46. *Energy Dissipation in Seismic Vibrations of an Eight-storied Structure.*

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(Read Sept. 15, 1936.—Received Sept. 21, 1936.)

1. *Mathematical theory.*

The problem of energy dissipation in the seismic vibrations of a structure that does not exceed 7-stories having already been studied, we shall now discuss a similar problem for an 8-storied sturcture, and in doing so, shall begin with the mathematical formulae that will necessarily be involved.

It is very important to know whether or not the floors are in a flexible condition. Our investigations[1] showed that were the formulae for extremely rigid floors used, the flexibility of the floors of virtually every building in Tokyo would be equivalent to a reduction to one-half the original effective value of $\sqrt{EI/ml^3}$.

The solutions and the boundary conditions of the problem for the case in which the floors are extremely rigid are shown in (32)$\sim$(41) of our previous paper[2]. When $E_1=E_2=\ldots$, $I_1=I_2=\ldots$, $m_1=m_2=\ldots$, the constants that belong to expressions for deflections of columns are as follows:

$$A_1=\phi\frac{2\Gamma}{\Phi}=\Bigg[(\gamma-12)^2\Big\{(\gamma-12)(\gamma-24)-144\Big\}\Big[-36\gamma(\gamma-24)(\gamma-36)$$
$$+\Big\{(\gamma-12)(\gamma-24)-144\Big\}^2\Big]-12\gamma\Big\{(\gamma-12)^2(\gamma-36)$$
$$-144(\gamma-24)\Big\}\Big\{(\gamma-12)(\gamma-36)^2-144(\gamma-24)\Big\}\Bigg]\frac{2\Gamma}{\Phi},$$

$$l^3D_1=\phi\frac{2\Gamma}{\Phi}=-2\Bigg[\gamma(\gamma-24)\Big\{(\gamma-24)^2-288\Big\}\Big[\Big\{(\gamma-24)^2$$

1) K. SEZAWA and K. KANAI, "The Effect of Stiffness of Floors on the Horizontal Vibrations of a Framed Structure", *Bull. Earthq. Res. Inst.*, **14** (1936), 367$\sim$376.

2) K. SEZAWA and K. KANAI, "Improved Theory of Energy Dissipation in Seismic Vibrations of a Structure", *Bull. Earthq. Res. Inst.*, **14** (1936), 164$\sim$188.

$$-288\Big\}^2-2.12^4\Big]\Big]\frac{2\Gamma}{\Phi},$$

$$A_2=-12(\gamma-12)\Big\{(\gamma-12)(\gamma-24)-144\Big\}\Big[-36\gamma(\gamma-24)(\gamma-36)$$

$$+\Big\{(\gamma-12)(\gamma-24)-144\Big\}^2\Big]\frac{2\Gamma}{\Phi},$$

$$l^3D_2=2.12\gamma\Big\{(\gamma-12)^2(\gamma-36)-144(\gamma-24)\Big\}\Big\{(\gamma-12)(\gamma-36)^2$$

$$-144(\gamma-24)\Big\}\frac{2\Gamma}{\Phi},$$

$$A_3=12^2\Big[\Big\{\gamma(\gamma-24)^2-12(\gamma-12)^2\Big\}^2-12\gamma(\gamma-12)^2(\gamma-36)^2\Big]\frac{2\Gamma}{\Phi},$$

$$l^3D_3=-2.12^2\gamma(\gamma-12)(\gamma-24)(\gamma-36)\Big\{(\gamma-24)^2-432\Big\}\frac{2\Gamma}{\Phi},$$

$$A_4=-12^3\Big[(\gamma-12)\Big\{(\gamma-12)^2-12\gamma\Big\}^2-12\gamma(\gamma-24)\Big[\Big\{(\gamma-12)^2$$

$$-12\gamma\Big\}+(\gamma-12)(\gamma-36)\Big]\Big]\frac{2\Gamma}{\Phi},$$

$$l^3D_4=2.12^3\gamma\Big\{(\gamma-12)(\gamma-24)-144\Big\}\Big\{(\gamma-24)(\gamma-36)-144\Big\}\frac{2\Gamma}{\Phi},$$

