7. *Determination of Elastic Constants of Soils by means of Vibration Methods.*

Part 2. Modulus of Rigidity and Poisson's Ratio.

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1. Introduction.

In the preceding paper[1], we reported our study of the elastic properties, such as longitudinal wave-velocities or Young's moduli as well as their solid viscosity coefficients of soils obtained by means of vibration method; these soils were taken from various depths in several sections of Tôkyô. From the results of our preceding experiments[2], we were able to ascertain that the elastic properties of these soils differ according to their kinds, their moisture contents, and their natural and recomposed states. To apply our results to geophysical problems, such as the propagation of seismic waves, especially in the most superficial layer of the earth's crust, we wished to carry out further experiments in this field, for the determination of other elastic properties of every kind of soil.

One of the objects of our present investigation was to find the elastic properties, such as the moduli of rigidity and Poisson's ratios[3], for the same kinds of soils as those adopted in the previous studies. Moreover we desired to ascertain whether or not the elastic constants of the soils vary according to their conditions.

The method of our present experiment is briefly described as follows; we continuously induced a variable frequency of torsional vibration at the foot of the soil specimen, and detected its fundamental resonance frequency. From that we then calculated its modulus of rigidity.

In order to obtain Poisson's ratio σ of the soil specimen, we mea-

1) M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **14** (1936), 632~657.

2) M. ISHIMOTO and K. IIDA, *ditto*. It is quoted in appropriate places.

3) These same elastic properties, especially for the rock specimens, were dynamically studied by John M. IDE. *Proc. Nat. Acad. Sci.*, **22** (1936), 482.

sured the transverse wave-velocity as well as the longitudinal wave-velocity for the same specimen, and then computed these constants from the following equation

$$\sigma=\frac{1}{2}\left\{\left(\frac{V_l}{V_t}\right)^2-2\right\},\ \text{or}\ \sigma=\frac{1}{2}\left\{\frac{E}{\mu}-2\right\},$$

$$V_l=\sqrt{\frac{E}{\rho}},\qquad V_t=\sqrt{\frac{\mu}{\rho}},$$

in which V_l is the longitudinal wave-velocity, V_t the transverse wave-velocity, E Young's modulus, μ the modulus of rigidity, and ρ the density of the soil specimen. We studied also the elastic constants of soils, considering the effects of moisture content. Moreover, in the present experiments we investigated two kinds of solid viscosity coefficients, namely, the normal and the tangential ones.

As we adopted the same kinds of soil as were employed in the experiments reported in the previous paper, we have therefore omitted dealing with their mechanical analysis here.

2. Experiments.

(a) Apparatus for the experiment.

To carry out our experiment, we adopted two apparatuses, one of

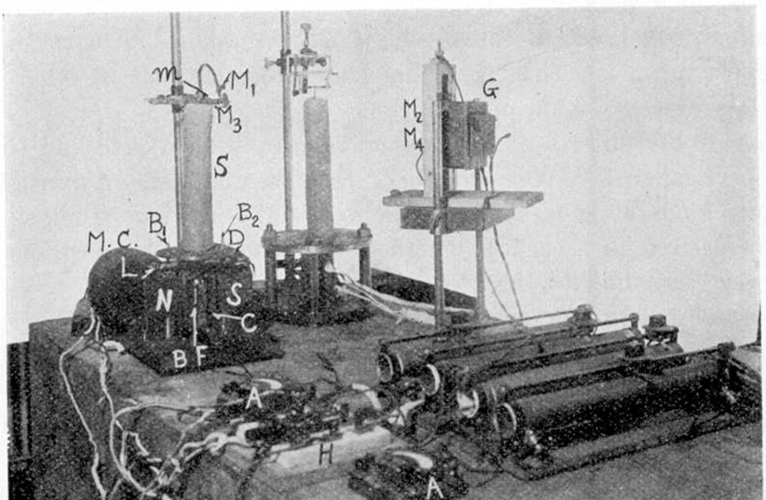


Fig. 1. Apparatuses for the experiment. (The symbols in this figure have the same meaning as in Fig. 2.)

which we had already used in our preceding study, whereas the other was newly constructed for the present purpose; the photograph of both is shown in Fig. 1. A schematic diagram of the arrangement of these apparatuses is shown in Fig. 2. It is necessary to describe the particular details of our new apparatus.

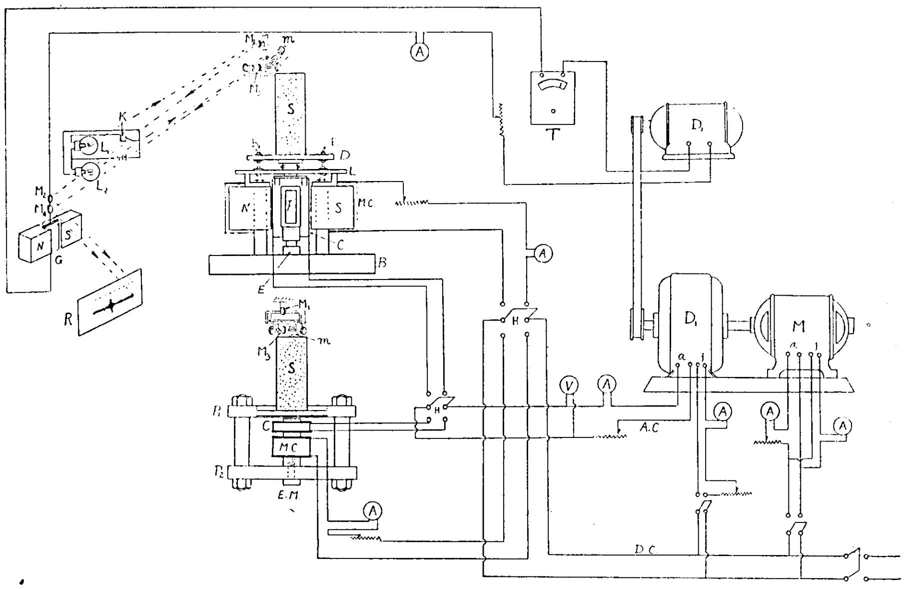


Fig. 2. Schematic diagram of arrangement of the apparatuses for producing the torsional and the longitudinal vibrations of soils.

a:	Armature terminal.	K:	Key.
A:	Ampere-meter.	L_1, L_2:	Light source.
$A.C.$:	Alternating current circuit.	m:	Magnet.
B:	Steel block.	M:	$D.C.$ motor.
B_1, B_2:	Brass bolt.	M_1, M_3:	Lens mirror.
C:	$A.C.$ coil.	M_2, M_4:	Plane mirror.
D:	Circular brass disc.	$M.C.$:	$D.C.$ coil for electromagnet.
D_1:	Alternating generator.	N, S:	Pole piece of electromagnet.
D_2:	Dynamo of the Weston tachometer.	P_1:	Circular iron disc for vibration.
$D.C.$:	Direct current circuit.	P_2:	Circular iron disc with electromagnet.
E:	End of the pivot.	R:	Bromide paper for record.
$E.M.$:	Electromagnet.	S:	Soil specimen.
F:	Solid cylindrical iron stick.	T:	Voltmeter of the Weston tachometer.
f:	Field terminal.		
G:	Galvanometer.		
H:	Switch.		

