9. *Notes on the Origins of Earthquakes.* (*Second paper.*)

By Win INOUYE,

Earthquake Research Institute.

(Read Oct. 20, 1936.—Received Dec. 21, 1936.)

1. *Introduction*

In his previous paper,[1] the author discussed the elastic waves radiated from a spherical cavity in a homogeneous isotropic elastic medium when the normal pressure on the surface of the cavity changed periodically; and two cases in which the azimuthal distribution of the normal pressure on the surface of the spherical cavity could be expressed either by $P_0(\cos\theta)$ or $P_2(\cos\theta)$ were studied theoretically.

In such a case, the normal component of displacement of the dilatational waves that originated from the single source ($P_0(\cos\theta)$) far exceeds that originated from the quadruple source ($P_2(\cos\theta)$), so long as the wave length of the dilatational waves is greater than the dimension of the spherical cavity, assuming that the range of pressure variation in the two cases are the same. Should the wave length be comparable with the dimension of the spherical cavity, the two normal components of the displacements in question become comparable to each other in magnitude.

Now, as the single source is the most simplest one and liable to occur accompanied with the rather complex quadruple source. Professor M. Ishimoto[2] has already shown that the geographical distribution of the initial motions of an earthquake could well be explained by assuming that the quadruple source co-acted with the single source at the hypocenter.

If so, then the wave lengths of the dilatational waves emanating from the seismic origin must be comparable with the dimension of the diameter of the origin in order that it shall accord with the observed distribution of the initial motions.

In this paper, the writer investigates the case in which the normal stress on the surface of the spherical cavity changes with different rapidities. In this case, the elastic waves of shock type are radiated

1) W. INOUYE, *Bull. Earthq. Res. Inst.*, **14** (1936), 582.
2) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **10** (1932), 449~471.

from the origin and the durations of the shocks of the dilatational and distortioal waves differ, the former usually being shorter than the latter.

2. The case in which there is no azimuthal difference in the normal pressures on the surface of the cavity, that is the one in which the azimuthal distribution of the normal pressure takes the form $P_0(\cos\theta)$, has been already studied by Professor K. Sezawa,[3] who found the expression for the displacement (normal) of the medium as follows:

The change in the pressure at $r=a$,

$$\left.\begin{array}{ll} P=o, & (r=a,\ t<o) \\ P=Ne^{-ct}(1-e^{-qt}), & (r=a,\ t>o) \end{array}\right\}$$

in which $c\to 0$, and q indicates the rapidity of the stress change.

The displacement at $r\to\infty$ can be determined from the following equation, written in convenient form by the writer,

$$u=\frac{a^3N}{4\mu r^2}+\frac{a^2NR\frac{\mathfrak{v}}{v}}{\mu r(4-4R\frac{\mathfrak{v}}{v}+R^2)}\left[e^{-R\tau}-e^{-2\frac{\mathfrak{v}}{v}\tau}\left\{\cos 2\sqrt{1-\left(\frac{\mathfrak{v}}{v}\right)^2}\tau\right.\right.$$

$$\left.\left.+\frac{2-\frac{v}{\mathfrak{v}}R}{2\sqrt{\left(\frac{v}{\mathfrak{v}}\right)^2-1}}\sin 2\sqrt{1-\left(\frac{\mathfrak{v}}{v}\right)^2}\tau\right\}\right],$$

where
$$R=\frac{aq}{\mathfrak{v}},\quad \tau=\frac{\mathfrak{v}}{a}\left(t-\frac{r-a}{v}\right),$$

v and $\mathfrak{v}$ are the velocities of the dilatational and the distortional waves respectively.

