45. *Notes on the Origin of Earthquakes. (Fourth paper.)*

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1. Introduction.

As is well known, there are two types of geographical distribution of pull and push waves at the initial phase of earthquake waves, namely, the conical and quadrant types.

The cases in which conical types were observed have been fully examined by Prof. M. Ishimoto[1] and Dr. T. Minakami,[2] while the cases in which the quadrant types were observed have been thoroughly treated by Dr. K. Honda.[3] According to these investigators, it seems to the writer that these cases could be satisfactorily explained by assuming certain simple forms of azimuthal differences in the stresses at a spherical surface taken at the seismic focus, such as $P_2(\cos\theta)$ for the conical type and $P_2^1(\cos\theta)$ for the quadrant type, these stresses undergo rapid changes.

The writer,[4] with the assistance of Mr. H. Kimura, tried to reproduce these types of distributions of initial motions by means of artificial earthquakes, but the results were negative.

The only representation available is the sound emitted by a tuning fork. In this case the compressional wave and the rarefactional wave are separated by two surfaces parabolic to each other. The solid angle of each of these paraboloides is about 120°. The true mechanism of the emission of sound waves in this case is not yet known. In practise, to comprehend the phenomena it is sufficient to consider a couple of doublets acting along a line in opposite phase and a sink when the two prongs of the tuning fork move outward, and a source when they move inward alternately between the two prongs.

When the writer fired an explosive placed in a short metal tube

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1) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **10** (1932), 449; *Proc. Imp. Acad. Tokyo*, **8** (1932), 36.

2) T. MINAKAMI, *Bull. Earthq. Res. Inst.*, **13** (1935), 114.

3) H. HONDA, *Geophys. Mag.*, **8** (1934), 153; **8** (1935), 327.

4) W. INOUYE and H. KIMURA, *Bull. Earthq. Res. Inst.*, **13** (1935), 194.

open at one end and closed and flanged at the other end, which telescoped into another similar tube (the whole placed under the ground), in all azimuths nothing but push waves was observed.[5] It seems, however, that if the lengths of the tubes are long enough to be comparable with the wave length (5~10 meters in our experiments), we should expect pull waves in some of the directions, seeing that this case may be regarded as corresponding to a couple of doublets acting along a line in opposite phase.

As just said, there are, in earthquakes, two types of distribution of push and pull waves, and in the majority of cases only these two types are observed.

These facts suggest that in the seismic focus, under the enormous hydrostatic pressure due to the weight of the overlying rock mass, certain special conditions of things must exist.

In the following paragraphs, the writer attempts to make clear the conditions prevailing within the seismic focus based on facts so far observed in connexion with seismic waves as already stated.

He is of the opinion that there are several ways of ascertaining these conditions. For example, we may be able to solve the problem by assuming that the material at the seismic focus is in a liquid state, that is, liquid magma, or by assuming that it is in a plastic state. In this study the writer avails himself of the ordinary elastic theory.

2. The Conical Type.

A spherical mass is taken for the seismic focus, and it is assumed that the elastic theory is applicable to this mass so long as the stresses are under the limit of the strength of the material.

We shall now consider the equilibrium of a spherical body.

The equations of equilibrium of elastic bodies in spherical coordinates, where the azimuthal component of the displacement is omitted, are expressed by

$$\left.\begin{aligned} &(\lambda+2\mu)\frac{\partial \Delta}{\partial r}-\frac{2\mu}{r}\frac{\partial \varpi}{\partial \theta}-\frac{2\mu}{r}\varpi\cot\theta=0 \\ &\frac{\lambda+2\mu}{r}\frac{\partial \Delta}{\partial \theta}+2\mu\frac{\partial \varpi}{\partial r}+2\mu\frac{\varpi}{r}=0 \end{aligned}\right\}, \qquad (1)$$

where u, v are radial and colatitudinal components of displacement, and

5) W. INOUYE, *Bull. Earthq. Res. Inst.*, **14** (1936), 582.

