4. *Further Studies on the Seismic Vibrations of a Gozyûnotô (Pagoda).*

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1. *Introduction.*

In our previous paper[1] we discussed the earthquake-proof properties of a gozyûnotô mainly from the standpoint of dissipation of the vibrational energy into the ground. The data of numerous experiments, due to Omori,[2] that we used in our calculation in the previous paper, were restricted to structural dimensions and the vibration frequency of the tower. Upon reexamining the problem, however, we found that application of such data implied the condition that the mass of the tower is disproportionately large, with the result that the dissipation of vibrational energy forms an important part of the damping phenomena. If, on the other hand, we were to take the most probable mass of the tower, the dissipation would be negligible, the nature of the problem thus differing entirely from that of a concrete structure of European style. We are therefore now in a position to reinvestigate the problem from a different point of view.

The deformation damping resistance of a structure as worked out by us[3] is not more effective on the damping phenomena of the structure here treated than in others. The conclusions arrived at after our inspection of actual towers, particularly of the Ueno Gozyûnotô, is that Coulomb's damping is the principal resistance and, furthermore, that the damping should increase with increase in the relative displacements between the respective adjacent roof truss parts for the reason that the structural members in every columnar part between the roof truss parts under consideration are connected by joggle joints, but not by simple slip joints. Aside from this fact, it is possible for the Coulomb damping in a vibrating body to increase with increase in the relative displacements in consequence of the diminution in the moving velocity with in-

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1) K. SEZAWA and K. KANAI, "On the Vibrations of a Gozyûnotô (Pagoda)", *Bull. Earthq. Res. Inst.*, **14** (1936), 525~533.

2) F. OMORI, *Bull. Earthq. Inv. Comm.*, **9** (1918~21), 110~150.

3) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **3** (1927), 50.

crease in the displacements in question. Under these considerations we assumed that the Coulomb damping approximately increases linearly as the relative displacements.

It should be borne in mind that, since not only gozyûnotôs but also almost all Japanese style structures, including a palace and a buddhist temple, are more or less similar in type, the correct treatment of Coulomb damping is extremely important over a wider range of problems. All we can add here is that this form of damping is very far-reaching in the case of the gozyûnotô.

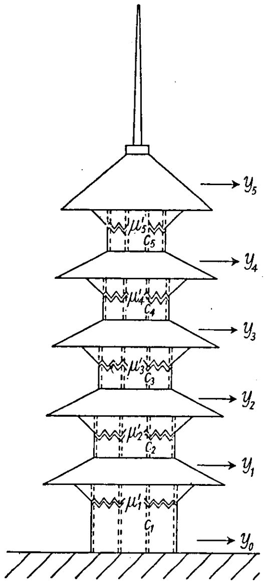

Fig. 1.

2. *The solution of the problem.*

Let $y_1, y_2, \cdots, y_5$ be horizontal displacements of the first, second,$\cdots$, fourth and fifth roof truss parts respectively, and y_0 the horizontal movement of the ground. If the elastic resistance is proportional to the relative displacements of adjacent roof truss parts and the damping resistance is a certain function of the relative velocities of the same roof truss parts, then the equations of vibrations are expressed by

$$\left.\begin{array}{l} M_1\ddot{y}_1+f_1(\dot{y}_1-\dot{y}_0)-f_2(\dot{y}_2-\dot{y}_1)+c_1(y_1-y_0)-c_2(y_2-y_1)=0\,, \\ M_2\ddot{y}_2+f_2(\dot{y}_2-\dot{y}_1)-f_3(\dot{y}_3-\dot{y}_2)+c_2(y_2-y_1)-c_3(y_3-y_2)=0\,, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ M_5\ddot{y}_5+f_5(\dot{y}_5-\dot{y}_4)+c_5(y_5-y_4)=0, \end{array}\right\} \quad (1)$$

where $M_1, M_2, \cdots, M_5, c_1, c_2, \cdots, c_5$ are masses concentrated on the respective roof truss parts and the stiffnesses of the columnar parts between the successive roof truss parts, whereas $f_1, f_2, \cdots, f_5$ are functions corresponding to the damping. Rewriting the above equations and putting $M_1=M_2=\cdots=M_5\equiv M$, $f_1=f_2=\cdots=f_5\equiv f$, $c_1=c_2=\cdots=c_5\equiv c$, we get

