

44. Notes on the Origin of Earthquakes. (Third paper.)

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1. Introduction

In his previous papers,¹⁾ the writer, under the same title as above, discussed theoretically the mechanisms of radiations of earthquake waves from a seismic origin, assuming their shape to be spherical. Should the pressure in the volcanic vent change, or should faults be newly formed or activated by certain endogenic forces, earthquake waves would emanate from these origins. In these cases, it is preferable to regard these waves as having originated from line sources than that they did so from spherical origins. Moreover, if, generally speaking, we regard the elastic waves as having originated from fissures opened in the earth's crust by the action of certain forces, it would seem to the author that all that is sufficient as a first step is to assume that they originated as the result of certain changes in the normal and tangential stresses at the boundaries of the line sources.

In the following paragraphs, the writer discusses the foregoing problem in two dimensions only, leaving the cases for three dimensions to a future opportunity.

2. Elliptic Origin

The equations of motion in an isotropic homogeneous elastic body in elliptic coordinates are given by

$$\left. \begin{aligned} \rho \frac{\partial^2 \vartheta}{\partial t^2} &= \frac{\lambda + 2\mu}{c^2(\cosh^2 \xi - \cos^2 \eta)} \left(\frac{\partial^2 \vartheta}{\partial \xi^2} + \frac{\partial^2 \vartheta}{\partial \eta^2} \right) \\ \rho \frac{\partial^2 \omega}{\partial t^2} &= \frac{\mu}{c^2(\cosh^2 \xi - \cos^2 \eta)} \left(\frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \eta^2} \right) \end{aligned} \right\},$$

where

ξ, η =curvilinear coordinates related to Cartesian coordinates (x, y) in the form,

1) W. INOUYE, *Bull. Earthq. Res. Inst.*, **14** (1936), 582; **15** (1937), 90.

$$\left. \begin{aligned} x &= c \cosh \xi \cos \eta \\ y &= c \sinh \xi \sin \eta \\ \Delta &= h_1^2 \left\{ \frac{\partial}{\partial \xi} \left(\frac{u}{h_1} \right) + \frac{\partial}{\partial \eta} \left(\frac{v}{h_1} \right) \right\} \\ 2\varpi &= h_1^2 \left\{ \frac{\partial}{\partial \xi} \left(\frac{v}{h_1} \right) + \frac{\partial}{\partial \eta} \left(\frac{u}{h_1} \right) \right\} \\ \frac{1}{h_1^2} &= c^2 (\cosh^2 \xi - \cos^2 \eta) \end{aligned} \right\},$$

where

u, v = components of displacement referred to curvilinear coordinates, ξ and η , respectively,

ρ = density of isotropic solid,

λ, μ = Lame's elastic constants.

According to Prof. K. Sezawa²⁾ the solutions of these equations are expressed by

$$\left. \begin{aligned} \Delta &= \sum_{n=0}^{\infty} B_n H_n(\xi, q) \left\{ \begin{array}{l} ce_n(\eta, q) \\ se_n(\eta, q) \end{array} \right\} e^{ipt} \\ 2\varpi &= \sum_{n=0}^{\infty} C_n H_n(\xi, q') \left\{ \begin{array}{l} ce_n(\eta, q') \\ se_n(\eta, q') \end{array} \right\} e^{ipt} \end{aligned} \right\}, \quad (1)$$

where $q = \frac{h^2 c^2}{32}$, $q' = \frac{k^2 c^2}{32}$, $\frac{\rho p^2}{\lambda + 2\mu} = h^2$, $\frac{\rho p^2}{\mu} = k^2$,

$H_n(\xi, q)$, $H_n(\xi, q')$ are the solutions of the following equations,

$$\left. \begin{aligned} \frac{d^2 H_n(\xi, q)}{d\xi^2} + (h^2 c^2 \cosh^2 \xi - n^2) H_n(\xi, q) &= 0 \\ \frac{d^2 H_n(\xi, q')}{d\xi^2} + (k^2 c^2 \cosh^2 \xi - n^2) H_n(\xi, q') &= 0 \end{aligned} \right\}. \quad (2)$$

