1. *Resonance Phenomena and Dissipation Waves in the Stationary Vibrations of a Semi-infinite Body.*

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1. *Introduction.*

It seems the prevalent belief that the surface vibration of a semi-infinit body caused by usual disturbances of a local type is present only in the form of propagated waves; the chances of general acceptance of the idea of resonance phenomena in such a vibration being rather remote. We shall however show that if certain periodic surface disturbances were distributed in a special way on a certain area of the surface of a semi-infinite body, it would excite a standing vibration of the body, at least within the area of disturbances, so long as the frequency of the disturbances is relatively low. The vibration under consideration naturally assumes the type of resonance at a certain frequency. Beyond another assigned frequency, however, the movement of the body changes from a state of standing vibration to one of wave motion. In some kinds of surface disturbances, on the other hand, the vibration never becomes stationary at any frequency, being present only if no surface wave can possibly exist. The various cases will be described in the sections to which they belong.

2. *Elementary solutions.*

The case in which the disturbances are distributed in the form of cylindrical coordinates will first be discussed. The solution is the same as that given in our preceding papers.[1]

When the surface vibrations are composed of both dilatational and distortional movements, the solutions assume the forms

$$\left.\begin{aligned} \Delta &= A_m J_m(kr)\, e^{-\alpha z - ipt} \cos m\theta\,, \\ u_1 &= -\frac{A_m}{h^2}\frac{\partial J_m(kr)}{\partial r}\, e^{-\alpha z - ipt} \cos m\theta\,, \end{aligned}\right|$$

1) K. SEZAWA, *Proc. Imp. Acad.*, **4** (1928), 267; *Bull. Earthq. Res. Inst.*, **6** (1929), 1—18.

$$\left.\begin{aligned} v_1 &= \frac{A_m}{h^2}\frac{J_m(kr)}{r}e^{-\alpha z-ipt}m\sin m\theta, \\ w_1 &= \frac{A_m}{h^2}\alpha J_m(kr)e^{-\alpha z-ipt}\cos m\theta, \end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} u_3 &= -\frac{C_m}{j^2}\frac{\partial J_m(kr)}{\partial r}e^{-\beta z-ipt}\cos m\theta, \\ v_3 &= \frac{C_m}{j^2}\frac{J_m(kr)}{r}e^{-\beta z-ipt}m\sin m\theta, \\ w_3 &= \frac{C_m}{j^2}\frac{k^2}{\beta}J_m(kr)e^{-\beta z-ipt}\cos m\theta, \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} u_2 &= \frac{B_m}{k^2}\frac{J_m(kr)}{r}e^{-\beta z-ipt}m\sin m\theta, \\ v_2 &= \frac{B_m}{k^2}\frac{\partial J_m(kr)}{\partial r}e^{-\beta z-ipt}\cos m\theta, \\ w_2 &= 0, \end{aligned}\right\} \tag{3}$$

where

$$k^2=\alpha^2+h^2=\beta^2+j^2, \quad h^2=\rho p^2/(\lambda+2\mu), \quad j^2=\rho p^2/\mu. \tag{4}$$

The surface disturbances may be classified into two kinds, the first of which is expressed by

$$\left.\begin{aligned} \widehat{zz} &= \lambda\Delta+2\mu\frac{\partial w}{\partial z} \equiv P_m J_m(kr)\cos m\theta e^{-ipt}, \\ \widehat{rz} &= \mu\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right) \equiv Q_m\frac{\partial J_m(kr)}{\partial(kr)}\cos m\theta e^{-ipt}, \\ \widehat{\theta z} &= \mu\left(\frac{1}{r}\frac{\partial w}{\partial\theta}+\frac{\partial v}{\partial z}\right) \equiv -Q_m\frac{J_m(kr)}{kr}m\sin m\theta e^{-ipt}, \end{aligned}\right\} \tag{5}$$

where $u=u_1+u_3$, $v=v_1+v_3$, $w=w_1+w_3$, and the second by

$$\left.\begin{aligned} \widehat{zz} &\equiv 0, \\ \widehat{rz} &= \mu\frac{\partial u_2}{\partial z} \equiv R_m\frac{J_m(kr)}{kr}m\sin m\theta e^{-ipt}, \\ \widehat{\theta z} &= \mu\frac{\partial v_2}{\partial z} \equiv R_m\frac{\partial J_m(kr)}{\partial(kr)}\cos m\theta e^{-ipt}. \end{aligned}\right\} \tag{6}$$

