *Prevalent Periods of Oscillation in Tidal Waves.*

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1. *Introduction.*

The tidal waves (tunami) that accompanied the earthquake of March 3, 1933, were transmitted through the Pacific Ocean and reached various stations along the coast surrounding that ocean. With respect to these tidal waves, there is, however, some uncertainty as to whether, in the train of waves, it is the beginning phase or the phase of the largest amplitude or any other, that was really transmitted with velocity $\sqrt{g\eta}$, η being the mean depth of the part of the ocean through which the tidal waves were transmitted.

The oscillatory features of sea waves have been discussed by a number of mathematiceans, such as Lord Kelvin,[1] Lord Rayleigh,[2] Lamb,[3] Pidduck,[4] and Terazawa,[5] but their problems mainly concerned the case wherein the waves are produced by a single impulse. Solutions of such a case are not likely to determine the relation between the amplitudes of surface waves due to periodic forces and the ratio of the length of the generated waves to the depth of the sea. Our last investigation[6] with respect to Love-waves suggests that the treatment used therein also applies, in principle, to the present case. It was thus possible for us to find the relation in the case of the tidal waves under consideration.

2. *Tidal waves caused by periodic change of surface pressure.*

Since the nature of tidal waves is almost the same irrespective of whether the disturbance is applied to the sea bed or to the free surface, we shall now consider the case in which a periodic pressure is applied to the free surface.

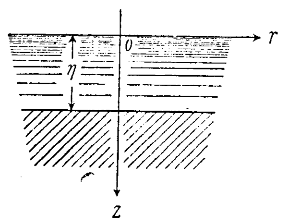

Fig. 1.

In the case of a two-dimensional propagation of gravity waves, the differential equa-

1) Lord KELVIN, *Proc. Roy. Soc.*, **42** (1887), 80~83.
2) Lord RAYLEIGH, *Phil. Mag.*, (6), **43** (1909), 1~6.
3) H. LAMB, *Proc. Math. Soc.*, (2), **2** (1904), 371~400; *Hydrodynamics*, IX.
4) F. B. PIDDUCK, *Proc. Roy. Soc.*, **83** (1910), 347~356.
5) K. TERAZAWA, *Proc. Roy. Soc.*, **92** (1915), 57~81.
6) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **15** (1937), 577~581.

tion assumes the form

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=0. \tag{1}$$

If the origin of z is on the free surface of the water, the solution of (1) is

$$\phi=Ce^{-ipt}\cosh f(z-\eta)J_0(fr), \tag{2}$$

where η is the depth of the water. Let u, w be the horizontal and vertical displacements of a water particle, then

$$\frac{\partial u}{\partial t}=-\frac{\partial\phi}{\partial r},\qquad \frac{\partial w}{\partial t}=-\frac{\partial\phi}{\partial z}, \tag{3}$$

whence

$$u=\frac{if}{p}e^{-ipt}C\cosh f(z-\eta)J_1(fr), \tag{4}$$

$$w=-\frac{if}{p}e^{-ipt}C\sinh f(z-\eta)J_0(fr). \tag{5}$$

Since

$$\frac{p_1}{\rho}=\frac{\partial\phi}{\partial t}-gw \tag{6}$$

on the free surface $z=0$, we have

$$p_1=-i\rho Ce^{-ipt}\frac{p^2\cosh f\eta-fg\sinh f\eta}{p}J_0(fr). \tag{7}$$

From Fouriers' integral

$$p_1=e^{-ipt}\int_0^\infty J_0(fr)fdf\int_0^\infty F(\alpha)J_0(f\alpha)\alpha d\alpha, \tag{8}$$

where $F(r)$ is the pressure distribution on the free surface. We shall take the case of a pressure distribution such that

$$e^{-ipt}F(r)=\frac{e^{-ipt}}{\sqrt{c^2+r^2}}, \tag{9}$$

then, since

$$\int_0^\infty\frac{J_0(f\alpha)\alpha d\alpha}{\sqrt{c^2+\alpha^2}}=\frac{e^{-fc}}{f},\text{ [7]} \tag{10}$$

