37. *Energy Dissipation in the Vibrations of a Bridge. II.*

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1. *Introduction.*

In the previous paper[1] we discussed the decay of the vibrations of a bridge through dissipation of its vibrational energy into its piers and adjacent spans. Whereas in the previous case the exciting force was at the middle point of a bridge span, we shall now consider the case in which the position of the exciting force remains fixed at any point in the same span.

2. *Mathematical treatments.*

The distances of the position of the exciting force from both ends of a given span are l_1, l_2 respectively, so that $l_1+l_2=l$. Let E_1I_1, ρ_1a_1, E_2I_2, ρ_2a_2, E_3I_3, ρ_3a_3 be the flexural stiffness and mass per unit length of the span in which the disturbing force is applied and those of the respective adjacent spans, and let also E_4a_4, ρ_4a_4, E_5a_5, ρ_5a_5 be the longitudinal stiffnesses and masses per unit length of the respective piers, the periodic force being of the type Fe^{ipt}. Taking the coordinates of x_1, x_4, x_2, x_5, x_3, x_6, and writing the corresponding displacements by y, y_1', y_2, y_2', u, u', we get the general solutions of the vibrations as follows

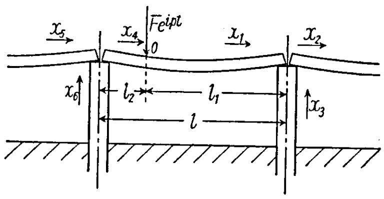

Fig. 1.

$$y_1=e^{ipt}\{A_1e^{i\sqrt{pc_1}x_1}+B_1e^{-i\sqrt{pc_1}x_1}+C_1e^{\sqrt{pc_1}x_1}+D_1e^{-\sqrt{pc_1}x_1}\}, \tag{1}$$

$$y_1'=e^{ipt}\{A_2e^{i\sqrt{pc_1}x_4}+B_2e^{-i\sqrt{pc_1}x_4}+C_2e^{\sqrt{pc_1}x_4}+D_2e^{-\sqrt{pc_1}x_4}\}, \tag{2}$$

$$y_2=e^{ipt}\{G_1e^{-i\sqrt{pc_2}x_2}+H_1e^{-\sqrt{pc_2}x_2}\}, \tag{3}$$

$$y_2'=e^{ipt}\{G_2e^{i\sqrt{pc_3}x_5}+H_2e^{\sqrt{pc_3}x_5}\}, \tag{4}$$

$$u=e^{ipt}\alpha_1e^{if_1x_3}, \tag{5}$$

$$u'=e^{ipt}\alpha_2e^{if_2x_6}, \tag{6}$$

1) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **15** (1937). 385~393.

where

$$\left.\begin{array}{lll} c_1=\left(\dfrac{\rho_1 a_1}{E_1 I_1}\right)^{\frac{1}{4}}, & c_2=\left(\dfrac{\rho_2 a_2}{E_2 I_2}\right)^{\frac{1}{4}}, & c_3=\left(\dfrac{\rho_3 a_3}{E_3 I_3}\right)^{\frac{1}{4}}, \\ p/f_1=\sqrt{E_4/\rho_4}, & p/f_2=\sqrt{E_5/\rho_5}. & \end{array}\right\}$$

The two ends of every span of the bridge are supported without moment of force, and any point of the middle span is subjected to a periodic force without moment of force. The boundary conditions are then expressed by

$$x_1=0, \quad x_4=0; \qquad y_1=y_1', \quad \frac{\partial y_1}{\partial x_1}=\frac{\partial y_1'}{\partial x_4}, \quad \frac{\partial^2 y_1}{\partial x_1^2}=\frac{\partial^2 y_1'}{\partial x_4^2}, \tag{7), (8), (9}$$

$$E_1 I_1\left(\frac{\partial^3 y_1}{\partial x_1^3}-\frac{\partial^3 y_1'}{\partial x_4^3}\right)=Fe^{ipt}, \tag{10}$$

$$x_1=l_1, \quad x_2=0, \quad x_3=0; \qquad y_1=u, \quad y_2=u, \quad \frac{\partial^2 y_1}{\partial x_1^2}=0, \quad \frac{\partial^2 y_2}{\partial x_2^2}=0, \tag{11), (12), (13), (14}$$

$$E_1 I_1\frac{\partial^3 y_1}{\partial x_1^3}-E_2 I_2\frac{\partial^3 y_2}{\partial x_2^3}=E_4 a_4\frac{\partial u}{\partial x_3}, \tag{15}$$

$$x_4=-l_2, \quad x_5=0, \quad x_6=0; \quad y_1'=u', \quad y_2'=u', \quad \frac{\partial^2 y_1'}{\partial x_4^2}=0, \quad \frac{\partial^2 y_2'}{\partial x_5^2}=0, \tag{16), (17), (18), (19}$$

$$E_3 I_3\frac{\partial^3 y_2'}{\partial x_5^3}-E_1 I_1\frac{\partial^2 y_1'}{\partial x_4^3}=E_5 a_5\frac{\partial u'}{\partial x_6}. \tag{20}$$