$$\left.\begin{aligned} \Delta &= \frac{\partial u}{\partial r} + \frac{2u}{r} - \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{v}{r}\cot\theta \\ 2\varpi &= \frac{\partial v}{\partial r} + \frac{v}{r} - \frac{1}{r}\frac{\partial u}{\partial \theta} \end{aligned}\right\}. \tag{2}$$

Eliminating u and v in (1) by means of (2), we get

$$\frac{\partial^2 \Delta}{\partial r^2} + \frac{2}{r}\frac{\partial \Delta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Delta}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial \Delta}{\partial \theta}\cot\theta = 0,$$

$$\frac{\partial^2 \varpi}{\partial r^2} + \frac{2}{r}\frac{\partial \varpi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varpi}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial \varpi}{\partial \theta}\cot\theta - \frac{\varpi}{r^2}(1+\cot^2\theta) = 0.$$

Solving these equations we get

$$\left.\begin{aligned} \Delta &= \left(A_n r^n + \frac{B_n}{r^{n+1}}\right) P_n(\cos\theta) \\ 2\varpi &= \left(A'_n r^n + \frac{B'_n}{r^{n+1}}\right) \frac{dP_n(\cos\theta)}{d\theta} \end{aligned}\right\}. \tag{3}$$

According to Prof. K. Sezawa and Dr. G. Nishimura,[6] the displacement (u_1, v_1) that answers to Δ in (3) and satisfies $\varpi=0$ is given by

$$\left.\begin{aligned} u_1 &= \left[\frac{A_n(n+2)}{2(2n+3)} r^{n+1} + \frac{B_n(n-1)}{2(2n-1)r^n}\right] P_n(\cos\theta) \\ v_1 &= \left[\frac{A_n}{2(2n+3)} r^{n+1} - \frac{B_n}{2(2n-1)r^n}\right] \frac{dP_n(\cos\theta)}{d\theta} \end{aligned}\right\},$$

in which $n=0, 1, 2, \ldots\ldots$ for A_n and $n=1, 2, 3, \ldots\ldots$ for B_n.

The displacement (u_2, v_2) derived from the value of ϖ in (3) under the condition, $\Delta=0$, is expressed by

$$\left.\begin{aligned} u_2 &= \left[\frac{A'_n n(n+1)}{2(2n+3)} r^{n+1} - B'_n \frac{n(n+1)}{2(2n-1)r^n}\right] P_n(\cos\theta) \\ v_2 &= \left[\frac{A'_n(n+2)}{2(2n+3)} r^{n+1} + \frac{B'_n(n-2)}{2(2n-1)r^n}\right] \frac{dP_n(\cos\theta)}{d\theta} \end{aligned}\right\},$$

in which $n=1, 2, \ldots\ldots$

The displacement (u_3, v_3) which satisfies $\Delta=0$, $\varpi=0$ is expressed by

6) K. SEZAWA and G. NISHIMURA, *Bull. Earthq. Res. Inst.*, **7** (1929), 389.

$$\left.\begin{aligned} u_3 &= \left[A''_n n r^{n-1} - \frac{B''_n(n+1)}{r^{n+2}}\right] P_n(\cos\theta) \\ v_3 &= \left[A''_n r^{n-1} + \frac{B''_n}{r^{n+2}}\right] \frac{dP_n(\cos\theta)}{d\theta} \end{aligned}\right\},$$

where $n=0, 1, 2, \ldots\ldots$.

Here we assume that just before the equilibrium is destroyed, resulting in an earthquake, the maximum shear stresses in the entire space within the spherical mass reach the limit of strength of the material, which means that the maximum shear stress is constant throughout the spherical body, and independent of the coordinates r, θ.

The condition of the maximum shear stress in the case with axial symmetry is expressed by

$$\widehat{r\theta}^2 + \left(\frac{\widehat{rr} - \widehat{\theta\theta}}{2}\right)^2 = K^2,$$

where

$$\widehat{rr} = \lambda\Delta + 2\mu\frac{\partial u}{\partial r},$$

$$\widehat{\theta\theta} = \lambda\Delta + 2\mu\left(\frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{u}{r}\right),$$

$$\widehat{r\theta} = \mu\left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial\theta}\right).$$

The displacement that satisfies this condition is limited to u_3, v_3, namely, the displacements independent of dilatation and rotation, which, moreover, is limited to terms containing A''_n.

