35. *Relation between the Thickness of a Surface Layer and the Amplitudes of Love-waves.*

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1. *Introduction.*

A few years ago we had occasion to conclude in a paper[1] that if the whole energy of seismic waves were transmitted along a surface layer in the form of Love-waves, their amplitudes would diminish with increase in the thickness of that layer, the discussion being however of an approximate character. Stoneley[2] recently found that, as expected from the approximate theory of waves, Love-waves of minimum group velocity are likely to predominate in seismograms and analysing such results, confirmed the existence of Jeffrey's discontinuity of about 480 km depth. Our previous conclusion just referred to therefore appears to differ somewhat from Stoneley's. On the other hand, one of us[3] had obtained mathematical formulae showing amplitude differences in Love-waves of different wave lengths that are generated from an internal source, by means of which it is now possible to know that the nature of the propagation is more in harmony with Stoneley's conclusion than with ours just mentioned.

2. *Love-waves generated from a source in the layer.*

We shall now deal with a three-dimensional problem. Let the thickness of the stratum and the depth of the seismic origin from the free surface be η and $\eta-\xi$ respectively, and let the densities and rigidities of the stratum and those of the subjacent medium be ρ, μ, ρ', μ', the axes of r, z being taken as shown in the sketch.

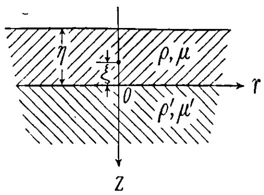

Fig. 1.

1) K. SEZAWA and K. KANAI, "Periods and Amplitudes of Oscillations in L- and M-phases," *Bull. Earthq. Res. Inst.*, **13** (1935), 18~38.

2) R. STONELEY, "Surface-waves associated with 20° Discontinuity," *M. N. R.A.S., Geophys. Suppl.*, **4** (1937), 39~43.

3) K. SEZAWA, "Love-waves Generated from a Source of a certain Depth," *Bull. Earthq. Res. Inst.*, **13** (1935), 1~17.

The solutions differ according as the source of disturbance is in the layer or in the subjacent medium. In the former case the displacement of waves generated from the origin is written in the form

$$v_o = e^{-ipt}\frac{\partial}{\partial(kr)}\frac{e^{ikR}}{kR}, \tag{1}$$

where $R^2=r^2+(z+\xi)^2$ and $k^2=\rho p^2/\mu$, $2\pi/k$ being the wave length within the layer. The amplitudes of waves at a relatively large distance is then

$$v=\frac{4\pi\kappa}{k^2}\cdot\frac{\partial H_o^{(1)}(\kappa r)}{\partial(\kappa r)}e^{-i\left(pt-\frac{\pi}{2}\right)}$$

$$\cdot\frac{\left\{\mu\sqrt{k^2-\kappa^2}\cos\sqrt{k^2-\kappa^2}\xi+\mu'\sqrt{\kappa^2-k'^2}\sin\sqrt{k^2-\kappa^2}\xi\right\}\cos\sqrt{k^2-\kappa^2}(\eta+z)}{\sqrt{k^2-\kappa^2}F'(\kappa)}$$

$$+\left(\frac{k'}{k}\right)^2 e^{i\left(k'r-pt+\frac{\pi}{2}\right)}\frac{\mu'}{\mu}\frac{\cos\sqrt{k^2-k'^2}(\eta-\xi)\cos\sqrt{k^2-k'^2}(\eta+z)}{(k^2-k'^2)\sin^2\sqrt{k^2-k'^2}\eta}\frac{1}{r^2}+\cdots, \tag{2}$$

where $k'^2=\rho'p^2/\mu'$, $2\pi/k'$ being the wave length of the same period in the subjacent medium, and κ is the first real root of

$$F(\kappa)=2\left\{\mu'\sqrt{\kappa^2-k'^2}\cos\sqrt{k^2-\kappa^2}\eta-\mu\sqrt{k^2-\kappa^2}\sin\sqrt{k^2-\kappa^2}\eta\right\}=0. \tag{3}$$

The first term in (2) gives Love-waves and the second the disturbance that is transmitted with the velocity of transverse waves in the layer.

The form of $F'(\kappa)$ is as follows

$$F'(\kappa)=2\left\{\left(\mu\kappa\eta+\frac{\mu'\kappa}{\sqrt{\kappa^2-k'^2}}\right)\cos\sqrt{k^2-\kappa^2}\eta\right.$$

$$\left.+\left(\mu\kappa+\mu'\kappa\eta\sqrt{\kappa^2-k'^2}\right)\frac{\sin\sqrt{k^2-\kappa^2}\eta}{\sqrt{k^2-\kappa^2}}\right\}. \tag{4}$$