The second kind of disturbances never gives rise to vertical displacement, nor normal surface stress, nor even volume change of body.

Solving (5), (6) we obtain

$$\left.\begin{aligned} -\frac{\mu A_m}{h^2} &= \frac{1}{k^2}\left\{\frac{P_m(2-j'^2)+Q_m 2\beta'}{(2-j'^2)^2-4\alpha'\beta'}\right\}, \\ \frac{\mu C_m}{j^2} &= \frac{1}{k^2}\left\{\frac{P_m 2\alpha'\beta'+Q_m\beta'(2-j'^2)}{(2-j'^2)^2-4\alpha'\beta'}\right\}, \end{aligned}\right\} \tag{7}$$

$$-\frac{B\mu_m}{k^2} = \frac{R_m}{\beta}, \tag{8}$$

where $j'=j\ k$, $\alpha'=\alpha/k$. $\beta'=\beta/k$.

3. *Vibration of the first kind.*

From disturbances of the first kind we obtain the vibration of the first kind, in which case the displacement is expressed by

$$u=u_1+u_3, \quad v=v_1+v_3, \quad w=w_1+w_3. \tag{9}$$

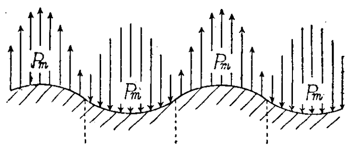

Fig. 1.

We shall assume that the nodal planes with respect to the vibrations or the nodal lines of disturbances are of a given distribution, whereas the frequency of the vibrational disturbances are varied, the vibrational character of different ranges of j', namely of frequency p, being considered separately.

(a) $$0 < j' < 1 \qquad \left(0 < \sqrt{\frac{\rho}{\mu}}\frac{p}{k} < 1\right);$$

The vibration is of the surface type and standing under the periodic surface disturbance.

(b) $(2-j'^2)^2-4\alpha'\beta'=0$, namely $j'=0{\cdot}9194$ for $\lambda=\mu$;

The displacement becomes such that

$$u \to \infty, \quad v \to \infty, \quad w \to \infty,$$

the standing vibration is in resonance condition.

(c) $$1 < j' < \sqrt{\frac{\lambda+2\mu}{\mu}} \qquad \left(1 < \sqrt{\frac{\rho}{\mu}}\frac{p}{k} < \sqrt{\frac{\lambda+2\mu}{\mu}}\right);$$

In this case u_1, v_1, w_1 form standing surface vibrations, whereas u_3, v_3, w_3 become waves transmitted downwards with velocity

$$\sqrt{\frac{\mu}{\rho}}\frac{1}{\sqrt{1-\frac{\mu}{\rho}\frac{k^2}{p^2}}}.$$

This results directly from the condition in (4).

(d) $$\sqrt{\frac{\lambda+2\mu}{\mu}}<j' \qquad \left(\sqrt{\frac{\lambda+\mu}{\mu}}<\sqrt{\frac{\rho}{\mu}}\frac{p}{k}\right);$$

u_1, v_1, w_1 and u_3, v_3, w_3 both form waves that are transmitted downwards with velocities

$$\sqrt{\frac{\lambda+2\mu}{\rho}}\frac{1}{\sqrt{1-\frac{\lambda+2\mu}{\rho}\frac{k^2}{p^2}}}, \quad \sqrt{\frac{\mu}{\rho}}\frac{1}{\sqrt{1-\frac{\mu}{\rho}\frac{k^2}{p^2}}}$$

respectively in consequence of conditions in (4).