We shall consider now only that case in which the Poisson's ratio $\sigma=\frac{1}{4}$, that is $\lambda=\mu$, and in which the displacement at $r\to\infty$ can be expressed by

$$u=\frac{a^3N}{4\mu r^2}+\frac{a^2NR}{1{\cdot}732\mu r(4-2{\cdot}309R+R^2)}$$

$$\times\left[e^{-R\tau}-e^{-1{\cdot}155\tau}\left\{\cos 1{\cdot}633\tau+(0{\cdot}7072-0{\cdot}6124R)\sin 1{\cdot}633\tau\right\}\right].$$

3) K. SEZAWA, and K. KANAI, *Bull. Earthq. Res. Inst.*, **14** (1936), 10~17.

We will consider two cases in which $\frac{aq}{\mathfrak{v}}=\infty$ and $\frac{aq}{\mathfrak{v}}=1$. The rapidities of changes in the pressure on the surface of the spherical cavity are shown graphically in Fig. 1, from which it will be seen that in the case in which $\frac{aq}{\mathfrak{v}}=1$, the pressure almost attains to its final value in the period during which the dilatational wave travels a distance that is three or four times longer than the radius of the cavity.

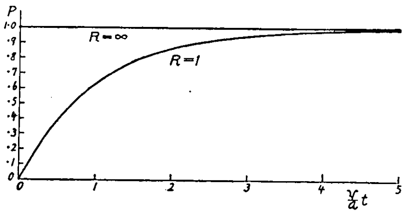

Fig. 1.

The displacement variation in time in the case $r\to\infty$, due to stress change in two different rapidities at $r=a$, is shown in Fig. 2, from which it will be seen that the displacements manifest shock types.

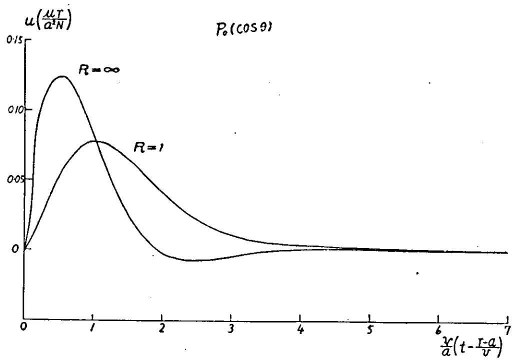

Fig. 2.

3. The general case in which the azimuthal distribution of normal stresses on the spherical cavity is expressed by a certain spherical function has already been treated by Dr. H. Kawasumi,[4] according to whose result, under the conditions at $r=a$

$$\widehat{rr}=N\varphi(t)P_n^m(\cos\theta)\left.\begin{matrix}\cos m\varphi\\ -\sin m\varphi\end{matrix}\right\}$$

$$\widehat{r\theta}=T\varphi(t)\frac{dP_n^m(\cos\theta)}{d\theta}\left.\begin{matrix}\cos m\varphi\\ -\sin m\varphi\end{matrix}\right\}$$

$$\widehat{r\varphi}=-mT\varphi(t)\frac{P_n^m(\cos\theta)}{\sin\theta}\left.\begin{matrix}\sin m\varphi\\ \cos m\varphi\end{matrix}\right\},$$

the normal, co-latitudinal, and longitudinal components of displacements being given respectively by

4) H. KAWASUMI and R. YOSIYAMA, *Jisin*, 7 (1935), 359.

$$u=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left\{\Delta_1\frac{1}{dr}\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{\sqrt{r}}+n(n+1)\Delta_2\frac{H^{(2)}_{n+\frac{1}{2}}(kr)}{r^{3/2}}\right\}e^{ipt}dp$$

$$\times\int_{-\infty}^{\infty}\varphi(w)e^{-ipw}dwP_n^m(\cos\theta)\quad\begin{matrix}\cos m\varphi\\-\sin m\varphi\end{matrix}\Bigg\}$$

$$v=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\Delta_1\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}}+\Delta_2\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\right\}\right]e^{ipt}dp$$

$$\times\int_{-\infty}^{\infty}\varphi(w)e^{-ipw}dw\frac{dP_n^m(\cos\theta)}{d\theta}\quad\begin{matrix}\cos m\varphi\\-\sin m\varphi\end{matrix}\Bigg\}$$