Whence the components of displacement of the dilatational wave that answers to Δ and satisfies $\varpi=0$ in (1) are given by

$$u_1 = - \sum_{n=0}^{\infty} \frac{B_n}{h^2 c_1 / \cosh^2 \xi - \cos^2 \eta} \frac{\partial H_n(\xi, q)}{\partial \xi} \left\{ \begin{array}{l} ce_n(\eta, q) \\ se_n(\eta, q) \end{array} \right\} e^{ipt}$$

2) K. SEZAWA, Bull. Earthq. Res. Inst., 5 (1928), 59.

$$v_1 = - \sum_{n=1}^{\infty} \frac{B_n}{h^2 c \sqrt{\cosh^2 \xi - \cos^2 \eta}} H_n(\xi, q) \left\{ \begin{array}{l} \frac{\partial ce_n(\eta, q)}{\partial \eta} \\ \frac{\partial se_n(\eta, q)}{\partial \eta} \end{array} \right\} e^{ipt} \quad .$$

The components of displacement of the distortional wave that satisfies $\mathcal{A}=0$ are given by

$$u_2 = - \sum_{n=1}^{\infty} \frac{C_n}{k^2 c \sqrt{\cosh^2 \xi - \cos^2 \eta}} H_n(\xi, q') \left\{ \begin{array}{l} \frac{\partial ce_n(\eta, q')}{\partial \eta} \\ \frac{\partial se_n(\eta, q')}{\partial \eta} \end{array} \right\} e^{ipt} \quad ,$$

$$v_2 = - \sum_{n=1}^{\infty} \frac{C_n}{k^2 c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \frac{\partial H_n(\xi, q')}{\partial \xi} \left\{ \begin{array}{l} ce_n(\eta, q') \\ se_n(\eta, q') \end{array} \right\} e^{ipt}$$

Since on the surface, $\xi=\xi_0$, the two equations

$$\lambda \left[\mathcal{A} \right]_{\xi=\xi_0} + 2\mu \left[\frac{1}{c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \frac{\partial (u_1 + u_2)}{\partial \xi} \right. \\ \left. + \frac{v_1 + v_2}{c^2 (\cosh^2 \xi - \cos^2 \eta)} \frac{\partial}{\partial \eta} \left(c \sqrt{\cosh^2 \xi - \cos^2 \eta} \right) \right]_{\xi=\xi_0} = \text{normal stress},$$

$$\mu \left[\frac{\partial}{\partial \xi} \left(\frac{v_1 + v_2}{c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \right) + \frac{\partial}{\partial \eta} \left(\frac{u_1 + u_2}{c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \right) \right]_{\xi=\xi_0} = \text{tangential stress}$$

must hold.

3. Line Origin

If now we consider a line origin, we may put $\xi_0=0$, and for simplicity, put $\lambda=\mu$.

The normal stress at the origin is then given by
normal stress ($\equiv \phi(\eta) e^{ipt}$)

$$= \mu e^{ipt} \left[\sum_{n=0}^{\infty} B_n H_n(\xi, q) ce_n(\eta, q) \right. \\ \left. + 2 \left[\frac{1}{c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \left\{ - \sum_{n=0}^{\infty} \frac{-B_n \cosh \xi \sinh \xi}{h^2 c (\cosh^2 \xi - \cos^2 \eta)^{\frac{3}{2}}} \frac{\partial H_n(\xi, q)}{\partial \xi} ce_n(\eta, q) \right. \right. \right. \\ \left. \left. \left. - \sum_{n=0}^{\infty} \frac{B_n}{h^2 c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \frac{\partial^2 H_n(\xi, q)}{\partial \xi^2} ce_n(\eta, q) \right\} \right] \right]$$

