22. *The Same Stationary Vibration of an Origin Accompanying Different Types of Disturbances Therefrom.*

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1. In a previous paper[1] we discussed the probability of resonance phenomena in the stationary vibration of the surface of a spherical cavity, and concluded that, owing to the vibrational energy being dissipated towards infinity, even under resonance conditions, the vibration of the cavity scarcely assumes an infinitely large amplitude for a finite value of n. In the case of smaller values of n the amplitudes of the stationary vibration as well as of the dissipation waves were not sensibly large even under resonance conditions. With a view to ascertaining the most probable resonance frequencies in such a case, we assumed a disturbance at the origin of a type differing entirely from that of the previous case. Since in the case without dissipation, the vibration type of a body under resonance could be uniquely determined provided its vibrational mode were specified, the investigation of the separate cases in which the same mode of vibration are excited under disturbances assumed to be different would give a better answer to the determination of the true resonance conditions just mentioned. In contrast to the previously assumed normal force at the surface of the cavity, we shall now consider the case in which the surface at the spherical cavity is subjected to a periodically changing shearing force.

2. The solutions of the problem being the same as those in the previous paper,[2] the boundary conditions are such that

$$\widehat{rr}=0, \quad \widehat{r\theta}=p_s\frac{dP_n(\cos\theta)}{d\theta}e^{ipt} \tag{1}$$

at $r=a$. In virtue of the relations

1) K. SEZAWA and K. KANAI, "Resonance Phenomena and Dissipation Waves in the Stationary Vibration of the Surface of a Spherical Cavity," *Bull. Earthq. Res. Inst.*, **15** (1937), 13~20.

2) *loc. cit.* 1).

$$\widehat{rr}=\lambda\Delta+2\mu\frac{\partial u}{\partial r},\quad \widehat{r\theta}=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right), \tag{2}$$

where $u=u_1+u_2$, $v=v_1+v_2$, we find

$$\left.\begin{aligned}
&2A_n\left\{-\frac{(n-1)}{h^2r^2}H^{(2)}_{n+\frac{1}{2}}(hr)+\frac{1}{hr}H^{(2)}_{n+\frac{3}{2}}(hr)\right\}\\
&\quad+B_n\left\{-\frac{2(n^2-1)}{j^2r^2}H^{(2)}_{n+\frac{1}{2}}(jr)+\frac{2n+1}{jr}H^{(2)}_{n+\frac{3}{2}}(jr)-H^{(2)}_{n+\frac{5}{2}}(jr)\right\}=\frac{p_s\sqrt{r}}{\mu},\\
&A_n\left[\left\{\left(\frac{\lambda}{\mu}-\frac{2n(n-1)}{h^2r^2}\right)H^{(2)}_{n+\frac{1}{2}}(hr)+2\left\{\frac{2n+1}{hr}H^{(2)}_{n+\frac{3}{2}}(hr)-H^{(2)}_{n+\frac{5}{2}}(hr)\right\}\right]\right.\\
&\qquad+2B_nn(n+1)\left\{-\frac{n-1}{j^2r^2}H^{(2)}_{n+\frac{1}{2}}(jr)+\frac{1}{jr}H^{(2)}_{n+\frac{3}{2}}(jr)\right\}=0
\end{aligned}\right\} \tag{3}$$

at $r=a$. When $\lambda=\mu$, we have

$$\left.\begin{aligned}
&A_n\Phi=\frac{p_s\sqrt{a}}{\mu}\frac{2n(n+1)}{ja}\left\{\frac{n-1}{ja}H^{(2)}_{n+\frac{1}{2}}(ja)-H^{(2)}_{n+\frac{3}{2}}(ja)\right\},\\
&B_n\Phi=\frac{p_s\sqrt{a}}{\mu}\left[\left\{1-\frac{2n(n-1)}{h\,a^2}\right\}H^{(2)}_{n+\frac{1}{2}}(ha)\right.\\
&\qquad\qquad\left.+2\left\{\frac{2n+1}{ha}H^{(2)}_{n+\frac{3}{2}}(ha)-H^{(2)}_{n+\frac{5}{2}}(ha)\right\}\right],\\
&\Phi=\left[\left\{1-\frac{2n(n-1)}{h^2a^2}\right\}H^{(2)}_{n+\frac{1}{2}}(ha)+2\left\{\frac{2n+1}{ha}H^{(2)}_{n+\frac{3}{2}}(ha)-H^{(2)}_{n+\frac{5}{2}}(ha)\right\}\right]\\
&\qquad\cdot\left\{-\frac{2(n+1)(n-1)}{j^2a^2}H^{(2)}_{n+\frac{1}{2}}(ja)+\frac{2n+1}{ja}H^{(2)}_{n+\frac{3}{2}}(ja)-H^{(2)}_{n+\frac{5}{2}}(ja)\right\}\\
&\qquad+\frac{4n(n+1)}{hja^2}\left\{\frac{n-1}{ja}H^{(2)}_{n+\frac{1}{2}}(ja)-H^{(2)}_{n+\frac{3}{2}}(ja)\right\}\\
&\qquad\qquad\cdot\left\{-\frac{n-1}{ha}H^{(2)}_{n+\frac{1}{2}}(ha)+H^{(2)}_{n+\frac{3}{2}}(ha)\right\}.
\end{aligned}\right\} \tag{4}$$

