5. *Some Experiments on the Generation and Propagation of Elastic Waves.**

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1. Introduction.

A number of investigations on the generation and the propagation of elastic waves have been accomplished in the field of mathematical physics, and the results are already applied in the elucidation of seismic waves generated and propagated in the crust of our planet, throwing remarkable light on the nature and the origin of earthquakes. The investigations so far attained in these branches could sufficiently explain the regional distribution of the initial dilatation and compression as well as the magnitude of the first impulsion, though the character of the first impulsion, whether it is of shock type or not, is still to be explained.

As to experimental studies of elastic waves, we have only a few works as compared with those on theoretical studies. The experimental difficulties owing to the high velocity of propagation seems to have prevented the developement of studies in this branch of investigation. But in view of the bearing of the experimental studies in the elucidation of seismic waves and the verification of theoretical results, some step forward is to be made in the domain of experimental studies no matter how difficult the realisation may be.

With this in view, we have first studied, for simplicity's sake, the generation and propagation of elastic waves along metallic wires. We certainly aware that the nature of waves propagated in three dimensions is not immeadiately inspected from the mere knowledge of that of one dimensional waves, and the extension of our experiments to more general cases will be made in future.

The results of our preliminary study, however, contains points of some importance as will be seen from the descriptions in the following.

2. Experimental Arrangement.

Experiments were made on the generation and propagation of distor-

* Communicated by Prof. M. Ishimoto.

tional waves along metallic wires attached with a number of wooden pieces in equal distances.[1] (See Fig. 1. PL. I.) This disposition is not what we have aimed at, but this contrivance reduced the velocity of propagation to so small a value that we can easily observe the phenomena in the laboratory. The dimensions of wooden pieces used are 11·94 cm × 1·65 cm × 1·20 cm, and the weight of which were 13·0 gr in the mean, the deviation being within ±1 gr.

Two kinds of wires were used in the present experiment, the one being a brass wire with a diameter 0·059 cm, and the other steel wire of 0·064 cm dia. The length of wire between consecutive wooden pieces were 1·89 cm in each experiment. The wire was hung from the ceiling and clumped at the lower end. The lengths of wire between wooden pieces and the upper and the lower clumps were also made 1·89 cm, equal to the distance of consecutive wooden pieces.

The motion of a wooden piece was recorded on smoked paper (arround a drum driven by a synchronous moter) with an indicator of straw arm which is directly connected to the wooden piece. This recording device is the same with those used in the Ishimoto acceleration seismometer and Hagiwara velocity seismograph, with the credit of the smallness of the weight of the arm and the friction at the indicator end. The driving rate of the smoked paper was 0·94 cm/sec.

At both ends of a certain wooden piece small iron pieces so light that they exerted no appreciable influence were attached in order to pull by electromagnets in case of giving impulsive couple to the suspended wire or clumping the wooden piece at required positions. But in some cases wooden pieces move to and fro through the position of electromagnets in the above arrangement. In such cases wooden pieces were connected by silk thread with iron pieces, which after leaving the electromagnets did not hinder the motion of wooden pieces.

The forces applied were couples in each case, and lateral vibrations of the wire were never excited when the positions of the magnets were properly adjusted.

3. Measurement of Velocity of Propagation.

First using the brass wire model, an impulsive couple was given at the lower end of the wire, and travel times were observed at various points. From the travel time curve (Fig. 2) the velocity of propagation was obtained as 35·9 sections/sec. Since the length of one section consisting of the wire of length $l=1{\cdot}89$ cm and wooden piece of breadth $l'=$

1) This contrivance was invented by Prof. M. Ishimoto and a model was constructed by him.

1·65 cm is 3·54 cm the velocity comes out $35{\cdot}9\times3{\cdot}54=127$ in cm/sec.