$$\left.\begin{array}{l} f(\dot{y}_1-\dot{y}_0)+c(y_1-y_0)=-M(\ddot{y}_1+\ddot{y}_2+\cdots+\ddot{y}_5)\,, \\ f(\dot{y}_2-\dot{y}_1)+c(y_2-y_1)=-M(\ddot{y}_2+\ddot{y}_3+\cdots+\ddot{y}_5)\,, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ f(\dot{y}_5-\dot{y}_4)+c(y_5-y_4)=-M\ddot{y}_5\,. \end{array}\right\} \quad (2)$$

The expressions of types (2) for a special case, $f=0$, was obtained by

Martel.[4)]

The simplest way to express f is to write

$$f(\dot{y}_n-\dot{y}_{n-1})=\mu'(\dot{y}_n-\dot{y}_{n-1}) \tag{3}$$

where μ' is a constant depending on the frequency of the vibration. By adjusting the form of μ', it is possible to have a damping force in agreeing with Coulomb's friction. From the condition assumed we have

$$f(\dot{y}_n-\dot{y}_{n-1})=Fi(y_n-y_{n-1}), \tag{4}$$

where F is a constant independent of the vibrational frequency. The nature of the damping resistance and that of the elastic resistance differ only in the term of i. Let

$$y_0=A_0e^{ipt},\quad y_1=A_1e^{ipt},\ \cdots\cdots,\quad y_5=A_5e^{ipt}. \tag{5}$$

Equating (3), (4) by means of (5), we get

$$\mu'=F/p. \tag{6}$$

Now substituting (5) in (2) under the condition $\mu'=F/p$, we get

$$\begin{aligned}\frac{A_1\Phi}{A_0}=&\left[\left\{-1+10\left(\frac{p\mu'}{c}\right)^2-5\left(\frac{p\mu'}{c}\right)^4\right\}+10\left(\frac{Mp^2}{c}\right)\left\{1-6\left(\frac{p\mu'}{c}\right)^2+\left(\frac{p\mu'}{c}\right)^4\right\}\right.\\&\left.+15\left(\frac{Mp^2}{c}\right)^2\left\{-1+3\left(\frac{p\mu'}{c}\right)^2\right\}+7\left(\frac{Mp^2}{c}\right)^3\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}-\left(\frac{Mp^2}{c}\right)^4\right]\\&+i\left(\frac{p\mu'}{c}\right)\left[\left\{-5+10\left(\frac{p\mu'}{c}\right)^2-\left(\frac{p\mu'}{c}\right)^4\right\}+40\left(\frac{Mp^2}{c}\right)\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}\right.\\&\left.+15\left(\frac{Mp^2}{c}\right)^2\left\{-3+\left(\frac{p\mu'}{c}\right)^2\right\}+14\left(\frac{Mp^2}{c}\right)^3-\left(\frac{Mp^2}{c}\right)^4\right],\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{A_2\Phi}{A_0}=&\left[\left\{-1+10\left(\frac{p\mu'}{c}\right)^2-5\left(\frac{p\mu'}{c}\right)^4\right\}+6\left(\frac{Mp^2}{c}\right)\left\{1-6\left(\frac{p\mu'}{c}\right)^2+\left(\frac{p\mu'}{c}\right)^4\right\}\right.\\&\left.+5\left(\frac{Mp^2}{c}\right)^2\left\{-1+3\left(\frac{p\mu'}{c}\right)^2\right\}+\left(\frac{Mp^2}{c}\right)^3\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}\right]\\&+i\left(\frac{p\mu'}{c}\right)\left[\left\{-5+10\left(\frac{p\mu'}{c}\right)^2-\left(\frac{p\mu'}{c}\right)^4\right\}+24\left(\frac{Mp^2}{c}\right)\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}\right.\\&\left.+5\left(\frac{Mp^2}{c}\right)^2\left\{-3+\left(\frac{p\mu'}{c}\right)^2\right\}+2\left(\frac{Mp^2}{c}\right)^3\right],\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{A_3\Phi}{A_0}=&\left[\left\{-1+10\left(\frac{p\mu'}{c}\right)^2-5\left(\frac{p\mu'}{c}\right)^4\right\}+3\left(\frac{Mp^2}{c}\right)\left\{1-6\left(\frac{p\mu'}{c}\right)^2+\left(\frac{p\mu'}{c}\right)^4\right\}\right.\\&\left.+\left(\frac{Mp^2}{c}\right)^2\left\{-1+3\left(\frac{p\mu'}{c}\right)^2\right\}\right]+i\left(\frac{p\mu'}{c}\right)\left[\left\{-5+10\left(\frac{p\mu'}{c}\right)^2-\left(\frac{p\mu'}{c}\right)^4\right\}\right.\end{aligned}$$