In this case, the displacement (u_3, v_3) is expressed by

$$\left.\begin{aligned} u_3 &= A''_n n r^{n-1} P_n(\cos\theta) \\ v_3 &= A''_n r^{n-1} \frac{dP_n(\cos\theta)}{d\theta} \end{aligned}\right\}.$$

The condition of the maximum shear stress at a point (r, θ) is then given by

$$\mu^2 A''^2_n r^{2n-4}\left[4(n-1)^2\left(\frac{dP_n(\cos\theta)}{d\theta}\right)^2 + \left\{\frac{d^2P_n(\cos\theta)}{d\theta^2} - n(n-2)P_n(\cos\theta)\right\}^2\right] = K^2.$$

This condition is satisfied throughout the spherical body only when $n=2$.

In this case, the equation becomes

$$9\mu^2 A_n''^2 = K^2 \,,$$

which contains no term involving the coordinates, so that the displacement is given by

$$\left.\begin{aligned} u_3 &= \frac{2K}{3\mu} r P_2(\cos\theta) \\ v_3 &= \frac{K}{3\mu} r \frac{dP_2(\cos\theta)}{d\theta} \end{aligned}\right\} .$$

The normal stress due to the displacement at the boundary of the sphere independent of its radius is given by

$$\widehat{rr} = \frac{4}{3} K P_2(\cos\theta) \,,$$

and the tangential stress by

$$\widehat{r\theta} = \frac{2K}{3} \frac{dP_2(\cos\theta)}{d\theta} \,.$$

Seismic waves may be generated by certain rapid changes in these stresses caused by destruction of the equilibrium within the seismic focus by some causes.

The initial motions of the seismic waves in this case correspond to the conical type.

3. Quadrant Type.

We shall next take the general case of the equilibrium of a spherical body.

Let r, θ, ϕ be spherical polar coordinates, and let u, v, w stand for the components of the displacement in the direction of the radius, colatitude, and azimuth, when the equations of equilibrium of the body may be expressed by

$$\left.\begin{aligned} &(\lambda+2\mu)\frac{\partial\Delta}{\partial r} - \frac{2\mu}{r\sin\theta}\frac{\partial(\varpi_\phi \sin\theta)}{\partial\theta} + \frac{2\mu}{r\sin\theta}\frac{\partial\varpi_\theta}{\partial\phi} = 0 \\ &(\lambda+2\mu)\frac{1}{r}\frac{\partial\Delta}{\partial\theta} - \frac{2\mu}{r\sin\theta}\frac{\partial\varpi_r}{\partial\phi} + \frac{2\mu}{r}\frac{\partial(\varpi_\phi r)}{\partial r} = 0 \\ &(\lambda+2\mu)\frac{1}{r\sin\theta}\frac{\partial\Delta}{\partial\phi} - \frac{2\mu}{r}\frac{\partial(\varpi_\theta r)}{\partial r} + \frac{2\mu}{r}\frac{\partial\varpi_r}{\partial\theta} = 0 \end{aligned}\right\} , \qquad (7)$$

where

$$
\left.\begin{aligned}
\Delta&=\frac{1}{r^2\sin\theta}\left[\frac{\partial(ur^2\sin\theta)}{\partial r}+\frac{\partial(vr\sin\theta)}{\partial\theta}+\frac{\partial(wr)}{\partial\phi}\right]\\
2\varpi_r&=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}(wr\sin\theta)-\frac{\partial}{\partial\phi}(vr)\right]\\
2\varpi_\theta&=\frac{1}{r\sin\theta}\left[\frac{\partial u}{\partial\phi}-\frac{\partial(wr\sin\theta)}{\partial r}\right]\\
2\varpi_\phi&=\frac{1}{r}\left[\frac{\partial(vr)}{\partial r}-\frac{\partial u}{\partial\theta}\right]
\end{aligned}\right\}. \qquad (8)
$$