On the other hand, the equation of motion refered to x-axis taken in the direction of wire and measured by the length of one section $(l+l')$ is

$$I\frac{\partial^2\xi}{\partial t^2}=\tau\frac{\partial^2\xi}{\partial x^2},\ \left(\tau=\frac{\pi n a^4}{2l}\right) \quad (1)$$

where ξ represents angular displacement, τ the moment of couple of the wire of length l when the one end of the wire is rotated through one radian, and n and a denote the rigidity and radius of wire, while I is the moment of inertia arround the axis of rotation of one section and practically equal to that of the wooden piece as the part due to the wire is negligibly small. Then $c=\sqrt{\tau/I}$ is the velocity of propagation and $v=c(l+l)$ is the velocity in c. g. s. unit. Introducing the dimensions already mensioned and the value of $n=3{\cdot}43\times10^{11}$ c. g. s. as determined experimentally, we have $v=132$ cm/sec in good accord with the observed value 127 cm/sec.

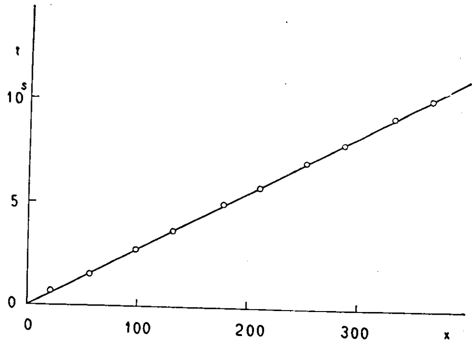


Fig. 2. Time distance curve for brass wire model.

Similar determination with the steel wire model gave (Fig. 3) $c=62{\cdot}35$ sections/sec or $v=220{\cdot}7$ cm/sec.

4. Energy of waves Generated from the Release of Statical Strains.[2]

The upper and the lower end of the wire as well as the fourth wooden piece from the lower end were clumped in strainless positions. Then the lower end was rotated through a certain angle θ, so that strain energy was stored between the lower end and the fourth wooden piece. (Fig. 4 represents the angular displacement of the wire.)

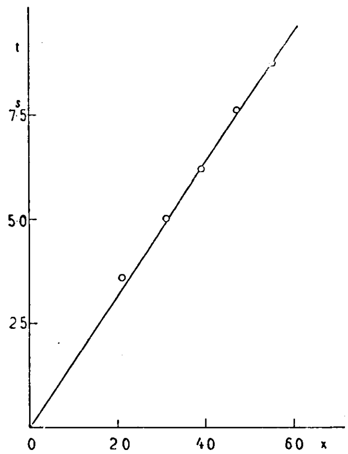


Fig. 3. Time distance curve for steel wire model.

2) The possibility of the generation of earthquakes from the release of statical strain has often been suggested since the beginning of seismology, and recently Prof. Ishimoto called attension of this fact and Messrs. H. Kawasumi and K. Yosiyama [Bull. Earshq. Res. Inst., **13** (1935), 496~503, & Disin, **7** (1935), 359~375.] could for the first time prove it theoretically although Prof. K. Sezawa and Mr. K. Kanai [Bull. Esrthq. Res.Inst., **13** (1935), 740~749; **14** (1936), 10~18.] contested against this conclusion.

When the couple which has kept the fourth wooden piece in its

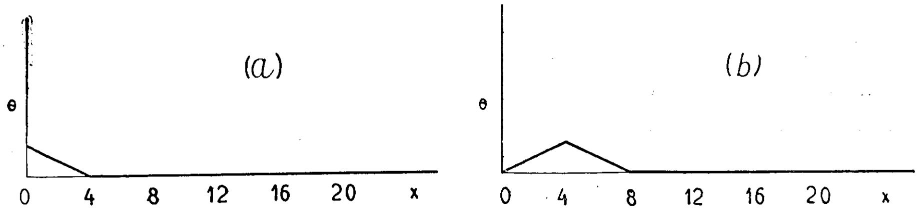


Fig. 4. Initial strain. (a): Type A. (b): Type B.

original position is taken off, the statical strain energy stored in the lower part is transported by the elastic waves then generated and propagated upward. Fig. 5 (PL.II) is the record of this wave at 36th piece from the lower end. From this record we see a wave with permanent displacement in one direction. The small undulation following the large displacement seems to be the free oscilation of one section, since the calculated period $2\pi/\sqrt{\tau/I}=0{\cdot}17$ sec is equal to the observed period.