The final solutions are such that

$$u_1=\frac{A_n\sqrt{r}}{hr}\left\{-\frac{n}{hr}H^{(2)}_{n+\frac{1}{2}}(hr)+H^{(2)}_{n+\frac{3}{2}}(hr)\right\}P_n(\cos\theta)e^{ipt},$$

$$
\left.\begin{aligned}
v_1 &= -\frac{A_n\sqrt{r}}{h^2r^2}H^{2}_{n+\frac{1}{2}}(hr)\frac{dP_n(\cos\theta)}{d\theta}e^{ipt},\\
u_2 &= -\frac{B_n\sqrt{r}}{j^2r^2}n(n+1)H^{(2)}_{n+\frac{1}{2}}(jr)P_n(\cos\theta)e^{ipt},\\
v_2 &= \frac{B_n\sqrt{r}}{jr}\left\{-\frac{n+1}{jr}H^{(2)}_{n+\frac{1}{2}}(jr)+H^{(2)}_{n+\frac{3}{2}}(jr)\right\}\frac{dP_n(\cos\theta)}{d\theta}e^{ipt},
\end{aligned}\right\} \qquad (5)
$$

the values of the constants A_n, B_n being shown in (4). Using these equations we shall calculate the displacements u, v at the surface $r=a$ as well as those of dissipation waves u_1, v_2 at $r=\infty$ for three cases of n, namely $n=1$, 2, 4; u_1, v_2 being radial displacement of dilatational waves and transverse displacement of distortional waves respectively. The results for the three cases are shown in Figs. 1～6.

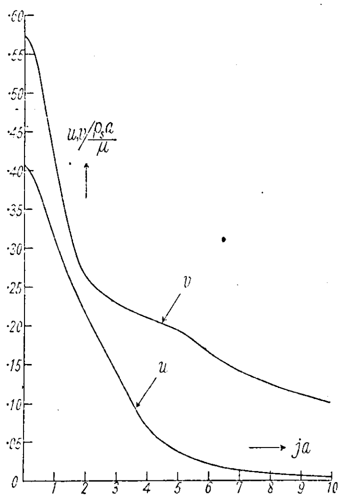

Fig. 1. Displacements at $r=a$; $n=1$.

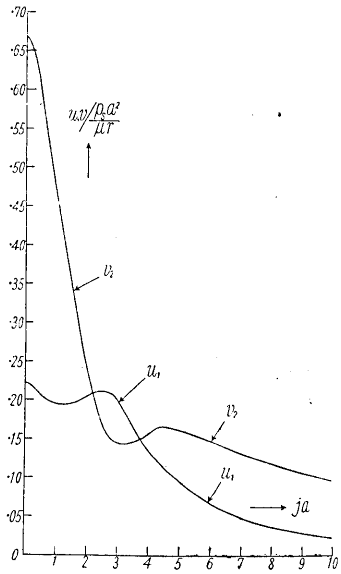

Fig. 2. Amplitudes of dissipation waves; $n=1$.

In the present results, $P_n(\cos\theta)$ in u or $dP_n(\cos\theta)/d\theta$ in v was again conventionally replaced by unity.

The solutions of the case $n=\infty$ are easily obtained in the same manner as those described in the previous paper, which however are omitted here.