7) K. TERAZAWA, *loc. cit.* 6).

we have

$$p_1 = e^{-ipt}\int_0^\infty e^{-fc}J_0(fr)\,df. \tag{11}$$

From (7) and (11)

$$C = -\frac{p_1}{i\rho}\int_0^\infty \frac{e^{-fc}df}{p^2\cosh f\eta - fg\sinh f\eta}. \tag{12}$$

From (4), (5), (12) we have

$$u = \frac{-e^{-ipt}}{\rho}\int_0^\infty \frac{fe^{-fc}\cosh f(z-\eta)}{p^2\cosh f\eta - fg\sinh f\eta}J_1(fr)\,df, \tag{13}$$

$$w = \frac{e^{-ipt}}{\rho}\int_0^\infty \frac{fe^{-fc}\sinh f(z-\eta)}{p^2\cosh f\eta - fg\sinh f\eta}J_0(fr)\,df. \tag{14}$$

3. *An alternative method of deducing the displacement expression, and additional results.*

The result shown in the preceding section may be obtained alternatively. Let an idealised centre of applied pressure be at $r=0$, $z=-c$. The pressure that diminishes inversely as the radial distance from that centre is then expressed by

$$p_0 = \frac{e^{-ipt}}{\sqrt{r^2+(z+c)^2}}. \tag{15}$$

(i) In the case wherein the pressure is applied to the free surface, we have $c>0$, the expression of the pressure on that surface then being

$$p_0 = \frac{e^{-ipt}}{\sqrt{r^2+c^2}}, \tag{16}$$

which is of the same form as that in (9).

(ii) In the case of a subaqueous disturbance, $0>c>-\eta$. Then, putting $c=-c_1$, we obtain

$$p_0 = \frac{e^{-ipt}}{\sqrt{r^2+(z-c_1)^2}}. \tag{17}$$

(iii) When the pressure is applied to the sea bed, $-\eta>c$ and p_0

is again of the same form as (17).

We shall first consider the case (i). In this case

$$\frac{e^{-ipt}}{\sqrt{r^2+(z+c)^2}}=\frac{e^{-ipt}}{R}=e^{-ipt}\int_0^\infty e^{-f(z+c)}J_0(fr)\,df. \tag{18}$$

The pressure transmission in water is, in idea, nothing more than wave transmission. If (15) or (17) be the initial waves, there are an infinite number of successive reflected waves (namely, the pressure) formed at the free surface as well as on the sea bed. Thus in Case (i) we obtain

$$p_0'=e^{-ipt}\int_0^\infty\Big[e^{-f(z+c)}+e^{f(z-c-2\eta)}-e^{-f(z+c+2\eta)}-e^{f(z-c-4\eta)}+\cdots\Big]J_0(fr)\,df$$

$$=e^{-ipt}\int_0^\infty\frac{e^{-f(z+c)}+e^{f(z-c-2\eta)}}{1+e^{-2f\eta}}J_0(fr)\,df=e^{-ipt}\int_0^\infty\frac{e^{-fc}\cosh f(z-\eta)}{\cosh f\eta}J_0(fr)\,df. \tag{19}$$

In case (ii) we have

$$\left.\begin{aligned}p_0'=\frac{e^{-ipt}}{\sqrt{r^2+(z-c_1)^2}}&=e^{-ipt}\int_0^\infty e^{-f(z-c_1)}J_0(fr)\,df, \quad [z-c_1>0]\\ &=e^{-ipt}\int_0^\infty e^{f(z-c_1)}J_0(fr)\,df, \quad [z-c_1<0]\end{aligned}\right\} \tag{20}$$

whence

$$p_0'=e^{-ipt}\int_0^\infty\Big[e^{-f(z-c_1)}+e^{f(z+c_1-2\eta)}-e^{-f(z-c_1+2\eta)}-e^{f(z+c_1-4\eta)}+\cdots$$