$$\begin{aligned}
 & + \sum_{n=1}^{\infty} \frac{-C_n \cosh \xi \sinh \xi}{k^2 c (\cosh^2 \xi - \cos^2 \eta)^{\frac{3}{2}}} H_n(\xi, q') \frac{\partial s e_n(\eta, q')}{\partial \eta} \\
 & + \sum_{n=1}^{\infty} \frac{C_n}{k^2 c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \left\{ \frac{\partial H_n(\xi, q')}{\partial \xi} \frac{\partial s e_n(\eta, q')}{\partial \eta} \right\} \\
 & + \left\{ - \sum_{n=1}^{\infty} \frac{B_n}{h^2 c^3 (\cosh^2 \xi - \cos^2 \eta)^{\frac{3}{2}}} H_n(\xi, q) \frac{\partial c e_n(\eta, q)}{\partial \eta} \right. \\
 & \left. - \sum_{n=1}^{\infty} \frac{C_n}{h^2 c^3 (\cosh^2 \xi - \cos^2 \eta)^{\frac{3}{2}}} \frac{\partial H_n(\xi, q')}{\partial \xi} s e_n(\eta, q') \right\} \\
 & \times c (\cosh^2 \xi - \cos^2 \eta)^{-\frac{1}{2}} \cos \eta \sin \eta \Bigg] \Bigg]_{\xi=0}.
 \end{aligned}$$

The upper equation of (2) for a small value of ξ may be written in the form,³⁾

$$\frac{d^2 H_n(\xi, q)}{d\xi^2} + \left[(h^2 c^2 - n^2) + h^2 c^2 \left(\xi^2 + \frac{\xi^4}{3} + \dots \right) \right] H_n(\xi, q) = 0.$$

Putting $H_n(\xi, q)$ in the expanded form,

$$H_n(\xi, q) = a_0 + a_1 \xi + a_2 \xi^2 + \dots$$

and neglecting small quantities of the second order, we obtain,

$$H_n(\xi, q) = e^{-i\sqrt{h^2 c^2 - n^2} \xi}.$$

In the same manner we obtain,

$$H_n(\xi, q') = e^{-i\sqrt{h^2 c^2 - n^2} \xi}.$$

Thus the normal stress at the boundary reduces to

$$\begin{aligned}
 \phi(\eta) e^{ip\eta} &= \mu e^{ip\eta} \left[\sum_{n=0}^{\infty} B_n c e_n(\eta, q) \right. \\
 &+ \frac{1}{\sin^2 \eta} \left\{ - \sum_{n=0}^{\infty} \alpha_n B_n c e_n(\eta, q) + \sum_{n=1}^{\infty} \beta_n C_n \frac{\partial s e_n(\eta, q')}{\partial \eta} \right\} \\
 &\left. + \frac{\cos \eta}{\sin^3 \eta} \left\{ - \sum_{n=1}^{\infty} \gamma B_n \frac{\partial c e_n(\eta, q)}{\partial \eta} - \sum_{n=1}^{\infty} \beta_n C_n s e_n(\eta, q') \right\} \right],
 \end{aligned}$$

3) K. SEZAWA, Bull. Earthq. Res. Inst., 2 (1927), 29.

where

$$\left. \begin{aligned} \alpha_n &= -2\left(1 - \frac{n^2}{k^2 c^2}\right) \\ \beta_n &= \frac{-2i\sqrt{k^2 c^2 - n^2}}{k^2 c^2} \\ \gamma &= \frac{2}{h^2 c^2} \end{aligned} \right\}$$

The tangential stress at the origin is given by tangential stress ($\equiv \phi'(\eta) e^{ip\theta}$)

