40. *Relations between Gravity Values and Corresponding Subterranean Mass Distribution.*

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Let AA in Fig. 1 be the plane of the earth's surface along which the gravity values are known from point to point, and BB another plane which is at a depth d from the surface and on which are distributed surface densities responsible for these gravity values. Assume further that the surface densities along BB and, consequently the gravity values along AA, do not change in the direction that extends from $+\infty$ to $-\infty$ perpendicularly to the plane of this paper, so that they can be expressed by $\rho(y)$ and $g(x)$ respectively, x and y being the abscissae along AA and BB, with line OO as the common origin. The gravity $g(x)$ at $x=x$ due to $\rho(y)$ is then given by

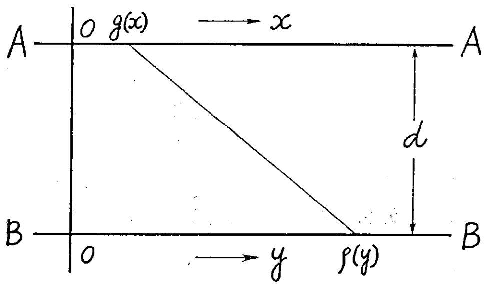

Fig. 1.

$$g(x)=2k^2d\int_{-\infty}^{+\infty}\frac{\rho(y)}{(y-x)^2+d^2}dy.$$

The direct analytical solution of this integral equation for an actually observed $g(x)$ is usually not possible, and it has been for this reason that problems of this kind have been studied in the other direction, viz., by searching such $\rho(y)$ by trial and error method that will give gravity values which agree best with the observed. The numerical solution of the equation, though theoretically simple, becomes almost insuperably complex, were more or less accurate results required.

If $\rho(y)$ were a harmonic function of y, then matters would be greatly simplified.[1)]

Thus, if

1) It has been informed that a similar idea was proposed by H. RAINBOW.

$$\rho(y) = \rho_n \cos ny, \tag{1a}$$

then

$$g(x) = 2k^2 d\rho_n \int_{-\infty}^{+\infty} \frac{\cos ny}{(y-x)^2 + d^2} dy.$$

Putting

$$p = y - x,$$

we get

$$g(x) = 2k^2 d\rho_n \int_{-\infty}^{+\infty} \frac{\cos n(p+x)}{p^2 + d^2} dp$$

$$= 2k^2 d\rho_n \left\{ \cos nx \int_{-\infty}^{+\infty} \frac{\cos np}{p^2 + d^2} dp - \sin nx \int_{-\infty}^{+\infty} \frac{\sin np}{p^2 + d^2} dp \right\}$$

$$= 2\pi k^2 \rho_n e^{-nd} \cos nx, \tag{1b}$$

making use of the integrals

$$\int_{-\infty}^{+\infty} \frac{\cos np}{p^2 + d^2} dp = \frac{\pi}{d} e^{-nd}$$

and

$$\int_{-\infty}^{+\infty} \frac{\sin np}{p^2 + d^2} dp = 0.$$

Conversely if

$$g(x) = g_n \cos nx, \tag{2a}$$

then

$$\rho(y) = \frac{g_n}{2\pi k^2} e^{nd} \cos ny. \tag{2b}$$

That there is no other density distribution at depth d that produces the same gravity values as $\rho(y)$ can be proved in the following way. If $\rho'(y)$ be any one of such distributions, then the gravity values on the earth's surface due to $\{\rho'(y) - \rho(y)\}$ must be zero from point to point. Therefore $\{\rho'(y) - \rho(y)\}$ cannot be other than zero, in other words $\rho'(y)$ and $\rho(y)$ are equal everywhere.

From the relations (1) and (2), we see that

(1) $g(x)$ and $\rho(y)$ are harmonic functions which are in phase with each other, and the ratio of amplitudes is given by $2\pi k^2 e^{-nd}$,

(2) for given ρ_n and d, g_n decreases if n increases,

(3) for given ρ_n and n, g_n decreases if d increases,

(4) for given g_n and d, ρ_n increases if n increases,
(5) for given g_n and n, ρ_n increases if d increases.