Eliminating u, v, w in (7) by means of (8), we get

$$
\left.\begin{aligned}
&\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Delta}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Delta}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Delta}{\partial\phi^2}=0\\
&\frac{\partial^2\varpi_r}{\partial r^2}+\frac{4}{r}\frac{\partial\varpi_r}{\partial r}+\frac{2}{r^2}\varpi_r+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varpi_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi_r}{\partial\phi^2}=0\\
&\frac{1}{r}\frac{\partial^2(\varpi_\theta r)}{\partial r^2}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varpi_\theta}{\partial\phi^2}-\frac{1}{r^2\sin^2\theta}\frac{\partial^2(\varpi_\phi\sin\theta)}{\partial\phi\partial\theta}-\frac{1}{r}\frac{\partial^2\varpi_r}{\partial r\partial\theta}=0\\
&\frac{1}{r}\frac{\partial^2(\varpi_\phi r)}{\partial r^2}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial(\varpi_\phi\sin\theta)}{\partial\theta}-\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial\varpi_\theta}{\partial\phi}-\frac{1}{r\sin\theta}\frac{\partial^2\varpi_r}{\partial r\partial\phi}=0
\end{aligned}\right\}.
$$

Solving these equations we obtain

$$
\left.\begin{aligned}
\Delta&=\left(A_{mn}r^n+\frac{A'_{mn}}{r^{n+1}}\right)P_n^m(\cos\theta)\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
2\varpi_r&=\left(B_{mn}r^{n-1}+\frac{B'_{mn}}{r^{n+2}}\right)P_n^m(\cos\theta)\begin{matrix}\sin\\ -\cos\end{matrix}\Big\}m\phi\\
2\varpi_\theta&=\Bigg[\left(D_{mn}r^n+\frac{D'_{mn}m}{r^{n+1}}\right)\frac{P_n^m(\cos\theta)}{\sin\theta}\\
&\quad+\left(\frac{B_{mn}}{n}r^{n-1}-\frac{B'_{mn}}{(n+1)r^{n+2}}\right)\frac{dP_n^m(\cos\theta)}{d\theta}\Bigg]\begin{matrix}\sin\\ -\cos\end{matrix}\Big\}m\phi\\
2\varpi_\phi&=\Bigg[\left(D_{mn}r^n+\frac{D'_{mn}}{r^{n+1}}\right)\frac{dP_n^m(\cos\theta)}{d\theta}\\
&\quad+\left(\frac{B_{mn}}{n}r^{n-1}-\frac{B'_{mn}}{(n+1)r^{n+2}}\right)\frac{P_n^m(\cos\theta)}{\sin\theta}\Bigg]\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi
\end{aligned}\right\}, \qquad (9)
$$

in which A_{mn}, A'_{mn}, B_{mn}, B'_{mn}, D_{mn}, D'_{mn} are arbitrary constants.

The displacement (u_1, v_1, w_1) answering to Δ in (9) and satisfying $\varpi_r=\varpi_\theta=\varpi_\phi=0$ is expressed by[7]

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7) K. SEZAWA and G. NISHIMURA, *loc. cit.*

$$\left.\begin{aligned}
u_1&=\left[\frac{A_{mn}(n+2)}{2(2n+3)}r^{n+1}+\frac{A'_{mn}(n-1)}{2(2n-1)r^n}\right]P_n^m(\cos\theta)\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
v_1&=\left[\frac{A_{mn}}{2(2n+3)}r^{n+1}-\frac{A'_{mn}}{2(2n-1)r^n}\right]\frac{dP_n^m(\cos\theta)}{d\theta}\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
w_1&=-\left[\frac{A_{mn}m}{2(2n+3)}r^{n+1}-\frac{A'_{mn}}{2(2n-1)r^n}\right]\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{matrix}\sin\\ -\cos\end{matrix}\Big\}m\phi
\end{aligned}\right\}.$$