$$w=-\frac{m}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\Delta_1\frac{H^{(2)}_{n+\frac{1}{2}}(hr)}{r^{3/2}}+\Delta_2\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}\,H^{(2)}_{n+\frac{1}{2}}(kr)\right\}\right]e^{ipt}dp$$

$$\times\int_{-\infty}^{\infty}\varphi(w)e^{-ipw}dw\frac{P_n^m(\cos\theta)}{\sin\theta}\quad\begin{matrix}\sin m\varphi\\\cos m\varphi\end{matrix}\Bigg\},$$

where

$$\Delta=-\left(\lambda h^2\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{\sqrt{a}}-2\mu\frac{d^2}{da^2}\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{\sqrt{a}}\right)$$

$$\times\mu\left[\frac{d}{da}\left(\frac{1}{a}\frac{d}{da}\left\{\sqrt{a}\,H^{(2)}_{n+\frac{1}{2}}(ka)\right\}\right)-\frac{1}{a^2}\frac{d}{da}\left\{\sqrt{a}\,H^{(2)}_{n+\frac{1}{2}}(ka)\right\}\right.$$

$$\left.+n(n+1)\frac{H^{(2)}_{n+\frac{1}{2}}(ka)}{a^{5/2}}\right]-2\mu^2n(n+1)\left\{\frac{d}{da}\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{a^{3/2}}\right.$$

$$\left.-\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{a^{5/2}}+\frac{1}{a}\frac{d}{da}\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{\sqrt{a}}\right\}\frac{d}{da}\frac{H^{(2)}_{n+\frac{1}{2}}(ka)}{a^{3/2}},$$

$$\Delta_1=-N\mu\left[\frac{d}{da}\left(\frac{1}{a}\frac{d}{da}\left\{\sqrt{a}\,H^{(2)}_{n+\frac{1}{2}}(ka)\right\}\right)-\frac{1}{a^2}\frac{d}{da}\left\{\sqrt{a}\,H^{(2)}_{n+\frac{1}{2}}(ka)\right\}\right.$$

$$\left.+n(n+1)\frac{H^{(2)}_{n+\frac{1}{2}}(ka)}{a^{5/2}}+\right]-2T\mu n(n+1)\frac{d}{da}\frac{H^{(2)}_{n+\frac{1}{2}}(ka)}{a^{3/2}},$$

$$\Delta_2=+N\mu\left\{\frac{d}{da}\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{a^{3/2}}-\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{a^{5/2}}+\frac{1}{a}\frac{d}{da}\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{\sqrt{a}}\right\}$$

$$-T\left(\lambda h^2\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{\sqrt{a}}-2\mu\frac{d^2}{da^2}\frac{H^{(2)}_{n+\frac{1}{2}}(ha)}{\sqrt{a}}\right).$$

4. We shall next take the case $n=2$, $m=0$, tangential stresses $\widehat{r\theta}=0$, $\widehat{r\varphi}=0$, and the change in the normal stress at $r=a$

$$\left.\begin{array}{ll}\widehat{rr}=0 & (r=a,\ t<0)\\ \widehat{rr}=Ne^{-ct}(1-e^{-qt})P_2(\cos\theta), & (r=a,\ t>0)\end{array}\right\}$$

in which $c\rightarrow 0$.

The normal component of displacement of the dilatational wave at a point distant from the origin as compared with the radius of the origin is given by

$$u=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Delta_1}{\Delta}\frac{d}{dr}\frac{H^{(2)}_{\frac{5}{2}}(hr)}{\sqrt{r}}e^{ipt}dp\int_{-\infty}^{\infty}\varphi(w)e^{-ipw}dwP_2(\cos\theta)$$

$$=\frac{1}{2\pi}\frac{N\mathfrak{v}^4}{\mu ar^2}P_2(\cos\theta)i\left[\int_{-\infty}^{\infty}\frac{f(p)e^{ip\left(t-\frac{r-a}{v}\right)}}{(p-ic)g(p)}dp\right.$$