Vibrating disc. We employed a circular brass disc D for a vibrating disc, with a diameter of 14 cm and a thickness of 0·5 cm. On this vibrating disc, the soil column S stood upright. In order to avoid the slip, certain to occur during its torsional vibration, at the contact surface of this disc D and the soil specimen S, we notched its disc-surface so that it was like a rasp. There was a coil C attached firmly beneath the vibrating disc D, which was connected with the circuit of a generator D_1 so as to pass an alternating electric current through it. As this coil was situated in the magnetic field between the pole-pieces (N, S) of an electromagnet, it was set in torsional vibration when we applied the alternating current to it. By such a method the disc D was forced to vibrate in torsional manner; the frequency of it was the same as that of the alternating electric current. To be sure, the vibrating part of the apparatus was not the disc alone, but a vibrating system; there was, as a matter of fact, a proper period of vibration in it. This consisted of the moment of inertia of the disc, etc., and of the restitutive force introduced into it. To drive away this period of vibration from the period found in the present experiment, we utilized a pair of bolts B_1 and B_2 to fasten the disc D firmly to the fixed brass plate L. We adjusted the diameter of these bolts as well as their length, so as to obtain the suitable proper period of this system. Needless to say, when we used bolts which were too stiff, the torsional sensibility of the disc became very small. We therefore made a choice of bolts for this system, having its proper period of vibration just at the edge of the field in which we wished to carry out our experiment.

To succeed in obtaining the pure torsional vibration of this disc D, we controlled the vibration with the part of a pivot, its sharpened end E touching a conical support on the brass plate L, through which the shaft of this disc rotated freely during torsional vibrations.

The magnetic field, in which we inserted the coil C, was regulated by the amount of the direct current which passed through the exciting coil $M.C.$ of the electromagnet.

Magnification of vibration amplitude of the top of soil specimen. In the present experiment, as we intended to determine the amplitude of the torsional vibration of the soil specimen, we adopted its magnification device as being almost the same as that described in our preceding paper. However, we changed this device to fit our purpose. The vibration amplitude of the top of the soil specimen attained its maximum, when the vibration frequency of this vibrating disc reached a condition to provoke a stationary wave in the soil specimen. Even in this case, the vibration of its top was, of course, perceptible to the tip of one's finger; but we had to magnify it by some device in order to record continuously the variation of its vibration amplitude and to determine accurately the position of its maximum amplitude. We therefore adopted the same device, a pair of small pivot and small magnet m, to get a sufficient magnification of it. A part of the frame of this small pivot was inserted into one side of the top of the soil specimen S. As the weight of this small pivot and its frame amounted only to about 5 gr, it seemed to us that it did not disturb the vibration of the soil specimen. The pivot of 1·77 mm in diameter was always in contact with the small magnet m, which rotated easily upon its own axis. In this way, the pivot could be rotated with the smallest possible degree of friction. On this pivot there was a small lens-mirror M_3 with the focal length of about 50 cm, which reflected a light-beam radiated from a slit of the light-source L_2 so as to magnify the vibration amplitude of the top of soil on the bromide paper as much as about 2200 times. We could then register the continuous variation of the amplitude on the photographic sensitive bromide paper R, by means of the same device as that which has been already described in the preceding paper. We had, in addition, a fixed plane-mirror M_1 which served as the scale for the accurate measure of the resonance frequency.

Motor-generator, tachometer, and galvanometer. These apparatuses for the experiments are the same as we have already mentioned in our preceding paper. Therefore it will not necessary to repeat here a description of them.

(b) Testing of the apparatus.

Proper period of the vibrating disc. As the effect of the proper period of the vibrating disc on resonance frequency of the soil specimen is always considerable, we endeavoured to eliminate the proper period from the field of our experiment.

In our apparatus preliminary constructed, we noticed that at the frequency of about 30 vibrations per second the vibration amplitude of the disc became to the maximum. This was the natural effect of the proper period of the disc, together with the vibrating system consisting of the moment of inertia of the disc; etc., and the restitutive force mainly due to the bolts which connected the disc with the fixed brass plate L. In order to discount the effect of this proper period from the present experiment, we selected a pair of bolts one after another, until at last we decided upon those with a diameter of 0·6 cm and a length of 1·5 cm. By the employment of these two bolts we conclusively proved that the proper period of this vibrating system was eliminated from the field of our use. We found, at any rate, that this fundamental vibration frequency of our disc system was about 270 vibrations per second. In order to find whether or not this proper period might be varied by the moment of inertia of a dead weight of metal loaded on the vibrating disc, we repeated the same experiment with several different sizes of metal, the weights of which were up to the limit of 2 kg. From the results of these experiments, we also ascertained that the proper period is scarcely affected by the additional moment of inertia. We may, therefore, conclude from the above experiments that no effect was found which needs to be taken into consideration. We can thus pick out the pure characteristics of the soil specimens.

Calibration of the tachometer. In the present study we also employed the same calibration curve as that shown in our preceding paper. We have taken into consideration the correction of the readings of the Weston tachometer.

(c) Methods of the experiment.

The methods of the present experiment were also about the same as those employed in the previous experiments. We experimented on the soil specimens in the natural state and sometimes in the recomposed state; the form of which was a cylinder with the initial height of 20 cm$\sim$40 cm, and the diameter of about 5 cm. At that time we took the same kind of commercial rubber as that used in the previous experiment, for the purpose of comparing its elastic properties with those of the soils. The dimension of this specimen was the same as before. We measured the density and the moisture content of the soil specimen by the same process as before.

In order to determine Poisson's ratio of the soil specimen, we employed both apparatuses together, one being the longitudinal-vibration apparatus which had already been adopted in our previous experiment. First of all, setting the soil specimen on the vibrating disc of the new torsional-vibration apparatus, we made a photographic record while applying the forced transverse vibration to the soil up to the limit; and then setting this same specimen on the vibrating plate of the longitudinal-vibration apparatus, we again made a photographic record while applying the forced longitudinal vibration up to the same limit. To apply the direct and the alternating electric currents, one after the other, to the two different apparatuses, we employed two switches H which served for our purpose very well. We repeated the procedure on the identical material arranged in various heights that were diminished in equal amounts. We thus got the reducing curves in the two cases of the longitudinal and the torsional vibrations, for soils of the same condition. We succeeded finally in computing the two constants of elasticity and of Poisson's ratio of the tested soils.

3. Results of the Experiments.

(a) Photographic records.

Actual photographs thus obtained of the two kinds of soil specimens collected at Hongô and at Maru-no-uti in Tôkyô are shown in Figs. 3 and 4. The photographs are shown in pairs, according to the two kinds of vibrations, namely, the torsional and the longitudinal, in the different heights of the soil specimens. As will be seen from these figures, the characteristics of these resonance curves are almost identical; and the frequency, corresponding to the resonance position in the case of the torsional vibration, is generally smaller than that in the case of the longitudinal vibration. Merely from this appearance we have reached the conclusion that the transverse wave-velocity is always less than that of the longitudinal wave-velocity. In some of these photographs, we can also detect the resonance of higher orders; this is due to the size and condition of the soil specimens, such as their height, or their moisture content. We observe also the fact that the resonance curves tended to flatten when the height or the moisture content was diminished.

(b) Determination of two kinds of wave-velocities, Poisson's ratio, and two kinds of solid viscosity coefficients.