The displacement (u_2, v_2, w_2) that answers to ϖ_r together with the second terms in the expression of ϖ_θ, ϖ_ϕ given in (9) under the condition that $\Delta=0$, is expressed by

$$\left.\begin{aligned}
u_2&=0\\
v_2&=\left[\frac{mB_{mn}}{n(n+1)}r^n+\frac{mB'_{mn}}{n(n+1)r^{n+1}}\right]\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
w_3&=-\left[\frac{B_{mn}}{n(n+1)}r^n+\frac{B'_{mn}}{n(n+1)r^{n+1}}\right]\frac{dP_n^m(\cos\theta)}{d\theta}\begin{matrix}\sin\\ -\cos\end{matrix}\Big\}m\phi
\end{aligned}\right\}.$$

The displacement (u_4, v_4, w_4) derived from the values of the first terms of ϖ_θ, ϖ_ϕ in (9) fulfilling the conditions, $\Delta=\varpi_r=0$, is written

$$\left.\begin{aligned}
u_4&=\left[\frac{D_{mn}n(n+1)}{2(2n+3)}r^{n+1}+\frac{D'_{mn}n(n+1)}{2(2n-1)r^n}\right]P_n^m(\cos\theta)\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
v_4&=\left[\frac{D_{mn}(n+3)}{2(2n+3)}r^{n+1}+\frac{D'_{mn}(n-2)}{2(2n-1)r^n}\right]\frac{1}{m}\frac{dP_n^m(\cos\theta)}{d\theta}\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
w_4&=-\left[\frac{D_{mn}m(n+3)}{2(2n+3)}r^{n+1}+\frac{D'_{mn}m(n-2)}{2(2n-1)r^n}\right]\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{matrix}\sin\\ -\cos\end{matrix}\Big\}m\phi
\end{aligned}\right\}.$$

The displacement (u_3, v_3, w_3) that satisfies $\Delta=\varpi_r=\varpi_\theta=\varpi_\phi=0$, is expressed by

$$\left.\begin{aligned}
u_3&=\left[C_{mn}nr^{n-1}-\frac{C'_{mn}(n+1)}{r^{n+2}}\right]P_n^m(\cos\theta)\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
v_3&=\left[C_{mn}r^{n-1}+\frac{C'_{mn}}{r^{n+2}}\right]\frac{dP_n^m(\cos\theta)}{d\theta}\begin{matrix}\cos\\ \sin\end{matrix}\Big\}m\phi\\
w_3&=-m\left[C_{mn}r^{n-1}+\frac{C'_{mn}}{r^{n+2}}\right]\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{matrix}\sin\\ -\cos\end{matrix}\Big\}m\phi
\end{aligned}\right\}.$$

Here we assume, as in the previous case, that the maximum shear stresses are constant throughout the spherical body attaining the limit of ultimate strength of the material against breaking down of the resistance immediately preceding an earthquake.

The condition of maximum shear stress is expressed by[8]

$$\frac{1}{2}\left[\left(\frac{\widehat{rr}-\widehat{\theta\theta}}{2}\right)^2+\left(\frac{\widehat{\theta\theta}-\widehat{\phi\phi}}{2}\right)^2+\left(\frac{\widehat{\phi\phi}-\widehat{rr}}{2}\right)^2\right]+\frac{3}{4}(\widehat{r\theta}^2+\widehat{\theta\phi}^2+\widehat{\phi r}^2)=K^2,$$

where

$$\left.\begin{aligned}
\widehat{rr}&=\lambda\Delta+2\mu\frac{\partial u}{\partial r}\\
\widehat{\theta\theta}&=\lambda\Delta+2\mu\left(\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right)\\
\widehat{\phi\phi}&=\lambda\Delta+2\mu\left(\frac{1}{r\sin\theta}\frac{\partial w}{\partial\phi}+\frac{v}{r}\cot\theta+\frac{u}{r}\right)\\
\widehat{r\theta}&=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right)\\
\widehat{\theta\phi}&=\mu\left\{\frac{1}{r}\left(\frac{\partial w}{\partial\theta}-w\cot\theta\right)+\frac{1}{r\sin\theta}\frac{\partial v}{\partial\phi}\right\}\\
\widehat{\phi r}&=\mu\left(\frac{1}{r\sin\theta}\frac{\partial u}{\partial\phi}+\frac{\partial w}{\partial r}-\frac{w}{r}\right)
\end{aligned}\right\}.$$

The displacements that satisfy this condition are restricted to u_3, v_3, w_3 (taking only the first terms containing C_{mn}), that is displacements independent of dilatation and rotation.