$$\left.-\int_{-\infty}^{\infty}\frac{f(p)e^{ip\left(t-\frac{r-a}{v}\right)}}{(p-iq)g(p)}dp\right],$$

where

$$\varphi(w)=e^{-ct}(1-e^{-qt})\ \text{in which}\ c\rightarrow 0,$$

$$f(p)=\left[5\frac{a}{\mathfrak{v}}p^2-48\frac{\mathfrak{v}}{a}+i\left\{\frac{a^2}{\mathfrak{v}^2}p^3-21p+48\frac{\mathfrak{v}^2}{a^2p}\right\}\right]$$

$$\left[-\frac{1}{\sqrt{3}}\frac{r}{\mathfrak{v}}p^2+9\sqrt{3}\frac{\mathfrak{v}}{r}+i\left\{4p-27\frac{\mathfrak{v}^2}{r^2p}\right\}\right]$$

$$\doteqdot\frac{r}{\sqrt{3}}\left(-i\frac{a^2}{\mathfrak{v}^3}p^5-5\frac{a}{\mathfrak{v}^2}p^4+i21\frac{1}{\mathfrak{v}}p^3+48\frac{1}{a}p^2-i48\frac{\mathfrak{v}}{a^2}p\right)$$

$$-i4\left(-i\frac{a^2}{\mathfrak{v}^2}p^4-5\frac{a}{\mathfrak{v}}p^3+i21p^2+48\frac{\mathfrak{v}}{a}p-i48\frac{\mathfrak{v}^2}{a^2}\right),$$

$$g(p)=p^6-i12{\cdot}505\frac{\mathfrak{v}}{a}p^5-81{\cdot}67\frac{\mathfrak{v}^2}{a^2}p^4+i326\frac{\mathfrak{v}^3}{a^3}p^3$$

$$+760\frac{\mathfrak{v}^4}{a^4}p^2-i872\frac{\mathfrak{v}^5}{a^5}p-552\frac{\mathfrak{v}^6}{a^6}$$

$$\doteq\left\{p-(i1{\cdot}800+4{\cdot}185)\frac{\mathfrak{v}}{r}\right\}\left\{p-(i1{\cdot}800-4{\cdot}185)\frac{\mathfrak{v}}{a}\right\}$$

$$\left\{p\quad(i3{\cdot}640+1{\cdot}432)\frac{\mathfrak{v}}{a}\right\}\left\{p-(i3{\cdot}640-1{\cdot}432)\frac{\mathfrak{v}}{a}\right\}$$

$$\left\{p-(i0{\cdot}807+1{\cdot}040)\frac{\mathfrak{v}}{a}\right\}\left\{p-(i0{\cdot}807-1{\cdot}040)\frac{\mathfrak{v}}{a}\right\}.$$

The co-latitudinal component of displacement of the distortional wave at $r\to\infty$ is expressed by

$$v=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left\{\Delta_2\frac{1}{r}\frac{d}{dr}\left\{\sqrt{r}\,H_{\frac{5}{2}}^{(3)}(kr)\right\}e^{ipt}dp\right.$$

$$\times\int_{-\infty}^{\infty}\varphi(w)e^{-ipw}dw\frac{dP_2(\cos\theta)}{d\theta}$$

$$=-\frac{1}{2\pi}\frac{2N\mathfrak{v}^4}{\mu a r^2}\frac{dP_2(\cos\theta)}{d\theta}i\left[\int_{-\infty}^{\infty}\frac{f'(p)e^{ip\left(t-\frac{r-a}{\mathfrak{v}}\right)}}{(p-ic)g(p)}dp\right.$$

$$\left.-\int_{-\infty}^{\infty}\frac{f'(p)e^{ip\left(t-\frac{r-a}{\mathfrak{v}}\right)}}{(p-iq)g(p)}dp\right],$$