The energy flux through a point is $\tau\dfrac{\partial\xi}{\partial x}\dfrac{\partial\xi}{\partial t}$, and the total energy E transported across a point is

$$E=\int\tau\frac{\partial\xi}{\partial x}\frac{\partial\xi}{\partial t}dt\doteqdot\frac{\tau}{c}\int\left(\frac{\partial\xi}{\partial t}\right)^2dt\,, \tag{2}$$

because of the fact that $\xi=f(x\text{-}ct)$. On the other hand the initial strain energy

$$V=\frac{1}{2}\tau'\theta^2=\frac{1}{2}\frac{\tau}{4}\theta^2=\frac{\tau}{8}\theta^2\,, \tag{3}$$

where τ' is the coefficient of couple due to the rotation of 4 sections, while θ is the initial angular displacement at the lower end. Then the ratio of energies

$$\frac{E}{V}=\frac{1}{c}\int\left(\frac{\partial\xi}{\partial t}\right)^2dt\Big/\frac{\theta^2}{8}\,. \tag{4}$$

Since the result of comparison (Fig. 6) of the deflections ξ by waves with θ shows a linear inter-relation, the wave energy E and the strain energy V seem also to be linearly proportional through the relation (4).

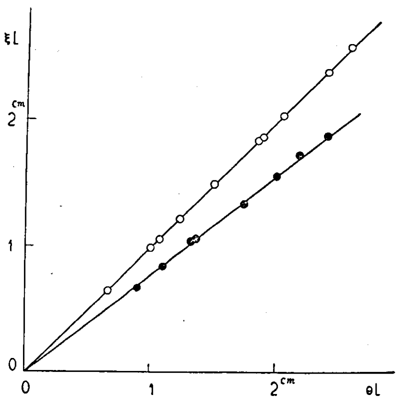


Fig. 6. Correlation between the angular displacement ξ and initial strain θ. (L=15 cm is the length of indication arm.)
● Brass wire model.
○ Steel wire model.

Then in order to calculate the energy of wave by the formula (2) the original record was enlarged photographically and the deflections were read off at every 1/200 sec. As the indicator end moves on a circular arc, due corrections to the amplitude and time are applied. Full line curve in Fig. 7 represents the corrected curve, while the curve

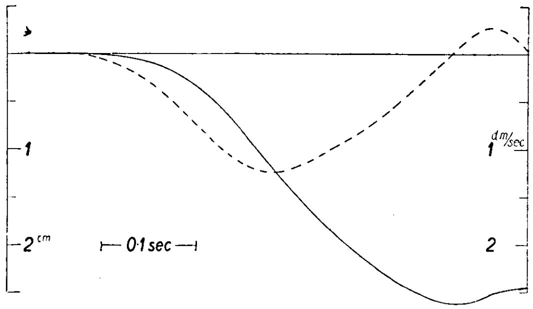


Fig. 7. Corrected displacement ξL (full line curve) and its time derivative $\frac{d\xi L}{dt}$ (broken line curne) due to waves generated by the initial strain of type A. (Brass wire model).

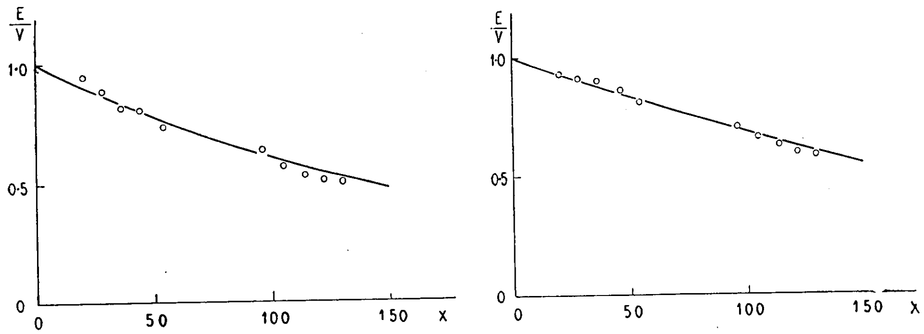
in broken line is the corresponding velocity obtained by numerical differentiation by means of the formula[3]