4) R. R. MARTEL, "The Dynamic Behaviour of Some Simple Bents Subjected to Established Simple Harmonic Motion", *World Eng. Congr. Tokyo*, 1929, 3, Paper No. 490.

$$+12\left(\frac{Mp^2}{c}\right)\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}+\left(\frac{Mp^2}{c}\right)^2\left\{-3+\left(\frac{p\mu'}{c}\right)^2\right\}\Bigg], \quad (9)$$

$$\frac{A_4\Phi}{A_0}=\left[\left\{-1+10\left(\frac{p\mu'}{c}\right)^2-5\left(\frac{p\mu'}{c}\right)^4\right\}+\left(\frac{Mp^2}{c}\right)\left\{1-6\left(\frac{p\mu'}{c}\right)^2+\left(\frac{p\mu'}{c}\right)^4\right\}\right]$$

$$+i\left(\frac{p\mu'}{c}\right)\left[\left\{-5+10\left(\frac{p\mu'}{c}\right)^2-\left(\frac{p\mu'}{c}\right)^4\right\}+4\left(\frac{Mp^2}{c}\right)\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}\right], \quad (10)$$

$$\frac{A_5\Phi}{A_0}=\left\{-1+10\left(\frac{p\mu'}{c}\right)^2-5\left(\frac{p\mu'}{c}\right)^4\right\}+i\left(\frac{p\mu'}{c}\right)\left\{-5+10\left(\frac{p\mu'}{c}\right)^2-\left(\frac{p\mu'}{c}\right)^4\right\}, \quad (11)$$

where

$$\Phi=P+iQ$$

$$=\Bigg[\left\{-1+10\left(\frac{p\mu'}{c}\right)^2-5\left(\frac{p\mu'}{c}\right)^4\right\}+15\left(\frac{Mp^2}{c}\right)\left\{1-6\left(\frac{p\mu'}{c}\right)^2+\left(\frac{p\mu'}{c}\right)^4\right\}$$

$$+35\left(\frac{Mp^2}{c}\right)^2\left\{-1+3\left(\frac{p\mu'}{c}\right)^2\right\}+28\left(\frac{Mp^2}{c}\right)^3\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}-9\left(\frac{Mp^2}{c}\right)^4$$

$$+\left(\frac{Mp^2}{c}\right)^5\Bigg]+i\left(\frac{p\mu'}{c}\right)\Bigg[\left\{-5+10\left(\frac{p\mu'}{c}\right)^2-\left(\frac{p\mu'}{c}\right)^4\right\}+60\left(\frac{Mp^2}{c}\right)\left\{1-\left(\frac{p\mu'}{c}\right)^2\right\}$$

$$+35\left(\frac{Mp^2}{c}\right)^2\left\{-3+\left(\frac{p\mu'}{c}\right)^2\right\}+56\left(\frac{Mp^2}{c}\right)^3-9\left(\frac{Mp^2}{c}\right)^4\Bigg\}. \quad (12)$$