While the relations (1) and (2) hold for the simple case with a single value of n, those for different n's may be superposed by mere addition in order to get corresponding relations which hold for more general and complicated cases. Thus, if

$$\rho(y)=\sum_n A_n \cos ny+\sum_n B_n \sin ny,$$

then

$$g(x)=2\pi k^2\left\{\sum A_n e^{-nd} \cos nx+\sum_n B_n e^{-nd} \sin nx\right\},$$

and conversely if

$$g(x)=\sum_n \alpha_n \cos nx+\sum_n \beta_n \sin nx,$$

then

$$\rho(y)=\frac{1}{2\pi k^2}\left\{\sum_n \alpha_n e^{nd} \cos ny+\sum_n \beta_n e^{nd} \sin ny\right\}.$$

Although the summations in the above expressions do not necessarily imply that the n's are successive integers as in the case of Fourier series, this constitutes the most important field for application of the above relations to actual problems.

If a distribution of gravity gradients along the earth's surface is given instead of that of gravity values, there is a similar method for finding the subterranean density distribution that is responsible for these gradient values. Thus, if

$$\frac{\partial g}{\partial x}=\sum_n A_n \cos nx+\sum B_n \sin nx,$$

then

$$g(x)=\sum_n \frac{A_n}{n} \sin nx-\sum_n \frac{B_n}{n} \cos nx+G,$$

so that

$$\rho(y)=\frac{1}{2\pi k^2}\left\{\sum_n \frac{A_n}{n} e^{nd} \sin ny-\sum_n \frac{B_n}{n} e^{nd} \cos ny\right\}+P.$$

Before proceeding to apply the present method to practical problems, investigations will be made of the range of its applicability and of the errors that may arise therefrom. In the above theories, each elementary strip of the subterranean plane has been assumed to extend from $+\infty$ to $-\infty$ and to be of uniform density in the direction perpendicular to the paper, which condition, however, is never met with in practice. Errors would therefore arise in the values of the gravity if an elementary strip of a subterranean plane that is actually of a finite length were regarded as infinitely long. Let the length of the strip be $2l$ and the linear density ρ, then the gravity at point P, which is at a vertical distance d and a horizontal distance s from the middle point of the strip, is given by

$$g=\frac{2k^2\rho d}{d^2+s^2}\left(1+\frac{d^2+s^2}{l^2}\right)^{-\frac{1}{2}}$$

$$=g_\infty\left(1+\frac{d^2+s^2}{l^2}\right)^{-\frac{1}{2}}.$$

The value of l which gives the same ratio, $\frac{g}{g_\infty}$, varies with the values of s as shown by the curves in Fig. 2. If both l and s are several times d, then the ratio is greater than 0·90, so that in these cases the errors in question are less than 10%.

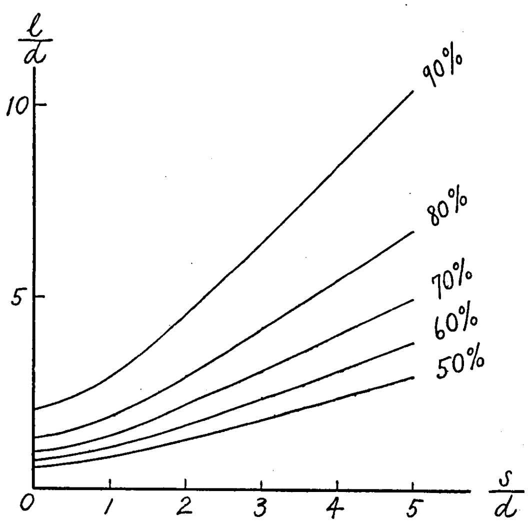

Fig. 2.

In the second place, the plane on which the densities are distributed has been assumed to extend from $+\infty$ to $-\infty$ in y-direction also. If the width of this plane is finite, say $2y$, the gravity due to it at the point above its centre is given by

$$g=4k^2\rho\tan^{-1}\frac{y}{d}$$

which tends to

$$g_\infty=2\pi k^2\rho$$

if y tends to infinity. The ratio $\frac{g}{g_\infty}$ is therefore given by

$$\frac{g}{g_\infty}=\frac{2}{\pi}\tan^{-1}\frac{y}{d}$$

the values of which are plotted in Fig. 3. If $\frac{y}{d}$ is greater than 6, errors arising from regarding the plane as of infinite width will be less than 10%.