$$\begin{aligned} &= \mu e^{ip\theta} \left[\frac{\partial}{\partial \xi} \left[- \sum_{n=1}^{\infty} \frac{B_n}{h^2 c^2 (\cosh^2 \xi - \cos^2 \eta)} H_n(\xi, q) \frac{\partial ce_n(\eta, q)}{\partial \eta} \right] \right. \\ &\quad + \frac{\partial}{\partial \xi} \left[- \sum_{n=1}^{\infty} \frac{C_n}{h^2 c^2 (\cosh^2 \xi - \cos^2 \eta)} \frac{\partial H_n(\xi, q')}{\partial \xi} se_n(\eta, q') \right] \\ &\quad + \frac{\partial}{\partial \eta} \left[- \sum_{n=0}^{\infty} \frac{B_n}{h^2 c^2 (\cosh^2 \xi - \cos^2 \eta)} \frac{\partial H_n(\xi, q)}{\partial \xi} ce_n(\eta, q) \right] \\ &\quad \left. + \frac{\partial}{\partial \eta} \left[\sum_{n=1}^{\infty} \frac{C_n}{h^2 c^2 (\cosh^2 \xi - \cos^2 \eta)} H_n(\xi, q') \frac{\partial se_n(\eta, q')}{\partial \eta} \right] \right]_{\xi=0} \\ &= \mu e^{ip\theta} \left[- \frac{\alpha'_o B_o}{\sin^2 \eta} \frac{\partial ce_o(\eta, q)}{\partial \eta} - 2 \sum_{n=1}^{\infty} \frac{\alpha'_n B_n}{\sin^2 \eta} \frac{\partial ce_n(\eta, q)}{\partial \eta} \right. \\ &\quad - \sum_{n=0}^{\infty} \frac{-\alpha'_n B_n \sin 2\eta}{\sin^4 \eta} ce_n(\eta, q) - \sum_{n=1}^{\infty} \frac{\beta'_n C_n}{\sin^2 \eta} se_n(\eta, q') \\ &\quad \left. + \sum_{n=1}^{\infty} \left\{ \frac{-\gamma' C_n \sin 2\eta}{\sin^4 \eta} \frac{\partial se_n(\eta, q')}{\partial \eta} + \frac{\gamma' C_n}{\sin^2 \eta} \frac{\partial^2 se_n(\eta, q')}{\partial \eta^2} \right\} \right], \end{aligned}$$

where

$$\left. \begin{aligned} \alpha'_n &= \frac{-i\sqrt{h^2 c^2 - n^2}}{h^2 c^2} \\ \beta'_n &= \frac{-(k^2 c^2 - n^2)}{h^2 c^2} \end{aligned} \right\}$$

$$\gamma' = \frac{1}{k^2 c^2} \quad \left. \right|$$

Since we cannot proceed further in the general manner, we must content ourselves by solving the problem approximately. We accordingly use Mathieu functions of the 0th to 6th orders, and, neglect the higher terms in q .

The Mathieu functions⁴⁾ are then given by

$$\left. \begin{aligned} ce_0 &= 1 + 4q \cos 2\eta \\ ce_1 &= \cos \eta + q \cos 3\eta \\ ce_2 &= \cos 2\eta + q \left(\frac{2}{3} \cos 4\eta - 2 \right) \\ ce_3 &= \cos 3\eta + q \left(-\cos \eta + \frac{1}{2} \cos 5\eta \right) \\ ce_4 &= \cos 4\eta + q \left(-\frac{2}{3} \cos 2\eta + \frac{2}{5} \cos 6\eta \right) \\ ce_5 &= \cos 5\eta + q \left(-\frac{1}{2} \cos 3\eta + \frac{1}{3} \cos 7\eta \right) \\ ce_6 &= \cos 6\eta + q \left(-\frac{2}{5} \cos 4\eta + \frac{2}{7} \cos 8\eta \right) \\ \\ se_1 &= \sin \eta + q \sin 3\eta \\ se_2 &= \sin 2\eta + \frac{2}{3} q \sin 4\eta \\ se_3 &= \sin 3\eta + q \left(-\sin \eta + \frac{1}{2} \sin 5\eta \right) \\ se_4 &= \sin 4\eta + q \left(-\frac{2}{3} \sin 2\eta + \frac{2}{5} \sin 6\eta \right) \\ se_5 &= \sin 5\eta + q \left(-\frac{1}{2} \sin 3\eta + \frac{1}{3} \sin 7\eta \right) \\ se_6 &= \sin 6\eta + q \left(-\frac{2}{5} \sin 4\eta + \frac{2}{7} \sin 8\eta \right) \end{aligned} \right\}$$

4) B van der POL and M.J.O. STRUTT, *Phil. Mag.*, **5** (1928), 18;
M.J.O. STRUTT, *Ann. d. Phys.*, **84** (1927), 485.