We now write

$$\left.\begin{array}{l} \sqrt{pc_1}\,l_1=\gamma_1, \quad \sqrt{pc_2}\,l_2=\gamma_2, \quad \dfrac{F}{(\sqrt{pc_1})^3 E_1 I_1}=2\eta, \quad \dfrac{E_2 I_2 c_2^3}{E_1 I_1 c_1^3}=\xi_1, \\ \dfrac{E_3 I_3 c_3^3}{E_1 I_1 c_1^3}=\xi_2, \quad \dfrac{E_4 a_4 f_1}{E_1 I_1(\sqrt{pc_1})^3}=\dfrac{\nu_1}{\gamma_1}, \quad \dfrac{E_5 a_5 f_2}{E_1 I_1(\sqrt{pc_1})^3}=\dfrac{\nu_2}{\gamma_1}, \end{array}\right\} \tag{21}$$

and take a special case such that

$$\left.\begin{array}{ll} E_1 I_1=E_2 I_2=E_3 I_3=EI, & \rho_1 a_1=\rho_2 a_2=\rho_3 a_3=\rho a, \\ E_4 a_4=E_5 a_5=E'a', & \rho_4 a_4=\rho_5 a_5=\rho' a', \\ \xi_1=\xi_2=\xi, & \nu_1=\nu_2=\nu. \end{array}\right\} \tag{22}$$

Substituting the solutions of (1)∼(6) in the boundary conditions (7)∼(20) for the special case cited above, we get

$$A_1\Phi=\gamma_1 e^{i\gamma_2}\Big[-\gamma_1\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+\mathrm{sh}\gamma_2\Big\}+i\Big\{\gamma_1\sin\gamma_2+2(\gamma_1+\nu)\,\mathrm{sh}\,\gamma_2\Big\}\Big]$$
$$+\gamma_1 e^{-i\gamma_1}\mathrm{ch}\,\gamma_0\Big[\gamma_1\Big\{\cos\gamma_2+\mathrm{ch}\,\gamma_2\Big\}-2\sin\gamma_2-\mathrm{sh}\,\gamma_2\Big\}$$
$$+i\Big\{(\gamma_1+4\nu)\sin\gamma_2+2\nu\,\mathrm{sh}\,\gamma_2\Big\}\Big]$$
$$+e^{-i\gamma_1}\mathrm{sh}\,\gamma_0\Big[\Big\{\gamma_1^2(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+2(-\gamma_1^2+\nu\gamma_1+2\nu^2)\sin\gamma_2-\gamma_1^2\mathrm{sh}\,\gamma_2\Big\}$$
$$+i\gamma_1\Big\{-2\nu(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+(\gamma_1+4\nu)\sin\gamma_2\Big\}\Big], \quad (23)$$

$$B_1\Phi=\gamma_1 e^{-i\gamma_2}\Big[\gamma_1\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+\mathrm{sh}\,\gamma_2\Big\}+i(\gamma_1\sin\gamma_2-2\nu\,\mathrm{sh}\,\gamma_2)\Big]$$
$$+\gamma_1 e^{i\gamma_1}\mathrm{ch}\,\gamma_0\Big[\gamma_1\Big\{-(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+2\sin\gamma_2+\mathrm{sh}\,\gamma_2\Big\}$$
$$-i\Big\{(3\gamma_1+4\nu)\sin\gamma_2+2(\gamma_1+\nu)\,\mathrm{sh}\,\gamma_2\Big\}\Big]$$
$$+e^{i\gamma_1}\mathrm{sh}\,\gamma_0\Big[\Big\{-\gamma_1^2(\cos\gamma_2+\mathrm{ch}\,\gamma_2)-2\nu(3\gamma_1+2\nu)\sin\gamma_2+\gamma_1^2\mathrm{sh}\,\gamma_2\Big\}$$
$$-i\gamma_1\Big\{2(\gamma_1+\nu)(\cos\gamma_2+\mathrm{ch}\,\gamma_2)-(3\gamma_1+4\nu)\sin\gamma_2\Big\}\Big], \quad (24)$$

