59. *Revisions of my "Notes on the Origins of Earthquakes."*

By Win INOUYE,

Earthquake Research Institute.

(Read Sept. 22, 1937.—Received Sept. 21, 1937.)

1. *Introduction*

In the paper[1] entitled "Notes on the Origins of Earthquakes", the writer made a theoretical study of the elastic waves generated from a seismic origin in which the normal pressure on the surface of the spherical cavity could be expressed by the sum of two zonal harmonics $P_0(\cos\theta)$ and $P_2(\cos\theta)$. It was also shown that, assuming the radius of the seismic origin to be constant, the azimuthal distributions of displacement of the dilatational wave at a distant point vary with the wave length.

Since then Prof. K. Sezawa[2], after a thorough study of wave generation from a spherical cavity, called attention to certain errors in the writer's calculations.

With the kind assistance of Dr. G. Nishimura, the writer revised his calculations with results as here shown.

2.

The solution of the equation of motions in spherical coordinates assumes the forms

$$\Delta = A_n \frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}} P_n(\cos\theta) e^{ipt},$$

$$\left.\begin{aligned} u_{1,n} &= \frac{-A_n}{h^2} \frac{d}{dr} \frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}} P_n(\cos\theta) e^{ipt}, \\ v_{1,n} &= \frac{-A_n}{h^2} \frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}} \frac{dP_n(\cos\theta)}{d\theta} e^{ipt}, \end{aligned}\right\}$$

---

1) W. INOUYE, *Bull. Earthq. Res. Inst.*, **14** (1936), 582.

2) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **14** (1936) 506;
K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **15** (1937), 13.

$$\left.\begin{aligned} u_{2,n}&=\frac{-n(n+1)}{k^2}B_n\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{3/2}}P_n(\cos\theta)e^{ipt},\\ v_{2,n}&=\frac{-B_n}{k^2}\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\right\}\frac{dP_n(\cos\theta)}{d\theta}e^{ipt}, \end{aligned}\right\}$$

where $h^2=\dfrac{\rho p^2}{\lambda+2\mu}$, $k^2=\dfrac{\rho p^2}{\mu}$,

u, v=radial and colatitudinal component of displacement respectively.

We shall take a special case in which it is assumed that the surface $r=a$ is free from tangential traction and is under normal pressure $p_nP_n(\cos\theta)e^{ipt}$.

Thus the equations

$$\widehat{rr}=\lambda\Delta+2\mu\frac{\partial u}{dr}=p_nP_n(\cos\theta)e^{ipt},$$

$$\widehat{r\theta}=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right)=0,$$

where $u=u_{1,n}+u_{2,n}$, $v=v_{1,n}+v_{2,n}$, must hold on that surface.

Satisfying the above conditions, we have

$$2A_n\left\{-\frac{(n-1)}{h^2r^2}H^{(2)}_{n+\frac{1}{2}}(hr)+\frac{1}{hr}H^{(2)}_{n+\frac{3}{2}}(hr)\right\}$$

$$+B_n\left\{-\frac{2(n^2-1)}{k^2r^2}H^{(2)}_{n+\frac{1}{2}}(kr)+\frac{2n+1}{kr}H^{(2)}_{n+\frac{3}{2}}(kr)-H^{(2)}_{n+\frac{5}{2}}(kr)\right\}=0,$$

$$A_n\left\{\left(\frac{\lambda}{\mu}-\frac{2n(n-1)}{h^2r^2}\right)H^{(2)}_{n+\frac{1}{2}}(hr)+2\left(\frac{2n+1}{hr}H^{(2)}_{n+\frac{3}{2}}(hr)-\mathrm{H}^{(2)}_{n+\frac{5}{2}}(hr)\right)\right\}$$

$$+2B_nn(n+1)\left\{-\frac{n-1}{k^2r^2}H^{(2)}_{n+\frac{1}{2}}(kr)+\frac{1}{kr}H^{(2)}_{n+\frac{3}{2}}(kr)\right\}=\frac{p_n\sqrt{r}}{\mu}$$

at $r=a$.