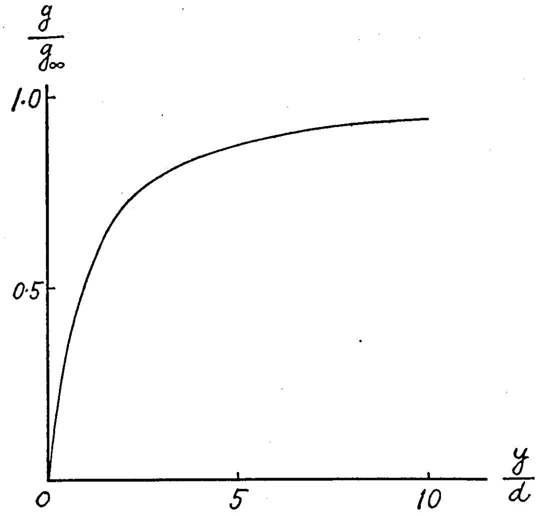

Fig. 3.

Since in both the above calculations, the surface density on the subterranean plane has been assumed to be uniform throughout, estimations of errors are subject to certain changes, were the variation in density along this plane taken into account.

The third problem to be investigated is on errors arising from substituting the actual masses that are distributed in space by those that are hypothetical and which are condensed on a single plane at a certain depth.

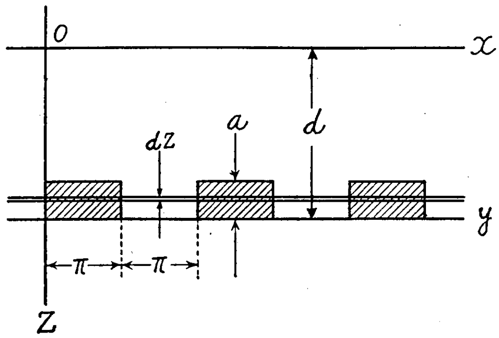

Fig. 4.

In Fig. 4, let the shaded rectangular portions be cross-sections of masses of thickness a, width π, and density 1 which are infinitely long in the direction perpendicular to the paper and which are distributed at a regular periodic interval π. The density distribution in y-direction along a layer at depth z and of thickness dz is

$$\begin{aligned}\rho(y) &= dz \qquad 0<y<\pi \\ &=0 \qquad \pi<y<2\pi.\end{aligned}$$

As the Fourier expression for $\rho(y)$ is

$$\rho(y)=\left\{\frac{1}{2}+\frac{2}{\pi}\sum\frac{1}{2n+1}\sin(2n+1)y\right\}dz,$$

the corresponding surface gravity is given by

$$dg(x)=2\pi k^2\left\{\frac{1}{2}+\frac{2}{\pi}\sum\frac{e^{-(2n+1)z}}{2n+1}\sin(2n+1)x\right\}dz.$$

Integrating this throughout the whole thickness of the masses from $(d-a)$ to d, we have finally

$$g(x)=2\pi k^2\left[\frac{a}{2}+\frac{2}{\pi}\sum\frac{1}{(2n+1)^2}\left\{e^{-(2n+1)(d-a)}-e^{-(2n+1)d}\right\}\sin(2n+1)x\right].$$

On the other hand, the surface gravities due to the same masses if they were respectively condensed at depths d, $\left(d-\frac{a}{2}\right)$ and $(d-a)$ would be

$$g_d(x)=2\pi k^2\left\{\frac{a}{2}+\frac{2a}{\pi}\sum\frac{1}{2n+1}e^{-(2n+1)d}\sin(2n+1)x\right\},$$

$$g_{d-\frac{a}{2}}(x)=2\pi k^2\left\{\frac{a}{2}+\frac{2a}{\pi}\sum\frac{1}{2n+1}e^{-(2n+1)\left(d-\frac{a}{2}\right)}\sin(2n+1)x\right\}$$

and

$$g_{d-a}(x)=2\pi k^2\left\{\frac{a}{2}+\frac{2a}{\pi}\sum\frac{1}{2n+1}e^{-(2n+1)(d-a)}\sin(2n+1)x\right\}.$$

These values of gravity are compared in Tables I, II.