$$C_1\Phi=\gamma_1 e^{\gamma_2}\Big[-(\gamma_1+2\nu)\sin\gamma_2+i\gamma_1\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)-2\sin\gamma_2-\mathrm{sh}\gamma_2\Big\}\Big]$$
$$+\gamma_1 e^{-\gamma_1}\cos\gamma_0\Big[-(\gamma_1+2\nu)(\sin\gamma_2+2\,\mathrm{sh}\,\gamma_2)$$
$$-i\gamma_1\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+\mathrm{sh}\gamma_2\Big\}\Big]$$
$$+e^{-\gamma_1}\sin\gamma_0\Big[\gamma_1(\gamma_1+2\nu)\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+\mathrm{sh}\gamma_2\Big\}$$
$$-i\Big\{\gamma_1^2\sin\gamma_2+2(\gamma_1^2+2\nu\gamma_1+2\nu^2)\,\mathrm{sh}\,\gamma_2\Big\}\Big], \quad (25)$$

$$D_1\Phi=\gamma_1 e^{-\gamma_2}\Big[(\gamma_1+2\nu)\sin\gamma_2-i\gamma_1\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+\mathrm{sh}\,\gamma_2\Big\}\Big]$$
$$+\gamma_1 e^{\gamma_1}\cos\gamma_0\Big[(\gamma_1+2\nu)(\sin\gamma_2+2\,\mathrm{sh}\,\gamma_2)$$
$$+i\gamma_1\Big\{(\cos\gamma_2+\mathrm{ch}\,\gamma_2)+2\sin\gamma_2+3\mathrm{sh}\gamma_2\Big\}\Big]$$

$$+e^{\gamma_1}\sin\gamma_0\Big[-\gamma_1(\gamma_1+2\nu)\Big\{(\cos\gamma_2+\mathrm{ch}\gamma_2)+3\,\mathrm{sh}\gamma_2\Big\}$$

$$+i\Big\{-2\gamma_1^2(\cos\gamma_2+\mathrm{ch}\gamma_2)+\gamma_1^2\sin\gamma_2+4\nu(\gamma_1+\nu)\,\mathrm{sh}\gamma_2\Big\}\Big], \quad (26)$$

$$\frac{\eta\Phi}{4}=\gamma_1(\gamma_1+2\nu)(-\cos\gamma_0\,\mathrm{sh}\gamma_0+\sin\gamma_0\,\mathrm{ch}\gamma_0+\sin\gamma_0\,\mathrm{sh}\gamma_0)$$

$$+i\Big[\gamma_1^2-\gamma_1^2\cos\gamma_0(\mathrm{ch}\gamma_0+\mathrm{sh}\gamma_0)+\sin\gamma_0\Big\{\gamma_1^2\mathrm{ch}\gamma_0-2\nu(\gamma_1+\nu)\,\mathrm{sh}\gamma_0\Big\}\Big], \quad (27)$$

$$H_1\Phi=2\gamma_1\Big[\Big\{-(\gamma_1+2\nu)(\sin\gamma_0\mathrm{sh}\gamma_2+\mathrm{sh}\gamma_0\sin\gamma_2)+i\gamma_1\Big\{(\cos\gamma_0-\mathrm{ch}\gamma_0)(\sin\gamma_2+\mathrm{sh}\gamma_2)$$

$$-(\sin\gamma_0-\mathrm{sh}\gamma_0)(\cos\gamma_2+\mathrm{ch}\gamma_2)-(\sin\gamma_0\,\mathrm{sh}\gamma_2+\mathrm{sh}\gamma_0\sin\gamma_2)\Big\}\Big], \quad (28)$$

$G_1=H_1$, $G_2=H_2=R''+iS''$ (which appears presently), (29), (30)

where

$$\gamma_1=\sqrt{pc_1}l_1,\quad \gamma_2=\sqrt{pc_2}l_2,\quad \gamma_0=\gamma_1+\gamma_2=\sqrt{pc}l,$$

$$\nu=\nu_1=\nu_2=\sqrt{\frac{E'\rho'a'^2l_1^2}{EI\rho a}},\quad \eta=\frac{Fl_1^3}{2\gamma_1^3EI}. \quad (31)$$

Although not given here, similar results were obtained for the general case.