4. *Vibration of the second kind.*

Vibrations of this kind correspond to disturbances of the second kind. The nature of the vibration for different ranges of j' is

(a) $$0<j'<1 \qquad \left(0<\sqrt{\frac{\rho}{\mu}}\frac{p}{k}<1\right);$$

These vibrations are of surface type and standing under the periodic surface disturbances.

(b) Without resonance conditions.

(c) $$1<j' \qquad \left(1<\sqrt{\frac{\rho}{\mu}}\frac{p}{k}\right);$$

u_2, v_2 form waves that are transmitted downwards with velocity

$$\sqrt{\frac{\mu}{\rho}}\frac{1}{\sqrt{1-\frac{\mu}{\rho}\frac{k^2}{p^2}}}.$$

5. *Vibration of the second kind in a stratified body.*

While the vibration of the second kind in a semi-infinite body does not show any resonance phenomenon, the same vibration in a stratified body has resonance conditions. Let the thickness of the layer be H, and let the axis of z be directed vertically downwards from the upper surface of the layer. The displacement u_2, v_2 in the layer and the displacement u_2', v_2' in the subjacent medium are such that

$$\left.\begin{aligned} u_2 &= \frac{-R_m}{\mu k\beta}\frac{J_m(kr)}{r}\frac{\{(\mu\beta+\mu'\beta')e^{\beta(H-z)}+(\mu\beta-\mu'\beta')e^{-\beta(H-z)}\}}{\{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}\}}m\sin m\theta e^{-ipt},\\ v_2 &= \frac{-R_m}{\mu k\beta}\frac{\partial J_m(kr)}{\partial r}\frac{\{(\mu\beta+\mu'\beta')e^{\beta(H-z)}+(\mu\beta-\mu'\beta')e^{-\beta(H-z)}}{\{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}\}}\cos m\theta e^{-ipt},\\ w_2 &= 0, \end{aligned}\right\}\quad(10)$$

$$\left.\begin{aligned} u_2' &= \frac{-2R_m}{k}\frac{J_m(kr)}{r}\frac{e^{\beta'(H-z)}}{\{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}\}}m\sin m\theta e^{-ipt},\\ v_2' &= \frac{-2R_m}{k}\frac{\partial J_m(kr)}{\partial r}\frac{e^{\beta'(H-z)}}{\{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}\}}\cos m\theta e^{-ipt},\\ w_2' &= 0, \end{aligned}\right\}\quad(11)$$

where $k^2=j^2+\beta^2=j_1^2+\beta'^2$, $j^2=\rho p^2/\mu$, $j_1^2=\rho' p^2/\mu'$; ρ, ρ', μ, μ' being densities and rigidities of the surface layer and the subjacent medium. We shall now discuss the problem of different frequencies of vibrations.

(a) $0<j_1'<1 \quad \left(0<\sqrt{\frac{\rho'}{\mu'}}\frac{p}{k}<1\right)$, j_1' being j_1/k;

This vibration is of surface type, and standing.

(b) $(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}=0$;

This standing vibration is under resonance condition.

(c) $1<j' \quad \left(1<\sqrt{\frac{\rho'}{\mu'}}\frac{p}{k}\right)$;

u_2', v_2' are waves transmitted vertically downwards with velocity

$$\sqrt{\frac{\mu'}{\rho'}}\frac{1}{\sqrt{1-\frac{\mu'}{\rho'}\frac{k^2}{p^2}}}.$$