Rewriting these expressions we get

$$\left(\frac{p\mu'}{c}\right)^2=\alpha\,, \quad \frac{Mp^2}{c}=\beta; \quad (13)$$

$$\frac{A_1\Phi}{A_0}=\left\{(-1+10\alpha-5\alpha^2)+10\beta(1-6\alpha+\alpha^2)+15\beta^2(-1+3\alpha)+7\beta^3(1-\alpha)-\beta^4\right\}$$

$$+i\sqrt{\alpha}\left\{(-5+10\alpha-\alpha^2)+40\beta(1-\alpha)+15\beta^2(-3+\alpha)+14\beta^3-\beta^4\right\}, \quad (7')$$

$$\frac{A_2\Phi}{A_0}=\left\{(-1+10\alpha-5\alpha^2)+6\beta(1-6\alpha+\alpha^2)+5\beta^2(-1+3\alpha)+\beta^2(1-\alpha)\right\}$$

$$+i\sqrt{\alpha}\left\{(-5+10\alpha-\alpha^2)+24\beta(1-\alpha)+5\beta^2(-3+\alpha)+2\beta^3\right\}, \quad (8')$$

$$\frac{A_3\Phi}{A_0}=\left\{(-1+10\alpha-5\alpha^2)+3\beta(1-6\alpha+\alpha^2)+\beta^2(-1+3\alpha)\right\}$$

$$+i\sqrt{\alpha}\left\{(-5+10\alpha-\alpha^2)+12\beta(1-\alpha)+\beta^2(-3+\alpha)\right\}, \quad (9')$$

$$\frac{A_4\Phi}{A_0}=\left\{(-1+10\alpha-5\alpha^2)+\beta(1-6\alpha+\alpha^2)\right\}$$

$$+i\sqrt{\alpha}\left\{(-5+10\alpha-\alpha^2)+4\beta(1-\alpha)\right\}, \quad (10')$$

$$\frac{A_5\Phi}{A_0}=(-1+10\alpha-5\alpha^2)+i\sqrt{\alpha}\,(-5+10\alpha-\alpha^2)\,, \tag{11'}$$

$$\Phi=\Big\{(-1+10\alpha-5\alpha^2)+15\beta(1-6\alpha+\alpha^2)+35\beta^2(-1+3\alpha)+28\beta^3(1-\alpha)-9\beta^4+\beta_5\Big\}+i\sqrt{\alpha}\Big\{(-5+10\alpha-\alpha^2)+60\beta(1-\alpha)+35\beta^2(-3+\alpha)+56\beta^3-9\beta^4\Big\}\,. \tag{12'}$$

3. *Some numerical examples.*

Omori's report[5)] contains, though not always perfectly accurate, a good deal of experimental data regarding free vibrations as well as artificial forced vibrations of the gozyûnotôs, from which it is possible to estimate roughly the values of the damping constants peculiar to the vibrations of various towers. In the case of the Ueno Gozyûnotô, the amplitude of the free vibration diminishes to about one-half its initial value after three or four cycles of vibrations, whereas the amplitude of the forced vibration at a frequency of about two-thirds of the natural one is nearly one-tenth or one-fifteenth of the amplitude under resonance conditions. After a number of tentative calculations we found that the following two examples represent extreme cases of the mostly probable conditions which actual towers of the natural period 1 sec should assume; the case of Ueno Gozyûnotô seeming to correspond to the first of the examples.

(i) $M/c=0{\cdot}002052\,\mathrm{sec}^2$, $F/Mg=0{\cdot}02547\,\mathrm{cm}^{-1}$;

(ii) $M/c=0{\cdot}002052\,\mathrm{sec}^2$, $F/Mg=0{\cdot}2487\,\mathrm{cm}^{-1}$.

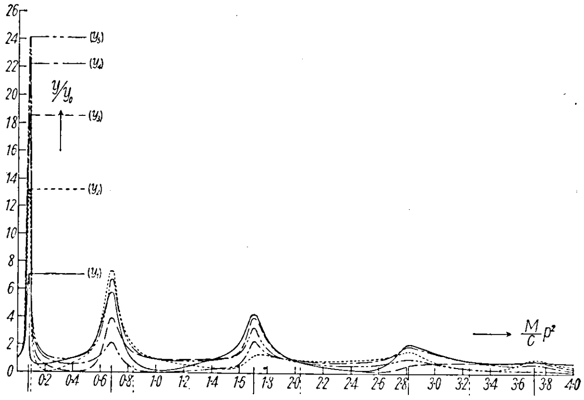

Fig. 2. Displacements. $M/c=0{\cdot}002052\,\mathrm{sec}^2$, $F/Mg=0{\cdot}02547\,\mathrm{cm}^{-1}$. Full, dotted, broken, chain, and double chain lines represent the displacements of the first, second, third, forth, and fifth roof truss parts respectively.