As was already set forth in our preceding paper, the equation of motion for the longitudinal wave in the soil specimen is expressed by

$$\rho\frac{\partial^2\zeta}{\partial t^2}=E\frac{\partial^2\zeta}{\partial z^2}+\gamma_l\frac{\partial^3\zeta}{\partial z^2\partial t}, \tag{1}$$

where ζ is the displacement in the direction of the height z, t the time, ρ the density, E the Young's modulus, and γ_l the coefficient of normal solid viscosity. We have also the similar equation of motion for the transverse wave in the soil specimen such as

$$\rho\frac{\partial^2\xi}{\partial t^2}=\mu\frac{\partial^2\xi}{\partial z^2}+\gamma_t\frac{\partial^3\xi}{\partial z^2\partial t}, \tag{2}$$

in which ξ is the displacement in the direction of the horizontal, μ the modulus of rigidity, and γ_t the coefficient of tangential solid viscosity.

The solutions of (2) in the case of the free and the forced vibrations of the soil specimen are the same as those obtained in our previous paper, provided μ and γ_t are in the place of E and γ_l respectively.

Namely, the solution in the case of free vibration of the soil specimen is given by

$$\left.\begin{aligned}&\xi=Ae^{-\frac{\gamma_t f^2}{2\rho}t}\cos\left\{\sqrt{\frac{\mu f^2}{\rho}-\left(\frac{\gamma_t f^2}{2\rho}\right)^2}\,t-\alpha\right\}\sin fz,\\ &f=\frac{2r-1}{2h},\quad r=1,\ 2,\ldots\ldots\end{aligned}\right\}\quad(3)$$

We have also the following solution in the case of their forced vibration

$$\left.\begin{aligned}&\xi=C(\xi_1+i\xi_2)e^{ipt},\\ &\xi_1=\cos f_1z\cosh f_2z+\frac{1}{\vartheta}(\sin f_1h\cos f_1h\sin f_1z\cosh f_2z\\ &\qquad-\cos f_1z\sinh f_2z\sinh f_2h\cosh f_2h),\\ &\xi_2=\sin f_1z\sinh f_2z-\frac{1}{\vartheta}(\sin f_1h\cos f_1h\cos f_1z\sinh f_2z\\ &\qquad+\sin f_1z\cosh f_2z\sinh f_2h\cosh f_2h),\end{aligned}\right\}\quad(4)$$

where, $\vartheta=\cos^2 f_1h\cosh^2 f_2h+\sin^2 f_1h\sinh^2 f_2h,$

$$f_1-if_2=\frac{fp}{(n^4+4p^2\varepsilon^2)^{\frac{1}{4}}}\left\{\sqrt{\frac{1}{2}\left(1+\frac{n^2}{\sqrt{n^4+4p^2\varepsilon^2}}\right)}-i\sqrt{\frac{1}{2}\left(1-\frac{n^2}{\sqrt{n^4+4p^2\varepsilon^2}}\right)}\right\},$$

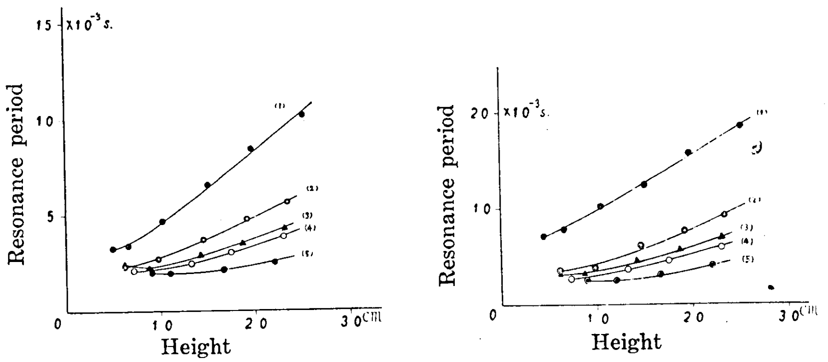
and $\varepsilon=\dfrac{1}{2}\dfrac{\gamma_t}{\rho}f^2,\quad n^2=\dfrac{\mu}{\rho}f^2.$

The resonance period T_t in the case of the torsional vibration is given by

$$T_t=\frac{4h}{(2r-1)\sqrt{\dfrac{\mu}{\rho}-\dfrac{(2r-1)^2\pi^2\gamma_t^2}{16\rho^2h^2}}},\quad r=1,\ 2,\ 3,\ldots\ldots\quad(5)$$

This formula showing the relation between the resonance period T_t and the height h of the soil specimen are almost the same as before.

The process of determining the transverse wave-velocity and the tangential solid viscosity coefficient is also the same as that set forth in our previous paper. The diagrams showing the relation between the fundamental resonance period T_t or T_l (resonance period in the case of longitudinal vibration) and the height h of the soil specimens are shown in Figs. 5~9, in which the pepiod, T_t or T_l is



(a) Longitudinal vibration. (b) Torsional vibration.

Fig. 5. Relation between the fundamental resonance period and the height of soil in the natural state taken from Maru-no-uti.
(1) Moisture content 48·7%. (2) 42·6%. (3) 39·4%. (4) 37·6%. (5) 31·8%.

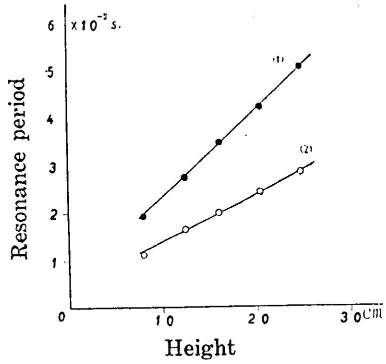
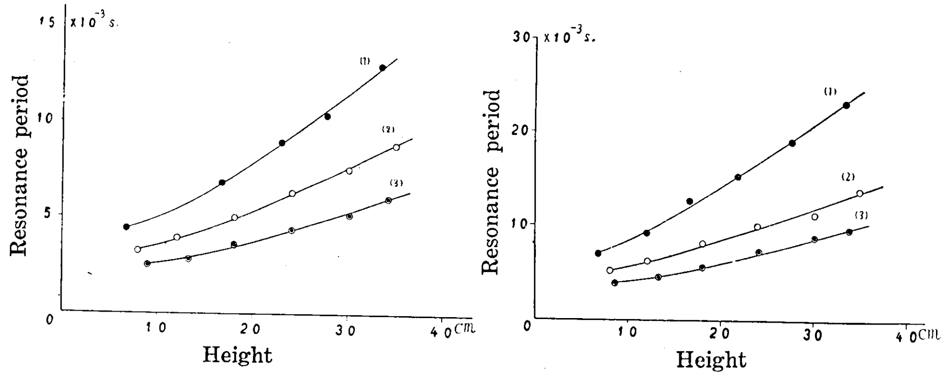


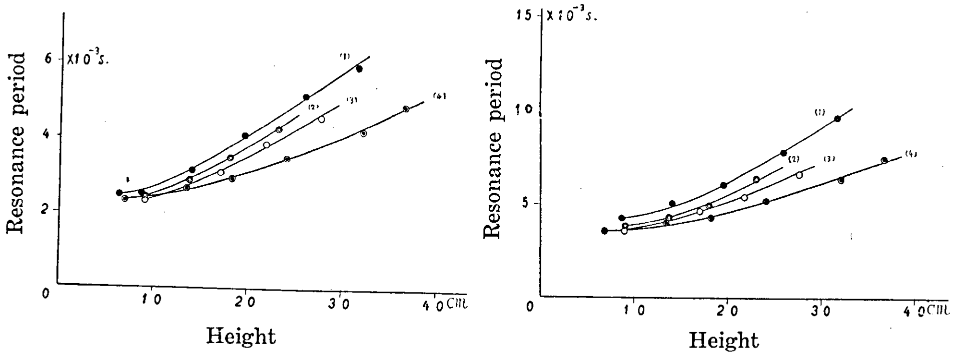
Fig. 6. Relation between the fundamental resonance period and the height of soil in the recomposed state taken from Maru-no-uti.
Moisture content 47·7%.
(1) Case of torsional vibration.
(2) 〃 longitudinal vibration.

taken as ordinate and the height h as abscissa. These figures are the reducing curves, by which the transverse and the longitudinal wave-velocities and the normal and the tangential solid viscosity coefficients are all determined. Also the reducing curves of rubber are shown in Fig. 10. In every experiment it is also believed that the mode of vibration of the soil specimens corresponds to that of the clamped- and free-end bar, the reason for which will be explained later. As the mode



(a) Longitudinal vibration. (b) Torsional vibration.