$$\dot{y}_n = \frac{2}{3}(y_{n+1} - y_{n-1}) - \frac{1}{12}(y_{n+2} - y_{n-2}) , \tag{5}$$

where y's are deflections at equal (unit) time interval. This is the first derivative of a cubic equation determined by the method of least squares using five consecutive values at the central point. Although the displacements and velocities required are angular displacements and velocities, all the arms of indicators being of equal length the readings from the records were used as they were without reducing to the angular ones. From these values of velocities, energy of wave E was calculated by the equation (2) by means of Sympson's method of numerical integration. The comparision of the result with the initial strain energy shows that they are nearly equal in magnitude although the wave energy is a little less than the strain energy. This may naturally be attributed to the loss in the excitation of the free oscillations of wooden pieces as well as damping of surrounding air. To make clear of this relation, observations were made at five positions. The results of calculation from five observations of the primary wave as well as five values corres-

3) This formula was obtained by Mr. H. Kawasumi.

ponding to the reflected wave at the upper end are plotted in Fig. 8 (a) aginst the distances from the lower end. From this diagram we see that the wave energy is dissipated nearly exponentially with distance from the initial values which is equal to the initial strain energy.



(a): Brass wire model. (b): Steel wire model.

Fig. 8. Dissipation of wave energy E with distance of the wave generated by the initial strain of type A (Fig. 4). The ordinates are measured in fraction of initial strain energy V.

Fig. 8 (b) for the steel wire model also indicates the same relation.

Next, the upper and the lower ends as well as the 8th piece from the lower end were clumped at strainless positions and the fourth one was rotated through a certain angle. Fig. 4 b represents the angular displacement of the wire (Type B). Then the couples which were acting to the fourth and the 8th pieces were suddenly taken off at the same time. In this case the strain energy stored between 8th piece and the lower end is transmitted upward. The wave form is shown in Fig. 9 (PL. III). With this wave, the calculation of energy were done as in the former example. Curves in Fig. 10 represent corrected displacement and velocity. Ratios of energies (E/V) at five positions were found for primary as well as the reflected waves. Fig. 11 (a) and 11 (b) represent the dissipations of waves as they are propagated along the brass and steel wires respectively.

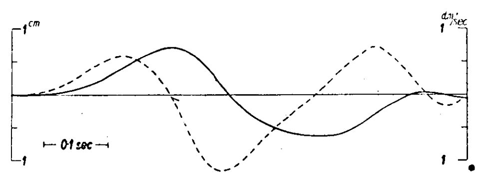
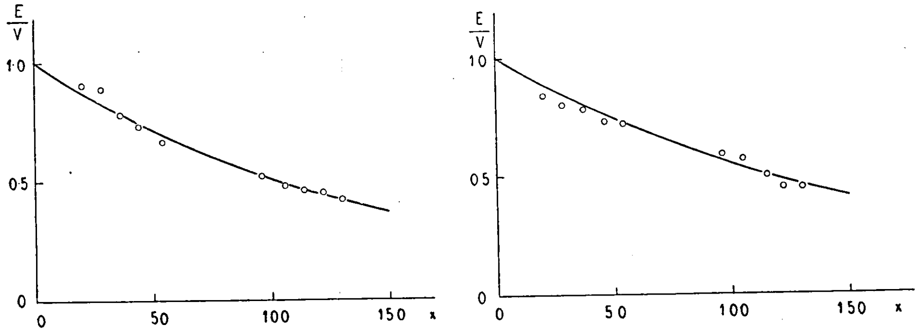


Fig. 10. Corrected displacement ξL (full line curve) and its time derivative $\frac{d\xi L}{dt}$ (broken line curve) due to waves generated by the initial strain of type B. (Brass wire model.)

Thus we see from Figs. 8 (a), 8 (b), 11 (a), and 11 (b) that the wave Energy E is dissipated nearly exponentially, and extrapolation to $x=0$ shows in each case that the wave energy is practically equall to the energy released from the initial statical strain. We may, therefore,



(a): Brass wire model. (b): Steel wire model.