$$-e^{-f(z+c_1)}-e^{f(z-c_1-2\eta)}+e^{-f(z+c_1+2\eta)}+e^{f(z-c_1-4\eta)}-\cdots\Big]J_0(fr)\,df$$

$$=e^{-ipt}\int_0^\infty\frac{2\sinh fc_1\cosh f(\eta-z)}{\cosh f\eta}J_0(fr)\,df, \quad [z-c_1>0]$$

$$p_0'=e^{-ipt}\int_0^\infty\Big[e^{f(z-c_1)}-e^{-f(z+c_1)}-e^{f(z-c_1-2\eta)}+e^{-f(z+c_1+2\eta)}+e^{f(z-c_1-4\eta)}-\cdots$$

$$+e^{f(z+c_1-2\eta)}-e^{-f(z-c_1+2\eta)}-e^{f(z+c_1-4\eta)}+\cdots\Big]J_0(fr)\,df$$

$$=e^{-ipt}\int_0^\infty\frac{2\sinh fz\cosh f(\eta-c_1)}{\cosh f\eta}J_0(fr)\,df. \quad [z-c_1<0] \tag{21}$$

In Case (iii) we have

$$p_0'=e^{-ipt}\int_0^\infty\left[e^{f(z-c_1)}-e^{-f(z+c_1)}-e^{f(z-c_1-2\eta)}+e^{-f(z+c_1+2\eta)}+\cdots\right]J_0(fr)df$$

$$=e^{-ipt}\int_0^\infty\frac{e^{-(c_1-\eta)}\sinh fz}{\cosh f\eta}J_0(fr)df. \quad (22)$$

We shall particularly consider Case (i). From (6), gravity waves whose surface pressure is not zero, may assume the same forms as (4), (5). The surface pressure in such waves is then

$$p_1=-i\rho Ce^{-ipt}\frac{p^2\cosh f\eta-fg\sinh f\eta}{p}J_0(fr), \quad (7')$$

which must be equal to p_0' in (18), namely,

$$p_1=p_0', \quad (23)$$

so that

$$C=-\frac{p}{i\rho}\int_0^\infty\frac{e^{-fc}df}{p^2\cosh f\eta-fg\sinh f\eta}. \quad (12')$$

Whence it follows that

$$u=-\frac{e^{-ipt}}{\rho}\int_0^\infty\frac{fe^{-fc}\cosh f(z-\eta)}{p^2\cosh f\eta-fg\sinh f\eta}J_1(fr)df, \quad (13')$$

$$w=\frac{e^{-ipt}}{\rho}\int_0^\infty\frac{fe^{-fc}\sinh f(z-\eta)}{p^2\cosh f\eta-fg\sinh f\eta}J_0(fr)df. \quad (14')$$

(13′), (14′) are thus equivalent to (13), (14) respectively.

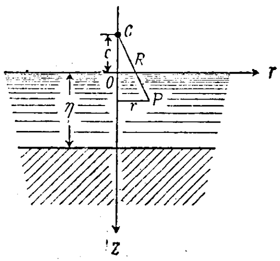

Fig. 2.

For the case in which the disturbance is applied to the sea bed or at any depth beneath the water surface, (4), (5) should be substituted in Bernoulli's equation, the expression of pressure thus formed being equated to that in (21) or (22) for a specified surface, say, $z=\eta$. A more convenient method of obtaining the same solution is, however, the one shown in one[8] of our previous papers.

8) K. SEZAWA, "Formation of Shallow-water Waves due to Subaqueous Shocks", *Bull. Earthq. Res. Inst.*, 7 (1929), 15∼40.