TABLE I.

$2\pi=40$ km. $d=\frac{\pi}{2}=10$ km.

$a=\frac{\pi}{4}=5$ km. unit=milligal.

x	g	g_d	$g_{d-\frac{a}{2}}$	g_{d-a}	g_d-g	$g_{d-\frac{a}{2}}-g$	$g_{d-a}-g$
90°	290	264	288	324	−26	−2	+34
120°	284	258	280	314	−26	−4	+30
150°	255	237	253	279	−18	−2	+24
180°	209	209	209	209	0	0	0
210°	163	180	166	140	+17	+3	−23
240°	135	161	138	105	+26	+3	−30
270°	128	154	130	95	+26	+2	−33

TABLE II.

$2\pi = 40$ km. $\quad d = \frac{\pi}{2} = 10$ km.

$a = \frac{\pi}{4} = 2{\cdot}5$ km. $\quad$ unit=milligal.

x	g	g_d	$g_{d-\frac{a}{2}}$	g_{d-a}	$g_d - g$	$g_{d-\frac{a}{2}} - g$	$g_{d-a} - g$
90°	138	132	137	145	−6	−1	+7
120°	134	128	134	140	−6	0	+6
150°	123	119	122	126	−4	−1	+3
180°	104	104	104	104	0	0	0
210°	87	90	87	82	+3	0	−5
240°	75	80	75	69	+5	0	−6
270°	71	77	71	64	+6	0	−7

It will be seen from the tables that the errors are very small if the masses are condensed on their middle plane at depth $\left(d - \frac{a}{2}\right)$.

Consider in the second place the case in which two layers of different densities are bounded by a sinusoidal cylindrical surface of amplitude a. If the difference in density is 1, to find the gravity values due to this mass distribution is equivalent to finding the same due to the mass distribution shown in Fig. 5.

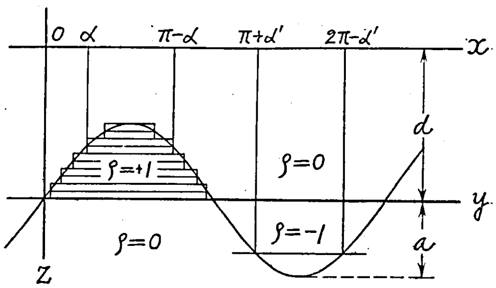

Fig. 5.

As in the preceding example, consider a layer at depth z and of thickness dz. The density distribution in y-direction along this layer is,

for the upper half,

$$\rho(y) = 0 \qquad 0 < x < \alpha, \quad \pi - \alpha < x < 2\pi$$

$$\rho(y) = dz \qquad \alpha < x < \pi - \alpha,$$

and for the lower half,

$$\rho(y) = 0 \qquad 0 < x < \pi + \alpha', \quad 2\pi - \alpha' < 2\pi$$

$$\rho(y) = -dz \qquad \pi + \alpha' < x < 2\pi - \alpha'.$$

The Fourier expressions for $\rho(y)$ is,

for the upper half,

$$\rho(y)=\left\{\frac{\pi-2\alpha}{2\pi}-\frac{2}{\pi}\sum\frac{\sin 2n\alpha}{2n}\cos 2ny\right.$$

$$\left.+\frac{2}{\pi}\sum\frac{\cos(2m+1)\alpha}{2m+1}\sin(2m+1)y\right\}dz$$

and for the lower half,

$$\rho(y)=\left\{-\frac{\pi-2\alpha'}{2\pi}+\frac{2}{\pi}\sum\frac{\sin 2n\alpha'}{2n}\cos 2ny\right.$$

$$\left.+\frac{2}{\pi}\sum\frac{\cos(2m+1)\alpha'}{2m+1}\sin(2m+1)y\right\}dz.$$

The surface gravity which corresponds to this elementary layer is, for the upper half,

$$dg(x)=2\pi k^2\left\{\frac{\pi-2\alpha}{2\pi}-\frac{2}{\pi}\sum\frac{\sin 2n\alpha}{2\pi}e^{-2nz}\cos 2nx\right.$$

$$\left.+\frac{2}{\pi}\sum\frac{\cos(2m+1)\alpha}{2m+1}e^{-(2m+1)z}\sin(2m+1)x\right\}dz$$

and for the lower half,

$$dg(x)=2\pi k^2\left\{-\frac{\pi-2\alpha'}{2\pi}+\frac{2}{\pi}\sum\frac{\sin 2n\alpha'}{2n}e^{-2nz}\cos 2nx\right.$$

$$\left.+\frac{2}{\pi}\sum\frac{\cos(2m+1)\alpha'}{2m+1}e^{-(2m+1)z}\sin(2m+1)x\right\}dz.$$