Thus, the amplitudes of waves on the free surface $z=-\eta$ at a relatively large epicentral distance are

$$v=\left\{2\pi\frac{\partial H_o^{(1)}(\kappa r)}{\partial(\kappa r)}e^{-i\left(pt-\frac{\pi}{2}\right)}\right\}$$

$$\cdot\left[\frac{\left(\frac{\kappa}{k}\right)^2\mu\cos\sqrt{k^2-\kappa^2}\xi+\mu'\sqrt{\frac{\kappa^2-k'^2}{k^2-\kappa^2}}\sin\sqrt{k^2-\kappa^2}\xi}{\left(\mu\kappa\eta+\frac{\mu'\kappa}{\sqrt{\kappa^2-k'^2}}\right)\cos\sqrt{k^2-\kappa^2}\eta+\left(\mu'\kappa\eta\sqrt{\frac{\kappa^2-k'^2}{k^2-\kappa^2}}+\frac{\mu\kappa}{\sqrt{k^2-\kappa^2}}\right)\sin\sqrt{k^2-\kappa^2}\eta}\right]$$

$$+\left(\frac{k'}{k}\right)^2 e^{i\left(k'r-pt+\frac{\pi}{2}\right)}\frac{\mu'}{\mu}\frac{\cos\sqrt{k^2-k'^2}(\eta-\xi)}{(k^2-k'^2)\sin\sqrt{k^2-k'^2}\eta}\frac{1}{r^2}+\cdots. \quad (5)$$

3. *Love-waves generated from a source in the subjacent medium.*

In this case the depth of the seismic origin from the free surface is $\eta+\xi$, so that $R^2=r^2+(z-\xi)^2$. The primary waves in this case are written in the form

$$v_0=e^{-ipt}\frac{\partial}{\partial(k'r)}\frac{e^{ik'R}}{k'R}. \quad (6)$$

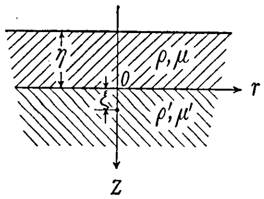

Fig. 2.

Since it has been specified that the amplitudes of the primary waves are invariably the same for both cases in the position of the source, Love-waves generated from the origin in the subjacent medium have unduly large amplitudes.

The amplitudes of waves on the free surface due to the disturbance (6) are expressed by

$$v=\left\{\frac{2\pi k^2}{k'^2}\frac{\partial H_0^{(1)}(\kappa r)}{\partial(\kappa r)}e^{-i\left(pt-\frac{\pi}{2}\right)}\right\}$$

$$\cdot\frac{\left(\frac{\kappa}{k}\right)^2\mu' e^{-\sqrt{\kappa^2-k'^2}\xi}}{\left(\mu\kappa\eta+\frac{\mu'\kappa}{\sqrt{\kappa^2-k'^2}}\right)\cos\sqrt{k^2-\kappa^2}\eta+\left(\mu'\kappa\eta\sqrt{\frac{\kappa^2-k'^2}{k^2-\kappa^2}}+\frac{\mu\kappa}{\sqrt{k^2-\kappa^2}}\right)\sin\sqrt{k^2-\kappa^2}\eta}$$

$$+e^{-i\left(k'r-pt+\frac{\pi}{2}\right)}\frac{\mu'}{\mu}\frac{\mu'\cos\sqrt{k^2-k'^2}\eta-\mu\sqrt{k^2-k'^2}\xi\sin\sqrt{k^2-k'^2}\eta}{\mu(k^2-k'^2)\sin^2\sqrt{k^2-k'^2}\eta}\frac{1}{r^2}+\cdots. \quad (7)$$

The solutions shown in the preceding section and in this were obtained in our previous paper.[4)]

4. *Application of the solutions to a layer of 480 km thickness.*

The layer discussed by Stoneley[5)] is 480 km thick and the constants assumed by him relating to that layer are such that

$$\rho=3{\cdot}5,\quad \rho'=4{\cdot}11,\quad \sqrt{\mu/\rho}=4{\cdot}7\,\text{km/s},\quad \sqrt{\mu'/\rho'}=5{\cdot}66\,\text{km/s},$$

$$\mu=7{\cdot}73.10^{11},\quad \mu'=13{\cdot}2.10^{11}.$$

4) K. SEZAWA, *loc. cit.* 3).
5) R. STONELEY, *loc. cit.* 2).

The relations between k/κ and $\kappa\eta$ as shown in Stoneley's paper, are

$k/\kappa=$	1·00	1·02	1·04	1·05	1·06	1·08	1·10	1·12	1·14	1·16	1·18	1·1866
$\kappa\eta=$	∞	6·7280	4·3745	3·6	3·3009	2·6342	2·1530	1·7693	1·4374	1·1248	0·7940	0

Substituting these values in (5) and (7), we have obtained the relation between $\kappa\eta$ and v for Love-waves for various cases of ξ/η, namely, (i) $\xi/\eta=0{\cdot}9$, (ii) $\xi/\eta=0{\cdot}5$, (iii) $\xi/\eta=0{\cdot}1$, (iv) $\xi/\eta=-0{\cdot}1$ (v) $\xi/\eta=-1$, (vi) $\xi/\eta=-2$, the results being shown in Figs. 3, 4. The symbols K_1, K_2 in Figs. 3, 4 are written in place of the first factors of the respective first terms on the right-hand side of the equations (5), (7). For the reason given in the last section, the amplitudes of radiated Love-waves in the cases (iv), (v), (vi) are unnaturally large.