$$A_5=12^4(\gamma-12)\Big\{\gamma(\gamma-36)^2-12^3\Big\}\frac{2\Gamma}{\Phi},$$

$$l^3D_5=-2.12^4\gamma(\gamma-24)\Big\{(\gamma-24)^2-288\Big\}\frac{2\Gamma}{\Phi},$$

$$A_6=-12^5\Big\{\gamma(\gamma-24)(\gamma-36)-12^3\Big\}\frac{2\Gamma}{\Phi},$$

$$l^3D_6=2.12^5\gamma(\gamma-12)(\gamma-36)\frac{2\Gamma}{\Phi},$$

$$A_7=12^6\Big\{(\gamma-12)(\gamma-24)-144\Big\}\frac{2\Gamma}{\Phi},$$

$$l^3D_7=-2.12^6\gamma(\gamma-24)\frac{2\Gamma}{\Phi},$$

$$A_8 = -12^7(\gamma - 12)\frac{2\Gamma}{\Phi},$$

$$l^3 D_8 = 2.12^7 \gamma \frac{2\Gamma}{\Phi},$$

$$B_s = 0, \qquad C_s = \frac{3}{2} l D_s, \quad (s = 1, 2, \ldots . 8) \tag{1}$$

where

$$\left.\begin{aligned}
&\Phi = \phi\Gamma + \frac{18Ej^2\varepsilon}{\mu l^3}\phi\Lambda, \\
&\Gamma = \Gamma_1 + i\Gamma_2, \qquad \Lambda = \Lambda_1 + i\Lambda_2, \\
&\Gamma_1 = 3\left(\frac{\lambda}{\mu} + 2\right) + \nu\gamma\left(\frac{\lambda}{\mu} - 3\sqrt{\frac{\lambda}{\mu} + 2}\right), \\
&\Gamma_2 = \sqrt{\nu\gamma}\left\{3\left(\frac{\lambda}{\mu} + 2 + \sqrt{\frac{\lambda}{\mu} + 2}\right) + \nu\gamma\left(\sqrt{\frac{\lambda}{\mu} + 2} - 2\right)\right\}, \\
&\Lambda_1 = \left(\frac{2\lambda}{\mu} + 5\right) - \nu\gamma, \\
&\Lambda_2 = \sqrt{\nu\gamma}\left(2\sqrt{\frac{\lambda}{\mu} + 2} + 1\right), \\
&\nu = \frac{E\rho I\varepsilon^2}{\mu m l^3}.
\end{aligned}\right\} \tag{2}$$

The values of the constants thus obtained should be substituted in the expressions for the deflections, namely,

$$y_s = (A_s + B_s x_s + C_s x_s^2 + D_s x_s^3)\, e^{ipt}. \qquad (s = 1, 2, \ldots, 8) \tag{3}$$

The ratio of bending moments in the columns on each floor to the product of ml due to the acceleration of the ground (on which no structure stands) is expressed by equations of the forms

$$EI\frac{d^2y_1}{dx_1^2}\Big/ 2p^2ml = 6(24-\gamma)\left\{(24-\gamma)^2 - 288\right\} \cdot \left[\left\{(24-\gamma)^2 - 288\right\}^2 - 2.12^4\right]\left(1 + \frac{2x_1}{l}\right)M,$$

$$EI\frac{d^2y_2}{dx_2^2}\Big/ 2p^2ml = 6.12\left\{(12-\gamma)^2(36-\gamma) - 144(24-\gamma)\right\}$$

$$\cdot\left\{(12-\gamma)(36-\gamma)^2-144(24-\gamma)\right\}\left(1+\frac{2x_2}{l}\right)M,$$

$$EI\frac{d^2y_3}{dx_3^2}\Big/2p^2ml=6.12^2(12-\gamma)(24-\gamma)(36-\gamma)$$

$$\cdot\left\{(24-\gamma)^2-432\right\}\left(1+\frac{2x_3}{l}\right)M,$$

$$EI\frac{d^2y_4}{dx_4^2}\Big/2p^2ml=6.12^3\left\{(12-\gamma)(24-\gamma)-144\right\}$$