We shall now write the solutions of the present case in their real forms. Thus, corresponding to the disturbance

$$F\cos pt \quad (32)$$

at $x_1=0$ $(x_4=l_2)$, we have

$$y_1=\left(\frac{Fl^3}{EI}\right)\left(\frac{l_1}{l}\right)^3\frac{1}{4\gamma_1^3}\sqrt{\frac{R^2+S^2}{P^2+Q_2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}+\tan^{-1}\frac{S}{R}\right), \quad (33)$$

where

$$P=\gamma_1(\gamma_1+2\nu_1)(-\cos\gamma_0\,\mathrm{sh}\gamma_0+\sin\gamma_0\,\mathrm{ch}\gamma_0+\sin\gamma_0\,\mathrm{sh}\gamma_0), \quad (34)$$

$$Q=\gamma_1^2-\gamma_1^2\cos\gamma_0(\mathrm{ch}\gamma_0+\mathrm{sh}\gamma_0)+\sin\gamma_0\Big\{\gamma_1^2\mathrm{ch}\gamma_0-2\nu_1(\gamma_1+\nu_1)\,\mathrm{sh}\gamma_0\Big\}, \quad (35)$$

$$R=\gamma_1(\gamma_1+2\nu_1)\Big[-2\sin\gamma_0\,\mathrm{sh}\,\gamma_0-2\sin\gamma_2\,\mathrm{sh}\,\gamma_2$$
$$+\mathrm{sh}\,\gamma_1\Big\{\cos\gamma_0(\sin\gamma_2+2\,\mathrm{sh}\,\gamma_2)-\sin\gamma_0(\cos\gamma_2+2\,\mathrm{sh}\,\gamma_2)\Big\}$$
$$+\sin\gamma_1\Big\{\mathrm{ch}\,\gamma_0(2\sin\gamma_2+\mathrm{sh}\,\gamma_2)+\mathrm{sh}\,\gamma_0(2\sin\gamma_2-\mathrm{ch}\,\gamma_2)\Big\}\Big], \tag{36}$$

$$S=4\nu_1(\gamma_1+\nu_1)(\sin\gamma_0\,\mathrm{sh}\,\gamma_1\,\mathrm{sh}\,\gamma_2-\mathrm{sh}\,\gamma_0\sin\gamma_1\sin\gamma_2)$$
$$+\gamma_1^2\Big[2(\cos\gamma_2\,\mathrm{sh}\,\gamma_2-\sin\gamma_2\,\mathrm{ch}\,\gamma_2-\sin\gamma_2\,\mathrm{sh}\,\gamma_2)$$
$$-2\sin\gamma_0\,\mathrm{sh}\,\gamma_0-\sin\gamma_0\,\mathrm{ch}\,\gamma_0+\cos\gamma_0\,\mathrm{sh}\,\gamma_0$$
$$+\cos\gamma_0\Big\{\mathrm{sh}\,\gamma_1(\cos\gamma_2+\sin\gamma_2+2\,\mathrm{sh}\,\gamma_2)+\mathrm{ch}\,\gamma_1\sin\gamma_2\Big\}$$
$$+\sin\gamma_0\Big\{\mathrm{sh}\,\gamma_1(-\cos\gamma_2+\sin\gamma_2+\mathrm{sh}\,\gamma_2)-\mathrm{ch}\,\gamma_1(\cos\gamma_2+\mathrm{ch}\,\gamma_2)\Big\}$$
$$+\mathrm{ch}\,\gamma_0\Big\{\sin\gamma_1(2\sin\gamma_2-\mathrm{ch}\,\gamma_2+\mathrm{sh}\,\gamma_2)-\cos\gamma_1\,\mathrm{sh}\,\gamma_2\Big\}$$
$$+\mathrm{sh}\,\gamma_0\Big\{\sin\gamma_1(\sin\gamma_2-\mathrm{ch}\,\gamma_2+\mathrm{sh}\,\gamma_2)+\cos\gamma_1(\cos\gamma_2+\mathrm{ch}\,\gamma_2)\Big\}\Big]. \tag{37}$$