6. *Periodic arbitrary disturbances.*

(i) For the arbitrary disturbance of the first kind on $z=0$

$$\widehat{zz}=f(r)\cos m\theta e^{-ipt},\quad \widehat{rz}=0,\quad \widehat{\theta z}=0,\qquad(12)$$

we generalize the result shown in Section 4 as follows:

$$u_1=\frac{\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{(2k^2-j^2)e^{-\alpha z}}{(2k^2-j^2)^2-4k^2\alpha\beta}\frac{\partial J_m(kr)}{\partial r}kdk\int_0^\infty f(R)J_m(kR)RdR,$$

$$v_1=\frac{-m\sin m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{(2k^2-j^2)e^{-\alpha z}}{(2k^2-j^2)^2-4k^2\alpha\beta}\frac{J_m(kr)}{r}kdk\int_0^\infty f(R)J_m(kR)RdR\,,$$

$$w_1=\frac{\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{\alpha(2k^2-j^2)e^{-\alpha z}}{(2k^2-j^2)^2-4k_2\alpha\beta}J_m(kr)kdk\int_0^\infty f(R)J_m(kR)RdR\,, \qquad (13)$$

$$u_3=\frac{-\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{2\alpha\beta e^{-\beta z}}{(2k^2-j^2)^2-4k^2\alpha\beta}\frac{\partial J_m(kr)}{\partial r}kdk\int_0^\infty f(R)J_m(kR)RdR\,,$$

$$v_3=\frac{m\sin m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{2\alpha\beta e^{-\beta z}}{(2k^2-j^2)^2-4k^2\alpha\beta}\frac{J_m(kr)}{r}kdk\int_0^\infty f(R)J_m(kR)RdR\,,$$

$$w_3=\frac{\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{2k^2\alpha e^{-\beta z}}{(2k^2-j^2)^2-4k^2\alpha\beta}J_m(kr)kdk\int_0^\infty f(R)J_m(kR)RdR\,. \qquad (14)$$

(ii) For arbitrary disturbances of the second kind on a semi-infinite body,

$$\left.\begin{aligned}\widehat{rz}&=\frac{\phi(r)}{kr}m\sin m\theta e^{-ipt}\,,\\ \widehat{\theta z}&=\frac{\partial\phi(r)}{\partial(kr)}\cos m\theta e^{-ipt}\,,\end{aligned}\right\} \qquad (15)$$

we get

$$\left.\begin{aligned}u_2&=\frac{m\sin m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{e^{-\beta z}}{\beta}\frac{J_m(kr)}{r}kdk\int_0^\infty \phi(R)J_m(kR)RdR\,,\\ v_2&=\frac{\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{e^{-\beta z}}{\beta}\frac{\partial J_m(kr)}{\partial r}kdk\int_0^\infty \phi(R)J_m(kR)RdR\,,\\ w_2&=0\,.\end{aligned}\right\} \qquad (16)$$

(iii) For arbitrary disturbances of the second kind on a stratified body we get

$$u_2=\frac{-m\sin m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{\{(\mu\beta+\mu'\beta')e^{\beta(H-z)}+(\mu\beta-\mu'\beta')e^{-\beta(H-z)}\}}{\beta\{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}\}}\Phi(k)\frac{J_m(kr)}{r}kdk\,,$$

$$v_2=\frac{-\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{\{(\mu\beta+\mu'\beta')e^{\beta(H-z)}+(\mu\beta-\mu'\beta')e^{-\beta(H-z)}\}}{\beta\{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}}\Phi(k)\frac{\partial J_m(kr)}{\partial r}kdk\,,$$

$$w_2=0\,, \qquad (17)$$

$$u_2' = \frac{-2m\sin m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{e^{\beta'(H-z)}}{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}}\Phi(k)\frac{J_m(kr)}{r}kdk,$$

$$v_2' = \frac{-2\cos m\theta e^{-ipt}}{\mu}\int_0^\infty \frac{e^{\beta'(H-z)}}{(\mu\beta+\mu'\beta')e^{\beta H}-(\mu\beta-\mu'\beta')e^{-\beta H}}\Phi(k)\frac{\partial J_m(kr)}{\partial r}kdk,$$