To find the gravity values due to the whole mass, these expressions must be integrated with respect to z throughout the full range of amplitude of the sinusoidal wave, but this is unfortunately not possible and a numerical integration has to be resorted to. Dividing the mass into ten elementary layers as in Fig. 5, the mass in each layer which is regarded as being rectangular in shape, is condensed on its respective middle plane and the gravity values due to all these are summed up. On the other hand, if the whole mass is condensed on the middle plane of the sinusoidal curve, the density distribution along this plane is given by

$$\rho(y)=a\cos y$$

and the corresponding gravity values by

$$g(x)=2\pi k^2 a e^{-d}\cos x.$$

The gravity values due to these two different mass distributions are compared in Table III. From these results, we see that if the

TABLE III.

x	$d=\frac{\pi}{2}=10$ km. $\lambda=2\pi=40$ km. $a=\frac{\pi}{8}=2{\cdot}5$ km.			$d=\frac{\pi}{2}=10$ km. $\lambda=2\pi=40$ km. $a=\frac{\pi}{8}=5{\cdot}0$ km.			$d=\frac{\pi}{2}=15$ km. $\lambda=2\pi=40$ km. $a=\frac{\pi}{8}=2{\cdot}5$ km.			$d=\frac{\pi}{2}=15$ km. $\lambda=2\pi=40$ km. $a=\frac{\pi}{8}=1{\cdot}25$ km.		
	Actual	Cond.	Diff.	Actual	Cond.	Diff.	Actual	Cond.	Diff.	Actual	Cond.	Diff.
90°	48	44	− 4	106	87	− 19	110	96	− 14	51	48	− 3
120°	40	38	− 2	86	76	− 10	90	83	− 7	43	41	− 2
150°	22	22	0	45	44	− 1	43	48	+ ·5	23	24	+ 1
180°	− 2	0	+ 2	− 9	0	+ 9	− 9	0	+ 9	− 2	0	+ 2
210°	− 24	− 22	+ 2	− 47	− 44	+ 3	− 53	− 48	+ 5	− 25	− 24	+ 1
240°	− 38	− 38	0	− 77	− 76	+ 1	− 81	− 83	− 2	− 41	− 41	0
270°	− 44	− 44	0	− 87	− 87	0	− 91	− 96	− 5	− 46	− 48	− 2

amplitude of the sinusoidal curve is less than one-fourth its mean depth, the errors arising from condensing the masses on the middle plane are negligible.

The fourth problem to be investigated is on the convergency of the Fourier series used. The differentiation of a convergent Fourier series, term by term, does not always give another convergent series. Since in practical problems, it will suffice to treat them with a Fourier series of finite term number, convergency of the series is out of question. It has been shown that if

$$g(x)=\sum\alpha_n \cos nx+\sum\beta_n \sin nx$$

then

$$\rho(y)=\frac{1}{2\pi k^2}\left\{\sum\alpha_n e^{nd} \cos ny+\sum\beta_n e^{nd} \sin ny\right\}.$$

If $g(x)$ is convergent, $\rho(y)$ is not necessarily also convergent, because the factor e^{nd}, which is increasing with increasing n, is multiplied to the α's and β's. If $\rho(y)$ thus obtained is found to be divergent, the physical interpretation is that this particular gravity distribution cannot be produced by a condensed density distribution at depths greater than d, and d must be less than that threshold value for which the series becomes convergent.

Applications of Fourier series as just described imply that the same distributions of gravity together with distributions of subterranean density are repeated periodically also beyond the range in which they are expressed in those series. Although this is evidently not actually the case, results obtained on this assumption will not greatly differ from the actual, because the mass distribution corresponding to the difference of the actual and the hypothetical Fourier density distributions beyond this range will produce no large gravity in the range in question.