The relation $M/c=0{\cdot}002052\,\mathrm{sec}^2$ gives the condition that the fundamental natural period of the tower is about 1 sec, whereas $F/Mg=0{\cdot}02547\,\mathrm{cm}^{-1}$ and $F/Mg=0{\cdot}2487\,\mathrm{cm}^{-1}$ indicate the conditions that the ratios of the tangential friction to the weight of a one story for relative displacements of 1 cm are 0·02547

5) F. Omori, *loc. cit.* 2)

and 0·2487 respectively. These ratios are rather too small when compared with the results of other kinds of experiments. The result of calculation for these two cases are shown in Figs. 2, 3. The full, dotted, broken, chain, and double-chain lines represent the displacement ratios of the first, second, third, fourth, and the fifth roof truss parts to the

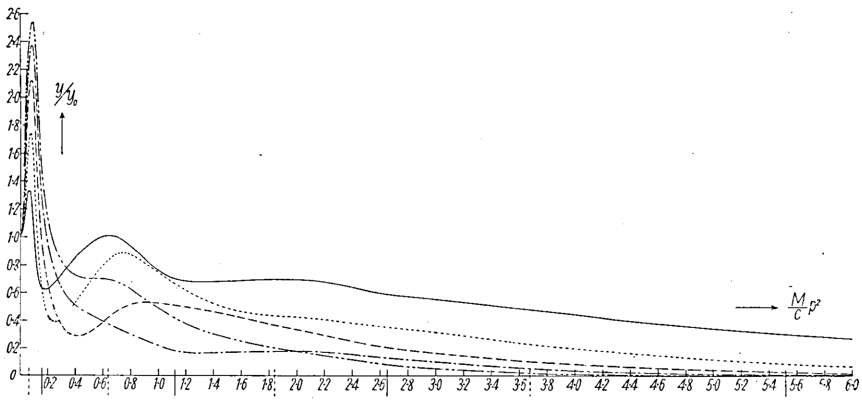

Fig. 3. Displacements. $M/c=0{\cdot}002052\,\text{sec}^2$, $F/Mg=0{\cdot}2487\,\text{cm}^{-1}$. Full, dotted, broken, chain and double chain lines represent the displacements of the first, second, third, forth, and fifth roof truss parts respectively.

ground vibration, namely y_1/y_0, y_2/y_0, y_3/y_0, y_4/y_0, y_5/y_0 respectively. The short vertical strips of full line and those of dotted line marked on every base line give the respective conditions, $P=0$, $Q=0$, that is to say, the real and imaginary parts of Φ become zero respectively; the values of β corresponding to $P=0$, $Q=0$ being shown below.

$$\text{Case (i)}\begin{cases} P=0;\ \beta=0{\cdot}0803,\ 0{\cdot}681,\ 1{\cdot}705,\ 2{\cdot}816,\ 3{\cdot}717; \\ Q=0;\ \beta=0{\cdot}0996,\ 0{\cdot}839,\ 2{\cdot}038,\ 3{\cdot}25. \end{cases}$$

$$\text{Case (ii)}\begin{cases} P=0;\ \beta=0{\cdot}159,\ 1{\cdot}123,\ 2{\cdot}65,\ 5{\cdot}521,\ -0{\cdot}45; \\ Q=0;\ \beta=0{\cdot}0659,\ 0{\cdot}638,\ 1{\cdot}839,\ 3{\cdot}679. \end{cases}$$

A root of $P=0$ in case (ii) assumes a negative value, showing that the natural vibration corresponding to this, should it exist, is aperiodic.

It will be seen that the resonance condition in case (i) takes place approximately when $P=0$ and the same condition in case (ii) approximately when $Q=0$. At all events, the frequency of the fundamental resonance was adjusted to 1 sec in both cases.

4. *Interpretation of the result.*

It will be seen from the results given here that, although in case (i), the amplitude of the uppermost roof truss part at frequency near the first resonance is about 24 times that of the ground movement, a similar amplitude in case (ii) is only about 2·5 times that of the ground movement.