$$w_2' = 0, \tag{18}$$

where

$$\Phi = \int_0^\infty \phi(R)J_m(kR)RdR. \tag{19}$$

7. *Movements of the body outside the region of disturbances.*

Although the vibrations of a body due to disturbances of local type are never standing outside the region of those disturbances, they become waves that are transmitted outwards, as will be seen upon evaluating the integrals (13) $\sim$ (19) for a large value of r. As, however, the matter of evaluating these integrals has frequently been discussed, we shall not go further into the subject. The conclusion, which may be arrived at very easily, is as follows:

(i) Disturbances of the first kind.

The waves that are transmitted outwards are composed of Rayleigh-waves, longitudinal waves, and transverse waves, the amplitudes of which at a large epicentral distance are respectively proportional to[2)]

$$\frac{1}{\sqrt{r}},\quad \frac{1}{r^2},\quad \frac{1}{r^2}.$$

(ii) Disturbances of the second kind on a semi-infinite body.

The transmitted waves are only bodily transverse waves, the amplitude of which at a long epicentral distance is proportional to $\frac{1}{r}$, the proof of which is very simple.[3)]

(iii) Disturbances of the second kind on a stratified body.

The transmitted waves are Love-waves and bodily transverse waves, the amplitudes of which waves at a great epicentral distance are re-

2) H. LAMB, *Phil. Trans. Roy. Soc.*, **203** (1904), etc.

3) One method of evaluating (16) is obtained by writing $\int_0^\infty \phi(R)J_m(kR)RdR = \Phi(k) = C$, and using formula $\int_0^\infty \frac{e^{-\beta z}}{\beta}J_m(kr)kdk = \frac{e^{-ijr_1}}{r_1}$ where $r_1^2 = r^2+z^2$.

spectively proportional to[4)]

$$\frac{1}{\sqrt{r}}, \quad \frac{1}{r^2}.$$

The velocity of transmission of the bodily waves is that of transverse waves in the subjacent medium.

8. *Movements of the ground within the region of disturbances.*

An important part of the present problem is the movement of the ground within the region of disturbance, particularly for the case wherein the disturbance is distributed in some periodic manner. Let the disturbance be distributed such that

$$\left.\begin{aligned} \widehat{zz} &= P_m J_m(Kr)\cos m\theta e^{-ipt}, \\ \widehat{rz} &= Q_m \frac{\partial J_m(Kr)}{\partial (Kr)}\cos m\theta e^{-ipt}, \\ \widehat{\theta z} &= -Q_m \frac{J_m(Kr)}{Kr} m \sin m\theta e^{-ipt}, \end{aligned}\right\} \quad (z=0;\quad r<a) \qquad (20)$$

$$\widehat{zz}=0, \quad \widehat{rz}=0, \quad \widehat{\theta z}=0 \qquad (z=0;\quad r>a) \qquad (21)$$

in the first kind; and

$$\left.\begin{aligned} \widehat{rz} &= R_m \frac{J_m(Kr)}{Kr} m \sin m\theta e^{-ipt}, \\ \widehat{\theta z} &= R_m \frac{\partial J_m(Kr)}{\partial (Kr)}\cos m\theta e^{-ipt}, \end{aligned}\right\} \quad (z=0,\quad r>a) \qquad (22)$$

$$\widehat{rz}=0, \quad \widehat{\theta z}=0 \qquad (z=0;\quad r>a) \qquad (22)$$

in the second kind, when from (7), (8)

$$\left.\begin{aligned} u_1 &= \frac{1}{\mu K^2}\left\{\frac{P_m(2-j'^2)+Q_m 2\beta'}{(2-j'^2)^2-4\alpha'\beta'}\right\}\frac{\partial J_m(Kr)}{\partial r}\cos m\theta e^{-ipt}, \\ v_1 &= \frac{-1}{\mu K^2}\left\{\frac{P_m(2-j'^2)+Q_m 2\beta'}{(2-j'^2)^2-4\alpha'\beta'}\right\}\frac{J_m(Kr)}{r} m \sin m\theta e^{-ipt}, \\ w_1 &= \frac{-\alpha}{\mu K^2}\left\{\frac{P_m(2-j'^2)+Q_m 2\beta'}{(2-j'^2)^2-4\alpha'\beta'}\right\} J_m(Kr)\cos m\theta e^{-ipt}, \end{aligned}\right\} \qquad (25)$$