Henceforth, we shall consider only the case $\lambda=\mu$.

(1) The single source, in which the normal pressure at the origin is expressed by $P_0(\cos\theta)e^{ipt}$.

The radial component of the displacement of the dilatational wave is given by

$$u_{1,0}=-\frac{p_0a^2}{\mu r}\frac{1}{x^2}\left[\left(\frac{\alpha_0}{hr}-\beta_0\right)\sin\left\{pt-h(r-a)\right\}\right.$$

$$\left.+\left(\alpha_0+\frac{\beta_0}{hr}\right)\cos\left\{pt-h(r-a)\right\}\right],$$

where
$$\alpha_0+i\beta_0=\left[\frac{3i}{x}+\frac{4}{x^2}-\frac{4i}{x^3}\right]^{-1}$$

and $x=ha$.

And at a distant point from the origin as compared with the wave length, we have

$$u_{1,0}=\frac{-p_0a^2}{\mu r}\frac{x}{\sqrt{9x^4-8x^2+16}}\cos\left\{pt-h(r-a)+\tan^{-1}\left(\frac{1}{x}-\frac{3}{4}x\right)\right\}.$$

(2) The quadruple source in which the normal pressure at the origin is expressed by $P_2(\cos\theta)e^{ipt}$.

The radial component of the displacement of the dilatational wave is given by

$$u_{1,2}=-\frac{p_2a^2}{\mu r}P_2(\cos\theta)\frac{1}{x^2}\left[\left\{\alpha_2\left(\frac{9}{h^2r^2}-1\right)+\beta_2\left(\frac{9}{h^3r^3}-\frac{4}{hr}\right)\right\}\cos\left\{pt-h(r-a)\right\}\right.$$
$$\left.-\left\{\beta_2\left(\frac{9}{h^2r^2}-1\right)-\alpha_2\left(\frac{9}{h^3r^3}-\frac{4}{hr}\right)\right\}\sin\left\{pt-h(r-a)\right\}\right],$$

and at a great focal distance, we have

$$u_{1,2}=-\frac{p_2a^2}{\mu r}P_2(\cos\theta)\frac{1}{x^2}\sqrt{\alpha_2^2+\beta_2^2}\cos\left\{pt-h(r-a)+\tan^{-1}\frac{\beta_2}{\alpha_2}\right\},$$

where

$$\alpha_2+i\beta_2=\left[\left\{\left(\frac{72}{x^4}-\frac{13}{x^2}\right)-i\left(\frac{72}{x^5}-\frac{37}{x^3}+\frac{3}{x}\right)\right\}\right.$$
$$\left.+12(\eta+i\zeta)\left\{-\left(\frac{12}{y^4}-\frac{1}{y^2}\right)+i\left(\frac{12}{y^5}-\frac{5}{y^3}\right)\right\}\right]^{-1},$$

$$\eta+i\zeta=\frac{\left(\frac{24}{x^4}-\frac{2}{x^2}\right)-i\left(\frac{24}{x^5}-\frac{10}{x^3}\right)}{\left(\frac{48}{y^4}-\frac{5}{y^2}\right)-i\left(\frac{48}{y^5}-\frac{21}{y^3}+\frac{1}{y}\right)},$$

and $x=ha$, $y=ka$.

As is well known, at a point distant from the origin, both dilatational and distortional waves have only the radial and the colatitudinal component of displacement, respectively.