In both examples, the lower the roof truss part the smaller the amplitude of the same part at relatively low frequency. For higher vibration frequencies, on the other hand, the amplitudes of the higher roof truss parts become smaller, the feature of the displacement distribution for a fairly high frequency being the very reverse of that of the fundamental resonance condition. In case (i), displacement of the first roof truss part becomes the largest at frequencies near the third resonance, whereas in case (ii) the same displacement becomes the largest one even at frequency below the second resonance.

It is a remarkable fact that, while the large amplitude under all resonance conditions is fairly pronounced in case (i), amplitude under any resonance higher than the third in case (ii) is scarcely noticeable, with the result that it is almost impossible to identify the resonance frequencies from the resonance curves.

It seems that the property of vibration damping in a structure of the present type is not only restricted to a gozyûnotô, but, is also found in the case of a buddhist temple, and even in a house of palace style of construction in Japan. It may however be concluded that the property under consideration is particularly pronounced in the case of the gozyûnotô.

In conclusion we wish to express our thanks to the Council of the Japan Society for the Promotion of Scientific Research, with whose aid progress in the present investigation was considerably furthered.

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4. 五重塔の耐震性に關する研究

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この前の報告に於て五重塔の耐震性に關する問題を論じて置いた. しかしその議論に必要なる數値的材料は主として大森博士が試みた振動週期についての試驗結果と塔の幾何學的寸法のみであつて，塔の重さについては直接考へて見なかつた爲に，五重塔の耐震性がコンクリートの建物

と同様に震動勢力の地中逸散といふことで見た所は相當に説明がついたのである．しかしその結果として塔の重量が非常に大きなものでないと成立たぬことに氣がついたので，その耐震性を更に見直して見ることにした譯である．

構造物中の粘性抵抗が五重塔の場合に限つて特に大きいとは考へられぬ．五重塔の構造をよくしらべて見ると Coulomb の固體摩擦がよく働いてをることがわかつた．而も之は塔の順々の屋根及斗組の間にある軸部に於ける結合部が主として嚙合接となつてをる爲に軸部の比較變性の大さに比例してこの摩擦が増加することがわかつたのである．接合せが嚙合でなく突附接の場合でも比較變位速度の増加と共に Coulomb の摩擦が減少するから，從て軸部の比較變位の大さと共に摩擦の増加するのは寧ろ一般的の性質のやうである．

このやうな考のもとに震動の理論的研究を試みたのであるが，その彈性的及び摩擦的常數は如何にして定めたかといふと，それには大森博士の五重塔の强制振動及び自由振動の試驗結果を比較して見た上でのことである．卽ち自由振動では3週期か4週期の繰返しの後に振幅が半減すること，强制振動では共振の振動數の約 2/3 の振動數に於ける振幅が同じ强制力のもとの共振に於ける振幅の 1/10 乃至 1/15 位であるといふことから推定したのである．

このやうにして實際の五重塔に最も可能でありさうな範圍の二つの極端な場合について共振曲線を出して見た．摩擦の少い方の極端では共振の振幅が零振動數の場合のそれの約 24 倍となり，摩擦の多い方の極端ではその値が約 2·5 倍となる．實際の塔はこの始めの方に近いやうである．それにしても摩擦力が力學的にむしろ少な過ぎる位であるにも拘らず，前回の論文では容易に近づき得なかつたやうな減衰振幅に到達し得たのである．

低い振動數では五重塔の下部程振幅が少くなり，稍高い振動數では逆に上部程振幅が少くなるのである．之は當然なこととはいひながら，五重塔の場合に特に目立つてをるやうに思はれる．このやうに上部程振幅が少くなることは摩擦力の少い方の例では第3共振以上の振動數で現れ，摩擦力の多い方の例では第2共振よりも低い振動數で既に現れるものである．

尙注目に値することは，摩擦の少い方の例では殆ど何れの共振も共振曲線中にはつきりと見えるのに，摩擦の多い方の例では第3共振以上はどれが共振であるか判別がつかぬ程度になる事柄である．

只今述べたやうな摩擦のための振動減衰性は五重塔にのみ限つたことでなく日本の寺院や御殿風の建等には何れにも存在するもののやうである．しかし柱や重量の割合から見て五重塔に特別に著しく現れるものであることが知られた次第である．心柱はそれに何か特別の摩擦裝置を與へぬ限り効力のないことは依然として同樣である．