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4) If we were to assume $\int_0^\infty \phi(R) J_m(kR) R dR = \Phi(k) = C$ in (17), (18), the integrals to be evaluated have the same form as those in the paper: K. SEZAWA, "Love-waves generated from a Source of certain Depth", *Bull. Earthq. Res. Inst.*, **13** (1935), 17; the result being the conclusion shown in (iii).

$$\left.\begin{aligned} u_3 &= \frac{-1}{\mu K^2}\left\{\frac{P_m 2\alpha'\beta' + Q_m\beta'(2-j'^2)}{(2-j'^2)^2-4\alpha'\beta'}\right\}\frac{\partial J_m(Kr)}{\partial r}\cos m\theta e^{-ipt}, \\ v_3 &= \frac{1}{\mu K^2}\left\{\frac{P_m 2\alpha'\beta' + Q_m\beta'(2-j'^2)}{(2-j'^2)^2-4\alpha'\beta'}\right\}\frac{J_m(Kr)}{r} m\sin m\theta e^{-ipt}, \\ w_3 &= \frac{1}{\beta}\left\{\frac{P_m 2\alpha'\beta' + Q_m\beta'(2-j'^2)}{(2-j'^2)^2-4\alpha'\beta'}\right\} J_m(Kr)\cos m\theta e^{-ipt}, \end{aligned}\right\} \tag{25}$$

$$\left.\begin{aligned} u_2 &= \frac{-R_m}{\mu\beta}\,\frac{J_m(Kr)}{r} m\sin m\theta e^{-ipt}, \\ v_2 &= \frac{-R_m}{\mu\beta}\,\frac{\partial J_m(Kr)}{\partial r}\cos m\theta e^{-ipt}, \\ w_2 &= 0, \end{aligned}\right\} \tag{26}$$

all within $r<a$ and on $z=0$, where

$$K^2=\alpha^2+h^2=\beta^2+j^2, \quad j'=j/K, \quad \alpha'=\alpha/K, \quad \beta'=\beta/K. \tag{27}$$

Although these expressions have been given without any proof, they are direct consequences of the relations (1), (2), (3), (20), (21), (22), (23), which could be shown by accurately evaluating the integrals (13) ~(19).

Now vibrations (u_1, v_1, w_1) and (u_3, v_3, w_3) for $r<a$, $z=0$ are in resonance condition when the frequency of the disturbance satisfies the condition

$$(2-j'^2)^2-4\alpha'\beta'=0. \tag{28}$$

Vibration (u_2, v_2, w_2), on the other hand, is never under resonance, even if $r<a$.

Although the answer to the question whether or not the movement in $r>a$ or $z\neq 0$ is in standing vibration cannot be strictly obtained with the aid of our present simple calculation, the probability is that movement in such a region is almost a wave motion.