What will now be given are some of the many examples of the applications of the present method.

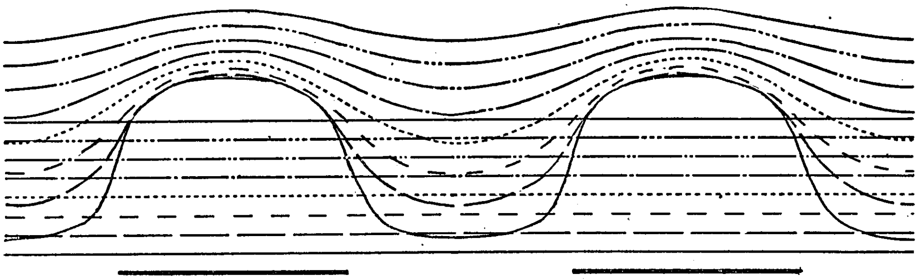

Fig. 6.

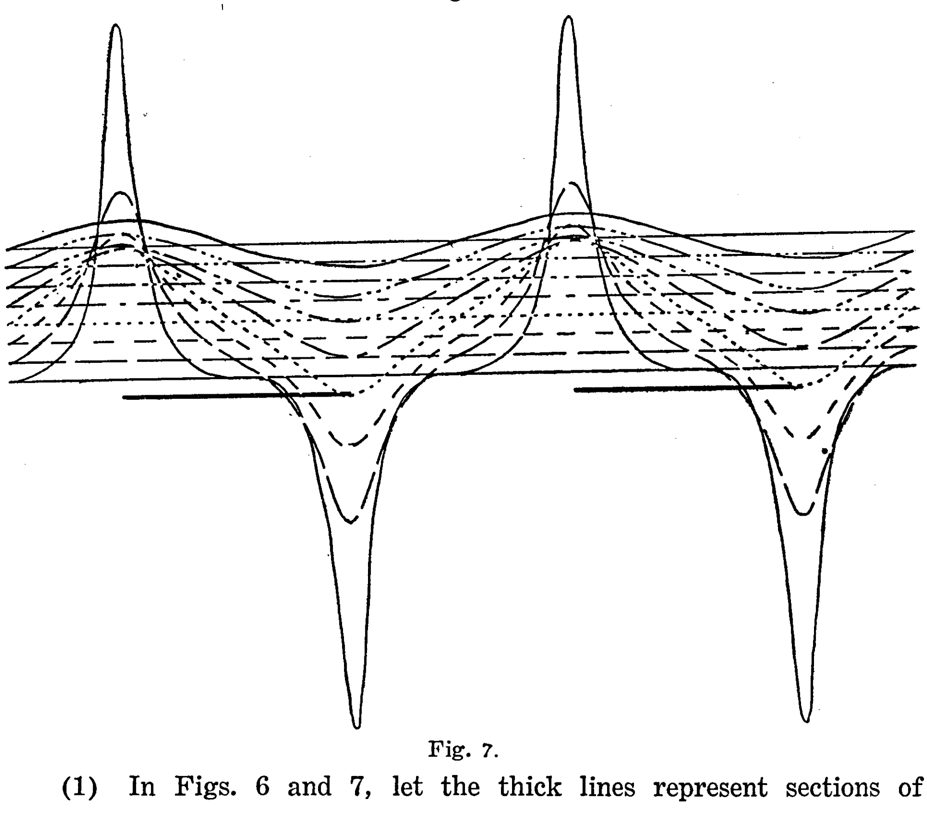

Fig. 7.

(1) In Figs. 6 and 7, let the thick lines represent sections of

strips of planes at depth d, with width π, and surface density 1. The density distribution along y-direction is

$$\rho(y)=\frac{1}{2}-\frac{2}{\pi}\sum_0^\infty\frac{1}{2n+1}\sin(2n+1)y$$

so that the surface gravity is

$$g(x)=2\pi k^2\left[\frac{1}{2}-\frac{2}{\pi}\sum\frac{1}{2n+1}e^{-(2n+1)d}\sin(2n+1)x\right]$$

and the gravity gradient is

$$\frac{\partial g}{\partial x}=-4k^2\sum e^{-(2n+1)d}\cos(2n+1)x.$$

The values of $e^{-(2n+1)d}$ are shown in Table IV for different n's and d's.