Introducing these equations into the boundary conditions given above, the normal and tangential stresses at the origin are expressed by

normal stress ($\equiv a_o + \sum_{n=1}^{\infty} a_n \cos n\eta$)

$$\begin{aligned}
 &= \mu \left[(\varepsilon_{1,0}B_0 + \varepsilon_{1,2}B_2 + \varepsilon_{1,4}B_4 + \varepsilon_{1,6}B_6 + \nu_{1,2}C_2 + \nu_{1,4}C_4 + \nu_{1,6}C_6) \right. \\
 &\quad + (B_1, B_3, B_5, C_1, C_3, C_5) \cos \eta \\
 &\quad + (\varepsilon_{2,0}B_0 + \varepsilon_{2,2}B_2 + \varepsilon_{2,4}B_4 + \varepsilon_{2,6}B_6 + \nu_{2,2}C_2 + \nu_{2,4}C_4 + \nu_{2,6}C_6) \cos 2\eta \\
 &\quad + (B_1, B_3, B_5, C_1, C_3, C_5) \cos 3\eta \\
 &\quad + (\varepsilon_{3,0}B_0 + \varepsilon_{3,2}B_2 + \varepsilon_{3,4}B_4 + \varepsilon_{3,6}B_6 + \nu_{3,2}C_2 + \nu_{3,4}C_4 + \nu_{3,6}C_6) \cos 4\eta \\
 &\quad + (B_1, B_3, B_5, C_1, C_3, C_5) \cos 5\eta \\
 &\quad + (\varepsilon_{4,0}B_0 + \varepsilon_{4,2}B_2 + \varepsilon_{4,4}B_4 + \varepsilon_{4,6}B_6) \frac{1}{\sin^2 \eta} \\
 &\quad \left. + (B_1, B_3, B_5, C_1, C_3, C_5) \frac{\cos \eta}{\sin^2 \eta} + \dots \right],
 \end{aligned}$$

tangential stress ($\equiv \sum_{n=1}^{\infty} b_n \sin n\eta$)

$$\begin{aligned}
 &= \mu \left[(B_1, B_3, B_5, C_1, C_3, C_5) \sin \eta \right. \\
 &\quad + (\varepsilon'_{1,2}B_2 + \varepsilon'_{1,4}B_4 + \varepsilon'_{1,6}B_6 + \nu'_{1,2}C_2 + \nu'_{1,4}C_4 + \nu'_{1,6}C_6) \sin 2\eta \\
 &\quad + (B_1, B_3, B_5, C_1, C_3, C_5) \sin 3\eta \\
 &\quad + (\varepsilon'_{2,2}B_2 + \varepsilon'_{2,4}B_4 + \varepsilon'_{2,6}B_6 + \nu'_{2,2}C_2 + \nu'_{2,4}C_4 + \nu'_{2,6}C_6) \frac{\cos \eta}{\sin \eta} \\
 &\quad + (\varepsilon'_{3,0}B_0 + \varepsilon'_{3,2}B_2 + \varepsilon'_{3,4}B_4 + \varepsilon'_{3,6}B_6 + \nu'_{3,2}C_2 + \nu'_{3,4}C_4 + \nu'_{3,6}C_6) \frac{\cos \eta}{\sin^3 \eta} \\
 &\quad \left. + \dots \right],
 \end{aligned}$$

where

$$\varepsilon_{1,0} = 1 + 8q\alpha_0$$

$$\varepsilon_{1,2} = -2q + 2\alpha_2 + \frac{8}{3}q\alpha_2 - 4r - \frac{64}{3}qr$$