In this case, the displacement (u_3, v_3, w_3) is given by

$$\left.\begin{aligned}
u_3&=C_{mn}nr^{n-1}P_n^m(\cos\theta)\cos m\phi\\
v_3&=C_{mn}r^{n-1}\frac{dP_n^m(\cos\theta)}{d\theta}\cos m\phi\\
w_3&=-mC_{mn}r^{n-1}\frac{P_n^m(\cos\theta)}{\sin\theta}\sin m\phi
\end{aligned}\right\}.$$

The condition of maximum shear stress at any point (r, θ, ϕ) is then given by

$$\mu^2C_{mn}^2r^{2n-4}\Bigg[\frac{1}{2}\cos^2 m\phi\Bigg[\left\{n(n-2)P_n^m(\cos\theta)-\frac{d^2P_n^m(\cos\theta)}{d\theta^2}\right\}^2$$
$$+\left\{\frac{d^2P_n^m(\cos\theta)}{d\theta^2}+\frac{m^2}{\sin^2\theta}P_n^m(\cos\theta)-\cot\theta\frac{dP_n^m(\cos\theta)}{d\theta}\right\}^2$$
$$+\left\{-m^2\frac{P_n^m(\cos\theta)}{\sin^2\theta}+\frac{dP_n^m(\cos\theta)}{d\theta}\cot\theta-n(n-2)P_n^m(\cos\theta)\right\}^2\Bigg]$$

8) HENCKY, *Zeits. f. A. M. M.*, 4 (1924), 323.

$$+3\cos^2 m\phi\,(n-1)^2\left(\frac{dP_n^m(\cos\theta)}{d\theta}\right)^2$$

$$+3\frac{\sin^2 m\phi}{\sin^2\theta}m^2\left[\left\{\cot\theta P_n^m(\cos\theta)-\frac{dP_n^m(\cos\theta)}{d\theta}\right\}^2\right.$$

$$\left.+(n-1)^2(P_n^m(\cos\theta))^2\right]\Bigg]=K^2\,.$$

The condition that this equation shall hold throughout the spherical body is fulfilled in cases in which $n=2$, $m=1$ and $n=2$, $m=2$.

In the former case the equation becomes

$$27\mu^2C_{1,2}^2=K^2\,,$$

and in the latter

$$108\,\mu^2C_{2,2}^2=K^2\,.$$

In the case in which $n=2$, $m=1$, the displacement is given by

$$\left.\begin{aligned} u_3&=\frac{3}{\sqrt{27}}\frac{K}{\mu}r\sin 2\theta\cos\phi\\ v_3&=\frac{3}{\sqrt{27}}\frac{K}{\mu}r\cos 2\theta\cos\phi\\ w_3&=-\frac{3}{\sqrt{27}}\frac{K}{\mu}r\cos\theta\sin\phi \end{aligned}\right\}.$$

The normal stress at the boundary of the sphere due to the displacement is given by

$$\widehat{rr}=\frac{6}{\sqrt{27}}K\sin 2\theta\cos\phi\,,$$

and the tangential stresses by

$$\left.\begin{aligned} \widehat{r\theta}&=\frac{6}{\sqrt{27}}K\cos 2\theta\cos\phi\\ \widehat{r\phi}&=-\frac{6}{\sqrt{27}}K\cos\theta\sin\phi \end{aligned}\right\}.$$