Eig. 7. Relation between the fundamental resonance period and the height of soil in the recomposed state of Komatugawa.
(1) Moisture content 27·0%. (2) 24·5%. (3) 21·2%.



(a) Longitudinal vibration. (b) Torsional vibration.

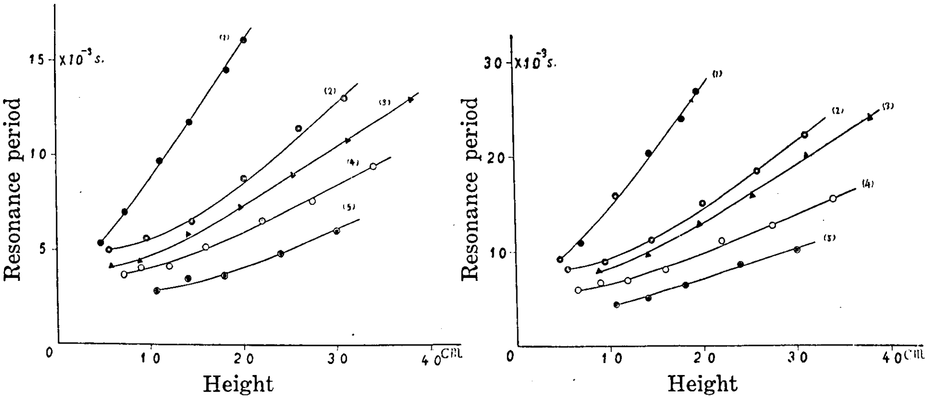
Fig. 8. Relation between the fundamental resonance period and the height of soil in the natural state taken from Hongô.
(1) Moisture content 48·2%. (2) 46·0%. (3) 43·0%. (4) 40·3%.

of vibration has been determined, we have succeeded in finding out from these reducing curves the values of the longitudinal and the transverse wave-velocities, namely V_l and V_t ; and the normal and the tangential solid viscosity coefficients, namely γ_l and γ_t, of the soil specimens. These values thus obtained are all tabulated in Table I.

The moduli of rigidity of the soil specimens were then computed from the equation $\mu=\rho V_t^2$, and their Young's moduli E were also calculated from the equation $E=\rho V_l^2$. As V_l and V_t, or E and μ are thus obtained, Poisson's ratio σ was then calculated from the equation $\sigma=$

$\frac{1}{2}\left\{\left(\frac{V_l}{V_t}\right)^2-2\right\}$ or $\sigma=\frac{1}{2}\left\{\frac{E}{\mu}-2\right\}$. These computed values of E, μ, and σ are all shown in Table II.

All of Young's moduli, the moduli of rigidity, and the solid viscosity coefficients of these soil specimens in the natural state show greater values



(a) Longitudinal vibration. (b) Torsional vibration.

Fig. 9. Relation between the fundamental resonance period and the height of a kind of clay in the recomposed state.
(1) Moisture content 32·6%. (2) 28·3%. (3) 27·8%. (4) 25·1%. (5) 22·0%.

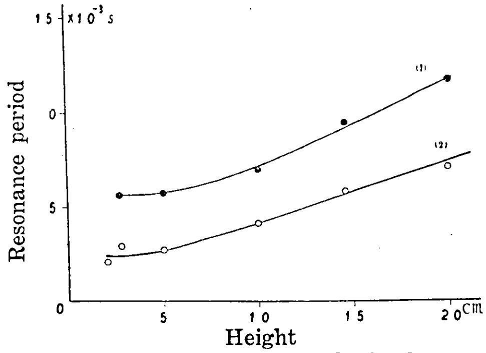


Fig. 10. The relation between the fundamental resonance period and the height of a kind of rubber, the result being obtained by torsional and longitudinal vibrations.
(1) Case of torsional vibration.
(2) 〃 longitudinal vibration.

than those in the recomposed state. These phenomena could be seen in the soils taken from Maru-no-uti, from Komatugawa, and in a kind of clay; these are shown in the reducing curves in Figs. 6~7 and 9 (a), (b) and in Tables I and II. It seems to us that Poisson's ratio of loam at Hongô is smaller than that of the silty-clay at Maru-no-uti within a certain moisture content limit.

Table I. Longitudinal and Transverse Wave-Velocities, Coefficients of Normal and Tangential Solid Viscosities, Density, and Moisture Content of Soils.

Kinds of soils	No.	Density ρ	Moisture content %	Longitudinal wave velocity m/sec	Transverse wave velocity m/sec	Coef. normal solid viscosity γ_l (c.g.s.)	Coef. tangential solid viscosity γ_t (c.g.s.)	$\frac{\gamma_l}{\gamma_t}$
Silty-clay (Maru-no-uti) (No. 1~5: natural state. No.6: recomposed state.)	1	1·50	48·7	98	58	$4{\cdot}58\times10^4$	$4{\cdot}02\times10^4$	1·14
	2	1·56	42·6	169	103	$1{\cdot}29\times10^5$	9·20 〃	1·40
	3	1·56	39·4	222	138	2·65 〃	$1{\cdot}62\times10^5$	1·64
	4	1·56	37·6	253	160	2·94 〃	1·67 〃	1·76
	5	1·59	31·8	364	237	4·90 〃	2·80 〃	1·75
	6	1·49	47·7	34·4	20·0	$5{\cdot}31\times10^3$	$6{\cdot}12\times10^3$	0·87
Silt (Komatugawa) (Recomposed state)	1	1·94	27·0	94	58	$1{\cdot}66\times10^5$	$1{\cdot}11\times10^5$	1·49
	2	1·89	24·5	163	104	2·34 〃	1·48 〃	1·58
	3	1·96	21·2	237	149	3·73 〃	2·21 〃	1·69
A kind of clay (Recomposed state)	1	1·89	32·6	51·0	29·6	$3{\cdot}33\times10^4$	$2{\cdot}90\times10^4$	1·15
	2	1·89	28·3	96·0	56·5	$1{\cdot}28\times10^5$	$9{\cdot}35\times10^4$	1·37
	3	1·89	27·8	116	68	1·38 〃	$1{\cdot}09\times10^5$	1·27
	4	1·90	25·1	150	91	2·18 〃	1·31 〃	1·66
	5	1·91	22·0	211	132	2·55 〃	1·60 〃	1·60
Loam (Hongô) (Natural state)	1	1·24	48·2	212	132	$2{\cdot}01\times10^5$	$1{\cdot}45\times10^5$	1·39
	2	1·19	46·0	232	148	2·13 〃	1·55 〃	1·38
	3	1·16	43·0	255	167	2·20 〃	1·56 〃	1·41
	4	1·09	40·3	303	200	2·18 〃	1·47 〃	1·48
Rubber		1·77		114·5	69·0	$1{\cdot}41\times10^5$	$1{\cdot}06\times10^5$	2·01

Table II. Young's Modulus, Modulus of Rigidity, Poisson's Ratio, and Moisture Content of Soils.