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$$\left. \begin{array}{l} \varepsilon_{1,4} = \frac{16}{15}q\alpha_4 + 4\alpha_4 - 32\gamma - \frac{31}{3}qr \\ \varepsilon_{1,6} = \frac{24}{35}q\alpha_6 + 6\alpha_6 - 108\gamma - \frac{2112}{35}qr \\ \nu_{1,2} = -2\beta_2 - \frac{16}{3}q'\beta_2 \\ \nu_{1,4} = -8\beta_4 - \frac{88}{15}q'\beta_4 \\ \nu_{1,6} = -\frac{208}{35}q'\beta_6 - 18\beta_6 \\ \varepsilon_{2,0} = 4q \\ \varepsilon_{2,2} = 1 + \frac{8}{3}q\alpha_2 - \frac{32}{3}qr \\ \varepsilon_{2,4} = -\frac{2}{3}q + 4\alpha_4 + \frac{16}{5}q\alpha_4 - 16\gamma - \frac{192}{5}qr \\ \varepsilon_{2,6} = 8\alpha_6 - 96\gamma + \frac{64}{35}q\alpha_6 - \frac{2656}{35}qr \\ \nu_{2,2} = -8q'\beta_2 \\ \nu_{2,4} = -12\beta_4 - \frac{64}{5}q'\beta_4 \\ \nu_{2,6} = -32\beta_6 - \frac{432}{35}q'\beta_6 \\ \varepsilon_{3,2} = \frac{2}{3}q \\ \varepsilon_{3,4} = 1 + \frac{8}{5}q\alpha_4 - \frac{48}{5}qr \\ \varepsilon_{3,6} = -\frac{2}{5}q + 4\alpha_6 + \frac{16}{7}q\alpha_6 - 24\gamma - \frac{256}{7}qr \\ \nu_{3,4} = -8q'\beta_4 \\ \nu_{3,6} = -20\beta_6 - \frac{96}{7}q'\beta_6 \end{array} \right\},$$

$$\left. \begin{array}{l}
 \varepsilon_{4,0} = -\alpha_0 - 4q\alpha_0 \\
 \varepsilon_{4,2} = -\alpha_2 - \frac{2}{3}q\alpha_2 + 4\gamma + \frac{32}{3}q\gamma \\
 \varepsilon_{4,4} = -\alpha_4 + 16\gamma + \frac{4}{15}q\alpha_4 + \frac{176}{15}q\gamma \\
 \varepsilon_{4,6} = -\alpha_6 + 36\gamma + \frac{4}{35}q\alpha_6 + \frac{416}{35}q\gamma \\
 \\
 \varepsilon'_{1,2} = -16q\alpha'_2 \\
 \varepsilon'_{1,4} = -24\alpha'_4 - \frac{128}{5}q\alpha'_4 \\
 \varepsilon'_{1,6} = -64\alpha'_6 - \frac{1536}{35}q\alpha'_6 \\
 \nu'_{1,2} = \frac{64}{3}q'\gamma' + \frac{8}{3}q'\beta'_2 \\
 \nu'_{1,4} = 32\gamma' + 4\beta'_4 + \frac{16}{5}q'\beta'_4 + \frac{192}{5}q'\gamma' \\
 \nu'_{1,6} = 8\beta'_6 + 96\gamma' + \frac{64}{35}q'\beta'_6 + \frac{1472}{35}q'\gamma' \\
 \\
 \varepsilon'_{2,2} = 4\alpha'_2 + \frac{32}{3}q\alpha'_2 \\
 \varepsilon'_{2,4} = 16\alpha'_4 + \frac{176}{15}q\alpha'_4 \\
 \varepsilon'_{2,6} = 36\alpha'_6 + \frac{416}{35}q\alpha'_6 \\
 \nu'_{2,2} = -2\beta'_2 - \frac{8}{3}q'\beta'_2 \\
 \nu'_{2,4} = -4\beta'_4 - \frac{16}{15}q'\beta'_4 \\
 \nu'_{2,6} = -6\beta'_6 - \frac{24}{35}q'\beta'_6
 \end{array} \right\},$$

$$\left. \begin{aligned}
 \varepsilon'_{3,0} &= 2\alpha'_0 + 8q\alpha'_0 \\
 \varepsilon'_{3,2} &= 2\alpha'_2 - \frac{8}{3}q\alpha'_2 \\
 \varepsilon'_{3,4} &= 2\alpha'_4 - \frac{8}{15}q\alpha'_4 \\
 \varepsilon'_{3,6} &= 2\alpha'_6 - \frac{8}{35}q\alpha'_6 \\
 \nu'_{3,2} &= -4\gamma' - \frac{16}{3}q'\gamma' \\
 \nu'_{3,4} &= -8\gamma' - \frac{32}{15}q'\gamma' \\
 \nu'_{3,6} &= -12\gamma' - \frac{48}{35}q'\gamma'
 \end{aligned} \right\}.$$