$$\cdot\left\{(24-\gamma)(36-\gamma)-144\right\}\left(1+\frac{2x_4}{l}\right)M,$$

$$EI\frac{d^2y_5}{dx_5^2}\Big/2p^2ml=6.12^4(24-\gamma)\left\{(24-\gamma)^2-288\right\}\left(1+\frac{2x_5}{l}\right)M,$$

$$EI\frac{d^2y_6}{dx_6^2}\Big/2p^2ml=6.12^5(12-\gamma)(36-\gamma)\left(1+\frac{2x_6}{l}\right)M,$$

$$EI\frac{d^2y_7}{dx_7^2}\Big/2p^2ml=6.12^6(24-\gamma)\left(1+\frac{2x_7}{l}\right)M,$$

$$EI\frac{d^2y_8}{dx_8^2}\Big/2p^2ml=6.12^7\left(1+\frac{2x_8}{l}\right)M, \qquad (4)$$

where

$$\left.\begin{aligned} M&=\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right),\\ P&=\phi\Gamma_1+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_1, \qquad Q=\phi\Gamma_2+\frac{18Ej^2\varepsilon}{\mu l^3}\psi\Lambda_2. \end{aligned}\right\} \qquad (5)$$

2. *Resonance and corresonance frequencies.*

$\phi=0$ gives the natural vibration frequencies in the usual sense, that is, the frequencies where the ground is infinitely rigid. γ satisfies the equation $\phi=0$ when

$$\gamma=0{\cdot}3521,\ 3{\cdot}6,\ 9{\cdot}2,\ 17{\cdot}435,\ 26{\cdot}2,\ 34{\cdot}7,\ 41{\cdot}76,\ 46{\cdot}38,$$

or when

$$\sqrt{\gamma}=0{\cdot}5933,\ 1{\cdot}897,\ 3{\cdot}034,\ 4{\cdot}176,\ 5{\cdot}12,\ 5{\cdot}89,\ 6{\cdot}46,\ 6{\cdot}81.$$

$\psi=0$ gives the natural vibration frequencies of the structure in the case where the ground is extremely soft. This we have called the corresonance condition. γ satisfies $\psi=0$ when

$$\gamma=1{\cdot}8269,\ 7{\cdot}0296,\ 14{\cdot}816,\ 24,\ 33{\cdot}185,\ 40{\cdot}9704,\ 46{\cdot}17,$$

or when

$$\sqrt{\gamma}=1{\cdot}3518,\ 2{\cdot}6515,\ 3{\cdot}85,\ 4{\cdot}899,\ 5{\cdot}759,\ 6{\cdot}399,\ 6{\cdot}795.$$

3. *Application of the theory to the Marunouti Building.*

The Marunouti Building, in Marunouti, Tokyo, is an 8-storied steel-framed reinforced concrete structure with a partial basement as well as a partial penthouse roof and a machine room roof, and let for shops and business offices by the owners, the Mitsubishi Co.. Although the building was completed in February 1923, some strengthening members were added to it soon after the Great Kwanto Earthquake of the same year. In the present prediction calculation, use has been made of data covering its present structural condition. The general view of the building and the general plan of the second floor are shown in Figs. 1, 2. (Fig. 1 was photographed by Mr. T. Takayama, whereas Fig. 2 was reproduced by Mr. K. Kaminaga from a sketch in the Architectural Engineering Pocket Book).

Fig. 1. A View from NE-corner.

The general and structural arrangements of floors higher than the 2nd and up to 7th, inclusive, are similar to the plan in Fig. 2. An arcade runs through the middle of the ground floor in a NS-direction. On the 1st floor are spaces over such parts of the arcade as correspond to the inner court yards. The positions of the columns are the same for every floor excepting the parts of the arcade just mentioned. The sizes of the columns and all the other structural members naturally diminish with their proximity to the roof floor.

The section of the steel frame in every column is of plate type, forming the built-up I in a plate and four angles, while that of some of the girders consists of two channels, so as to form box girders, the remainder being the simple I-shape, although the frame of every beam consists of a smaller I-bar.