Fig. 11. Dissipation of energy E with distance of the waves generated from the initial strain of type B (Fig. 4).

safely conclude that the whole statical strain energy is converted into wave energy and transmitted to distances, although a part of which is gradually dissipated in exciting the free oscillations of wooden pieces and owing to the damping effect of surrounding air. If the medium were perfectly homogeneous and there were no oscillators, the waves generated from the release of statical strains would keep their propagation without loss of energy but for the air damping.

These results are nothing but the experimental proof of the principle of the conservation of energy.

5. Forms of Waves Generated from the Release of Statical Strains or by Actions of Impulsive Forces.

Recent investigations revealed that the frequent occurrences of a kind of wave forms usually called a shock type at the very commencement of earthquakes, and methods of deducing actual amplitudes from the observed first impulsions on the seismograms have been worked out assuming the ground motion of such type. But no explanation on the mechanism of occurrence of waves of this type has been given. We have tried and succeeded to generate waves of such form propagated along metallic wires, as will be described in the following. Of course, as the writer has already stated, some precaution and restriction will be necessary in the application of the present result to the interpretation of seismic waves which are propagated in three dimensions.

In the experiments described in the above sections, the waves reflected at the lower end were superposed on the primary waves, and the results are not applicable to the present problem directly. In roder to avoid the boundary effect, impulsive couples or the initial strains were given in the present experiments at the central part of the wire. When a pair of impulsive couple were given (Fig. 12 I), the wave of permanent displacement to one direction was generated as shown in Fig. 13 (a), but when two pairs of couples were applied simultaneously in opposite directions as shown in Fig. 12 II, then the wave of shock type was generated. (Fig. 13 (b).) Other forms of initial strains and the waves therefrom are indicated in Fig. 12 III, IV, and Figs. 13 (c) and 13 (d) (PL. IV) respectively.

The waves generated by mechanisms of type I and type III are almost identical while those of type II and type IV are irrespectively of shock type. It is interesting to note that the number of couples required to obtain a type of wave form are always less than the number of couples required to obtain the statical strain which give rise the same type of waves when the strain is released. It is also to be noted that the directions of initial motions above and below the central part where the disturbances are given are the same in the mechanism of type I and IV, while it is oposite in types II and III. The consequence is that the motions by the impulses and the release of strain are not equal when the reflected waves arrived.

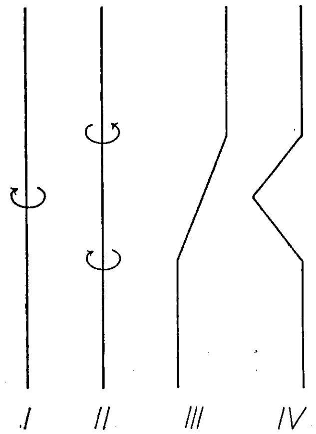


Fig. 12. Types of initial disturbances.
I, II: Impulsive couple(s).
III, IV: Initial strain.

By the superpositions of these mechanisms, waves of various types are to be generated. But the waves of complex types were not tried, as the adjustments of appratus were very difficult and such waves are of no practical value at present.

6. Conclusions.

In the present experiments, the distortional waves along metallic wires were studied. In order to diminish the velocity of propagation many wooden pieces were successfully attached to the wire, although the consequentious free oscillations of elementary parts were excited with some inconvenience. But the dissipation of the wave energy in this way was nearly exponential with the distance. Taking this into consideration following results were obtained.

1. Energy of wave generated from the release of statical strain is equal to the energy of the statical strain then released.