If now we consider the case in which uniform pressure changes periodically with time within the line origin, we may put the normal stress at the boundary of the origin as $a_0 e^{i\mu t}$ and the tangential stress as zero. Moreover we may put for symmetry,

$$B_1 \equiv B_3 \equiv B_5 \equiv C_1 \equiv C_3 \equiv C_5 \equiv 0.$$

We thus obtain the following simultaneous equations:

$$\left. \begin{aligned}
 \varepsilon_{1,0}B_0 + \varepsilon_{1,2}B_2 + \varepsilon_{1,4}B_4 + \varepsilon_{1,6}B_6 + \nu_{1,2}C_2 + \nu_{1,4}C_4 + \nu_{1,6}C_6 &= \frac{a_0}{\mu} \\
 \varepsilon_{2,0}B_0 + \varepsilon_{2,2}B_2 + \varepsilon_{2,4}B_4 + \varepsilon_{2,6}B_6 + \nu_{2,2}C_2 + \nu_{2,4}C_4 + \nu_{2,6}C_6 &= 0 \\
 0B_0 + \varepsilon_{3,2}B_2 + \varepsilon_{3,4}B_4 + \varepsilon_{3,6}B_6 + 0C_2 + \nu_{3,4}C_4 + \nu_{3,6}C_6 &= 0 \\
 \varepsilon_{4,0}B_0 + \varepsilon_{4,2}B_2 + \varepsilon_{4,4}B_4 + \varepsilon_{4,6}B_6 + 0C_2 + 0C_4 + 0C_6 &= 0 \\
 0B_0 + \varepsilon'_{1,2}B_2 + \varepsilon'_{1,4}B_4 + \varepsilon'_{1,6}B_6 + \nu'_{1,2}C_2 + \nu'_{1,4}C_4 + \nu'_{1,6}C_6 &= 0 \\
 0B_0 + \varepsilon'_{2,2}B_2 + \varepsilon'_{2,4}B_4 + \varepsilon'_{2,6}B_6 + \nu'_{2,2}C_2 + \nu'_{2,4}C_4 + \nu'_{2,6}C_6 &= 0 \\
 \varepsilon'_{3,0}B_0 + \varepsilon'_{3,2}B_2 + \varepsilon'_{3,4}B_4 + \varepsilon'_{3,6}B_6 + \nu'_{3,2}C_2 + \nu'_{3,4}C_4 + \nu'_{3,6}C_6 &= 0
 \end{aligned} \right\} \quad (3)$$

We obtain $B_0, B_2, B_4, B_6, C_2, C_4$, and C_6 from these equations.

Now when ξ is large, the asymptotic solution of Mathieu's equation (2) is obtained in the form⁵⁾

5) K. SEZAWA, loc. cit.

$$H_n(\xi, q) = \frac{e^{-i\xi_1} u}{\sqrt{\xi_1}},$$

where

$$\xi_1 = hc \sinh \xi, \quad u = 1 + \frac{a_1}{\xi_1} + \frac{a_2}{\xi_1^2} + \dots,$$

$$a_1 = -\frac{i}{2} (j^2 - M^2 + 1)$$

$$a_2 = -\frac{j^2}{2} + \frac{1}{8} (j^2 - M^2 + 1) (2 - j^2 + M^2)$$

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$$j = hc, \quad M^2 = a + \frac{j^2}{2}, \quad a = \frac{h^2 c^2}{2} - n^2$$