The colatitudinal component of the displacement of the distortional wave is given by

$$v_{2,2}=-\frac{p_2a^2}{\mu r}\frac{1}{y^2}(\alpha_2+i\beta_2)(\eta+i\zeta)\left\{i\left(-\frac{6}{k^3r^3}+\frac{3}{kr}\right)+\left(\frac{6}{k^2r^2}-1\right)\right\}$$
$$\times e^{i\{pt-k(r-a)\}}\frac{dP_2(\cos\theta)}{d\theta},$$

and at a great focal distance as compared with the wave length, we have

$$v_{2,2}=\frac{p_2a^2}{\mu r}\frac{1}{y^2}\left\{(\alpha_2^2+\beta_2^2)(\eta^2+\zeta^2)\right\}^{\frac{1}{2}}\frac{dP_2(\cos\theta)}{d\theta}$$
$$\times\cos\left\{pt-k(r-a)+\tan^{-1}\frac{\beta_2\eta+\alpha_2\zeta}{\alpha_2\eta-\beta_2\zeta}\right\}.$$

The amplitudes of the radial components of the displacement of the dilatational waves at a distant point generated from the $P_0(\cos\theta)$ and $P_2(\cos\theta)$ origins are plotted against $ha=2\pi\frac{a}{L}$ (where L is the wave length) in Fig. 1. As will be seen from the figure, the curves have

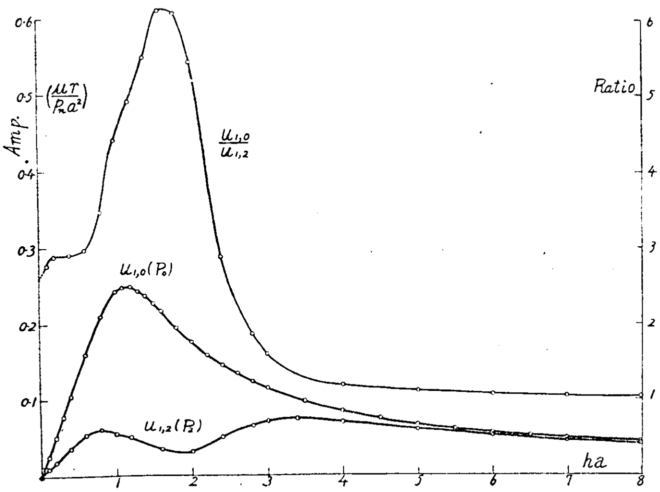

Fig. 1.

maximums at certain ha's, exhibiting spectrum-like features. The amplitudes of the dilational waves in the two cases vanish when $ha=0$, that is the waves are nil when the wave length are very long compared with the dimension of the origin, or in other words, waves do not propagate to a distant point from the point origins.

The amplitude of the dilatational wave generated from the P_2 origin becomes approximately equal to that from the P_0 origin when $ha>4$, which is due to the fact that the distortional waves appear less and less intensely than the dilatational waves when the wave length diminishes as compared with the size of the seismic origin, so that the dilatational wave is generated mainly from the P_2 origin. (see Fig. 2.)

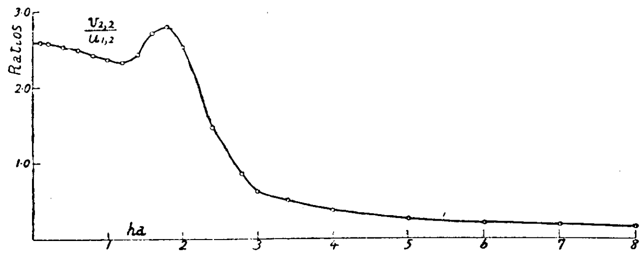

Fig. 2. The proportion of the displacement of the distortional to that of the dilatational wave.

(3) The case in which the normal pressure is expressed by two zonal harmonics $P_0(\cos\theta)$ and $P_2(\cos\theta)$.

In this case, the radial component of the displacement of the dilatational wave at a distant point is given by

$$u_{1,0}+u_{1,2}=-\frac{a^2p'}{\mu r}\frac{1}{x^2}\left[\left\{\alpha_0\varepsilon P_0(\cos\theta)-\alpha_2P_2(\cos\theta)\right\}^2\right.$$

$$\left.+\left\{\beta_0\varepsilon P_0(\cos\theta)-\beta_2P_2(\cos\theta)\right\}^2\right]^{\frac{1}{2}}\cos\left\{pt-h(r-a)\right.$$

$$\left.+\tan^{-1}\left(\frac{\beta_0\varepsilon P_0-\beta_2P_2}{\alpha_0\varepsilon P_0-\alpha_2P_2}\right)\right\},$$

where $p'=+p_2$, $\varepsilon=+\frac{p_0}{p_2}$.