where

$$f'(p)=\left[\frac{12v}{a}-\frac{ap^2}{v}+i\left(-\frac{12v^2}{a^2p}+5p\right)\right]\left[\frac{6\mathfrak{v}}{r}-\frac{rp^2}{\mathfrak{v}}+i\left(-\frac{6\mathfrak{v}^2}{r^2p}+3p\right)\right]$$

$$\doteq r\left(0{\cdot}5774\frac{a}{\mathfrak{v}^2}p^4-i5\frac{1}{\mathfrak{v}}p^3-20{\cdot}78\frac{1}{a}p^2+i36\frac{\mathfrak{v}}{a^2}p\right)$$

$$+3\left(-i0{\cdot}5774\frac{a}{\mathfrak{v}}p^3-5p^2+i20{\cdot}78\frac{\mathfrak{v}}{a}p+36\frac{\mathfrak{v}^2}{a^2}\right).$$

Upon evaluating the integrations by applying the theory of residues, the final results for the normal component of displacement of the dilatational wave and the co-latitudinal component of displacement of the distortional wave are given respectively by u and v as follows:

$$u=-0{\cdot}3478\frac{Na^3}{\mu r^2}P_2(\cos\theta)$$

$$+\frac{Na^2}{\sqrt{3}\,\mu r}P_2(\cos\theta)$$

$$\times\left[\frac{(R^5-5R^4+21R^3-48R^2+48R)e^{-R\tau}}{-R^6+12{\cdot}505R^5-81{\cdot}67R^4+326R^3-760R^2+872R-552}+2\frac{e^{-1{\cdot}800\tau}}{8\;370}\right.$$

$$\times\left\{\frac{(1{\cdot}225\times10^6-8{\cdot}487\times10^4R)\cos(4{\cdot}185\tau)-(-1{\cdot}057\times10^5+2{\cdot}559\times10^5R)\sin(4{\cdot}185\tau)}{(1564-137{\cdot}5R)^2+(9{\cdot}5+314{\cdot}4R)^2}\right.$$

$$\left.-\frac{1{\cdot}225\times10^6\cos(4{\cdot}185\tau)+1{\cdot}057\times10^5\sin(4{\cdot}185\tau)}{2{\cdot}446\times10^6}\right\}$$

$$+2\frac{e^{-3{\cdot}640\tau}}{2{\cdot}864}$$

$$\times\left\{\frac{(-1{\cdot}689\times10^5+7{\cdot}050\times10^4R)\cos(1{\cdot}432\tau)-(-3{\cdot}237\times10^5+6{\cdot}121\times10^4R)\sin(1{\cdot}432\tau)}{(821{\cdot}5-190{\cdot}2R)^2+(56{\cdot}1-90{\cdot}24R)^2}\right.$$

$$\left.-\frac{-1{\cdot}689\times10^5\cos(1{\cdot}432\tau)+3{\cdot}237\times10^5\sin(1{\cdot}432\tau)}{6{\cdot}780\times10^5}\right\}$$

$$+2\frac{e^{-0{\cdot}807\tau}}{2{\cdot}080}$$

$$\times\left\{\frac{(3490+2214R)\cos(1{\cdot}040\tau)-(-6397+5073R)\sin(1{\cdot}040\tau)}{(52{\cdot}41+121{\cdot}3R)^2+(242{\cdot}8-144{\cdot}5R)^2}\right.$$

$$\left.\left.-\frac{3490\cos(1{\cdot}040\tau)+6397\sin(1{\cdot}040\tau)}{61697}\right\}\right],$$

$$v=-0{\cdot}3912\frac{Na^3}{\mu r^2}\frac{dP_2(\cos\theta)}{d\theta}$$

$$-\frac{2Na^2}{\mu r}\frac{dP_2(\cos\theta)}{d\theta}$$

$$\times\left[\frac{(0{\cdot}5774R^4-5R^3+20{\cdot}78R^2-36R)e^{-R\tau}}{-R^6+12{\cdot}505R^5-81{\cdot}67R^4+326R^3-760R^2+872R-552}\right.$$