In the case in which $n=2$, $m=2$, the displacement is given by

$$\left.\begin{aligned} u_3&=\frac{6}{\sqrt{108}}\frac{K}{\mu}r\sin^2\theta\cos 2\phi\\ v_3&=\frac{3}{\sqrt{108}}\frac{K}{\mu}r\sin 2\theta\cos 2\phi\\ w_3&=-\frac{6}{\sqrt{108}}\frac{K}{\mu}r\sin\theta\sin 2\phi \end{aligned}\right\}.$$

The normal stress at the boundary of the sphere is given by

$$\widehat{rr}=\frac{12}{\sqrt{108}}K\sin^2\theta\cos 2\phi\,,$$

and the tangential stresses by

$$\left.\begin{aligned}\widehat{r\theta}&=\frac{6}{\sqrt{108}}K\sin 2\theta\cos 2\phi\\ \widehat{r\phi}&=-\frac{24}{\sqrt{108}}K\sin\theta\sin 2\phi\end{aligned}\right\}.$$

As in the former case, seismic waves may be generated by some rapid changes undergone in these stresses.

The initial motions of the seismic waves in these cases belong in the quadrant type category.

4. Remarks.

In the preceeding two articles, the writer treated the problem without taking into consideration the outer medium of the spherical mass, assuming that the elasticity of the outer medium differs from that of the spherical mass, a conception that may be permissible if we assume that the spherical portion in the medium is locally weakened by certain causes, such as local heating, etc.

If we assume that the elasticity of the outer medium differs from that within the spherical mass, we can satisfy the boundary conditions at the spherical surface, that is the displacements and the stresses are continuous at the spherical surface $r=a$, by using all the various displacements in the outer medium in contrast with displacement u_3, v_3, w_3, that is, the displacement independent of dilatation and rotation, within the spherical mass. It must be remembered, moreover, that we are considering only the state of things within the seismic focus, leaving out of consideration both the external and internal forces that bring about the earthquake.

If we assume that the material within the sphere to be of plastic nature, the case in which the stresses are independent of the azimuth, namely the case $P_0(\cos\theta)$, is also possible to exist.[9]

5. Summary.

In the majority of earthquakes, we observe two types of distribu-

9) K. SEZAWA, *Bull, Earthq. Res. Inst.*, **9** (1931), 398.

tions of push and pull waves at the initial phase of earthquake waves, namely, conical and quadrant, although we cannot yet represent any one of these types by means of artificial earthquakes.

These facts suggest the existence of certain special conditions in the seismic focus.

The writer investigated the conditions prevailing within the seismic focus with the above stated facts as basis, and arrived at the conclusion that the maximum shear stresses might be constant within the seismic focus reaching the limit of ultimate strength of the material immediately preceding an earthquake.

In conclusion, the writer's cordial thanks are due to Prof. M. Ishimoto, Prof. K. Sezawa, Prof. Ch. Tsuboi, and Prof. N. Miyabe for their kind advices.

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45. 發震機構に就いて (第4報)

地震研究所 井上宇胤

初動の"壓し波""引き波"の地理的分布に就いては多數の研究がなされてゐる．此等の研究によつて初動の地理的分布には二種類ある事が分つた．卽ち在來から知られてゐた四象限型こ石本所長の研究されたコニカル型こである．

然るに理論的には他の多くの型を考へる事が可能であるこ共に他方今日迄の人工地震によつては以上の二つの型の孰れをも表現し得ないのである．

此等の事實から筆者は震源に於ては何事か特殊な事情が存在するのではなからうかこの考へを以て以上の二つの型のみ生起する樣な震源に於ける狀態を考察してみたのである．

其の結果震源に於ける物質を彈性體こ考へるならば，地震の發生する直前に於ては震源の全域に亘り，一樣に最大剪應力が其の彈性體の破壞限度に達してゐるこ云ふ事になつた．

勿論此の樣な狀態から如何にして地震波が發生するか或は又，如何なる力によつて斯の如き狀態を引き起したかは今は問題こしてゐない．

只單に此の樣な狀態に於ける應力が或る早さをもつて消失或は單に變化したならば以上の二つの型に相當する初動が觀測されるであらうこ云ふのである．