(a) We shall first take the case in which a pair of the doublets act along an axis.

Here the normal pressure on the surface of the spherical cavity is given by

$$\left\{-p'P_2(\cos\theta)-\frac{1}{2}p'P_0(\cos\theta)\right\}e^{ipt}.$$

The change in the azimuthal distributions of the amplitudes of the dilatational wave as the value of ha varies is shown in Fig. 3. In

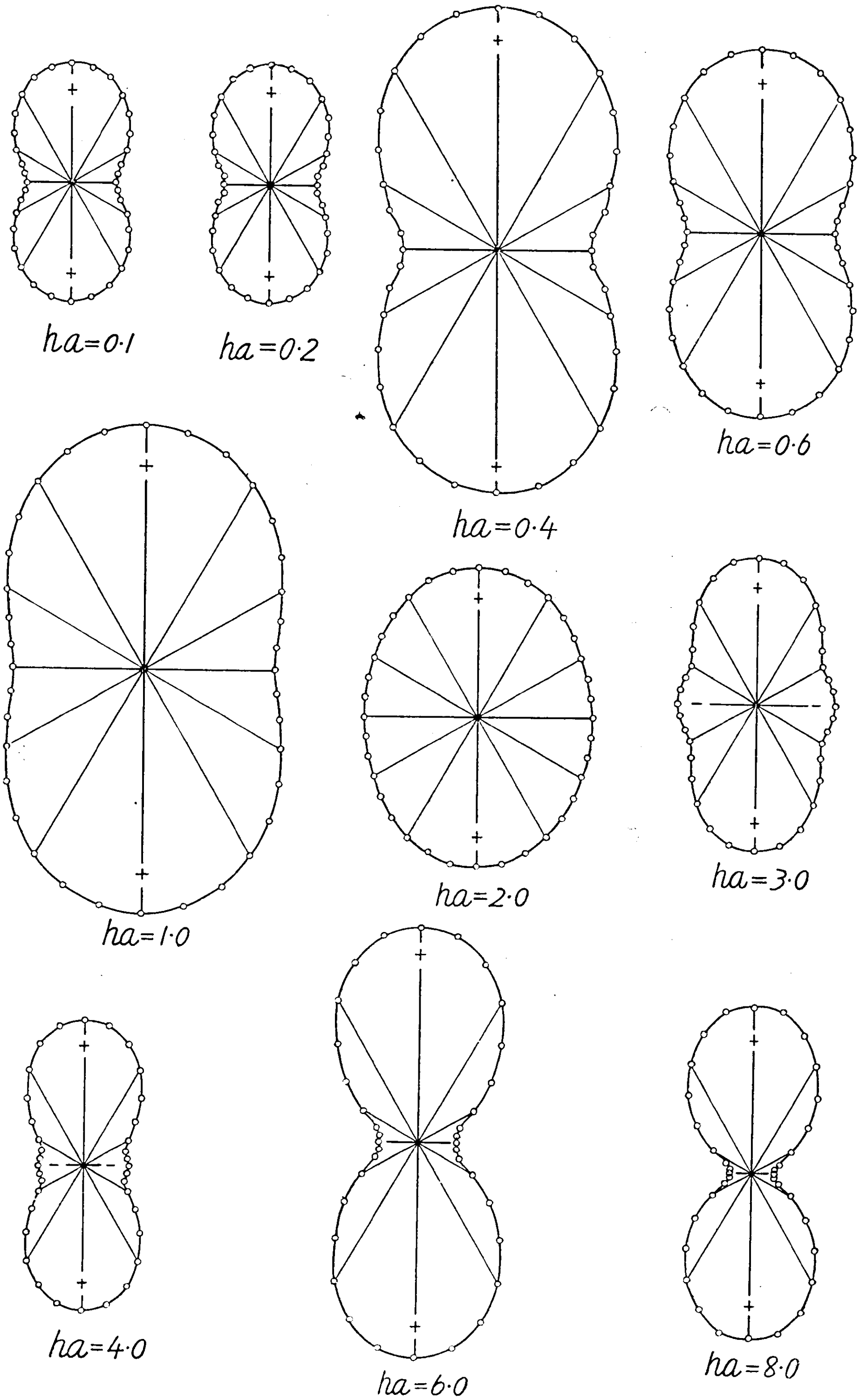

Fig. 3. The case in which a pair of the doublets act along an axis.