Kinds of soils	No.	Moisture Content %	Young's modulus E (c.g.s.)	Modulus of rigidity μ (c.g.s.)	Poisson's ratio σ
Silty-clay (Maru-no-uti) (No. 1~5: natural state. No. 6: recomposed state.)	1	48·7	$1{\cdot}44\times10^8$	$5{\cdot}05\times10^7$	0·43
	2	42·6	4·46 〃	$1{\cdot}65\times10^8$	0·35
	3	39·4	7·69 〃	2·97 〃	0·30
	4	37·6	$1{\cdot}00\times10^9$	3·99 〃	0·25
	5	31·8	2·11 〃	8·93 〃	0·19
	6	47·7	$1{\cdot}76\times10^7$	$5{\cdot}96\times10^6$	0·48

(to be continued.)

Table II. (*continued*).

Kinds of soils	No.	Moisture Content %	Young's modulus E (c.g.s.)	Modulus of rigidity μ (c.g.s.)	Poisson's ratio σ
Silt (Komatugawa) (Recomposed state)	1	27·0	$1·71\times10^8$	$6·52\times10^7$	0·32
	2	24·5	5·02 〃	$2·04\times10^8$	0·28
	3	21·2	$1·10\times10^9$	4·35 〃	0·26
A kind of clay (Recomposed state)	1	32·6	$5·02\times10^7$	$1·66\times10^7$	0·48
	2	28·3	$1·74\times10^8$	6·03 〃	0·47
	3	27·8	2·54 〃	7·45 〃	0·41
	4	25·1	4·27 〃	$1·57\times10^8$	0·35
	5	22·0	8·50 〃	3·28 〃	0·29
Loam (Hongô) (Natural state)	1	48·2	$5·48\times10^8$	$2·16\times10^8$	0·30
	2	46·0	6.41 〃	2·60 〃	0·22
	3	43·0	7·56 〃	3·23 〃	0·17
	4	40·3	$1·01\times10^9$	4·36 〃	0·15
Rubber			$2·35\times10^8$	$8·43\times10^7$	0·38

(c) Effect of moisture content on the constants of elasticity and solid viscosity.

To observe the effect of moisture content on the physical properties

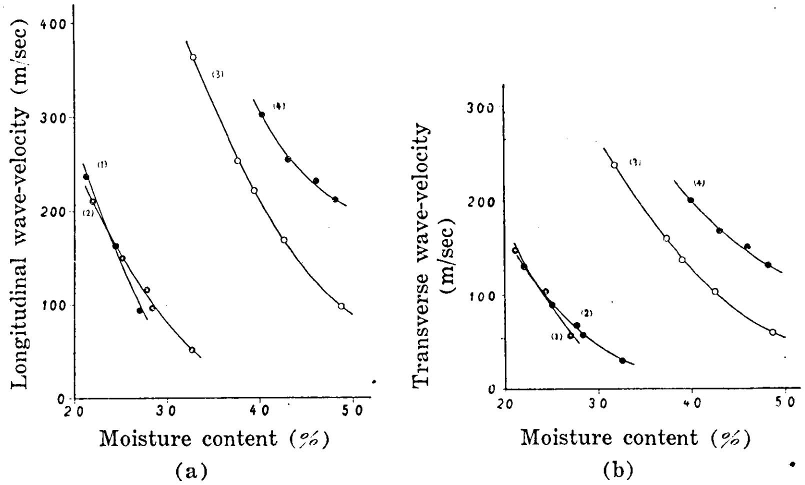


Fig. 11. (a) Relation between the longitudinal wave-velocity and the moisture content.
(b) Relation between the transverse wave-velocity and the moisture content.
(1) Silt of Komatugawa. (2) A kind of clay. (3) Silty-clay of Maru-no-uti. (4) Loam of Hongô.

of soils, we plotted the computed values (Tables I and II) from the results of the experiments shown in several diagrams, such as in Figs.

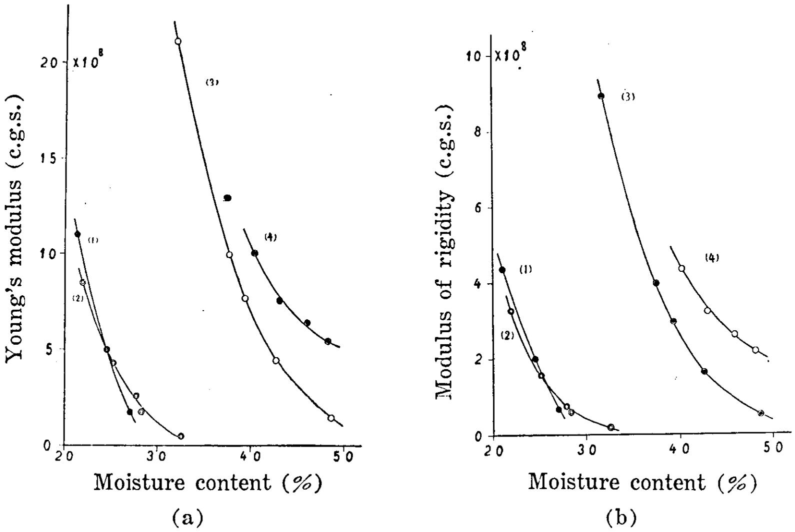


Fig. 12. (a) Relation between Young's modulus and the moisture content.

(b) Relation between the modulus of rigidity and the moisture content.

(1) Silt of Komatugawa. (2) A kind of clay. (3) Silty-clay of Maru-no-uti. (4) Loam of Hongô.

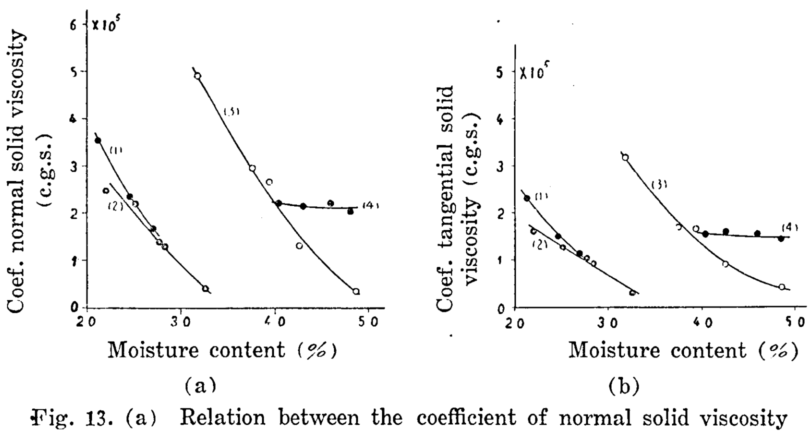


Fig. 13. (a) Relation between the coefficient of normal solid viscosity and the moisture content.

(b) Relation between the coefficient of tangential solid viscosity and the moisture content.

(1) Silt of Komatugawa. (2) A kind of clay. (3) Silty-clay of Maru-no-uti. (4) Loam of Hongô.