$$+2\frac{e^{-1{\cdot}860\tau}}{8{\cdot}370}$$

$$\times\left\{\frac{(1{\cdot}088\times10^5+1{\cdot}477\times10^4R)\cos(4{\cdot}185\tau)-(-1{\cdot}200\times10^5+3{\cdot}235\times10^4R)\sin(4{\cdot}185\tau)}{(1564-137{\cdot}5R)^2+(9{\cdot}5+314{\cdot}4R)^2}\right.$$

$$\left.-\frac{1{\cdot}088\times10^5\cos(4{\cdot}185\tau)+1{\cdot}200\times10^5\sin(4{\cdot}185\tau)}{2{\cdot}446\times10^6}\right\}$$

$$+2\frac{e^{-3{\cdot}640\tau}}{2{\cdot}864}$$

$$\times\left\{\frac{(-16290+6091R)\cos(1{\cdot}432\tau)-(-23650+4101R)\sin(1{\cdot}432\tau)}{(821{\cdot}5-190{\cdot}2R)^2+(56{\cdot}1-90{\cdot}24R)^2}\right.$$

$$\left.-\frac{-16290\cos(1{\cdot}432\tau)+23650\sin(1{\cdot}432\tau)}{6{\cdot}780\times10^5}\right\}$$

$$+2\frac{e^{-0{\cdot}807\tau}}{2{\cdot}080}$$

$$\times\left\{\frac{(399-4671R)\cos(1{\cdot}040\tau)-(7476-3241R)\sin(1{\cdot}040\tau)}{(52{\cdot}41+121{\cdot}3R)^2+(242{\cdot}8-144{\cdot}5R)^2}\right.$$

$$\left.\left.-\frac{399\cos(1{\cdot}040\tau)-7476\sin(1{\cdot}040\tau)}{61697}\right\}\right].$$

In both above two equations, the first term is the displacement that remains after the passage of the elastic waves as elastic deformation, and the second the oscillatory part that travels as elastic waves. It must be remembered that the terms representing the elastic deformation decrease as the inverse square of the distance from the origin, while those representing the propagating waves diminish in inverse ratio to the distance.

The displacements u (P phase) and v (S phase) in the above equations at $r \to \infty$ in the two cases,

$$R = \frac{aq}{\mathfrak{b}} = \infty \quad \text{and} \quad R = 1,$$

are shown respectively in Figs. 3, 4, from which it will be seen that the waves are of shock type, the duration of shock of distortional waves being usually longer than those of dilatational waves.

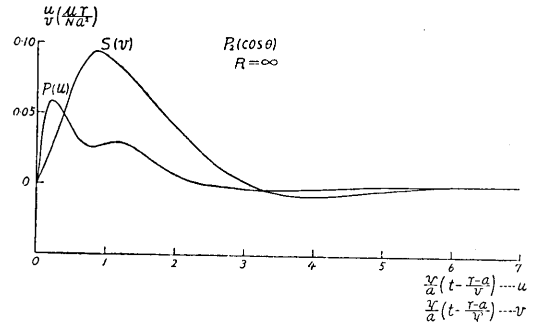

Fig. 3.

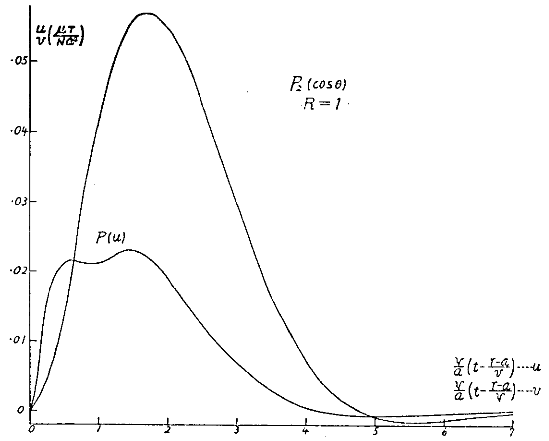

Fig. 4.