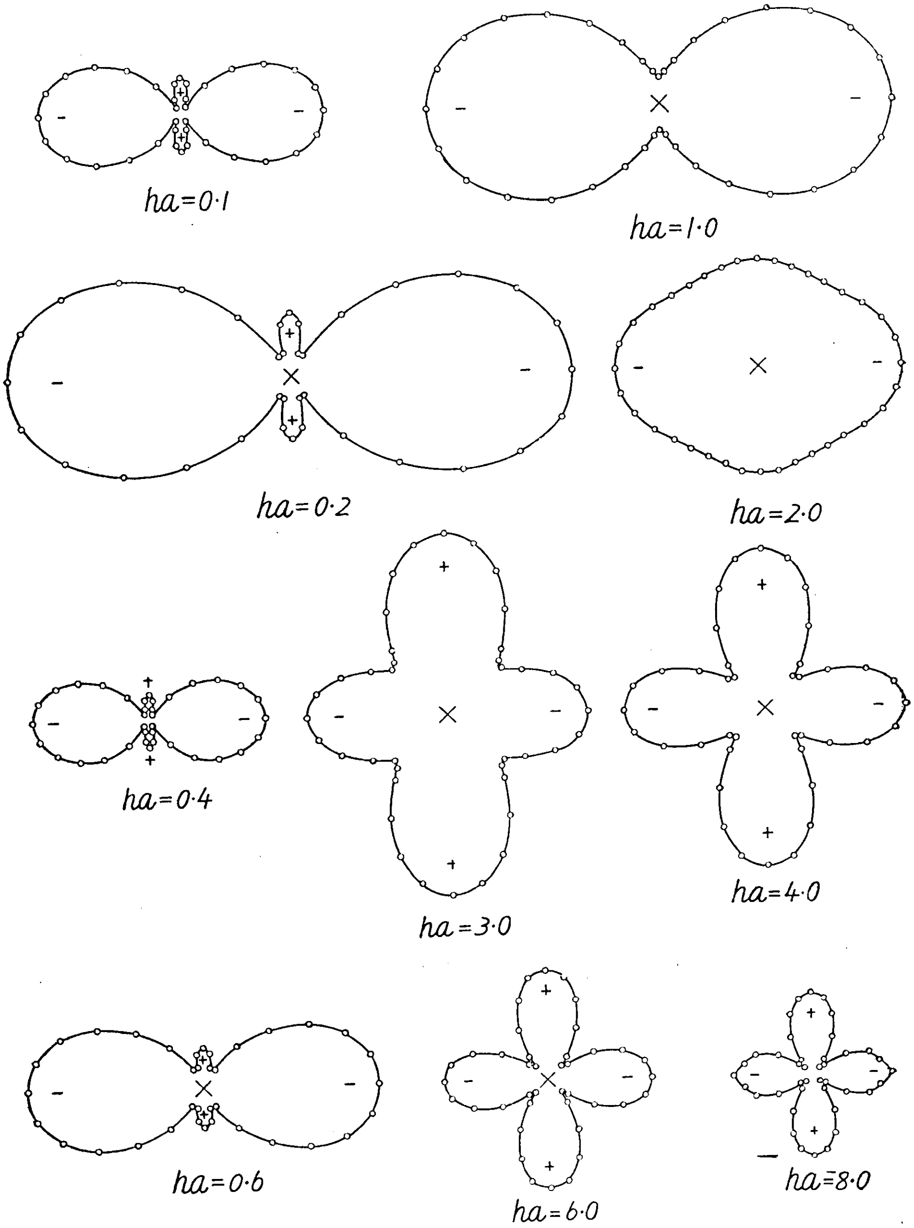

Fig. 4. The case in which the normal pressure on the surface of the spherical cavity is expressed by cos 2θ.

the figure, the axis $\theta=0$ is taken upwards and the amount of the displacement is represented by the vector from the centre. The push and the pull waves are represented by (+) and (−) respectively.