The displacements of waves in adjacent spans and in the piers are

$$y_2(\text{waves})=-\left(\frac{Fl^3}{EI}\right)\left(\frac{l_1}{l}\right)^3\frac{1}{4\gamma_1^2}\sqrt{\frac{R'^2+S'^2}{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}+\tan^{-1}\frac{R'}{S'}\right), \tag{38}$$

$$y_2'(\text{waves})=\left(\frac{Fl^3}{EI}\right)\left(\frac{l_1}{l}\right)^3\frac{1}{4\gamma_1^2}\sqrt{\frac{R''^2+S''^2}{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}+\tan^{-1}\frac{R''}{S''}\right), \tag{39}$$

$$u=2y_2,\qquad u'=2y_2', \tag{40}$$

where

$$R'=(\gamma_1+2\nu)(\sin\gamma_0\,\mathrm{sh}\,\gamma_2+\mathrm{sh}\,\gamma_0\sin\gamma_2), \tag{42}$$

$$S'=\gamma_1\Big\{-(\cos\gamma_0-\mathrm{ch}\,\gamma_0)(\sin\gamma_2+\mathrm{sh}\,\gamma_2)+(\sin\gamma_0-\mathrm{sh}\,\gamma_0)(\cos\gamma_2+\mathrm{ch}\,\gamma_2)$$
$$+(\sin\gamma_0\,\mathrm{sh}\,\gamma_2+\mathrm{sh}\,\gamma_0\sin\gamma_2)\Big\}, \tag{43}$$

$$R''=(\gamma_1+2\nu)\Big\{\cos\gamma_0\,\mathrm{sh}\,\gamma_0\sin\gamma_2+\sin\gamma_0\,\mathrm{ch}\,\gamma_0\,\mathrm{sh}\,\gamma_2-\sin\gamma_0\,\mathrm{sh}\,\gamma_0(\cos\gamma_2+\mathrm{ch}\,\gamma_2)\Big\}, \tag{44}$$

$$S'' = \gamma_1 \Big\{ (\cos\gamma_0 \operatorname{ch}\gamma_0 - 1)(\sin\gamma_2 - \operatorname{sh}\gamma_2) + \cos\gamma_0 \operatorname{sh}\gamma_0 (\cos\gamma_2 + \sin\gamma_2 + \operatorname{ch}\gamma_2)$$
$$+ \sin\gamma_0 \operatorname{ch}\gamma_0 (-\cos\gamma_2 - \operatorname{ch}\gamma_2 + \operatorname{sh}\gamma_2)$$
$$+ \sin\gamma_0 \operatorname{sh}\gamma_0 (-\cos\gamma_2 + \sin\gamma_2 - \operatorname{ch}\gamma_2 + \operatorname{sh}\gamma_2) \Big\}. \tag{45}$$

When ν_1 is fairly large, $\sqrt{P^2+Q^2} \rightarrow 2\nu_1^2 \sin\gamma_0 \operatorname{sh}\gamma_0$, so that the approximate resonance conditions are expressed by

$$\sin\gamma_0 = 0, \qquad \left\{ \sin(\rho a l^4 p^2/EI)^{\frac{1}{4}} = 0 \right\}. \tag{46}$$

In our example which will appear in the next section, $\nu_1 = 19{\cdot}025$. However, in resonance conditions, even should no damping force exist in the structure of the span, the amplitude of the vibrations will not assume an infinitely large value because of the condition that the denominator of the actual expression for the amplitude is

$$\sqrt{P^2+Q^2}\,, \tag{47}$$

each of P and Q being of a certain finite value.

3. *A numerical example and its interpretation.*

We shall take the case of a deck girder bridge of the same size and weight as those shown in the preceding paper, but with the difference that the exciting force is at $l/4$ from one end ($\nu_1 = 19{\cdot}025$).

Using the relations already obtained, we calculated $y_{1x_1=0}$, $y_{2\max}$, $y'_{2\max}$, $u_{\max}$, $u'_{\max}$, for different values of γ_0, namely $(\rho a l^4 p^2/EI)^{\frac{1}{4}}$, the result being shown in Fig. 2. The ordinates corresponding to $y_{2\max}$, $y'_{2\max}$ are shown on a magnified scale in Fig. 3. Every ordinate of these figures may be assumed to indicate the relative value of any one of $y_{1x_1=0}$, $y_{2\max}$, or $y'_{2\max}$ for a given intensity of f in the disturbing force

$$F = f\gamma_0^4 \left(= f\frac{\rho a l^4 p^2}{EI} \right), \tag{48}$$

but for different vibrational frequencies. It is evident that, in the case of a locomotive or of an exciting machine, the condition that f is a constant, must exist.