3. Figs. 1, 3, 5 show that the value of u/v, namely, the ratio of the radial to transverse displacements at the origin invariably tends to

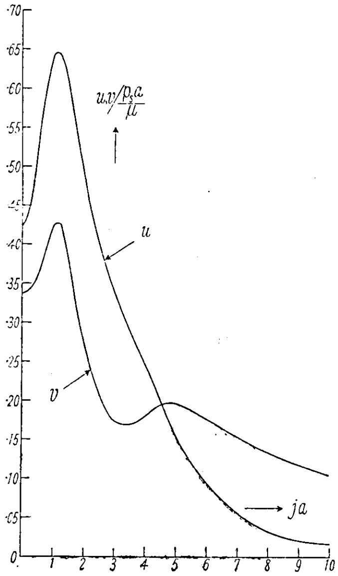

Fig. 3. Displacement at $r=a$; $n=2$.

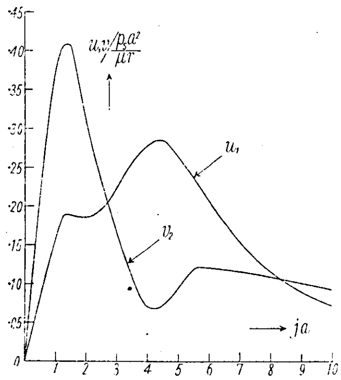

Fig. 4. Amplitutes of dissipation waves; $n=2$.

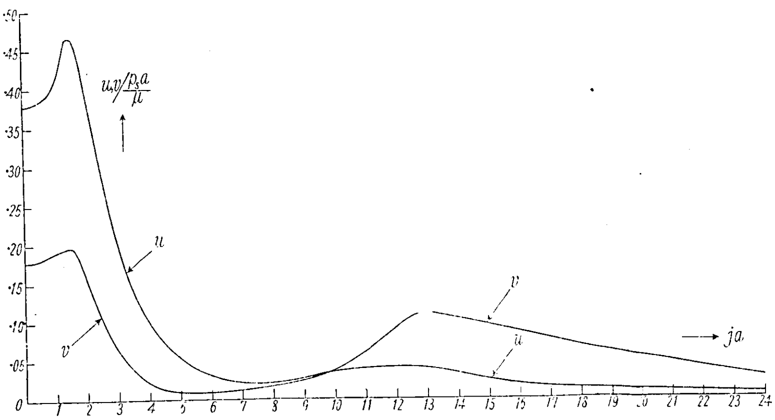

Fig. 5. Displacement at $r=a$; $n=4$.

zero for infinitely large values of ja, that is to say, the movement of the surface at the origin is purely transverse for extremely high vibrational frequencies. Figs. 3, 5 also show that both u and v assume maximum values at $ja=1{\cdot}15$, $1{\cdot}5$, $0{\cdot}9194\,n$ for $n=2$, $n=4$, $n=\infty$

respectively.

From Figs. 2, 4, 6 it will be seen that, while the relation that $u_1/v_2 \to 0$ for $ja \to \infty$ also holds, the same ratio at $ja \to 0$ assumes the

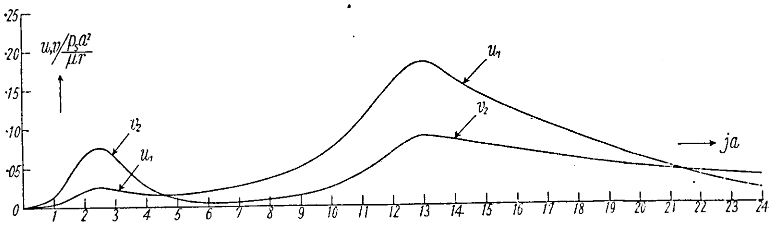

Fig. 6. Amplitudes of dissipation waves; $n=4$

values $\mu/(\lambda+2\mu)$, $2\{\mu/(\lambda+2\mu)\}^{3/2}$, $(3/10)\{\mu/(\lambda+2\mu)\}^{3/2}$ for $n=1$, 2, 4 respectively. The first maxima of u_1, v_2 arise at $ja=2{\cdot}5$ and $4{\cdot}5$ respectively for $n=1$; at $ja=1{\cdot}4$ for $n=2$; and at $ja=2{\cdot}5$ for $n=4$. The presence of the second maxima obeys the condition such that $ja=4{\cdot}4$ (for u_1), $5{\cdot}7$ (for v_2) for $n=2$, and $ja=13{\cdot}5$ for $n=4$.

Almost all the properties mentioned above are immediately connected with those of the previous case, namely, the case in which the spherical origin is subjected to disturbance of normal pressure type, both the results being arranged in Table I for comparison.

Table I.