2. Forms of waves generated from the release of statical strains or

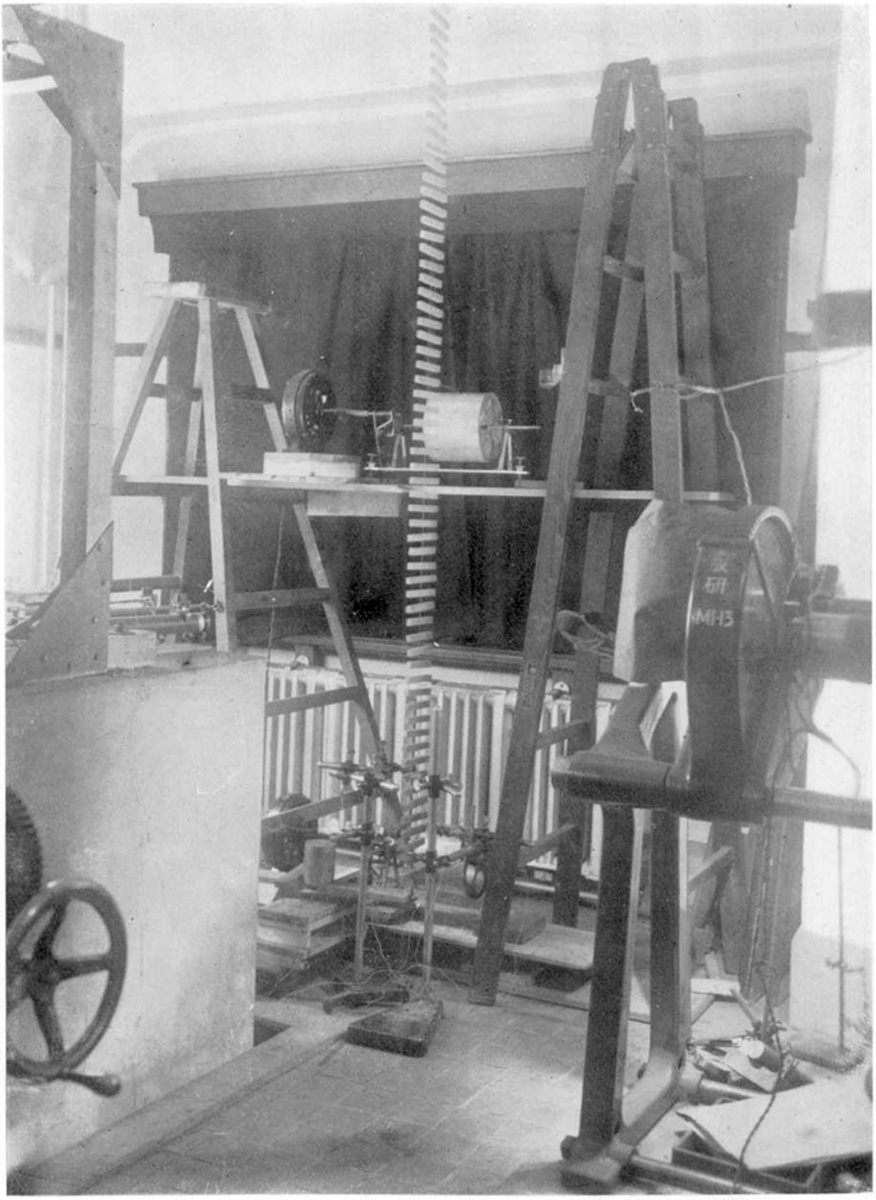


Fig. 1. Experimental arrangement.