4. *Evaluation of the integrals for the waves, with results.*

We shall now evaluate (13), (14). Since

$$\left.\begin{aligned} J_0(fr) &= \frac{-i}{\pi}\int_0^\infty (e^{ifr\cosh k} - e^{-ifr\cosh k})\,dk, \\ J_1(fr) &= -\frac{1}{\pi}\int_0^\infty (e^{ifr\cosh k} + e^{-ifr\cosh k})\cosh k\,dk, \end{aligned}\right\} \tag{24}$$

u, w become

$$u = \frac{e^{-ipt}}{\pi\rho}\int_0^\infty \cosh k\,dk\int_0^\infty \frac{e^{-fc}(e^{ifr\cosh k}+e^{-ifr\cosh k})f\cosh f(z-\eta)}{p^2\cosh f\eta - fg\sinh f\eta}\,df, \tag{25}$$

$$w = -\frac{ie^{-ipt}}{\pi\rho}\int_0^\infty dk\int_0^\infty \frac{e^{-fc}(e^{ifr\cosh k}-e^{-ifr\cosh k})f\sinh f(z-\eta)}{p^2\cosh f\eta - fg\sinh f\eta}\,df, \tag{26}$$

so that

$$u_{z=0} = \frac{e^{-ipt}}{\pi\rho}\int_0^\infty \cosh k\,dk\int_0^\infty \frac{e^{-fc}(e^{ifr\cosh k}+e^{-ifr\cosh k})\,fdf}{p^2 - fg\tanh f\eta}, \tag{27}$$

$$w_{z=0} = \frac{ie^{-ipt}}{\pi\rho}\int_0^\infty dk\int_0^\infty \frac{e^{-fc}(e^{ifr\cosh k}-e^{-ifr\cosh k})\,fdf}{p^2\coth f\eta - fg}. \tag{28}$$

We must now evaluate the integrals of the following types

$$\int_0^\infty \frac{e^{-ic+ifr\cosh k}\,fdf}{p^2 - fg\tanh f\eta},\quad \int_0^\infty \frac{e^{-fc+ifr\cosh k}\,fdf}{p^2\coth f\eta - fg}, \qquad (29A),\ (29B)$$

$$\int_0^\infty \frac{e^{-fc-ifr\cosh k}\,fdf}{p^2 - fg\tanh f\eta},\quad \int_0^\infty \frac{e^{-fc-ifr\cosh k}\,fdf}{p^2\coth f\eta - fg}. \qquad (30A),\ (30B)$$

The integrals in (29A), (29B) are taken round the upper contour in Fig. 3 and those in (30A), (30B) round the lower contour in the same figure

Let the solutions of $p^2 - fg\tanh f\eta$ be $f=\alpha$, $f=i\beta_1$, $f=i\beta_2, \cdots$, then remembering that

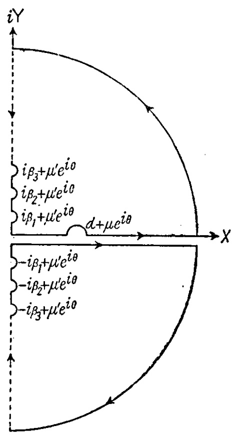

Fig. 3.

$$F(\alpha+\mu e^{i\theta})=F(\alpha)+\frac{F'(\alpha)}{1!}\mu e^{i\theta}+\frac{F''(\alpha)}{2!}(\mu e^{i\theta})^2+\cdots, \tag{31}$$

we obtain

$$(29\text{A})+(29\text{B})=\frac{-2\pi i\alpha e^{-\alpha c+i\alpha r\cosh k}}{g(\tanh\alpha\eta+\alpha\eta\,\text{sech}^2\alpha\eta)}-\int_0^\infty\frac{e^{-fr\cosh k}fdf}{p^2+fg\tan f\eta}2\cos fc$$
$$+\sum_n\frac{\pi\beta_n e^{-\beta_n r\cosh k}2i\sin\beta_n c}{g(\tan\beta_n\eta+\beta_n\eta\sec^2\beta_n\eta)}. \tag{32}$$

Although the second term of the right-hand side tends to zero for $r\to\infty$, yet with a view to obtaining its solution in more accurate form, its mathematical evaluation will now be performed.