5. *Interpretation of the result.*

Figs. 3, 4 show that, generally speaking, the greater the depth of the origin, the greater the increase in the amplitudes of Love-waves

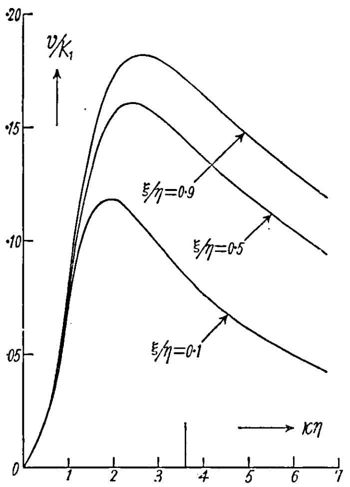

Fig. 3. Amplitudes of Love-waves.

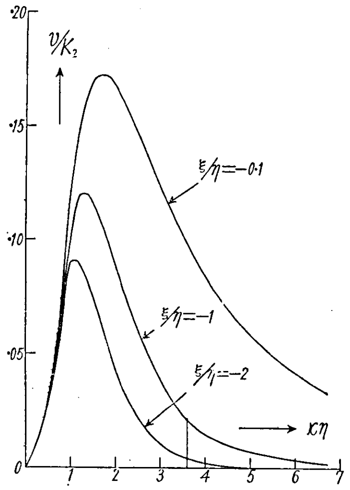

Fig. 4. Amplitudes of Love-waves.

for every vibrational frequency. The most important fact gleaned from the present study is that the amplitude of Love-waves of a given wave length assumes a maximum value for a certain thickness η of the layer. The amplitude of the same waves diminishes in the layer of a thickness that is either larger or smaller than the one just mentioned. The value of $\kappa\eta$ at which the amplitude of Love-waves is maximum, tends to increase with decrease in the depth of the seismic origin. It appears that $\kappa\eta=3{\cdot}6$ (shown by vertical strips in Fig. 3, 4), which defines the minimum group velocity of Love-waves, is likely to agree with the $\kappa\eta$ that corresponds to the maximum amplitude merely in the case of an extremely shallow seismic origin. Thus, the predominant period of the L-phase in seismograms is liable to augment with increase in the thickness of the layer—a fact virtually agreeing with that which naturally results from the idea of minimum group velocity.

The physical interpretation of the question why the amplitudes of Love-waves tend to diminish for any $\kappa\eta$, that is larger or smaller than its special one, is not difficult. For a larger value of $\kappa\eta$, the wave energy distributed in unit area of the layer becomes less in intensity, whereas for a smaller value of $\kappa\eta$, the major part of the same energy is scattered in the subjacent medium.

It is also possible to understand why surface waves, under damping resistance, hardly die away even with increase in epicentral distance.

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35. 地表層の厚さとラブ波の振幅との關係

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ラブ波の地表層中に於ける振動勢力はレーレー波と異なり，大體に於てその全體の厚さにわたつて分布してをるから，原勢力が一定であれば地表層の厚さが厚くなるに從てその振幅が減少するやうに考へ易いのである．著者の一人が以前に出して置いたところのその振幅に關する正確な公式を應用して計算して見ると，その結果は必しも前述の性質と一致しない事がわかつた．卽ち地表層の厚さが原波長のある割合の場合にラブ波の振幅が極大になり，それよりも厚くても薄くても振幅が減少するのである．その極大になる所は實驗的に波動の群速度が極少になるやうな場合であることが歐洲の地震學者によつて述べられてをるが，計算の結果によると少し違つた場合に極大値を取る．しかし傾向としては割合によく一致してをるやうに見える．

常識的には振幅が極大になる波長よりも短い波長のときには地表層の單位面積中の勢力が少くなる事であり，それよりも長い波長のときには地震勢力が固體波として主として下層中へ傳播する事を意味するものである．之については原點が地表層中にあつても下層中にあつても同じ事がいひ得るのである．

尙，震央距離が遠くなる程ラブ波の波長が長くなるのは分散性と減衰性に基づくとした所の我々の以前の考は少しく訂正する必要がある．卽ち實際の地表層は無數の層から成立つてをり，それぞれの層に特有なラブ波が傳はると考へて置き，遠方になるに從つて短い波長のものが減衰するとすればよいのである．卽ち單に一つの層とすると波動の減衰性があまりにも小さ過ぎる事が我々によつて研究してあつたが，その難點は之によつても解決できる譯である．又，英國の人が言つてをるやうに480粁もある表面層にもそれに特有なラブ波が有限振幅を以て傳播し得る事も了解できるのである．