9. *Numerical calculation of displacement distribution for a two-dimensional vibration of the first kind.*

With a view to ascertaining the numerical values of the displacements, we calculated a two-dimensional vibration of the first kind. Let the axis of x be drawn horizontally on the free surface and that of z vertically downwards. We then get

$$\left.\begin{aligned} \Delta &= Ae^{i(pt+\alpha z)}\cos kx, \\ u_1 &= \frac{kA}{h^2}e^{i(pt+\alpha z)}\sin kx, \\ w_1 &= \frac{\alpha A}{h^2}e^{i(pt+\alpha z)}\cos kx, \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned} u_2 &= \frac{\beta}{j^2}Be^{i(pt+\beta z)}\sin kx, \\ w_2 &= \frac{k}{j^2}Be^{i(pt+\beta z)}\cos kx, \end{aligned}\right\} \tag{30}$$

where $k^2=\alpha^2+h^2=\beta^2+j^2$. The boundary conditions in the present calculation are such that

$$\lambda\Delta+2\mu\frac{\partial w}{\partial z}=P\cos kxe^{ipt}, \quad \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=0 \tag{31}$$

at $z=0$ where $u=u_1+u_2$, $w=w_1+w_2$. These give the values of the constants, the result for the case $\lambda=\mu$ being

$$\left.\begin{aligned} \frac{\mu}{P}u_1 &= \frac{-(2-j'^2)}{k\Phi}e^{i(pt+\alpha z)}\sin kx, \\ \frac{\mu}{P}w_1 &= \frac{-\alpha'(2-j'^2)}{k\Phi}e^{i(pt+\alpha z)}\cos kx, \end{aligned}\right\} \tag{32}$$

$$\left.\begin{aligned} \frac{\mu}{P}u_2 &= \frac{2\alpha'\beta'}{k\Phi}e^{i(pt+\beta z)}\sin kx, \\ \frac{\mu}{P}w_2 &= \frac{2\alpha'}{k\Phi}e^{i(pt+\beta z)}\cos kx, \end{aligned}\right\} \tag{33}$$

where

$$\Phi=(2-j'^2)^2-4\alpha'\beta'. \tag{34}$$

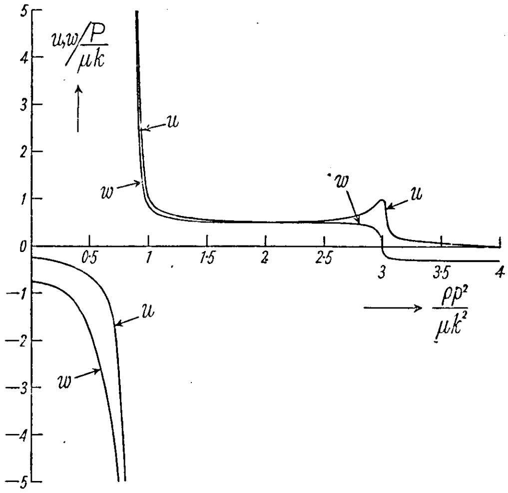

Fig. 2. Surface displacement.

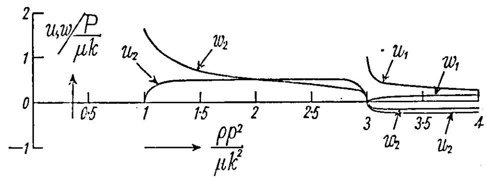

Fig. 3. Dissipation waves.

The nature of the problem is the same as that for the three-dimensional problem. Fig. 2 indicates the surface displacement, and Fig. 3 the amplitudes of the dissipation waves.

(a) If $0<\sqrt{\frac{\rho}{\mu}}\frac{p}{k}<1$, there exists a standing vibration without any wave dissipation.

(b) If $\sqrt{\frac{\rho}{\mu}}\frac{p}{k}=0.9194$, the standing vibration is under resonance, so that $u=u_1+u_2$, $w=w_1+w_2$ then become infinitely large as will be seen from Fig. 1.

(c) If $1<\sqrt{\frac{\rho}{\mu}}\frac{p}{k}<\sqrt{3}$, the transverse waves (u_2, v_2) are dissipating, the dilatational component of the displacement still forming stationary vibrations near the surface. The numerical relation will be seen from Fig. 1, 2.

(d) If $\sqrt{3}<\sqrt{\frac{\rho}{\mu}}\frac{p}{k}$, the longitudinal as well as the transverse waves are dissipating, without any standing wave on the surface.