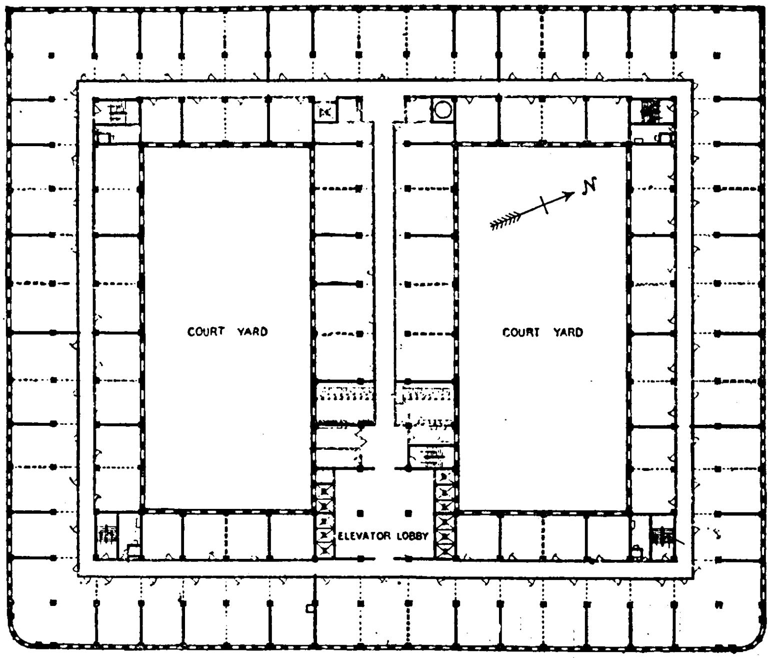

Fig. 2. General Plan of the Second Floor. Scale 1/580.

It was assumed in the numerical calculation that $E=2{\cdot}1.10^9$ kg m^2 $=2{\cdot}1.10^9.9{\cdot}8$ kg mass m/s^2/m^2, and that the floors are so rigid that the ends of the columns are always vertical. The calculated results of the important elements are shown in Table I, which may be used in almost any theoretical treatment of horizontal vibration of the structure under consideration. The moments of inertia of the columns in the table can be used for the case of vibration in a NS-direction.

By adjusting "the additional items" in the table, the total weight of the structure was made to be the same as that calculated by the designer. Since the weight due to the penthouse roof and the machine room roof is $73{\cdot}66.10^5$ kg, we added $73{\cdot}66.10^5$ kg/8 to the mean weight $67{\cdot}13.10^5$ kg of every floor from the lst floor to the roof. Thus $76{\cdot}33.10^5$ kg was used as the concentrated mass on every floor. The value of I was calculated as $21{\cdot}035$ m^4, but 1/2 of the value of $\sqrt{I}$ obtained here was used in the dissipation calculation to adjust the condition of slight flexibility of the floors.

To obtain the dissipation we put $\sqrt{\mu/\rho}=50$ m/s, which may be some-

Table I.