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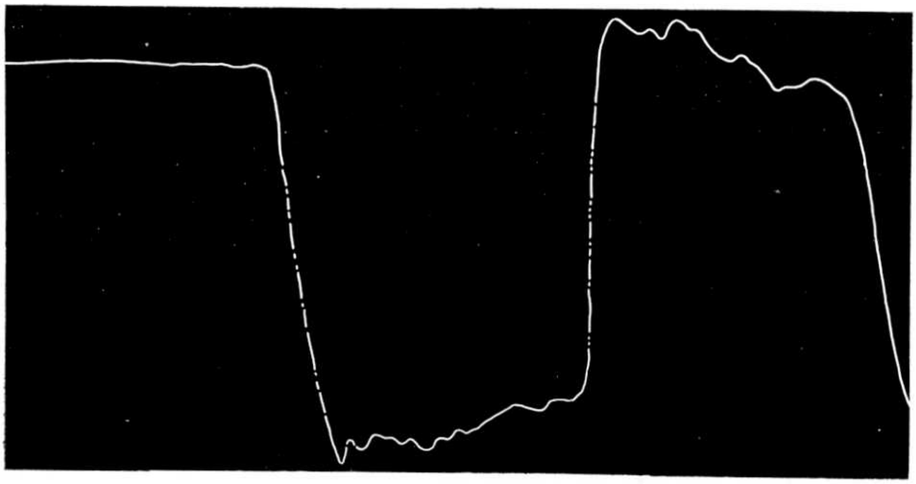
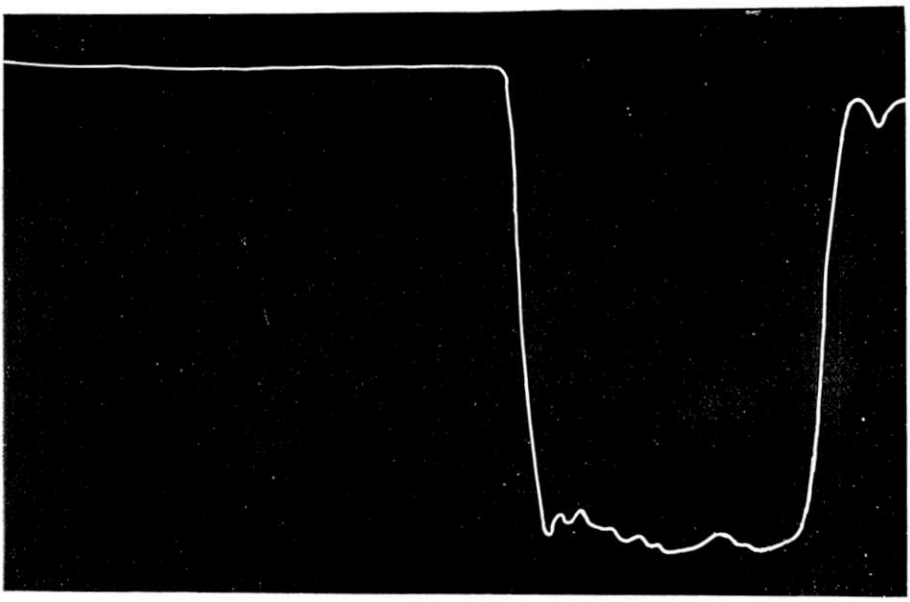
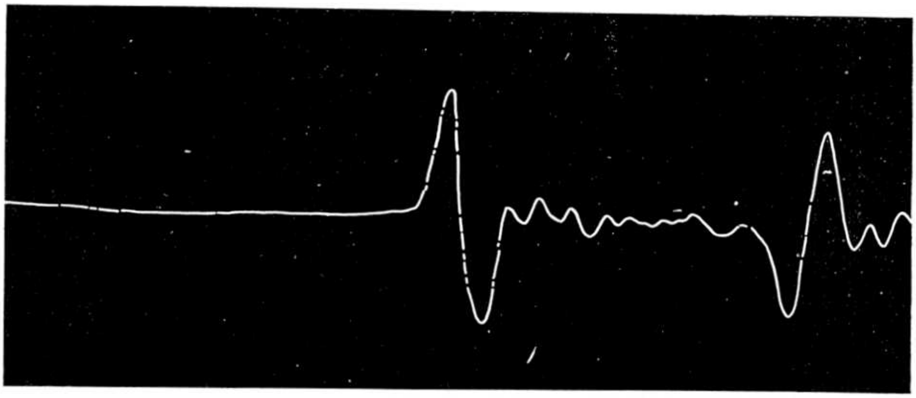


(a): Brass wire model.