11∼14. We took always the moisture content as abscissa in these diagrams; and the transverse wave-velocity, the longitudinal wave-velocity, Young's modulus, Poisson's ratio, and the tangential or the normal solid viscosity coefficient are all considered as ordinate in each case in these diagrams. As will be seen from these figures, all the values, such as V_l, V_t, E, and μ decrease somewhat rapidly with the increase of the moisture content, especially is the rate of this decrease of V_l and E greater than that of V_t and μ. Evidently the manner of the decrease differs according to each soil specimen. The degree of decrease of all these things in the soil taken from Maru-no-uti seems to be greater than in that of Hongô.

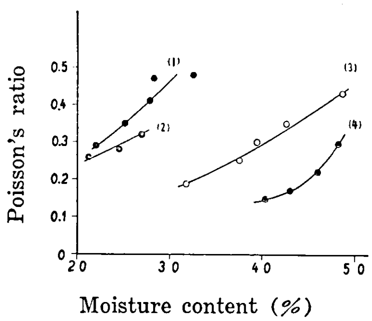


Fig 14. Relation between Poisson's ratio and the moisture content.
(1) A kind of clay. (2) Silt of Komatugawa. (3) Silty-clay of Maru-no-uti. (4) Loam of Hongô.

The solid viscosity coefficients of the finer grained soils, such as that of Maru-no-uti, Komatugawa, and a kind of clay, seem to decrease also somewhat rapidly with the increase of the moisture content; these coefficients are of the order of $10^4 \sim 10^5$ (c.g.s.) at the moisture content ranges of about 50%~20%; while in the case of coarse grained soils such as that of Hongô, the coefficients, of the order of which is 10^5 (c.g.s.), is somewhat constant even with the increase of the moisture content to a certain limit.

We have found that the normal solid viscosity coefficient γ_l of every soil specimen is greater than its tangential solid viscosity coefficient γ_t. The ratio of γ_l to γ_t seems to approach 1·0 when the moisture content of the soil increases, and *vice versa*.

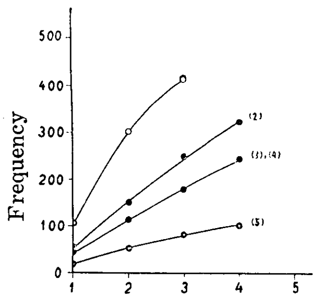
(d) Resonance frequencies of the higher orders of vibration. (Partials)

As already described in the preceding paper, the determination of the mode of vibration of the soil specimen is one of the most important matters of investigation. We, therefore, studied the resonance frequencies of the higher orders in every case. Five examples of the relation between the resonance frequencies of the torsional vibrations and the order of their successive maximum amplitude are shown in Table III and Fig. 15, in which we took the frequency as ordinate and the order as abscissa. We also found that these points are not in a straight line, owing to the effect of the solid viscosity of the soil speci-

Table III. Resonance Frequencies of the Fundamental and the Higher Orders.

Soil specimen		Height h	Transverse and longitudinal resonance frequencies.			
Kind	State		Fundamental	2nd order	3rd order	4th order
		cm	per sec	per sec	per sec	per sec
Silty-clay (Maru-no-uti)	Natural	25·1	54 93	149 280	247 468	330 ——
	Recomposed	24·7	19 35	53 98	84 162	110 214
Silt (Komatugawa)	Recomposed	33·2	43 73	112 198	187 327	263 ——
A kind of clay	Recomposed	38·0	42 77	111 213	187 352	288 ——
Loam (Hongô)	Natural	31·5	104 165	296 463	486 ——	

mens. As will be seen in Fig. 15 or Table III, the successive resonance frequencies of the higher orders are about an odd number of times that of the fundamental resonance frequency. We ascertained, therefore, that the observed vibration of the soil specimen is revealed by the transverse vibration in a clamped- and free- end bar, a fact which has already been pointed out in the preceding paper.



Order of the maximum amplitude.

Fig. 15. Relation between the resonance frequency and the order of resonance position. (1) Loam of Hongô in the natural state, $h=31{\cdot}5$ cm. (2) Silty-clay of Maru-no-uti in the natural state, $h=25{\cdot}1$ cm. (3) Clay in the recomposed state, $h=38$ cm. (4) Silt of Komatugawa in the recomposed state, $h=33{\cdot}2$ cm. (5) Silty-clay of Maru-no-uti in the recomposed state, $h=24{\cdot}7$ cm.

4. Remarks.

In the process of determining the velocity of elastic waves in the soil specimen, the investigation of the mode of its vibration is certainly one of the most important matters. If the mode of vibration of the soil specimen corresponds to that of the bar with both ends free, the loop of which is at both ends and the node at its center, then the elastic wave-velocity computed therefrom is, of course, reduced to half that computed from the conditions corresponding to the mode of vibration of a clamped-free bar. However, in most of our present examples, the mode of vibration of the soil specimen corresponds, unquestionably, to that of a clamped-free bar, because the resonance frequencies of the higher orders are about an odd number of times that of the fundamental frequency. In the process of determining the velocity of elastic waves in the soil specimen, we trace, therefore, a straight line, which is to be an asymptote to the reducing curve passing through the point of origin. This straight line represents the relation between the fundamental resonance period T_t and the height h, such as $T_t = \frac{4h}{\sqrt{\frac{\mu}{\rho}}}$, from which we can determine the transverse wave-velocity V_t.

When the soil specimen becomes hard by reason of the diminishing of its moisture content, it is, however, difficult to observe resonance frequencies of the higher orders within the limit of our field of experiment, owing to the increase of fundamental resonance frequency. In such a case the mode of vibration of the soil specimen is not to be determined, but, at any rate, we adopted the mode of vibration corresponding to the vibration in a clamped-free bar. Here, we remark that in the case of hard material, the boundary conditions must be changed. These problems are to be dealt with in future experiments.

In our laboratory, we endeavoured not to allow a decrease of moisture in the soil, so it is probable that moisture values were always constant in the interval of each of our experiments; since each allowed for no more than half an hour of exposure. We noticed during our other investigations that the finer grained soils, such as that of Maru-no-uti, have a tendency to great shrinkage in height with the decrease of moisture content and the density was almost constant; while in the case of coarse grained soils, such as that of Hongô, this tendency is not so conspicuous, but the density became small; this difference may be chiefly due to the state of the internal structure of the soils.

5. Summary and Conclusion.

1) We studied the elastic properties of the same kinds of soil taken from several sections in Tôkyô as those adopted in our preceding studies. The soils were taken out in the natural state from various depths underground in Hongô, Maru-no-uti, and Komatugawa, by means of the boring process. All of specimens, which were divided into 4 classes, namely, clay, silt, silty-clay, and loam, were made into the form of a cylinder, and tested in both the natural and the recomposed states.

2) We obtained the transverse wave-velocity in these soil specimens as well as the longitudinal wave-velocity, by means of the vibration

methods, and then computed their moduli of rigidity and Young's moduli from the following equations, respectively $\mu=\rho V_t^2$, and $E=\rho V_l^2$.

We calculated Poisson's ratio of these soils from the following equation

$$\sigma=\frac{1}{2}\left\{\left(\frac{V_l}{V_t}\right)^2-2\right\}, \text{ or } \sigma=\frac{1}{2}\left\{\frac{E}{\mu}-2\right\}.$$

The values of these constants are shown in Tables I and II.