5. The wave lengths calculated for cases in which the periods have been taken as shown in Fig. 5, the ratios of the amplitudes of the P phases due to a single source $P_0(\cos\theta)$ to those that have resulted from a quadruple source $P_2(\cos\theta)$ as well as the ratios of the amplitudes of the S phases to those of the P phases in the cases of a quadruple source, are all given in Table 1. As will be seen from the table, the wave lengths become longer as the velocity of change in pressure in the cavity decreases, and further that they are several times greater than the radius of the spherical cavity. The ratios of the amplitudes of displacement of the P phases due to a single source to those due to

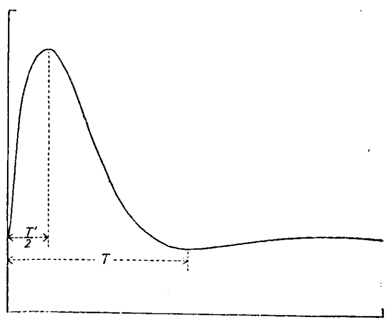

Fig. 5

Table I.

$R=\infty$					
Source	Phase	λ (VT)	λ' (VT′)	Up_0/Up_2	Vp_2/Up_2
$P_0(\cos\theta)$	P	$4{\cdot}15a$	$1{\cdot}73a$		
$P_2(\cos\theta)$	P	$5{\cdot}88a$	$0{\cdot}87a$	2·11	1·54
,,	S	$4{\cdot}0a$	$1{\cdot}7a$		
$R=1$					
Source	Phase	λ(VT)	λ' (VT′)	Up_0/Up_2	Vp_2/Up_2
$P_0(\cos\theta)$	P	—	$3{\cdot}47a$		
$P_2(\cos\theta)$	P	$8{\cdot}66a$	$4{\cdot}85a$	3·08	2·44
,,	S	$5{\cdot}5a$	$3{\cdot}4a$		

a quadruple source have the tendency to become greater as the wave lengths become longer, so that the displacements due to a single source come to prevail more and more over those due to a quadruple source as the velocities of pressure varitaions in the cavity decrease. The writer therefore considers it quite in order to assume that the velocity of the pressure variation in the cavity is fairly great, that is, for example, $\frac{aq}{\mathfrak{b}}$ takes some value between ∞ and 1. As just said, in the case in which $\frac{aq}{\mathfrak{b}}=1$, the pressure in the cavity almost attains to its final value in the period during which the longitudinal wave travels a distance three or four times that of the radius of the cavity.

6. We shall next consider the range of the pressure changes in the origins, assuming that the quadruple source in the case in which $\frac{aq}{\mathfrak{b}}=1$ corresponds to an actual earthquake.

According to Dr. H. Honda,[5] the normal component of the dilatational wave when r is large is given by

$$u_1=\mathfrak{A}\frac{1}{r}\sin 2\theta\cos\varphi\cos(pt-hr).$$

In the deep-focus earthquake of June, 2, 1929 (focal depth $\doteqdot$ 300 km),[6] the observed amplitudes in the P phases are well given by assuming

5) H. HONDA, *Geophys. Mag.*, 8 (1934), 153~164.
6) H. HONDA, *loc. cit.*

$$\mathfrak{A} = -7{\cdot}58 \times 10^5 \text{ cm}^2.$$

While, in our case and also in the case $P_2^1(\cos\theta)$ in which $\dfrac{aq}{\mathfrak{v}} = 1$,

$$\mathfrak{A} = 0{\cdot}02521 \frac{Na^2}{\mu},$$

whence

$$N = -7{\cdot}58 \times 10^5 \text{ cm}^2 \frac{\mu}{2{\cdot}521 \times 10^{-2} a^2}.$$

According to Dr. K. Sagisaka,[7] we assume $T' \doteqdot 1{\cdot}7$ sec.