The values of $u_{\max}$, $u'_{\max}$ are always twice those of $y_{2\max}$, $y'_{2\max}$ for any vibrational frequency. The reason why we may neglect the reflectional waves in the adjacent spans and in piers was explained in the last paper.

It will be seen from Figs. 2, 3 that, under the second resonance $(\rho al^4p^2/EI)^{\frac{1}{4}} \fallingdotseq 2\pi$, the amplitude of vibration is about twice that of first resonance $(\rho al^4p^2/EI)^{\frac{1}{4}} \fallingdotseq \pi$ or even that of the third resonance $(\rho al^4p^2/EI)^{\frac{1}{4}} \fallingdotseq 3\pi$. This results from the condition that the disturbance imparts its force at the loop $l/4$ from one end of the span for the second vibrational mode of the bridge span.

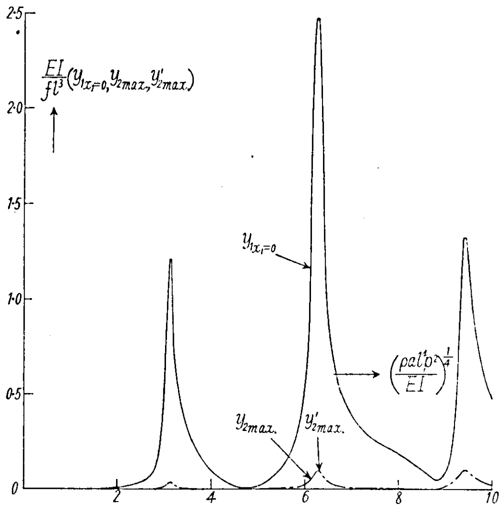

Fig. 2. Resonance curves of bridge vibrations.

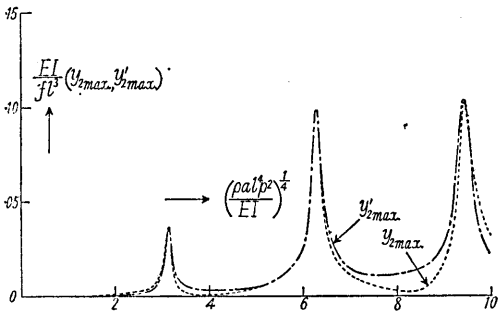

Fig. 3. Resonance curves of bridge vibrations. y_{2max}, y'_{2max} on a magnified scale.

From the vibrational nature of a bridge span, it is known that the most dangerous condition, if it be the case, among those in which velocities of moving disturbance on a bridge differ, is covered in the results of the present paper and the preceding one. While, generally speaking, the effect of the disturbance imparted by a locomotive upon a bridge of a finite span dies away before the application of a finite number of impacts, the cases which we discussed here, on the other hand, are such that vibrational disturbance continues to be effective for an infinitely long time.

It should be borne in mind that, since in our papers, we have discussed the damping from only one out of the numerous likely causes, such as viscous damping in the structure, energy dissipation in the form of elastic waves through neighbouring structural members, solid friction at the ends of every span, etc., the resonance curves in our calculation should be rather sharp compared with those obtained from vibration experiments on actual bridges.

37. 橋梁の振動に於ける勢力の逸散性（第2報）

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同じ題名の前回の報告では，橋梁の橋桁上に振動力が働く場合にその振動勢力が彈性波として隣の橋桁や橋脚に傳播する爲にこの橋桁の振動が減衰する事を示して置いた．實際に於て橋梁の振動減衰性の相當に大きな部分はこの逸散性が受持つてをるやうである．前回のものは振動力が橋桁の中央に働く場合であつたが，今回は任意の點にそれが働く場合を研究したのである．

普通の場合に振動力が振動體の節點に働くか，腹に働くかといふ事の結果としての傾向もこの研究結果に大いに現れてをるやうであるが，逸散性のためにその性質が少しく訂正される．しかし最も著しい特性は共振の振幅が相當に少くなり實際の觀測の數倍位になる事を示すことである．但しこの數倍といふ事は多くの減衰力の一つのみを取つてをるからである．

機關車が橋梁上を動くにしても，橋に最悪の影響を與へるのはその位置が動かずに靜止する場合である事は共振の力學から明かである．只今の論文はそのやうな場合を論じたといふ事によつて實際問題と結びつけ得るのである．しかしこの論文の別の目的は，地上にある建物の場合にその震動勢力の逸散性について誰にもわかるやうな解釋を與へにくかつたものが，今回の論文によつてその性質を容易に示し得たといふ點にある．