}

Hence the radial component of the dilatational wave at a distant point is given by

$$\begin{aligned} u_1 &= - \sum_{n=0}^{\infty} \frac{B_n}{h^2 c \sqrt{\cosh^2 \xi - \cos^2 \eta}} \frac{\partial H_n(\xi, q)}{\partial \xi} ce_n(\eta, q) e^{ipx} \\ &= - \sum_{n=0}^{\infty} \frac{B_n}{h^2 c \cosh \xi} \frac{\partial}{\partial \xi} \left(\frac{e^{-i h c \sinh \xi}}{\sqrt{h c \sinh \xi}} \right) ce_n(\eta, q) e^{ipx} \\ &= \sum_{n=0}^{\infty} \frac{i B_n}{h \sqrt{h c \sinh \xi}} ce_n(\eta, q) e^{i(pz - h c \sinh \xi)} \\ &= \sum_{n=0}^{\infty} \frac{i B_n}{h^{\frac{3}{2}} R^{\frac{1}{2}}} ce_n(\eta, q) e^{i(pz - hR)} \\ &= - \sum_{n=0}^{\infty} \frac{1}{h^{\frac{3}{2}} R^{\frac{1}{2}}} \left\{ A'_n ce_n(\eta, q) \sin(pt - hR) \right. \\ &\quad \left. + B'_n ce_n(\eta, q) \cos(pt - hR) \right\}, \end{aligned}$$

where $A'_n + iB'_n = B_n$ and R stand for the distance from the origin.

As an example, we put

$$q = 0.003333, \quad q' = 0.01,$$

whence we have $h^2 c^2 = 0.1066, \quad k^2 c^2 = 0.32$.

We thus obtain from the equations (3),

$$\left. \begin{array}{l} \mu A'_0 = -0.01065a_0, \quad \mu B'_0 = -0.0001858a_0 \\ \mu A'_2 = +0.007775a_0, \quad \mu B'_2 = +0.003823a_0 \\ \mu A'_4 = +0.001666a_0, \quad \mu B'_4 = -0.004619a_0 \\ \mu A'_6 = -0.0009628a_0, \quad \mu B'_6 = +0.001551a_0 \end{array} \right\}$$

The azimuthal distribution of the radial component (at a distant point) of the dilatational wave in this case is shown in Fig. 1. As will be seen from the figure, at the same epoch only one kind of wave, namely, either pull or push wave, may be observed throughout the azimuth at places equally distant from the origin.

We may here mention that in the case under consideration, the wave length of the dilatational wave is 9.64 times greater than the linear dimension of the origin, that is $2c$. If we change the wave length, we may obtain different distributions of the displacement of the radial component of the dilatational wave in azimuth.

The case in which the tangential stress at the boundary of the origin undergoes periodic changes will be treated in a future paper.

In conclusion, the writer's cordial thanks are due to Prof. M. Ishimoto, Prof. Ch. Tsuboi, Prof. R. Takahasi and Prof. F. Kishinouye for their kind advices.

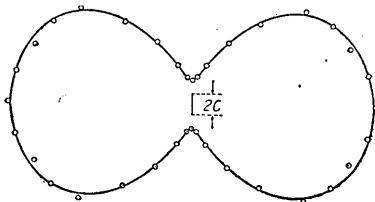


Fig. 1. The azimuthal distribution, at a distant point, of the radial component of displacement of the dilatational wave, the wave length of which is 9.64 times greater than $2c$.

44. 発震機構に就いて(第3報)

地震研究所 非上胤

第一報及び第二報に於ては震源を球形と考へ其の表面に働く垂直應力の變化によつて生ずる彈性波に就いて數理的研究を行つた。此處では火山の管形の火口内部に於ける壓力の變化によつて生ずる地震波の場合及び断層運動によつて生ずる地震波の場合等を論ずる爲に球形震源の代りに線形震源を考へ其の表面に働く應力の變化によつて生ずる彈性波の方位的特性を調査した。未だ極めて豫備的な研究であるが一般に割目の生成に伴つて生ずる彈性波の研究にも役立つものと思はれる。