Assuming

$$v = 8{\cdot}57 \text{ km/sec},$$

we obtain

$$\lambda' = 8{\cdot}57 \times 1{\cdot}7 = 14{\cdot}6 \text{ km},$$

while

$$\lambda' = 4{\cdot}85a,$$

whence

$$a = 3{\cdot}03 \text{ km}.$$

Assuming

$$\mu = 8{\cdot}4 \times 10^{11} \text{ dyne/cm}^2,$$

$$N = -7{\cdot}58 \times 10^5 \frac{8{\cdot}4 \times 10^{11}}{2{\cdot}521 \times 10^{-2}(3{\cdot}03 \times 10^5)^2}$$

$$= -2{\cdot}76 \times 10^8 \text{ dyne/cm}^2 \doteqdot -273 \text{ atms}.$$

In the deep-focus earthquake of Nov. 13, 1932 (focal depth $\doteqdot$ 300 km),[8] the observed amplitudes in the P phases are well expressed by assuming

$$\mathfrak{A} = -3{\cdot}22 \times 10^6 \text{ cm}^2.$$

The writer estimated $\dfrac{T'}{2} = 1{\cdot}92$ sec on the seismogram obtained by Dr. N. Nasu's vertical component seismometer installed at Tokyo. The proper period of this pendulum is 28 sec.

Whence we obtain $a = 6{\cdot}8$ km,

and

$$N = -2{\cdot}32 \times 10^8 \text{ dyne/cm}^2 = -228 \text{ atms}.$$

---

7) T. ISHIKAWA, *Kishô-Syûsi*, [ii], **10** (1932), 261.

8) H. HONDA, *Geophys. Mag.*, **8** (1934), 173.

The pressures thus obtained are quite of the same orders of magnitude as the pressure of gases (mainly water vapour), which are from 200 to 500 atms in the case of volcanic eruptions[9].

7. As already noted, the apparent periods of the S phases exceed those of the P phases (see Figs. 3, 4).

The writer thinks it of interest to refer to the observed periods in the P phases and the S phases of actual earthquakes.

According to Dr. T. Ishikawa,[10] in the deep-focus earthquake of Feb. 20, 1931, the observed periods of the S phases were in the mean 1·77 times greater than those of the P phases.

8. Summary.

In the present paper, the writer, following his previous investigations in which it is assumed that normal stress causes periodic changes, investigates the case in which the normal stress on the surface of a spherical cavity changes with different rapidities.

The main results are as follow:

1) In such a case, elastic waves of shock type emanate from the origin, the duration of shock in distortional waves being usually longer than those in dilatational waves.

2) It is plausible to assume that the origins send forth elastic waves, the wave lengths of which are comparable (several times the radius) with the dimension of the origins.

3) The range of pressure changes in the seismic origins (spherical cavity) may be of the same order of magnitudes as the pressure of gases in the case of volcanic eruptions, namely from 200 to 500 atms.

---

9) T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **11** (1933), 347.

10) T. ISHIKAWA, *loc. cit.*

9. 發震機構に就いて（第2報）

地震研究所 井上宇胤

第一報に於ては球形空窩の表面に週期的垂直應力が働く場合を數理的に研究したが此處では其の續きとして空窩の表面に働く垂直應力が種々なる早さを以て變化する場合を調査してある。

主なる結果を擧げれば次の如きものである。

（イ） 以上の樣な場合に於ては震源から遠い所に於ては衝擊性の波を記錄する事となり且橫波の週期の方が縱波の週期より相當延びる事となる，此れは旣に觀測されては居たが今日迄理論的の說明の附かなかつた事である。

（ロ） 震源から出る波の波長は震源を球形であると考へた時に其の半徑の數倍のものであると考へられる。

（ハ） 二つの深發地震に於て震源に於ける壓力變化の程度を吟味して見た所大略數百氣壓であれば觀測された縱波や橫波の說明が附く事を知つた，此の値は丁度火山爆發の際のガスの壓力と同程度のものである。