(b) We shall next consider the case in which the normal pressure on the surface of the spherical cavity is expressed by $\cos 2\theta$.

In this case, the normal pressure can be expressed by

$$\left\{-p'P_2(\cos\theta)+\frac{1}{4}p'P_0(\cos\theta)\right\}e^{ipt}.$$

The azimuthal distributions of both the push and pull waves are shown in Fig. 4. As will be seen from Figs. 3 and 4, so long as the wave length is greater than the radius of the seismic origin, the component due to the term $P_0(\cos\theta)$ is greater than that due to the term $P_2(\cos\theta)$. Should the wave length become comparable with the dimension of the seismic origin we shall see the azimuthal distribution of the dilatational wave quite resembling that of the pressure distribution at the origin.

3. *Summary*

The main revised results are as follows:

(1) The amplitudes of the radial component of the displacement of the dilatational waves at a distant point have maximum values at certain ha's in both single and quadruple sources.

(2) The amplitudes of the dilatational waves at a distant point in the two cases vanish when the wave lengths are very long compared with the dimension of the origin.

(3) The amplitude of the dilatational wave generated from the quadruple source becomes appoximately equal to that from the single source when $ha>4$, as the result of the fact that the distortional waves appear less and less intensely than the dilatational waves when the wave length diminishes as compared with the size of the seismic origin.

(4) In the case where the normal pressure at the origin is expressed by two zonal harmonics $P_0(\cos\theta)$ and $P_2(\cos\theta)$, so long as the wave length is greater than the radius of the seismic origin, the component due to the term $P_0(\cos\theta)$ is greater than that due to the term $P_2(\cos\theta)$. Should the wave length become comparable with the dimension of the seismic origin, we find that the azimuthal distribution of the displacement of the dilatational wave becomes quite similar to the azimuthal distribution in the pressure at the seismic origin.

(5) Although we cannot, from the mere results of these calcula-

tions, say anything conclusive regarding the wave length of such waves as might be generated from a seismic origin, the writer considers it preferable to think that waves might be generated having wave lengths comparable with the size of the origin than to think that the wave lengths of the waves are very much longer than the dimension of the seismic origin. (see summary of my "Notes on the Origins of Earthquakes, second paper.")

---

59. 發震機構に就いて(訂正)

地震研究所 井上宇胤

地震研究所彙報第十四卷に掲載した論文中に於て球形震源を考へ其の球面上の壓力變化の方位的分布が $P_0(\cos\theta)$ と $P_2(\cos\theta)$ の重合に依つて現はし得る場合を數理的に取扱つてあるが其の計算中に誤りがあつたのを訂正しておいた.

主なる結果は次の如きものである.

1. 單源から出る縱波も四重源の場合と同樣に波長と震源の大きさとの比率が或る値を取る時に最大の振幅を持つ事が分つた.

2. 單源から出る縱波も四重源から出る縱波も共に震源距離大なる所に於ては震源の大きさに比較して極めて大なる波長を有する波の振幅は零となる.

3. 四重源から出る橫波の振幅は波長が震源の大きさに比して小さく成れば成る程縱波の振幅に比して小となり從つて主として縱波のみ送り出す樣に成る.

4. 震源に於ける壓力變化が單源と四重源の重合に依つて現はし得る場合は波長が震源の大きさに比較して大なる間は單源に依る縱波の方が四重源に依る縱波より卓越して居るが波長が短かく成ると兩方が同程度と成る. 從つて震源に於ける壓力の方位的分布は同一であつても遠方に於て觀測される縱波の振幅の方位的分布は波長に依つて異なるものである.