Floor		1	2	3	4	5	6	7	Roof	Mean
Height of floor below l (m)		5·0325	3·5075	3·5075	3·5075	3·5075	3·5075	3·5075	3·5075	3·6981 +0·4194 =4·1175
Steel frames in columns	a (m^2) I (m^4)	96·77 0·946	96·77 0·946	69·40 0·5567	69·40 0·5567	49·20 0·3666	49·20 0·3666	32·90 0·2308	32·90 0·2308	62·07 0·5250
Remaining parts in columns	a (m^2) I (m^4)	169·05 25·97	158·95 21·53	143·75 13·60	140·52 13·09	139·02 12·56	133·53 11·78	129·17 11·03	129·17 11·03	142·89 15·07
Outside walls	a (m^2) I (m^4)	29·95 1·3525	37·61 1·2865	46·43 1·3897	43·48 1·3053	43·26 1·2900	39·44 1·1368	36·77 1·0572	36·77 1·0572	39·21 1·2344
Inside walls	a (m^2) I (m^4)	103·57 5·832	86·08 4·417	81·92 4·547	68·23 3·767	68·23 3·767	68·23 3·767	68·23 3·767	68·23 3·767	76·59 4·204
Totals	a (m^2) I (m^4)									320·8 21·04
Steel frames in columns	W(kg)	2·443.10^5	1·642.10^5	1·189.10^5	1·189.10^5	0·864.10^5	0·864.10^5	0·517.10^5	0·517.10^5	1·153.10^5
Remaining parts in columns	v (m^3) W(kg)	650·1 15·61.10^5	449·8 10·80.10^5	400·0 9·6.10^5	400·0 9·6.10^5	400·0 9·6.10^5	400·0 9·6·10^5	400·0 9·6.10^5	400·0 9·6.10^5	437·5 10·50.10^5
Outside walls	v (m^3) W (kg)	103·1 2·473.10^5	90·13 2·162.10^5	114·1 2·738.10^5	114·1 2·738.10^5	114.1 2·738.10^5	114·1 2·738.10^5	114·1 2·738.10^5	114·1 2·738.10^5	109·7 2·633.10^5
Inside walls	v (m^3) W (kg)	379·4 9·11.10^5	251·4 6·04.10^5	209·6 5·035.10^5	209·6 5·035.10^5	209·6 5·035.10^5	209·6 5·035.10^5	209·6 5·035.10^5	209·6 5·035.10^5	236·1 5·670.10^5
Floors	v (m^3) W (kg)	736·0 17·66.10^5	718·5 17·24.10^5	627·5 15·06.10^5	627·5 15·06.10^5	627·5 15·06.10^5	627·5 15·06.10^5	627·5 15·06.10^5	627·5 15·06.10^5	652·4 15·66.10^5
Frames in girders & beams	W (kg)	2·929.10^5	2·929.10^5	2·929.10^5	2·929.10^5	2·929.10^5	2·929.10^5	2·929.10^5	2·506.10^5	2·876.10^5
Remaining parts in girders & beams	v (m^3) W (kg)	443·3 10·64.10^5	443·3 10·64.10^5	443·3 10·64.10^5	443·3 10·64.10^5	443·3 10·64.10^5	443·3 10·64.10^5	443·3 10·64.10^5	459·0 11·016.10^5	445·3 10·687.10^5
Sum	W (kg)	60·87.10^5	51·45.10^5	47·19.10^5	47·19.10^5	46·87.10^5	46·87.10^5	46·52.10^5	46·47.10^5	49·18.10^5
Additional items	W (kg)	21·92.10^5	21·00.10^5	15·33.10^5	14·68.10^5	16·45.10^5	16·45.10^5	17·87.10^5	19·86.10^5	17·95.10^5
Total sum	W (kg)	82·79.10^5	72·45.10^5	62·52.10^5	61·87.10^5	63·33.10^5	63·33.10^5	64·39.10^5	66·33.10^5	67·13.10^5

what too small, and $\rho=2$, whence

$$\frac{18Ej^2\varepsilon}{\mu l^3}=6{\cdot}68\,,$$

$$\Gamma_1\quad 9-0{\cdot}02518\gamma, \qquad \Gamma_2=1{\cdot}0994\sqrt{\gamma}-0{\cdot}0001246\gamma^{3/2},$$

$$\Delta_1=7-0{\cdot}006\gamma, \qquad \Delta_2=0{\cdot}3458\sqrt{\gamma}\,.$$

The calculated result is shown in Figs. 3 a, 3 b[3].