Put

$$\frac{2\cos fc}{p^2+fg\tan f\eta}=\psi(f), \tag{33}$$

and write

$$\psi(f)=\psi(0)+\frac{\psi'(0)}{1!}f+\frac{\psi''(0)}{2!}f^2+\cdots, \tag{34}$$

where

$$\psi(0)=\frac{1}{p^2},\quad \psi'(0)=0,\quad \psi''(0)=\frac{-(c^2p^2+2g\eta)}{p^4},\dots\dots\dots\dots, \tag{35}$$

then

$$\int_0^\infty\psi(f)e^{-fr\cosh k}fdf=\int_0^\infty 2\left\{\frac{1}{p^2}f-\frac{(c^2p^2+2g\eta)}{2p^4}f^3+\cdots\right\}e^{-fr\cosh k}df. \tag{36}$$

By means of the formula[9)]

$$\int_0^\infty e^{-fy}f^x df=y^{-(x+1)}\Pi(x), \tag{37}$$

(36) reduces to

$$\int_0^\infty\psi(f)e^{-fr\cosh k}fdf=2\left[\frac{1}{(pr\cosh k)^2}-\frac{3(c^2p^2+2g\eta)}{(pr\cosh k)^4}+\cdots\right]. \tag{38}$$

Now, by means of the integral

9) E. JAHNKE u. F. EMDE, *Funktionentafeln*, S. 28.

$$\int_0^\infty \left\{ \frac{1}{(pr\cosh k)^2} - \frac{3(c^2p^2+2g\eta)}{(pr\cosh k)^4} + \cdots \right\} \cosh k dk$$

$$= \int_0^\infty \left\{ \frac{1}{(pr)^2} \frac{d\sinh k}{1+\sinh^2 k} - \frac{3(c^2p^2+2g\eta)}{(pr)^4} \frac{d\sinh k}{(1+\sinh^2 k)^2} + \cdots \right\}$$

$$= \frac{\pi}{2} \left\{ \frac{1}{(pr)^2} - \frac{3(c^2p^2+2g\eta)}{(pr)^4} + \cdots \right\}, \quad (39)$$

and

$$\left. \begin{aligned} &\int_0^\infty \cosh k e^{i\alpha r \cosh k} dk = -\frac{\pi}{2} H_1^{(1)}(\alpha r), \\ &\int_0^\infty \cosh k e^{-\beta_n r \cosh k} dk = K_1(\beta_n r), \end{aligned} \right\} \quad (40)^{10)}$$

we get

$$u_{z=0} = \frac{e^{-ipt}}{\rho} \left[\frac{i\pi\alpha e^{-\alpha c} H_1^{(1)}(\alpha r)}{g(\tanh\alpha\eta + \alpha\eta\,\mathrm{sech}^2\alpha\eta)} + \frac{2i}{g} \sum_n \frac{\beta_n \sin\beta_n c K_1(\beta_n r)}{\tan\beta_n\eta + \beta_n\eta\sec^2\beta_n\eta} \right.$$

$$\left. - \left\{ \frac{1}{(pr)^2} - \frac{3(c^2p^2+2g\eta)}{2(pr)^4} + \cdots \right\} \right]. \quad (41)$$

This is the final solution of $u_{z=0}$.

In the same way it is possible to evaluate the integral for $w_{z=0}$. After performing the contour integrals corresponding to (29B), (30B) and using the relations

$$\left. \begin{aligned} &\int_0^\infty e^{i\alpha r \cosh k} dk = \frac{\pi i}{2} H_0^{(1)}(\alpha r), \\ &\int_0^\infty e^{-\beta_n r \cosh k} dk = K_0(\beta_n r), \end{aligned} \right\} \quad (42)$$

we get

$$w_{z=0} = \frac{e^{-ipt}}{\rho} \left[\frac{i\pi\alpha e^{-\alpha c} H_0^{(1)}(\alpha r)}{p^2\eta\,\mathrm{cosech}^2\alpha\eta + g} - 2i \sum_n \frac{\beta_n \sin\beta_n c K_0(\beta_n r)}{g - p^2\eta\,\mathrm{cosec}^2\beta^n\eta} \right.$$

$$\left. + \frac{\eta}{r} \left\{ \frac{1}{(pr)^2} - \frac{9(c^2p^2+2g\eta)}{2(pr)^4} + \cdots \right\} \right]. \quad (43)$$

10) Theory of Bessel Functions, p. 180.