10. *Concluding remarks.*

It will be seen from these studies that, while the movement of the ground within the region of the applied disturbance is in stationary vibration or in wave motion according as the frequency of the vibratory disturbance is relatively low or relatively high for a given distribution of the same disturbance, movement outside the region under consideration is almost a wave motion for any frequency of the periodic disturbance. In that case wherein it is possible for the surface waves to exist outside the region of disturbance, there is such a frequency of periodic disturbances that the surface movement is under resonance conditions. On the other hand, in the case wherein no surface wave can possibly exist outside the region of disturbance, the surface movement is never under resonance condition for any periodic disturbance.

A number of writers have discussed with various conclusions whether microseismic movement, under certain weather conditions, is of wave motion or a standing vibration of the ground. Even should the movement under consideration be a standing vibration, its type would be of such surface vibration as has been shown in the present paper, it therefore being possible for resonance conditions to exist.

For resonance conditions to occur in the vibration of the ground surface, the presence of a superficial layer has no place in any important part of the problem unless the vibration of the second kind, namely, the case of pure shearing disturbance applied to the surface, is taken into consideration. The effect of surface layers on vibration under resonance in the case of the first kind of vibrations merely results some modification in the frequency of that resonance, as is the case for differences in velocity between Rayleigh-waves on a semi-infinite body and those on a stratified body.

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1. 半無限體の振動に於ける共振現象及び逸散波

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半無限體の表面にディスターバンスが加つてもその振動は波動の形としてだけ存在すると考へられるから，その共振現象などといふ事は簡單には想像できにくい．しかしながらディスターバンスが表面上に週期的に配列し且つそのディスターバンスの振動週期が極く緩い場合には固體の表面だけの定常振動が現れ，且つ共振の振動狀態にもなり得るものである．

ディスターバンスが固體に捩り變形の外に壓縮變形をも起すやうな振動を與へる場合には極めて低い振動週期，即ち $0<\sqrt{\rho/\mu}(p/k)<1$ に於て勢力が固體表面にのみ集中する定常振動を起し，且つ Rayleigh 波に相當する波長と振動數との關係のときに振動の共振現象が起るものである．振動週期が少しく急になり，$1<\sqrt{\rho/\mu}(p/k)<\sqrt{(\lambda+2\mu)/\mu}$ の狀態になると橫波が下方へ傳播するやうになる．しかし縱波の振動はやはり定常振動として表面近くに集つてゐるものである．$\sqrt{(\lambda+2\mu)/\mu}<\sqrt{\rho/\mu}(p\ k)$ の如き高い振動數になると定常振動は無くなり，縱波も橫波も傳擣する部分のみとなる．

ディスターバンスが捩り變形のみを起すやうな場合には，$0<\sqrt{\rho/\mu}(p/k)<1$ に於て表面の定常振動型のものが，$1<\sqrt{\rho/\mu}(p\ k)$ になると全部橫波として傳播するものである．この場合には共振が存在しない．

固體の表面に層のある場合にも同樣な性質があるが，ただディスターバンスが捩り變形を與へるやうな場合でも表面波の存在し得るときには共振現象のあることは非常に違つた點である．從て固體に表面波の存在し得る場合には必ず共振現象のあり得る譯である．

以上述べた事柄が成立つには必しも固體表面の全體に涉つてディスターバンスが分布してゐる必要がないのであつて，固體表面の一部分にディスターバンスがあつても，その部分の中だけについては前述の事柄が行はれる．しかしその部分の外では寧ろ波動現象のみが存在するものである．

地表面の脈動は波動であるか定常振動であるかといふ事が屢々問題となるが，脈動が大氣中でのディスターバンスによるとしても，それはこの論文の結果の如くその分布と振動週期如何によつて或は波動となり，或は定常振動となり，又特に共振振動ともなる事が知られるのである．