Source	r	ja	Type of disturbance	
			Normal force	Shearing force
$n=1$	$r=a$	∞	$v/u \to 0$	$u/v \to 0$
	$r=\infty$	∞	$v_2/u_1 \to 0$	$u_1/v_2 \to 0$
		0	$v_2/u_1=(\lambda+2\mu)/\mu$	$v_2/u_1=(\lambda+2\mu)/\mu$
		4·5	Max. of u_1	Max. of v_2
		2·5	Max. of v_2	Max. of u_1
$n=2$	$r=a$	∞	$v/u \to 0$	$u/v \to 0$
		1·15	Max. of u and v	Max. of u and v
	$r=\infty$	∞	$v_2/u_1 \to 0$	$u_1/v_2 \to 0$
		0	$v_2/u_1=\frac{1}{2}\left(\frac{\lambda+2\mu}{\mu}\right)^{\frac{3}{2}}$	$v_2/u_1=\frac{1}{2}\left(\frac{\lambda+2\mu}{\mu}\right)^{\frac{3}{2}}$
		1·4	1st max. of u_1 and v_2	1st max. of u_1 and v_2 (sub.)
		5·7	2nd max. of u_1	2nd max. of v_2
		4·4	2nd max. of v_2 (sub.)	2nd max. of u_1

(to be continued.)

Table I. (*Continued*)

$n=4$	$r=a$	∞	$v/u \to 0$	$u/v \to 0$
		1·5	Max. of u and v	Max. of u and v
	$r=\infty$	∞	$v_2/u_1 \to 0$	$u_1/v_2 \to 0$
		0	$v_2/u_1=\frac{10}{3}\left(\frac{\lambda+2\mu}{\mu}\right)^{\frac{3}{2}}$	$v_2/u_1=\frac{10}{3}\left(\frac{\lambda+2\mu}{\mu}\right)^{\frac{3}{2}}$
		2·5	1st max. of u_1 and v_2	1st max. of u_1 and v_2
		13·5	2nd max. of u_1 and v_2	2nd max. of u_1 and v_2

It now appears that the resonances for $n=2$ and $n=4$ are conditioned by $ja=1{\cdot}15, 1{\cdot}5$ respectively. The resonance condition of a solid vibrating sphere for $n=2$ is $ja=2{\cdot}64$, which obviously differs from that for a body having a spherical cavity of the same radius.

4. It is possible, at all events, to conclude that since the ratio of v_2/u_1 tends to increase with increase in n, the amplitudes of transverse waves would be unduly large conformably with the extent of the complexity presented at the origin by the distribution of the disturbances. It also appears that, for a given type of disturbance at the origin, the amplitudes assume relatively large values for a certain range of vibrational frequencies. That it is hardly possible for transverse waves of very high frequency to exist is also obvious.

Finally, it should be borne in mind that, since the present examples are here given as idealized models for explaining the nature of a fairly complex seismic origin, it is immaterial whether the actual seismic origin be of the doublet type, or a quadruplet type, or even of such diverse type as to be free from regular geometrical conditions.

22. 震源に於ける異型の發振に伴ふその同型の定常振動

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この前の報告で，球窩面が強制共振をなしても逸散波がある爲に無限大の振幅になり得ぬことを述べて置いた．而して何れの振動數が尤もらしい共振振動數であるかを正確にきめられぬことをも附加へて置いた．一體，逸散波さへなければ，強制力が如何に働いてもある定まつた型の振動は共振に於て一定の變位分布をなし，且つその共振週期も變る筈がないものである．然るに只

今の場合には逸散波があるからこの規則も完全には行はれ難い．しかし振動數の點では近似的に成立つであらうといふ考のもとにこの研究を試みたのである．

前回では球型源に壓力型の振動力を與へた代りに，今回は剪力型の振動力を與へて前と同じ型の振動を誘起さして見ることにした．計算の方法は前回と似たものである．

計算の結果によると前回の方法でも今回の方法でも，定常振動は定常振動同志に於て，逸散波は逸散波同志に於て，それぞれ一定の振動週期になると極大振動が現れ，それは原振動力の型には無關係といふことがわかつたのである．このやうにして所謂共振週期を見出した譯である．勿論之等は固形球の自由振動週期とは非常に違つてをる．

尙，振動週期の極めて長い場合には逸散波の傳播方向の變位と直角の方向の變位との比が振動力の型に無關係に一定値を取る．

振動週期の極く短い場合には，前回の振動型では半徑方向の變位が直角方向のそれよりも遙かに大きくなり，今回の振動型では逆になる．即ち原點を核として計算すると特別の場合しか出ないことがわかる．

この論文の計算はその儘實際にあてはまるとは考へないけれども，一つの理想的模型として取つたのに過ぎぬものである．