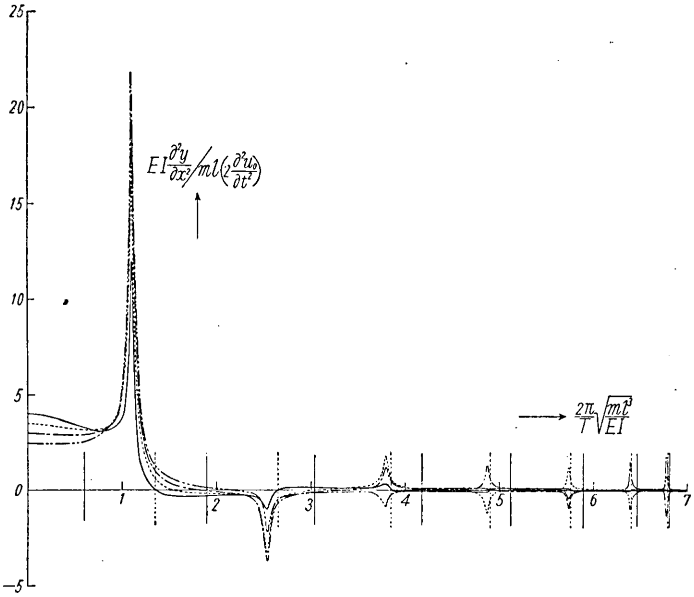

Fig. 3 a. Full, broken, chain, and double chain lines represent moments in columns of ground, first, second, and third floors respectively.

For the bending moment in every column, it is assumed that its maximum value at the frequency corresponding to $\sqrt{\gamma}=1{\cdot}10$, which is intermediate between the virtual first resonance and the first corresonance. The actual resonance period is

$$T=\frac{2\pi}{\sqrt{\gamma}}\sqrt{\frac{ml^3}{EI}}=\frac{2\pi}{1{\cdot}10}\sqrt{\frac{ml^3}{EI}}=0{\cdot}566\,\text{sec}\,.$$

The greatest value of the maximum bending moments is induced in columns between the third and fourth floors, the bending moments

3) The effect of such a deeply piled foundation as in the case of Marunouti Building is rather to greatly increase the energy dissipation owing to the fact that the area of contact surface between the piles and the ground soil is enormously increased. However, we are here discussing the problem of a hypothetical pileless structure wherein the dissipation is never very marked.

in all columns at this resonance period being of the magnitudes in Table II.

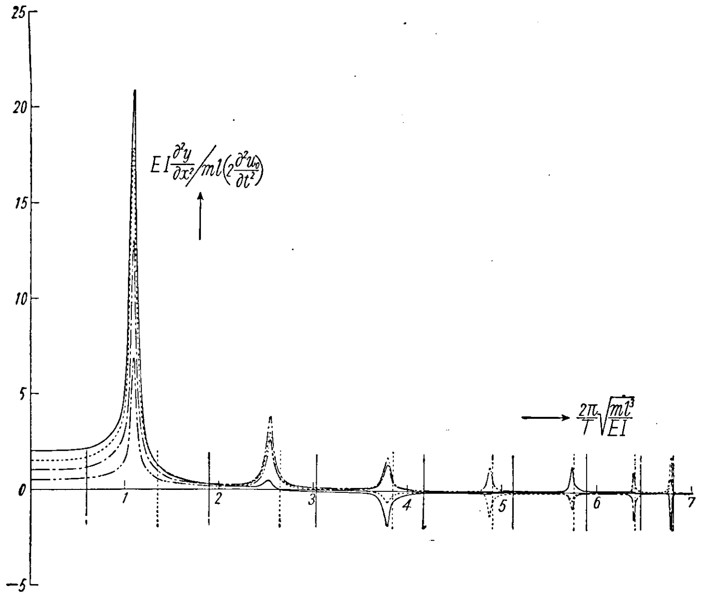

Fig. 3 b. Full, broken, chain, and double chain lines represent moments in columns of fourth, fifth, sixth, and seventh floors respectively.

Table II.

Column between floors	1∼0	2∼1	3∼2	4∼3	5∼4	6∼5	7∼6	8∼7
Number of moment value at resonance	7	5	3	1	2	4	6	8
$\dfrac{EI\frac{d^2y}{dx^2}}{ml\left(2\frac{\partial^2 u}{\partial t^2}\right)}$ at resonance	12·0	17·3	20·5	21·8	20·9	17·8	13·0	6·8
The same at zero frequency	4·0	3·5	3·0	2·5	2·0	1·5	1·0	0·5

The full and broken vertical lines along the abscissa in Figs. 3a, 3b represent the virtual resonance and corresonance conditions, namely, the condition wherein the ground is very rigid and one in which it is very soft.