Equations (41), (43) show that the movement of the surface consists of three parts. The first is the one due to gravity waves, whose velocity of transmission is related to the wave length as well as to the depth of the ocean, their amplitudes decaying in agreement with the law of two-dimensional propagation of waves. The second and third parts, on the other hand, lack the features of propagation waves, but represent oscillations like standing waves. While the amplitude corresponding to the second part varies as Bessel's functions of imaginary argument, that corresponding to the third one decays inversely as the second and third powers of the distance from the origin both for the respective u- and w-components of displacement. Strictly speaking, these amplitudes change furthermore with change in the ratio of the length of the generated waves to the depth of the sea, its nature being shown in the next section.

5. *The relation between the length of the tidal waves and sea depth.*

For a relatively large value of r, such asymptotic expansions as

$$H_0^{(1)}(\alpha r)=\sqrt{\frac{2}{\pi\alpha r}}e^{i\left(\alpha r-\frac{\pi}{4}\right)}, \qquad H_1^{(1)}(\alpha r)=\sqrt{\frac{2}{\pi\alpha r}}e^{i\left(\alpha r-\frac{3\pi}{4}\right)} \tag{44}$$

may preferably be used. The part of the tidal waves in the solutions of $u_{z=0}$, $w_{z=0}$ then assumes the forms

$$u_{z=0}=\left\{i\sqrt{\frac{2\pi\eta}{r}}\frac{e^{i\left(-pt+\alpha r-\frac{3\pi}{4}\right)}}{\rho g}\right\}\frac{\sqrt{\alpha\eta}e^{-\alpha c}}{\tanh\alpha\eta+\alpha\eta\,\mathrm{sech}^2\alpha\eta}, \tag{45}$$

$$w_{z=0}=\left\{i\sqrt{\frac{2\pi\eta}{r}}\frac{e^{i\left(-pt+\alpha r-\frac{\pi}{4}\right)}}{\rho g}\right\}\frac{\sqrt{\alpha\eta}e^{-\alpha c}}{\frac{p^2\eta}{g}\mathrm{cosech}^2\alpha\eta+1}. \tag{46}$$

From (45), (46) we have obtained the relation between η/λ, (λ being wave length) and the amplitude of tidal waves for three cases of c/η, namely (i) $c/\eta=1/10$, (ii) $c/\eta=1$, (iii) $c/\eta=10$, the results being shown in Fig. 4. The symbol K in Fig. 4 is written in lieu of the respective first factors of the expression $u_{z=0}$, $w_{z=0}$ in (41). The velocity of propagation of the waves (p/f) is also shown by broken lines in the same figure.

It will be seen from Fig. 4 that the vertical displacement of the wave motion assumes its maximum value for a certain ratio of η/λ. With increase of c/ξ, namely, with increase in the area of the applied disturbance, the value of η/λ tends to decrease. In other words, were the area of the applied disturbance large, the vertical amplitude of a relatively longer wave would then be maximum. The horizontal displacement of wave motion tends to decrease with increase in λ in almost every case of c/η, with the exception that c/η is very small, say 0·1, where u, which tends once to decrease with η/λ, again augments for a relatively large value of η/λ, say $\eta/\lambda > 0{\cdot}2$.

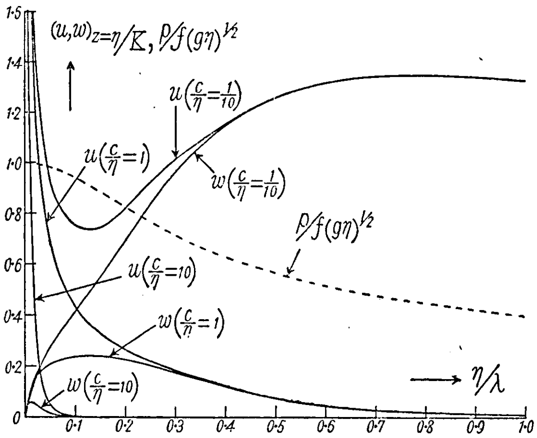

Fig. 4.