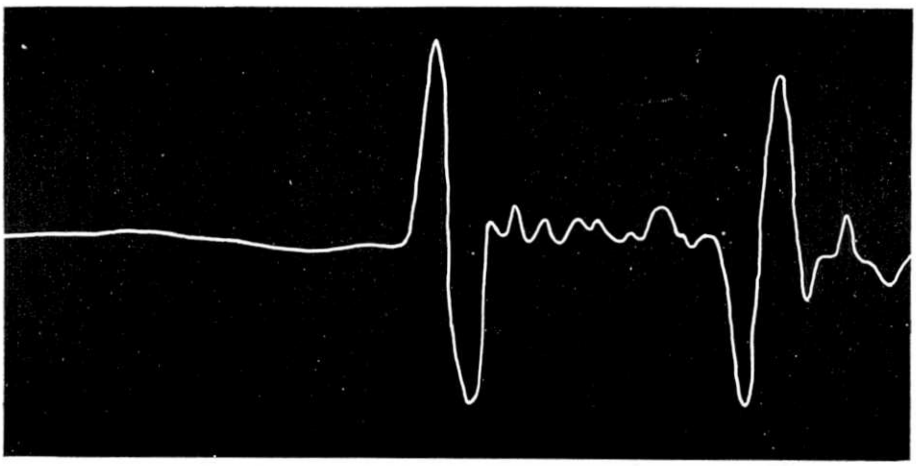


(b): Steel wire model.

Fig. 5.



(a): Brass wire model.



(b): Steel wire model.

Fig. 9.

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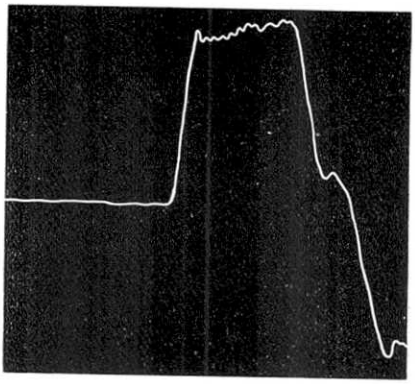
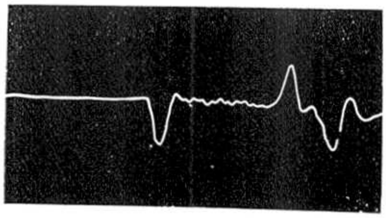
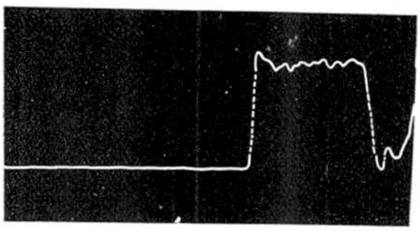


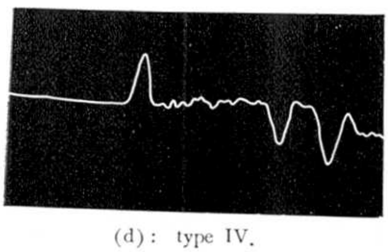
(a): type I.



(b): type II.



(c): type III.



(d): type IV.

Fig. 13. Waves due to various initial localized disturbances at the central part of the wire.
(a): (b), Initial impulsive couple(s).
(c): (d), Initial strain (See Fig. 9).

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by the actions of the impulsive forces at a part of one dimensional medium were found. Special attension was paid to the mechanism by which waves of shock-type were generated.

In conclusion, the writer wishes to express his best thanks to Prof. M. Ishimoto for his kind suggestions and guidance, and deep thanks are also due to Mr. H. Kawasumi for his valuable discussions.

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5. 彈性波の生成及び傳播に關する實驗

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彈性波の生成及び傳播に關する理論的研究は主として我國に於て最近著しい進歩を遂げたが，實驗的研究は未だ全く微々たるものである．此れは其の實驗の極めて困難なるに因るが，理論的に得られたる結果を實驗的に實證し，此れによつて理論の進展に資する事も又必要なるを以て，先づ簡單なる一次元的波動の生成及び傳播に關する實驗を行つて見た．

此の實驗に於ては針金に傳る捻れ波の傳播を實驗したが，多數の木片を等間隔に取り付ける事により傳播速度を小さくする事を得て實驗の困難の大部分は解消し，波動の傳播を肉眼にても見得る程度になつた．眞鍮の針金（直徑 0·059 cm）を用ひた場合には傳播速度は 132 cm/sec となり，鋼鐵線（直徑 0·064 cm）の場合には 221 cm/sec となつた．

種々の擾亂によつて起される波動を數箇所にて記錄測定し，波動の形を得，其れによりて運ばれる波動の勢力を計算した．例へば歪の潛勢力が波動を起し，其の際勢力の保存則があてはまる事を實證する事が出來た．

此の實驗は石本教授の指導鞭撻によつて出來たものである事を此處に銘記して厚く感謝の意を表する．