It was pointed out in the previous paper[4] that the vibrational frequency at which the bending moments become maximum, and the floor

4) K. SEZAWA and K. KANAI, "Energy Dissipation in Seismic Vibrations of Actual Buildings Predicted through Improved Theory", *Bull. Earthq. Res. Inst.*, **14** (1936), 377—386.

number at which the bending moments in the columns assume the greatest value at such vibrational frequency, depend on the ratio of $Ej^2\varepsilon/\mu l^3$. In the present case the first resonance condition in the sense that the bending moment becomes maximum at that condition takes place at a frequency that is not far from the one for the corresonance condition. It was furthermore ascertained that the higher the order of resonance, the more its frequency approaches that of the corresponding corresonance condition. The greatest bending moment at these higher resonance conditions are induced in different columns according as they differ in their order of resonance. But, it is easily confirmable from Figs. 3a, 3b that the greatest bending mements under consideration are fairly small for the second or any higher resonance condition.

It was also ascertained in writing this paper that in seismic vibrations, the greatest bending moment is not likely to be induced in the columns in the lower floors, such as the ground floor or the first, unless of course dissipation of the vibrational energy into the ground is neglected.

In conclusion we wish to express our thanks to the Council of the Foundation for the Promotion of Scientific and Industrial Research in Japan, by whose aid the present series of investigation was made, and also to the members of the staffs of the Mitsubishi Co., who kindly allowed us the use of the valuable data on the structure.

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46. 8階建築に於ける震動勢力の地中逸散性

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これまで7階以下の建築構造について震動勢力が地中に逸散する割合を算出して置いたが，その後研究が著しく進んで8階の場合を確めることができた．計算の理論は以前の考方であるけれども，8階の場合の結果を出すには相當の困難を伴つた．丸ノ内ビルディングの場合に計算をあてはめて見て大體尤もらしい結果が出た．構造中の粘性抵抗を入れずに振動勢力が基礎から彈性波として逃げるとして試みた計算の結果がこれ程迄によくあふといふ事は以前には全く豫期しなかつたところである．

7階の場合に既に述べたことであるが，$Ej^2\varepsilon/\mu l^3$ 卽ち建物の剛度と土地の剛度の比が高くなるに從つて最も破壞し勝ちの柱のある床が少しづゝ屋根の方へ移つて行くことは依然として同様

である．丸ビルの場合に土地の横波の速度を 50 m/s に取つて見るこ，第1共振に於ける柱の屈曲モーメントの大さの床についての順序，そのこきの屈曲モーメントの大さ，同じ加速度が働いて震動週期無限大のときの屈曲モーメントを表示するこ次の如くなる．

柱のある床	1階	2階	3階	4階	5階	6階	7階	8階
共振に於けるモーメント値の順	7	5	3	1	2	4	6	8
共振に於ける $EI\dfrac{d^2y}{dx^2}\Big/\left(ml2\dfrac{\partial^2u}{\partial t^2}\right)$	12·0	17·3	20·5	21·8	20·9	17·8	13·0	6·8
零振動數に於ける $EI\dfrac{d^2y}{dx^2}\Big/\left(ml2\dfrac{\partial^2u}{\partial t^2}\right)$	4·0	3·5	3·0	2·5	2·0	1·5	1·0	0·5

共振に於けるモーメント値の順は $Ej^2\varepsilon/\mu l^3$ の値によつて勿論變化する．尙，基礎の中へ深く入つてゐる杭がある爲に $Ej^2\varepsilon/\mu l^3$ の値が割合に小さくなりはしないかこいふ懸念があるかも知れぬが，實際は杭こ土地この接觸面が著しく增加し，μ が大きくなるここよりも ε の大きくなる方が急であり，從て實際の振動逸散性は玆に計算したのよりも寧ろ甚しいのではないかこ思はれる．この論文で之等の數値を正確に合さずに大體の定量的結果を出したここに對して或は議論の餘地があるかも知れぬけれごも，實はこのやうな研究は構造物に限らず殆ごすべての力學的問題に未だ何國にもやつてないから，この新しい物理的問題を提出して一般の正しい了解を得るのが先決問題の積りでやつてゐるのである．