It appears that for a fairly large value of η/λ the value of $|u|$ and that of $|w|$ are the same, indicating that the waves are then deep-sea waves, whereas for a smaller value of η/λ the ratio of $|w|/|u|$ becomes negligible, showing the condition that the waves are then long tidal waves.

It appears also that, provided the length of the waves is given, the vertical displacement assumes a relatively large value for a smaller value of c/η, that is to say, the more the pressure concentrated at the very vicinity of the central point of the source, the larger the amplitudes of the generated waves.

In the actual tidal waves (tunami), were the area of the original disturbance given, waves of a certain range of length would be predominant in their train. If, on the other hand, the vibrational frequency were given, the amplitudes of the waves would be more pronounced in the case of a sharp, concentrated original disturbance. It is now possible to conclude that the large amplitudes of tidal waves in the earthquake of 1933 were not the mere effect of a relatively broad area of the seismic origin. Those tidal waves were probably produced by earthquake shocks of certain suddeness occurring in a rather small area, the velocity of transmission of such waves being considerably smaller than $\sqrt{g\eta}$. The conclusion that the beginning phase of a wave train came from the boundary of some large area of the sea bed should now give way to our idea, namely, that the waves were generated from the origin of a relatively small area and transmitted with a velocity much less than $\sqrt{g\eta}$.

55. 振動性津浪の卓越周期

地震研究所 {妹澤克惟 / 金井清}

1933 年 3 月の地震に伴つた津浪は太平洋岸にある多くの驗潮儀に記錄された. たとひその津浪が振動性であつても傳播の途中や驗潮儀で減衰力の爲に鈍い振動性となつてしまふから實測の結果を直ちに理論と比較することはむづかしい. しかし單に理論から考へると, 原振動力が一定であつてもその振動週期によつて振幅が特に大きくなる場合があるのである. 且つその場合の津浪傳播速度は $\sqrt{g\eta}$ (η は海の平均深さ) よりも少いものである.

以上の事柄を正確な數理を以てしらべて見たところが, 以上の性質は原點に働く力の周期のみでなく, 力の働く面積によつて著しく異なることがわかつた. c^2 がこの面積を代表し, λ が波長を示すとすれば, 例へば $c/\eta=1/10$ のときには $\eta/\lambda=0{\cdot}5\sim1{\cdot}0$ の範圍で津浪の高さが著しくなり $c/\eta=10$ になると $\eta/\lambda=0{\cdot}01$ 位のところで津浪の高さが高くなり得ることがわかつた. 卽ち力の働く面積が大きい程長い波長の津浪が現れ易くなることが知られるのである.

津浪の水平流れは如何なる場合にも η/λ が大きくなる程減少する. 但し特別に $c/\eta=1/10$ の如き小なる c の場合には η/λ とともに水平流れが一度減少するけれども $\eta/\lambda=0{\cdot}2$ 以上では再び增加する傾向を持ち, その上で更に減少するものである.

一般に極端に長い λ の場合卽ち $\eta/\lambda\to0$ の場合には水平流れのみ大きく津浪の高さは殆ど無い程度であり, 逆に極端に短い波長の場合には津浪の高さとその水平の流れ (表面での) は同じ大きさなり, 深海波の性質を備へるに至るものである.

實際の津浪の場合に, 原力の働く面積が一定であればある範圍の波長 (又は周期) の波が津浪の中に卓越することになる. 又振動周期が一定の場合を考へると, 原面積が小さなもの程波高が高くなるものである. 以上の事から, 津浪の高いものが必しも廣い面積から起されるのでなく, 場合によつては寧ろ狹い面積のときに高さが高くなり, 且つそのときの傳播速度は $\sqrt{g\eta}$ よりも遙かに少いことがわかる. このやうにして出來た銳い振動のものが傳播の途中で鈍くなり, 結果として長波長のものが $\sqrt{g\eta}$ よりも遲い速度で傳つたやうに見えることもあるものであらう.