3) We determined the normal and the tangential solid viscosities, namely γ_l and γ_t, from the curved portions near the origin of each reducing curve, by means of the following equations

$$T_t=\frac{4h}{\sqrt{\frac{\mu}{\rho}-\frac{\pi^2\gamma_t^2}{16\rho^2h^2}}}, \text{ and } T_l=\frac{4h}{\sqrt{\frac{E}{\rho}-\frac{\pi^2\gamma_l^2}{16\rho^2h^2}}},$$

in which the symbols have the same meaning as before. The normal solid viscosity coefficients of these soils are greater than their tangential solid viscosity coefficients, and the greater the decrease of moisture content in the soil specimen, the less difference there is between normal and tangential viscosities. The ratio of γ_l to γ_t seems to be about 2·0 in the case of diminishing the moisture content of the soil, while it approches about 1·0 when the moisture content increases.

4) The transverse wave-velocity, the modulus of rigidity, and the tangential solid viscosity coefficient, as well as the londitudinal wave-velocity, Young's modulus, and the normal solid viscosity coefficients of the soils in the natural state are greater than those obtained in the recomposed state.

5) The elastic constants, such as Young's modulus and the modulus of rigidity, decrease somewhat rapidly with the increase of moisture content. These are of the order of $10^7\sim10^8$ (c.g.s.) at the moisture content ranges of about 50%∼30%; the solid viscosity coefficients of these soils are of the order of $10^4\sim10^5$ (c.g.s.); and also the transverse wave-velocity varies from about 60 m per sec to about 250 m per sec at the above moisture content ranges.

Poisson's ratio of these soils, however, increases with the increase of the moisture content; the variation of which is 0·43∼0·19 at the moisture content ranges of about 49%∼32% in the case of silty-clay at Maru-no-uti; while 0·30∼0·15 at the moisture content ranges of about 48%∼40% in the case of soils at Hongô. It seems to us that Poisson's ratio of loam at Hongô is smaller than that of silty-clay at Maru-no-uti within a certain moisture content limit. This difference may be chiefly due to the state of the internal structure of the soils.

6) The resonance frequencies of the second maximum amplitude attain about three times the frequency of the fundamental resonance, a fact of which was already pointed out in our preceding paper. The observed vibration of the soil specimen can be, therefore, revealed by the transverse vibration in a clamped- and free-end bar.

In conclusion, our sincerest thanks are due to Dr. T. Watanabe, member of Geotechinical Committee, Government Railways of Japan, who kindly gave us the soils of Mar-no-uti and a kind of clay adopted in our experiments. We also wish to express our thanks to Dr. N. Miyabe, member of our institute, who gave us a permission to study the soil specimen of Komatugawa which was recently collected himself.

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7. 振動方法による土の彈性測定

第2報 剛性率及びポアソン比の測定

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表面土中の縱波の傳播速度及び其のヤング率に關する報告は既に前號に示したが，其の實驗結果よりすれば，土の彈性は土の種々なる狀態，特に其の含水率によつて著しく異る事が判明した．

本論文は表面土の彈性に關する續研究にして，特に土中の橫波の傳播速度或は其の剛性率及びポアソン比等を取扱ひ，且つ此等の彈性係數が土の種々なる狀態によつて如何に變化するかをも調べたものである．試驗土は前回と全く同種類のものを採用し，其の試驗方法も亦前回と殆んど同樣である．實驗裝置は第1圖に示す如く電磁的方法によつて捩振動をなし，且つ每秒0〜700の範圍に連續的に振動數を變化せしめ得る振動盤を採用した．先づ土柱を其の上に乘せ，其の共鳴する振動數を求めて結局土中に於ける橫波の速度を測定した．次いで $V_t=\sqrt{\frac{\mu}{\rho}}$ の式より其の剛性率 μ を計算し，猶ほ同時に前回採用した實驗裝置を併用する事によつて同一物質の縱波の速度 V_l 及びヤング率 E を知り，斯くて次式 $\sigma=\frac{1}{2}\left\{\left(\frac{V_l}{V_t}\right)^2-2\right\}$ により其のポアソン比 σ を算出した．

實驗結果の主なるものを擧ぐれば次の如くである．

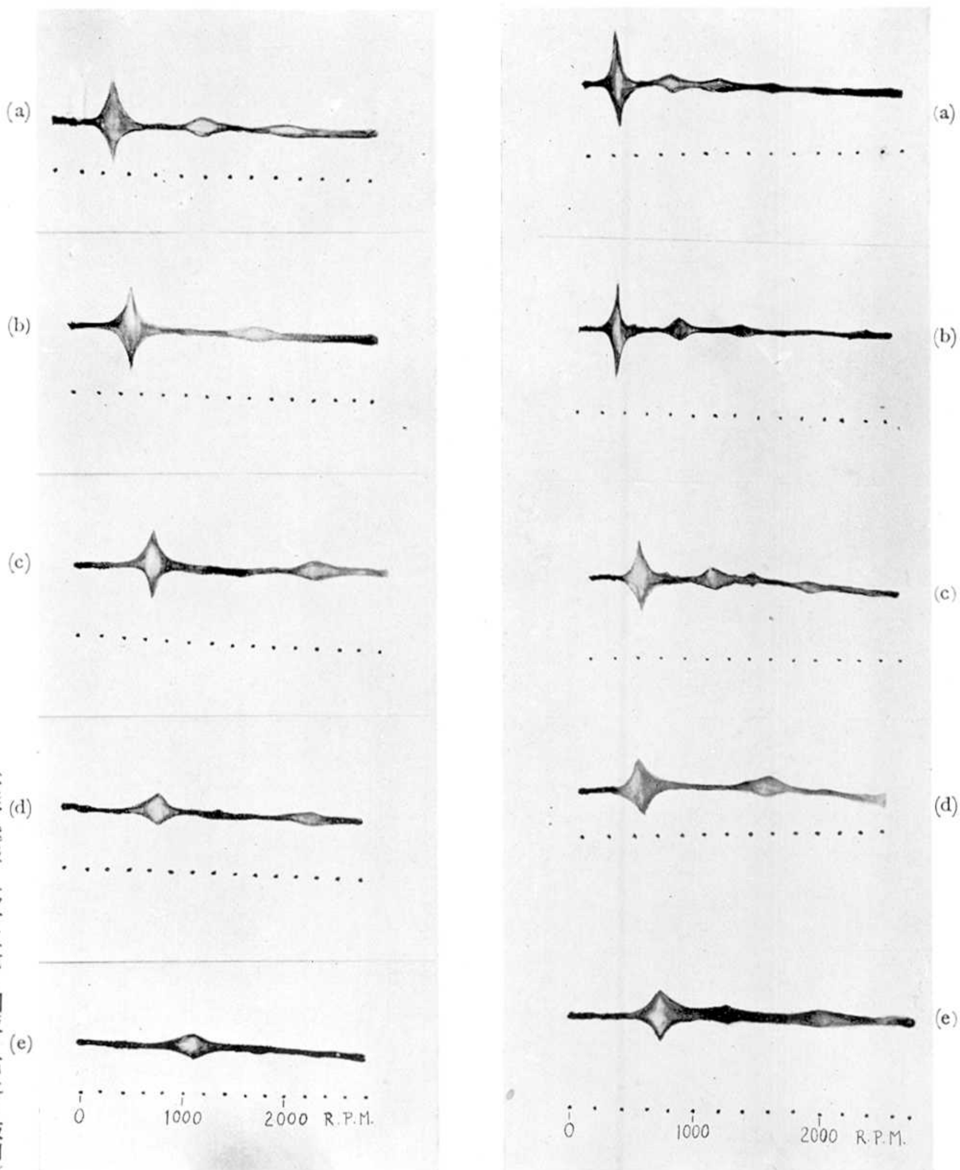
自然狀態に於ける土の縱波，橫波の傳播速度或はヤング率，剛性率及び固體粘性係數等は其のねり直し狀態のものより大なる値を示す事が判明した．又此等の諸量は含水量の增加に從つて減少する．含水率の變化範圍が約50%〜30%なる場合に，橫波の傳播速度は約60 m/sec〜250 m/sec程度，剛性率は 10^7〜10^8 (c.g.s.) 程度及び固體粘性係數は 10^4〜10^5 (c.g.s.) 程度の間に變化する．