TABLE IV.

n \ d	$\frac{\pi}{12}$	$2\times\frac{\pi}{12}$	$3\times\frac{\pi}{12}$	$4\times\frac{\pi}{12}$	$5\times\frac{\pi}{12}$	$6\times\frac{\pi}{12}$	$7\times\frac{\pi}{12}$	$8\times\frac{\pi}{12}$	$9\times\frac{\pi}{12}$
0	0·770	0·592	0·456	0·351	0·270	0·201	0·160	0·123	0·095
1	456	201	95	43	20	9	4	1	
2	270	73	20	5					
3	160	25	4						
4	95	9							
5	56	3							
6	33								
7	20								
8	12								
9	7								
10	4								
11	2								

The values of $g(x)$ and $\frac{\partial g}{\partial x}$ are plotted in Figs. 6 and 7, in which curves of different patterns correspond to quantities along the earth's surfaces represented respectively by lines of the same pattern. It will be seen that, as the depth of the mass increases, both $g(x)$ and $\frac{\partial g}{\partial x}$ rapidly approach the harmonic form. This is owing to the fact that the coefficient $e^{-(2n+1)d}$ rapidly tends to zero with increasing d, especially for larger values of n.

(2) The next problem is to find the subterranean mass distribu-

tion that will produce surface gravity of the form

$$g(x)=g_0e^{-x^2}.$$

If we use in this case Fourier integral instead of Fourier series, we have

$$g(x)=\frac{2g_0}{\pi}\int_0^\infty\int_0^\infty e^{-\lambda^2}\cos nx\cos n\lambda d\lambda dn,$$

and because

$$\int_0^\infty e^{-\lambda^2}\cos n\lambda d\lambda=\frac{\sqrt{\pi}}{2}e^{-\frac{n^2}{4}},$$

we have

$$g(x)=\frac{g_0}{\sqrt{\pi}}\int_0^\infty e^{-\frac{n^2}{4}}\cos nxdn.$$

The corresponding subterranean mass distribution is therefore

$$\rho(y)=\frac{g_0}{2k^2\pi^{\frac{3}{2}}}\int_0^\infty e^{nd-\frac{n^2}{4}}\cos nydn.$$

Because the coefficients $e^{nd-\frac{n^2}{4}}$ rapidly tends to zero as n increases, as shown in Table V, this integral may be substituted by a summation of finite terms with sufficient accuracy.

The values of $\rho(y)$ are plotted in Fig. 8. It is remarkable that in order to have surface gravity values of the type of a probability curve it is necessary to have a certain negative density.

TABLE V.

n \ d	0·5	1·0	1·5
0·5	1·206	1·548	1·988
1·5	1·206	2·552	5·403
2·5	731	2·552	8 908
3·5	269	1·548	8·905
4·5	60	569	5·403
5·5	18	127	1·988
6·5	1	17	444
7·5		1	60
8·5			5

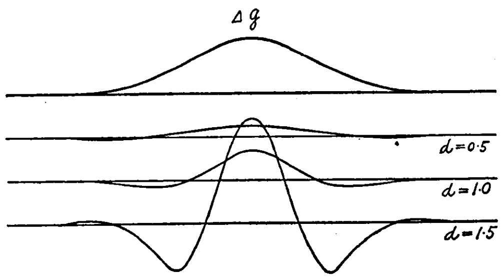

Fig. 8.

(3) The final example concerns the results of actual gravity measurements made by Vening Meinesz[2)] in the Dutch East Indies.

2) VENING MEINESZ, *Gravity Expeditions at Sea* 1923~1932., Vol II. (1934), Delft.

The regional isostatic anomalies along his profile No. 21, as shown by him are given in Table VI. Plotting these values according to distance as in Fig. 9, and connecting them by a smooth curve, the values for every 25 km were read off from the curve, of which there are 36 in all. As the curve seems to tend to 40 milligals toward both ends, this value was subtracted from all the observed values so as to get the very local gravity anomalies alone. With these 36 values, the gravity anomalies were subjected to harmonic analysis. The coefficients of the Fourier terms found were :

TABLE VI.

