41. *The Deflections of the Vertical, the Undulation of the Geoid, and Gravity Anomalies.*

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Let AA in Fig. 1 be the plane of the earth's surface, the gravity anomalies along which are known from point to point, and BB another plane which is at a depth d from the surface, and on which are distributed the surface densities responsible for these gravity anomalies. Assume further that the surface densities along BB, and consequently the gravity anomalies along AA, do not change in the direction that extends from $+\infty$ to $-\infty$ perpendicularly to the plane of this paper, so that they can be expressed by $\rho(y)$ and $\Delta g(x)$ respectively, x and y being the abscissae along AA and BB, with line OO as the common origin. It was shown in the preceding paper[1] that if

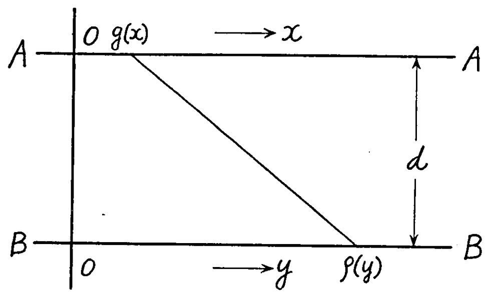

Fig. 1.

$$\rho(y) = \rho_n \cos ny\,,$$

then

$$\Delta g(x) = 2\pi k^2 \rho_n e^{-nd} \cos nx\,.$$

The horizontal component of attraction due to the same mass is given by

$$\frac{\partial V}{\partial x} = 2k^2 \rho_n \int_{-\infty}^{+\infty} \frac{(y-x)\cos ny}{(y-x)^2 + d^2}\,dy\,.$$

Putting

$$p = y - x\,,$$

we get

$$\frac{\partial V}{\partial x} = 2k^2 \rho_n \int_{-\infty}^{+\infty} \frac{p \cos n(p+x)}{p^2 + d^2}\,dp$$

1) C. TSUBOI and T. FUCHIDA, *Bull. Earthq. Res. Inst.*, **15** (1937), 636.

$$=2k^2\rho_n\left\{\cos nx\int_{-\infty}^{+\infty}\frac{p\cos np}{p^2+d^2}dp-\sin nx\int_{-\infty}^{+\infty}\frac{p\sin np}{p^2+d^2}dp\right\}$$

$$=-2\pi k^2\rho_n e^{-nd}\sin nx\,,$$

making use of the integrals

$$\int_{-\infty}^{+\infty}\frac{p\cos np}{p^2+d^2}dp=0$$

and

$$\int_{-\infty}^{+\infty}\frac{p\sin np}{p^2+d^2}dp=\pi e^{-nd}.$$

The deflection of the vertical $\Delta\theta$ at $x=x$ is given by

$$\Delta\theta=\frac{1}{g}\frac{\partial V}{\partial x}$$

$$=-\frac{2\pi k^2\rho_n}{g}e^{-nd}\sin nx\,.$$

The height of the geoid h at $x=x$ relative to that at $x=x_0$ is obtained by integrating the above expression with respect to x from x_0 to x, thus

$$h=\int_{x_0}^{x}\Delta\theta dx$$

$$=\frac{2\pi k^2\rho_n}{ng}e^{-nd}\cos nx-h_0\,.$$

Comparing this expression with that for Δg, we see that h and Δg are in phase with each other, a positive gravity anomaly being associated with up-warping of the geoid and a negative anomaly with down-warping.

If we omit the h_0 in the expression for h, which is a constant and combine it with that for Δg, we get

$$h=\frac{\Delta g}{ng}\,.$$

It is to be noted that h can be calculated from Δg with the aid of this expression without any previous knowledge of the depth of the subterranean mass $\rho(y)$.

Since

$$n=\frac{2\pi}{\lambda_n},$$

we have

$$h=\frac{\lambda_n}{2\pi}\frac{\Delta g}{g}.$$

If, for instance, we put

$$\lambda_n=100\text{ km},$$

$$|\Delta g|=0.100\text{ gal},$$

then we get

$$|h|=\frac{10^7\times 0.1}{2\pi\times 10^3}\doteqdot 170\text{ cm}.$$

The above relation between h and Δg can be extended to more general and complicated cases in which there is more than one harmonic component. Thus, if

$$\Delta g(x)=\sum A_n \cos nx+\sum B_n \sin nx,$$

then

$$h=\frac{a}{2\pi g}\left\{\sum\frac{A_n}{n}\cos nx+\sum\frac{B_n}{n}\sin nx\right\},$$

where a is the extent of the range in which Δg is expressed in the Fourier series. It is not necessary that all the harmonic components of Δg should be due to the mass at one and the same depth, the only requirement being that each component shall be due to the mass that is respectively horizontal.

Because each of the Fourier coefficients for h is n times smaller than the corresponding one for Δg, the relative variation in the height of the geoid will be much smaller than that in Δg.

The gravity anomalies found by Vening Meinesz along his profile No. 21 have already been discussed in the preceding paper. Utilizing the Fourier coefficients for Δg calculated in that paper, those for h were found to be as follows:

n	sin	cos	n	sin	cos	n	sin	cos
1	42·6	44·6	4	1·9	−1·0	7	−0·1	−0·2
2	−11·5	7·3	5	0·4	0·9	8	0·1	0·1
3	− 2·3	−4·5	6	−0·4	0·3			

The form of the geoid along this profile, which was obtained by synthesis of the harmonic components given in the table, is shown in Fig. 2.

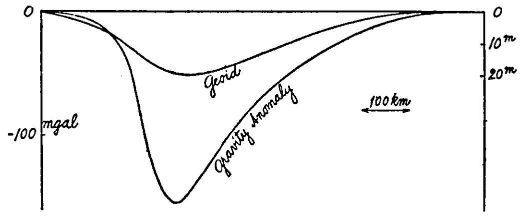

Fig. 2.

It is hoped that the method here proposed, which corresponds to that of Stokes, except that the latter applies to spherical surfaces, will find many applications in practical problems.

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41. 鉛直線偏倚，ジオイドの凹凸と重力異常

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適當な假定を設けると，表題の三者の間に簡單な關係がある事を示す．特に重力異常が與へられればジオイドの凹凸が解ると云ふ事は應用が廣いと考へられる．本文中には蘭領印度諸島附近に於けるマイネスの測定について應用例が示してある．