橫振動より求めた固體粘性係數は縱振動より求めたものに比して一般に小さく，其の比は含水量が小であると2に近いが，含水量の增加と共に1に接近する．

ポアソン比は含水量の增加と共に大きくなる．例へば丸の內の土（沈泥質粘土）に於ては含水率の變化の範圍が約49%〜32%に對して，ポアソン比は約0·43〜0·19の中に變化する．本鄉大學構內の土（赤土）に於ては含水率の變化の範圍が約48%〜40%に對して，ポアソン比の變化は約0·30〜0·15である．赤土が沈泥質粘土に對してポアソン比の小であるのは其等を形成する土の粒子の大小によるものと思はれる．

振動の mode は前論文に示せる如く，一端固定，他端自由な棒の振動樣式と見做してゐるが，高次の固有振動の周期からも其の差支へ無き事が證明される．

土の含水量を減ぜしめた場合，丸の內の沈泥質粘土は比重の變化は左程なく高さを減ずる．本鄉の土は高さは左程變化しないが，比重は非常に小さなり多孔質となるやうである．此の事は物質の內部構造組織卽ち海綿構造であるや否やの問題と思考される．



(i) Longitudinal vibration. (ii) Torsional vibration.

Fig. 3. Resonance curves of soils taken from Maru-no-uti. Moisture content 48·7% $\rho=1{\cdot}50$, h=height, N=frequency.

(i)	(a)	$h=25{\cdot}0$ cm, $N=98$ vib. per sec.		(ii)	(a)	$h=25{\cdot}1$ cm, $N=54$ vib. per sec.
	(b)	$h=19{\cdot}7$, $N=118$			(b)	$h=19{\cdot}7$, $N=64$
	(c)	$h=15{\cdot}0$, $N=153$			(c)	$h=15{\cdot}0$, $N=81$
	(d)	$h=10{\cdot}4$, $N=213$			(d)	$h=10{\cdot}4$, $N=98$
	(e)	$h=6{\cdot}6$, $N=298$			(e)	$h=6{\cdot}6$, $N=128$

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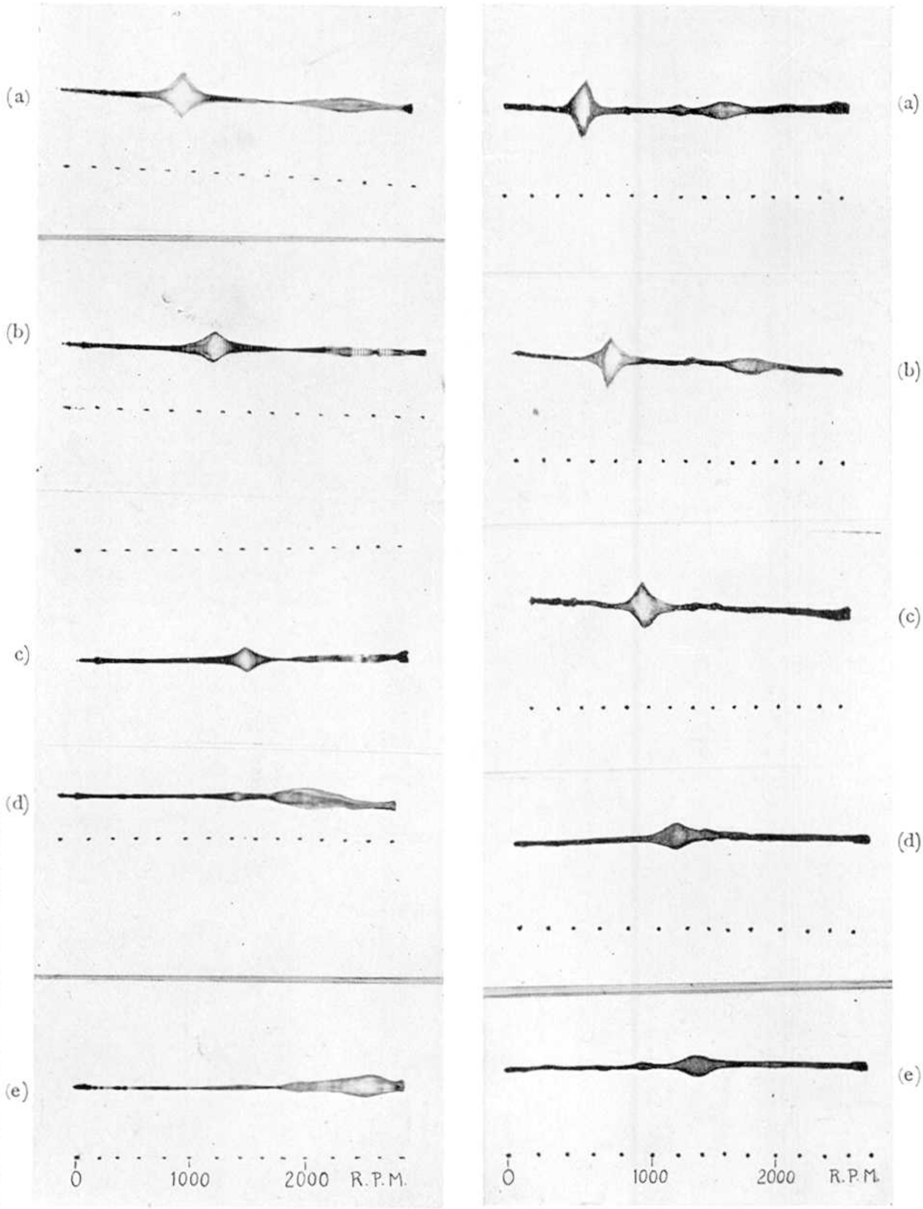

(i) Longitudinal vibration. (ii) Torsional vibration

Fig. 4. Resonance curves of soils taken from Hongô, Imperial University. Moisture content 48·2%, $\rho=1{\cdot}24$, h=height, N=frequency.

(i)
(a) $h=31{\cdot}4$ cm, $N=168$ vib. per sec.
(b) $h=25{\cdot}7$, $N=193$ ˮ
(c) $h=19{\cdot}5$, $N=243$ ˮ
(d) $h=14{\cdot}0$, $N=310$ ˮ
(e) $h=\ 8{\cdot}7$, $N=433$ ˮ

(ii)
(a) $h=31{\cdot}5$ cm, $N=104$ vib. per sec.
(b) $h=25{\cdot}8$, $N=128$ ˮ
(c) $h=19{\cdot}5$, $N=167$ ˮ
(d) $h=14{\cdot}0$, $N=198$ ˮ
(e) $h=\ 8{\cdot}7$, $N=236$ ˮ