Point	Reg. Isos. An.
No. 394	+ 22
395	+ 1
396	− 23
397	− 47
398	−104
29	+ 12
399	+ 33
400	+ 7

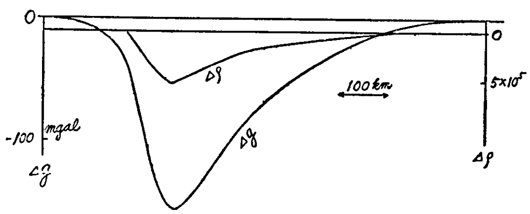

Fig. 9.

TABLE. VII.

n	sin	cos	n	sin	cos	n	sin	cos
0	—	46·1	7	1·0	1·3	14	− 0·2	0
1	−42·6	−44·6	8	− 0·7	0·7	15	− 0·1	− 0·2
2	22·9	−14·5	9	− 0·4	− 0·2	16	0	− 0·1
3	6·8	13·4	10	− 0·1	− 0·3	17	+ 0·2	− 0·3
4	− 7·5	4·1	11	0·2	− 0·2	18	—	0·1
5	− 2·0	− 4·4	12	0·4	0			
6	2·4	− 1·5	13	− 0·1	0·2			

If we assume with Vening Meinesz that the mass responsible for these gravity anomalies is at the depth of 25 km, which corresponds to $\frac{\pi}{18}$, then the Fourier coefficients for $2k^2\pi\rho(y)$ become :

TABLE VIII.

n	sin	cos	n	sin	cos	n	sin	cos
0	—	46·1	7	3·4	4·4	14	− 2·3	0
1	−50·7	−53·1	8	− 2·8	2·8	15	− 1·4	− 2·7
2	32·5	−20·6	9	− 1·9	− 1·0	16	0	− 1·6
3	11·5	22·6	10	− 0·6	− 1·7	17	4·2	− 6·2
4	−15·1	8·2	11	1·4	− 1·4	18	—	2·3
5	− 4·8	−10·5	12	3·2	0			
6	6·8	− 4·3	13	− 1·0	1·9			

This density distribution is also shown in Fig. 9. At the negative maximum

$$2\pi k^2\rho = 0.200,$$

therefore

$$\rho = \frac{0{\cdot}200}{2\times 3{\cdot}1\times 6{\cdot}7\times 10^{-8}} = 5\times 10^5.$$

This is equivalent to a mass of density 0·6 and thickness 8 km, which result is in exact agreement with Meinesz's assumption.

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40. 重力と地下構造との關係

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第1圖に於いて AA を地表面，BB を深さ d の面とし，BB の上に表面密度が分布して居るとする．AA 面上の重力及び BB 面上の密度は紙に垂直な方向には變化なく無限に延びて居て，夫々 $g(x)$ 及び $\rho(y)$で表し得るとする．若し

$$\rho(y) = \rho_n \cos ny$$

であるならば

$$g(x) = 2\pi k^2 \rho_n e^{-nd} \cos nx$$

である．逆に

$$g(x) = g_n \cos nx$$

ならば

$$\rho(y) = \frac{g_n}{2\pi k^2} e^{nd} \cos ny$$

でなければならない．此の關係を一般化すれば次の通りになる．即ち

$$\rho(y) = \sum A_n \cos ny + \sum B_n \sin ny$$

ならば

$$g(x) = 2\pi k^2 \{\sum A_n e^{-nd} \cos nx + \sum B_n e^{-nd} \sin nx\}$$

である．逆に

$$g(x) = \sum \alpha_n \cos nx + \sum \beta_n \sin nx$$

ならば

$$\rho(y) = \frac{1}{2\pi k^2}\left\{\sum \alpha_n e^{nd} \cos ny + \sum \beta_n e^{nd} \sin ny\right\}$$

である．是等の關係を利用すれば與へられた $\rho(y)$ から $g(x)$ を求めること，又逆に與へられた $g(x)$ から $\rho(y)$ を求めることが出來る．後の場合には d を假定しなければならないのは勿論の事である．

本文には上記の關係の導き方，その應用例數個と，此の方法がどの位の精度で適用されるかと云ふ吟味が述べてある．
